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Budget deficits and inflation feedback

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Abstract

This paper contributes to the literature on budget deficits and inflation in high inflation economies. The main finding is that recurrent outbursts of extreme inflation in these economies can be explained by a certain hysteresis effect associated with public finance. This interpretation meets the evidence that dramatic shifts between regimes of moderately high and extremely high (hyper-) inflation often occur without visible deterioration in public finance or abrupt shifts in fiscal or monetary policies. The existence of this hysteresis effect is explicitly explained by the action of two mechanisms: the arithmetic associated with the wrong side of the inflation tax Laffer curve and the Patinkin effect (the reverse of the much oftener cited Olivera-Tanzi effect). It is also shown that the division of the operational budget deficit into the part that is subject to negative inflation feedback and the part that is inflation-proof, has implications for both the discussion of the inflationary consequences of budget deficits and the proper design of stabilization policy.

Keywords: budget deficit, high inflation, the Patinkin effect.

JEL Classification: E41, E52, E61, E63

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Budget deficits and inflation feedback

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Abstract. This paper contributes to the literature on budget deficits and inflation in high inflation economies. The main finding is that recurrent outbursts of extreme inflation in these economies can be explained by a certain hysteresis effect associated with public finance. This interpretation meets the evidence that dramatic shifts between regimes of moderately high and extremely high (hyper-) inflation often occur without visible deterioration in public finance or abrupt shifts in fiscal or monetary policies. The existence of this hysteresis effect is explicitly explained by the action of two mechanisms: the arithmetic associated with the wrong side of the inflation tax Laffer curve and the Patinkin effect (the reverse of the much oftener cited Olivera-Tanzi effect). It is also shown that the division of the operational budget deficit into the part that is subject to negative inflation feedback and the part that is inflation-proof, has implications for both the discussion of the inflationary consequences of budget deficits and the proper design of stabilization policy.

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«...given that policymakers do not create inflation out of a clear blue sky, it is almost certain that countries with high inflation rates are countries that are already in trouble for fiscal or other reasons, and thus that it will be either impossible or extremely difficult to deal definitely with the issue of causation».

Fischer (1995, p.22)

1. Introduction

For most episodes of high inflation in developing countries, it can be said that the source of inflation is an imbalance in the fiscal sphere. However, is the causality between inflation and the deficit actually so clear? Despite the cross-country evidence of a strong positive relationship between the budget deficit and inflation during high inflation episodes (see, e.g., recent evidence in Fischer, Sahay and Vegh, 2002; and Catão and Terrones, 2005), case studies (see, e.g., Bruno, 1993) show that the relationship between the budget deficit, seigniorage and inflation is not always strong and positive. There are two concerns, among others, that complicate this relationship. Firstly, the economy may be operating either on the “efficient side” of the inflation tax Laffer curve (ITLC), where an increase in the budget deficit requires a higher steady state rate of inflation, or on the “wrong side”, where a higher budget deficit is associated with a lower steady state rate of inflation. Secondly, there are various channels through which inflation affects the budget deficit (either positively or negatively).

This paper provides a reconciliation between this mixed evidence and the ITLC model that takes into account the negative feedback from inflation to the real budget deficit. In particular, we show that recurrent outbursts of extreme inflation in modern high-inflation economies can be explained by a certain hysteresis effect associated with public finance. This interpretation meets the evidence that dramatic shifts between regimes of moderately high and extremely high (hyper-) inflation often occur without visible deterioration in public finance or abrupt shifts in fiscal or monetary policies. The existence of this hysteresis effect is explicitly explained by the action of two phenomena: the arithmetic associated with the “wrong side” of the ITLC and the negative inflation feedback to the budget deficit. It is also shown that the division of the operational budget deficit into the part that is subject to the inflation effect and the part that is inflation-proof, has implications for both the discussion of the inflationary consequences of budget deficits and the proper design of stabilization policy.

This paper focuses on several important stylized facts about fiscal policy and inflation dynamics in modern high-inflation economies. We do not provide a complete list of these facts and we narrow the analysis to a pure fiscal-monetary framework, leaving out other important issues such as inflation inertia, exchange rate dynamics, etc. Instead we concentrate on some of the

evidence that is not fully compatible with the standard monetary-fiscal models of inflation dynamics.

Modern extreme inflations versus classical hyperinflations

There are two types of hyperinflation processes known to economic history: “classical” interwar European hyperinflations (Germany, 1922-23; Austria, 1921-22; Hungary, 1923-24; among others)² and more recent hyperinflations experienced by chronic high-inflation countries (Argentina, 1989-90; Brazil, 1989-90; Bolivia, 1984-85; Congo (Dem.Rep.) 1991-92, 1993-94; Nicaragua, 1986-91; Zimbabwe, 2006-2008; etc.). All these episodes satisfy the Cagan (1956) definition of hyperinflation (a monthly inflation rate that remains above 50 percent for at least a year) and have rather similar characteristics of the dynamics of fiscal and monetary variables. However, they are different in one important aspect. In general, classical hyperinflations took place when a previously stable financial system was disrupted by extraordinary events (such as wars or economic transition), and these economies were stabilized and returned to their normal functioning under relatively low and stable inflation. Modern hyperinflations take place in chronically high inflation countries. In most cases after the end of the hyperinflation period, the economies reverted to high or moderate, but still very unstable inflation. This point can be highlighted further by considering the more general framework of the switching between two regimes: “moderately high” and “extremely high” inflation that is the true story of countries such as Argentina and Brazil over the last few decades. The question that we address in this paper is what specific economic forces determine this switch between “moderately high” and “extremely high” inflation regimes in these countries? And why is this indeed an abrupt switch, and not a gradual change?

Inflation effects on the budget deficit

The standard explanation of the end of “classical” hyperinflation (at almost no cost and with a permanent shift to stable low inflation) involves the assumption of rational expectations and credibility of future policy switches.³ Obviously, this mechanism may not work in economies that live with moderate or high inflation for decades and it cannot account for observed *recurrent* outbursts of hyperinflation.⁴ It is well known that these economies are prone to various *market* mechanisms (indexation schemes) that generate significant inflation inertia, making stabilization programs costly and difficult to implement. However, when a market economy creates mechanisms

² Hyperinflations following the transition to market economies in the early 1990’s may be also attributed to the first type. The Serbian hyperinflation of 1993-94 is the most dramatic example.

³ See the explanation of this logic in Sargent (1982) and the recent model by Barbosa, Cunha and Sallum (2006).

⁴ There are only a few exceptions, such as Israel’s credible stabilization in 1985-86.

that help it live with high inflation (these mechanisms actually act like “stabilizers” or “traps”) one should obviously expect the existence of certain “stabilizers” in highly unstable *public* finance.

In this paper we explore the role of one such “stabilizer”, the so-called “Patinkin effect”, which refers to the negative effect of inflation on the budget deficit. The term “Patinkin effect” was suggested by Cardoso (1998). In exploring Israel’s stabilization program of 1985, Patinkin (1993) stressed the importance of the negative effect of inflation on the *real* value of government spending. Cardoso (1998) states that this effect has been dominant in Brazil over the last few decades and discusses additional factors that reduce the real value of the budget deficit. Similarly, Gavrilencov (1995) notes a tendency in Russia for a softening of the budget deficit with an increase in inflation in the beginning of 1990s. If the Patinkin effect is present, it not only acts as a stabilizer of fiscal imbalance, but it also weakens the incentives for policy-makers to fight inflation.⁵ Indeed, decades of high inflation and numerous unsuccessful stabilization programs in some Latin American countries support this view.

Obviously, there are many channels through which inflation influences the real budget deficit.⁶ The most often cited channel is the “Olivera-Tanzi effect” (Olivera, 1967; Tanzi, 1977) that deteriorates real budget revenues through lags in tax collection. This effect destabilizes public finance. It should thus strengthen the incentives to stabilize high inflation. While the Olivera-Tanzi effect was reported for several high inflation economies, the opposite effect, known as the Patinkin effect, remains (in our opinion) out of focus. However, as long as both effects are partial, one can not be sure what the overall effect is. This implies that a deeper empirical investigation is required. From a theoretical point of view, there are two major reasons why the dominance of the Patinkin effect is an appealing assumption. First, as we show in this paper, adjusting the model to take the Olivera-Tanzi effect into account does not produce qualitatively new results, while incorporating the Patinkin effect does. Second, as just noted, the assumption that public finance in high inflation economies has some internal stabilizers is appealing, because it helps explain why high inflation may continue for decades.

The inflation tax Laffer curve and multiple equilibria

Various empirical investigations (Edwards and Tabellini, 1991; and more recently Bali and Thurston, 2000; and Fischer, Sahay and Vegh, 2002, among others) have found the so-called “Laffer curve effect”: the inflation tax, which is used to finance public spending when the

⁵ Thus the Patinkin effect may be considered to be an alternative explanation of delayed stabilization without explicit reference to political mechanisms (see, Drazen, 2000). Cardoso (1998) stresses that when inflation is reduced (or temporarily repressed), the disappearance of the factors that “stabilize” the budget deficit at high inflation throws down a challenge for the government to continue its stabilization efforts.

⁶ All of the factors in essence are distortional effects of inflation, determined by nominal state institutions. A brilliant overview of the real effects of inflation can be found in Fischer and Modigliani (1979).

government is unable to raise a sufficient amount of conventional taxes or to borrow from the public or abroad, is a limited source of finance. It increases with an increase in inflation only until money demand is inelastic on the so-called “efficient side”. When the inflation rate becomes significantly high, the economy begins to operate on the falling branch (the “wrong side”) of the ITLC.

The existence of the Laffer curve effect was challenged both on theoretical and empirical grounds for some historical episodes. Gutierrez and Vazquez (2004) show that ITLC arises in a cash-in-advance model only for an unrealistically high intertemporal substitution of consumption. The existence of ITLC in a money-in-the-utility function model also requires a specification of preferences that is similar to that studied by Calvo and Leiderman (1992). Eckstein and Leiderman (1992) and Bental and Eckstein (1997) stress the point that seigniorage revenue remained actually trendless despite a dramatic increase in inflation in Israel in 1979-1985. Kiguel and Neumeyer (1995) also indicate that Argentina remained on the “efficient” side of the ITLC in the *tabelita* and *pre-Austral* periods. Bali and Thurston (2000) provide an empirical estimation of the Laffer *surface*, explicitly taking into account the reserves ratio, the variation of which is important for inflation tax revenue. They report cases of high and even moderate inflation economies operating on the “wrong side”.

If one agrees that the ITLC is plausible (at least in some cases) both on theoretical and empirical grounds, then there are two levels of inflation (“low inflation” on the efficient side of the Laffer curve and “high inflation” on its wrong side) that can provide the same amount of inflation tax. In the literature on budget deficit monetization (Evans and Yarrow, 1981; Sargent and Wallace, 1987; Bruno and Fischer, 1990, among others) this problem is treated as a problem of dual equilibria. Assume that the budget deficit (or its part) must be financed by seigniorage. It was shown that if expectations are adaptive and the expected rate of inflation slowly adjusts to the actual rate of inflation, then the low inflation equilibrium is stable and the high inflation equilibrium is unstable. However, if expectations rapidly adopt to the actual rate of inflation or if expectations are rational (the case of perfect foresight), then low inflation is unstable and high inflation is stable. That is, the economy falls into the so-called “high inflation trap” (a Pareto-inefficient equilibrium). This result is puzzling. As long as the adaptive expectations hypothesis is usually rejected as an unappealing behavioral assumption allowing for a systematic prediction error, a high inflation trap is the outcome of the model. However, this means that the monetization of even a very low budget deficit will inevitably produce very high inflation and that an increase in the budget deficit will result in a decrease in inflation on the wrong side of the ITLC. This also means that if the budget deficit is higher than the maximum of the inflation tax, then the economy will face a hyperdeflation (not a hyperinflation!).

Relation to some recent contributions

The problem of dual equilibria in the ITLC has been extensively discussed in the literature. A prominent avenue of research is to use the more appealing assumption of adaptive learning instead of pure adaptive expectations. As was originally shown by Marcet and Sargent (1989), in this case the low inflation equilibrium is stable and, if the budget deficit exceeds the maximum inflation tax, then the economy is prone to explosive hyperinflation.⁷ In a response to general criticism of models with bounded rationality,⁸ Marcet and Nicolini (2003) provide a model with restricted learning mechanisms that allow expectations to be endogenous to policy switches. In their model, the low inflation equilibrium is also stable. However, even if one takes this more general assumption of bounded rationality, which guarantees the stability of the low inflation equilibrium, the basic model can not be used as a universal explanation of different economic outcomes. One needs a model that can imply a stable low inflation equilibrium when the economic environment is stable and the budget deficit is sufficiently low, and a stable (moderate or extremely) high inflation equilibrium on the wrong side of the ITLC when the financial system is unstable and the budget deficit is high.

To solve this problem Marcet and Nicolini (2003) consider a setup in which the government finances the budget deficit by seigniorage only when inflation is below some certain limit and establishes a fixed exchange rate regime otherwise in order to stabilize hyperinflation. They assume that expectations are not fully rational, but only small deviations from rationality in adaptive learning are allowed. The model implies that the low inflation equilibrium is locally stable but various shocks may push the inflation rate out of the region that allows the restricted adaptive learning mechanism to bring the economy to the low inflation equilibrium.⁹ While this approach meets some stylized facts on recurrent hyperinflations in high inflation economies, it relies on the assumption that the government can temporarily stabilize hyperinflation by setting an exchange rate peg that is rather arbitrary and does not deal with the underlying problem of budget deficit finance.¹⁰

⁷ Marimon and Sunder (1993, 1994) provide experimental evidence for the stability of the low inflation steady state under adaptive learning. Evans, Honkapohja and Marimon (2001) extend the model to account for heterogeneous learning rules and stress the role of constraints on fiscal policy in the convergence to low inflation equilibria.

⁸ Sargent (1993) criticizes models with adaptive learning procedures for their arbitrariness. Also, basic learning algorithms that do not take into account the reaction to changes in economic policy are subject to Lucas' critique.

⁹ Marcet and Nicolini (2005) evolve this approach by considering a switching regime for exogenous money growth. Building on Marcet and Nicolini (2003), Sargent, Williams and Zha (2005) develop and empirically evaluate a nonlinear general equilibrium model of hyperinflation. They show the importance of different shocks to seigniorage and agents' beliefs when average seigniorage is high. Contrary to Bruno and Fischer (1990) they show that fiscal anchors matter.

¹⁰ This methodology is in line with Michael Bruno's (Bruno, 1989) suggestion to solve the problem of "high inflation traps" by assuming that the speed of adaptation of expectations increases with inflation. Thus to escape from the trap and converge to low inflation equilibria, a policymaker can apply a price-wage freeze.

If the negative effect of inflation on the budget deficit (the Patinkin effect) dominates, as it was assumed in Cardoso (1998) and Smirnov (1997), then there could be three steady state levels of inflation in the ITLC model of budget deficit finance. Clearly, this approach outperforms the standard model, as it allows both low and extremely high inflation to be the stable steady state, while the middle equilibrium with moderately high inflation is unstable. In the paper we follow this insight.

There is another direction in economic literature, which treats the high inflation trap not as an outcome of an expectations formation mechanism associated with dual equilibria, but as an outcome of policy games. Zarazaga (1995) provides a game theoretic model in which several policymakers compete for seigniorage revenue to finance their specific spending under incomplete information. This competition results in “megainflation” outbursts. Heymann and Sanguinetti (1994) emphasize the distinction between the “target” and “observed” level of public expenditures. High target expenditures may provoke high inflation expectations. However, when high inflation is present, the inflation tax decreases along the wrong side of the ITLC, forcing the government to cut its expenditures. Thus, the observed low deficit and high inflation are consistent with each other. Moreover, to fight high inflation one must consider the “target” level of expenditures (and not the “observed” level!) that indicates “fiscal pressure”. In this paper we also explore the difference between “attempted” and “observed” fiscal policy, where the former is described by a “zero-inflation-deficit”, and the later is described by the actual deficit that is partially eroded by inflation under the Patinkin effect.

The rest of the paper is organized as follows. Section 2 gives an analysis of the relationship between inflation and the budget deficit under alternative assumptions about the form of inflation feedback. In Section 3 we analyze the inflationary consequences of budget deficits under the Patinkin effect. By dividing the budget deficit into the part that is subject to inflation feedback and the inflation-proof part, we reinterpret the evidence of the weak correlation between budget deficits and inflation. The final section contains a summary and discussion of the main results.

2. The model of budget deficit monetization

Consider the standard model that can be used to analyze inflation dynamics induced by budget deficit finance. The operational budget deficit, D , which in general includes the primary budget deficit and debt service, is financed by seigniorage, $S = \dot{M}/P$, where M is base money, and P is the price level.¹¹ Seigniorage may be further decomposed into two components, $S = \dot{M}/P = \dot{m} + m\pi$, the so-called *pure seigniorage*, \dot{m} (the increase in real money balances,

¹¹ All variables in the general case are functions of time. This is not shown in the text in order to simplify notation. A dot above a variable indicates a derivative with respect to time.

$m = M/P$), and the *inflation tax*, $m\pi$, where $\pi = \dot{P}/P$ is the rate of inflation. Thus, under the regime of fiscal dominance budget deficits determine the dynamics of real money balances and inflation:

$$(1) \quad \dot{m} + m\pi = D$$

It is usually assumed that the money market is always in equilibrium and that the demand for real money balances can be described by a Cagan-type function that is monotonically decreasing with respect to the expected rate of inflation: $M^S/P = m^D \equiv m(\pi^e)$, $m'(\bullet) < 0$. Furthermore, if the demand for real money balances has both elastic and inelastic parts, then, assuming that the expected inflation rate is equal to the actual inflation rate in the steady state, the inflation tax, $m\pi$, is a hump-shaped function of inflation represented in Fig. 1.

The next step is to assume some particular mechanism for forming expectations. The adaptive expectations hypothesis and the perfect foresight hypothesis have been most commonly used in the literature. The former mechanism is described by the equation:

$$(2) \quad \dot{\pi}^e = \theta(\pi - \pi^e),$$

where $\theta > 0$ stands for the speed of adaptation of expected inflation to its actual rate. The perfect foresight hypothesis, $\pi^e = \pi$, is a special case of the rational expectations hypothesis. It can be considered to be the extreme case as θ tends to infinity. Taking into account that $\dot{m} = m'_{\pi^e} \dot{\pi}^e$ we arrive at a differential equation for the inflation dynamics:

$$(3) \quad \dot{\pi} = \frac{\theta}{1 - \alpha\theta} \frac{D - \pi m(\pi)}{m(\pi)},$$

where $\alpha = -m'_{\pi^e}/m > 0$ is the semi-elasticity of money demand with respect to the expected inflation rate. This parameter is constant for the Cagan demand function that is very often assumed in the literature on high inflation.

For a given constant level of the operational budget deficit that does not exceed the maximum inflation tax, there are two steady state levels of inflation (see Fig. 1). If expectations are adaptive and $\alpha\theta < 1$ (i.e., expectations slowly adjust to the actual rate of inflation and the sensitivity of the demand for money to inflation is low), then low inflation is the stable steady state and high inflation is the unstable steady state.¹² However, if $\alpha\theta > 1$ or if expectations are rational (the case of perfect foresight, $\theta \rightarrow \infty$), then low inflation is the unstable steady state and high inflation is the stable steady state.

The assumption that the deficit is exogenous and independent of inflation – and therefore the accompanying assumption that the deficit finance by seigniorage is likewise independent of

¹² A similar result can be obtained under the assumptions of perfect foresight and the slow adjustment of the money market (see, e.g., Kiguel, 1989).

inflation, which was used implicitly above – is not always realistic. As we discussed in the Introduction, there are many factors that can bring about either a decrease or an increase in the *real* budget deficit d under inflation. Usually researchers mention the feedback from inflation to budget deficit simply to stress that inflation affects everything. However, is it simply an auxiliary assumption that does not change the principle mechanism of inflationary finance? To answer this question, let us assume that $D = D(\pi)$ in the ITLC model introduced earlier. If the Olivera-Tanzi effect is dominant in the economy, that is if $D'(\bullet) > 0$, then the budget deficit curve will have a positive slope. Fig. 1 illustrates this possible situation. Indeed in this case it seems most likely that the Olivera-Tanzi effect does not affect the principal result, namely that there are two steady states, the stability of which can be determined just as in the basic case.

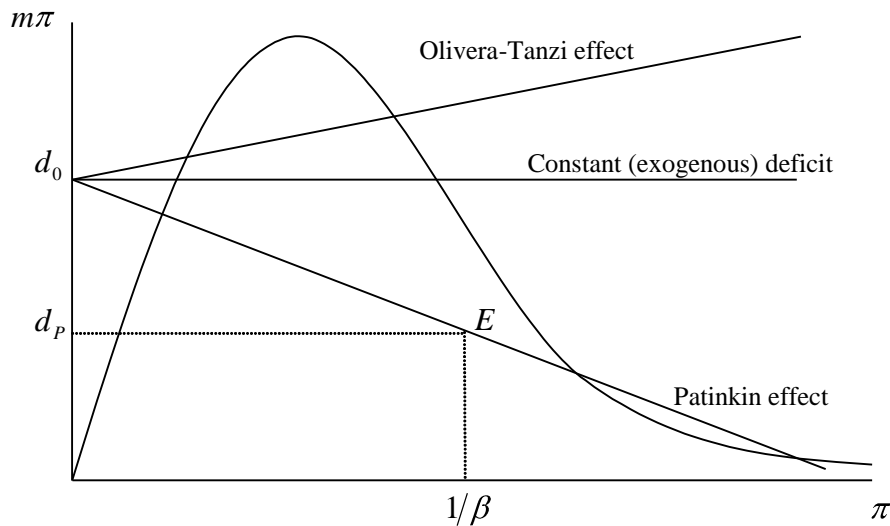


Fig. 1. Inflation tax Laffer curve

If the Patinkin effect has the greater impact, i.e. $D'(\bullet) < 0$, then the budget deficit curve will have a negative slope. For simplicity, we start by considering the following simple linear specification:

$$(4) \quad D(\pi) = d_E(1 - \beta\pi) + d_p.$$

Here, using Cardoso's terminology, $d_0 = d_E + d_p$ represents the “virtual deficit” that would have been observed in the case of zero inflation. However, while Cardoso (1998) and Smirnov (1997) consider the whole budget deficit to be subject to the Patinkin effect, it seems to be more realistic to assume that only a certain part of this zero-inflation deficit, namely d_E , may be affected by inflation. We will term it the “exposed deficit”. The other part, d_p , represents the “inflation-proof deficit” that is not subject to inflation feedback. In practice this division is determined by institutional arrangement. For example, d_E may consist of expenditures of particular ministries

whose bargaining power is too low to provide an indexation of their expenditures (as suggested by Patinkin, 1993). The inflation-proof deficit should include debt service in the case of foreign or indexed debt (which may be relatively large) among other items. Finally, the parameter β characterizes the strength of the Patinkin effect.

Fig. 1 shows that up to three steady states are possible. For instance, if the inflation expectations are adaptive or near-rational (with some appropriate mechanism of adaptive learning), the states with low or extremely high (hyper-) inflation are stable, while the equilibrium that corresponds to moderately high inflation is unstable.¹³

3. Inflation and budget deficits

One purpose of this paper is to reinterpret the evidence of the weak (at least in the short term) correlation between budget deficits and inflation. Thus, the next step is to see what the impact of fiscal policy is upon inflation under different regimes. In principle, increases in both the exposed and inflation-proof parts of the total deficit can be interpreted as fiscal expansion. However, as we will see, they operate in very different ways and thus they should be studied separately. Following the insight of Heymann and Sanguinetti (1994) we can state that changes in different items of the budget balance sheet may have very different effects on inflation (apart from their different effects on the real economy). If this is true, then the common practice of searching for the inflationary effects of fiscal policy using *aggregate* budgetary statistics may be somewhat misleading. This also has an important implication for the design of stabilization programs: in a situation when it is harmful to cut any spending items or to raise tax revenues, what expenditures should be cut first? In other words, is it exposed deficit or inflation-proof deficit that should be cut first to stabilize extremely high inflation? To explore this and other problems, we start with the formal analysis of the existence and stability of equilibria in a model of budget deficit finance that takes into account the Patinkin effect.

Proposition: If the system has up to three steady states for different values of the exposed budget deficit d_E and the inflation-proof budget deficit d_P , then the system has fold, transcritical or

¹³ The coexistence of two stable (and obviously Pareto-ranked) equilibria in this model opens up two prominent avenues of research that are beyond the scope of this paper. First, there could be room for the monetary anchors that act to bring the economy to the low inflation equilibria along the lines suggested by Bruno and Fischer (1990). Second, the situation when the economy falls into the Pareto-inferior equilibrium with extremely high inflation may be interpreted as the outcome of a specific “coordination failure”. It can be modeled as a policy game between the central bank that prefers low inflation and the government that is biased towards a high “virtual deficit” partially wiped out by inflation. Another possibility is to consider (following Zarazaga, 1995) the struggle between different budget agencies that have to either accept a low level of expenditures (and thus low inflation) initially, or to ask for the finance of high expenditures and watch the resulting high inflation erode them.

pitchfork bifurcation points in parameter d_E and it demonstrates a hysteresis effect with two fold bifurcations in parameter d_p .

Proof: See the Appendix.

3.1. Changes in the exposed budget deficit

We will consider first the effect of increasing the exposed deficit d_E , keeping the inflation-proof deficit constant. Note that an increase in d_E does not simply move up the line described by equation (6), but it pivots this line around point E with coordinates $(1/\beta, d_p)$ as shown in Fig. 1. Thus the number and stability of equilibria and the corresponding vector field crucially depend on the position of E with respect to the ITLC. Fig. 2 shows that there are eleven different cases. The corresponding bifurcation diagrams are described in Fig. 3.

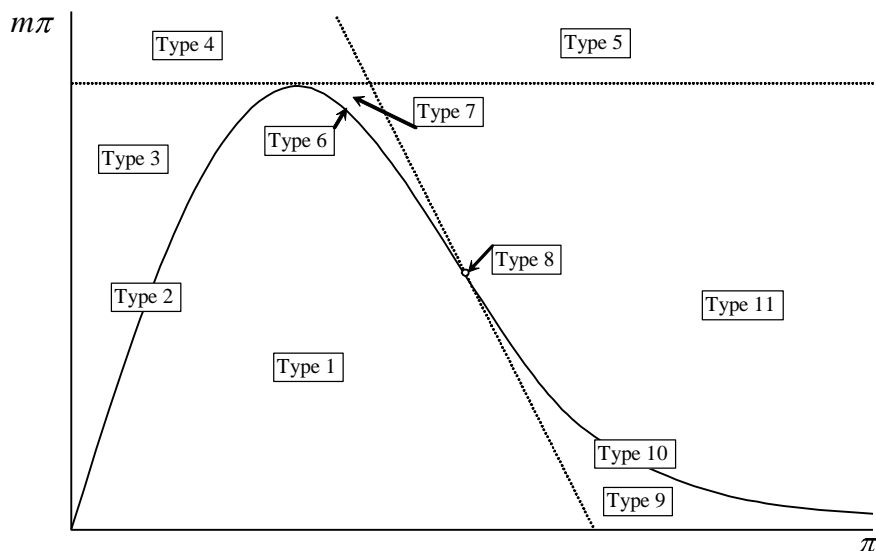


Fig. 2. Determination of different types of bifurcation

We can see that this rather simple model of inflationary finance gives rise to surprisingly many different bifurcation maps. Most of the diagrams demonstrate the coexistence of a fold bifurcation and a stable branch. Exceptions are shown on Fig. 3.5, where there is no bifurcation at all, and Fig. 3.8, which exhibits the case of a pitchfork bifurcation. Also, Fig. 3.6 and Fig. 3.10 demonstrate a combination of fold and transcritical bifurcations. Fig. 3.4, Fig. 3.7 and Fig. 3.10 indicate the existence of hysteresis (a double fold bifurcation). The main reason for this rich pattern is the combination of a downward sloping budget line and a hump shape of the ITLC. In particular, it is important that there is an inflection on its wrong side (the pitchfork bifurcation in Fig. 3.8 corresponds exactly to this point). The general results that we learn from this variety of bifurcations are the following.

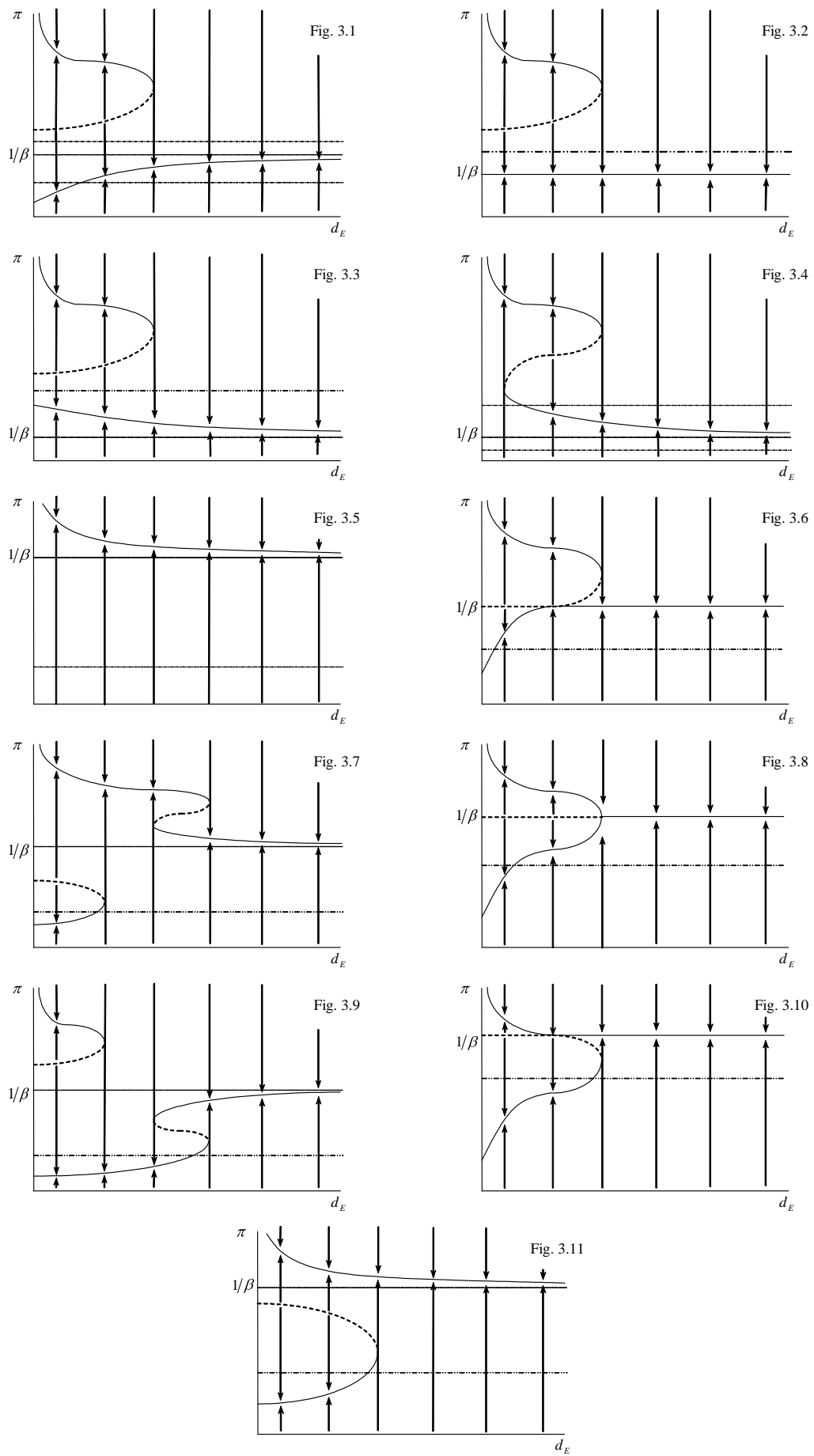


Fig. 3. Bifurcation diagrams for the change in parameter d_E

Multiplicity and stability of equilibria. The system may have up to three steady states. In fact, the existence of three steady states seems to be the most typical and plausible outcome. In this case the low inflation and extremely high (hyper-) inflation steady states are stable, while the intermediate steady state is unstable. In the basic model with exogenous (constant) budget deficit terms the “low inflation equilibrium” and the “high inflation equilibrium” correspond precisely to steady states that are on the increasing and decreasing branches of the ITLC respectively.¹⁴ However, in this model, as far as a declining deficit line is considered, the term “low inflation equilibrium” does not always correspond only to the increasing branch of the ITLC. The horizontal chain lines in Fig. 3 (and in Fig. 4 below as well) correspond to the rate of inflation that maximizes the inflation tax.¹⁵ The low inflation steady state may be on the decreasing branch of the ITLC as well. Only when point E is posited south-west of the peak of the ITLC (i.e. $1/\beta < \pi^*$ and $d_p < \pi^* m(\pi^*)$), will the low inflation steady state always be on the increasing branch of the ITLC.

For all positive values of the exposed deficit d_E , the system has a stable steady state with extremely high (hyper-) inflation. When d_E is very high this is the only steady state. In other words, this model rules out explosive hyperinflation (or hyperdeflation). This is because extremely high inflation can actually produce a negative value of $d_E(1 - \beta\pi)$, which is a partial surplus. Thus the operational balance $d_p + d_E(1 - \beta\pi)$ is sufficiently reduced to be financed by shrinking inflation tax revenues.

Except for the cases shown in Fig. 3.5 and (to a certain extent) in Fig. 3.4, a relatively low level of exposed deficit d_E guarantees the existence of a stable low-inflation steady state. In this respect we can treat the “high inflation trap” only as a local phenomenon (locally stable steady state). This result contrasts with the basic finding of the ITLC model with rational expectations, where a high-inflation steady state is globally stable.

How to reduce inflation by means of the exposed deficit? The natural question that arises here is whether it is possible to move the economy from the extremely high (hyper-) inflation stable steady state to a low inflation stable steady state if both of them exist. As can be seen from Fig. 3, an attempted fiscal contraction in the form of a reduction of the exposed deficit d_E cannot always produce this shift alone.¹⁶ In fact, if the economy is trapped in the extremely high (hyper-) inflation

¹⁴ As in the basic model, here we use the terms “low inflation equilibrium” or “low inflation branch” to denote a particular steady state with the lowest inflation rate among others. It does not mean literally that the inflation rate is low by international standards.

¹⁵ In the cases depicted in Fig. 3.1 and Fig. 3.4 there are two possible positions of this line with respect to the low inflation branch. For exposition, the scale of the ordinate line varies across the different cases depicted in Fig. 3.

¹⁶ Exceptions can be found in Fig. 3.9 and 3.10, but even in these cases the jump from high to low inflation occurs at relatively high levels of exposed deficit. When it is relatively low, its decrease cannot move inflation from the extremely high (hyper-) to the low inflation branch.

steady state, then a reduction in d_E leads to a further increase in inflation. There are two factors that explain this phenomenon. The first is the well-known effect associated with the wrong side of the ITLC, where a decrease in revenue requires an increase in the tax rate (inflation rate). The second is the specific arithmetic of the Patinkin effect. It is easy to see that in the extremely high (hyper-) inflation equilibrium, the inflation-proof deficit (that is, the ordinate of point E) is always higher than the inflation tax, $d_p > \pi m(\pi)$. This is only possible in the steady state if $d_E(1 - \beta\pi) < 0$. That is, the Patinkin effect creates a surplus, $1 - \beta\pi < 0$. However, as long as $d_E > 0$, its decrease must be balanced by an increase in the inflation rate.¹⁷

Surprisingly, Fig. 3 demonstrates that in most cases (except for those corresponding to Fig. 3.5, 3.10 and 3.11) the government has to increase (not decrease!) the exposed budget deficit d_E to move the economy from the extremely high (hyper-) inflation stable steady state to the low inflation stable steady state. The explanation of this seemingly paradoxical result lies in the logic described in the previous paragraph. An additional element that explains the bifurcation phenomenon consists in the following. Consider inflation as a function of exposed deficit given other parameters. Note that the absolute value of the expression $d_E(1 - \beta\pi(d_E))$ is a hump-shaped function of d_E . In other words, it demonstrates its own Laffer curve property. When d_E increases, the middle unstable steady state and the stable steady state with extremely high (hyper-) inflation move towards each other and eventually collapse in the bifurcation point. A further increase in d_E cannot provide sufficient partial surplus (equal to $d_E(1 - \beta\pi(d_E))$, which is negative and sufficiently high in absolute value), any more. The only remaining possibility for stable finance is the position on the low inflation branch, where $d_p < \pi m(\pi)$ and thus $d_E(1 - \beta\pi) > 0$ (the Patinkin effect still works but does not create a surplus).¹⁸

To summarize the findings of the two previous paragraphs: if one considers the exposed deficit as the sole stabilization tool, then its effect is associated with a certain type of *hysteresis*. A government that is attempting to stop extremely high (hyper-) inflation finds itself trapped in a situation when it cannot simply give up the exploitation of the Patinkin effect (this creates a partial surplus) by reducing the exposed deficit once this mechanism is in place. The only way to switch to the other regime (where the Patinkin effect does not create a partial surplus) is to exploit the previous regime to its limit.

¹⁷ We are not concerned with an unstable branch of equilibria. However, it is interesting to note that due to the existence of the Patinkin effect an increase in the exposed deficit does not always lead to a decrease in the intermediate steady state inflation rate despite the arithmetic of the wrong side of ITLC. Indeed, the unstable intermediate steady state is always on the wrong side. However, this unstable branch may be either decreasing or increasing.

¹⁸ The same logic may also explain why the cases illustrated in Fig. 3.4 and Fig. 3.5 do not have a low-inflation stable equilibrium.

Is there an unambiguous relationship between the (exposed) deficits and inflation? The relationship between d_E and inflation is negative (not positive!) when the economy is in the extremely high (hyper-) inflation steady state posited on the wrong side of ITLC¹⁹. Moreover, Fig. 3 demonstrates that this relationship is ambiguous even when the economy is on the efficient side of the ITLC (Fig. 3.3 and Fig. 3.4 demonstrate the decreasing low inflation branch). At the same time, as long as the *actual* deficit should be equal to the inflation tax in the steady state, the relationship between the *actual* budget deficit and inflation is always positive on the efficient side of ITLC and negative on its wrong side. However, unlike the basic model with an exogenous budget deficit, here the arithmetic of the wrong side provides only a partial explanation for the negative relationship between the budget deficit and inflation. An exhaustive explanation would require the Patinkin effect to be taken into account.

There are two considerations that underline this ambiguity. First, it is necessary to clearly distinguish the *virtual* (zero-inflation) and *actual* budget deficits, since the former mostly indicates the attempted fiscal policy, while the latter characterizes the actual result that takes into account inflation feedback. Furthermore, due to the “wrong side effect” and the Patinkin effect, neither the attempted nor the actual fiscal contraction is always able to reduce inflation. Second, there is a general problem of steady state analysis and the usage of discrete data from the budget balance sheet. The Patinkin effect does not decrease the budget deficit all at once, but rather gradually over time. Thus, an analysis across steady states and the empirical search for a relationship between the budget deficit and inflation may be misleading in determining how important the Patinkin effect is.

3.2. Changes in the inflation-proof budget deficit

Let us now turn to the analysis of the consequences of changes in the part of the monetized operational deficit that is not deteriorated by inflation. This part can consist of different items. Obviously, in high inflation countries the debt service (assuming that the real value of the debt is indexed or that it is denominated in foreign currency) deserves special attention. In general, public debt is a dynamic variable, yet in our reduced model we can consider debt service to be a bifurcation parameter. Changes in debt service may be caused by changes in fiscal rules, according to which a certain part of the operational deficit is monetized if, for example, the government can not roll over its debt further in the face of a confidence crisis. Alternatively, these changes may be due to shifts in the exogenous interest rate, which may arise for the same reason: a confidence crisis, when investors are ready to buy new debt only at a higher interest rate. Another important possibility is that the government may guarantee private debt that represents implicit liabilities in

¹⁹ Fig. 3.9 provides a more subtle case, where there are actually two branches of extremely high (hyper) inflation for small and large values of d_E . For the latter, the extremely high (hyper) inflation branch is increasing.

good times, but becomes an explicit part of public debt in time of crisis, and this leads to a discrete increase in debt service. On the other hand, debt service may decrease if there is a restructuring of debt or debt relief.

Changes in the inflation-proof deficit d_p result in a parallel shift of the budget deficit line described by equation (4). There are two cases that correspond to two different bifurcation diagrams in Fig. 4. If the budget deficit line is flatter than the slope of the ITLC in its inflection point at the wrong side, then depending on the size of d_p there could be one, two, or three steady states. The bifurcation diagram (Fig. 4.1) shows that there is a hysteresis with two fold bifurcations. However, if the budget deficit line is steeper than the slope of the ITLC at the inflection point, then for any d_p there is always one steady state rate of inflation (Fig. 4.2). The slope of the budget deficit line is equal to $-\beta d_E$. Thus, Fig. 4.1 corresponds to the case when the exposed deficit d_E is relatively small and the Patinkin effect is relatively weak, while Fig. 4.2 corresponds to the case when the exposed deficit is relatively large and the Patinkin effect is relatively strong.

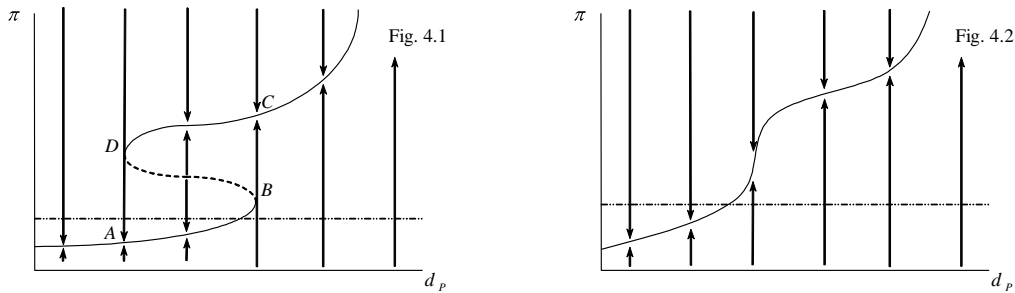


Fig. 4. Bifurcation diagrams for the change in parameter d_p

Comparing these different cases, we see that in the case of a relatively large exposed deficit and a strong Patinkin effect, changes in the size of the inflation-proof deficit d_p do not have a strong stabilizing or destabilizing effect. However, if the exposed deficit is small enough and the Patinkin effect is weak, then an increase in d_p over a critical level leads to an abrupt change in the inflationary regime, moving the economy from high inflation to hyperinflation. If the economy suffers from extremely high (hyper-) inflation, then a sufficiently large decrease in the size of the inflation-proof deficit may stop extreme inflation (hyperinflation) and move the economy to the low inflation branch. There are three major conclusions.

Inflation-proof deficit and inflation. While the relationship between the exposed budget deficit and a stable inflation rate is ambiguous, the relationship between the inflation-proof deficit and a stable inflation rate is always positive, though nonlinear. Furthermore, this is true no matter what side of the Laffer curve the economy is operating on. Actually, the part of the low inflation

branch in Fig. 4.1 falls on the wrong side, while the entire extremely high (hyper-) inflation branch is on the wrong side.

Recurrent hyperinflations. This model can explain the well-known phenomena that (chronically) high inflation economies often fall into the hyperinflation regime for some period and then return to moderate inflation. These catastrophic (rapid) changes in the regime may occur even without visible deterioration or an improvement in economic conditions. So far as even small changes in the size of the inflation-proof deficit (in particular, in the size of debt service) can lead to abrupt changes in the regime, various things such as publicly guaranteed debt (implicit government liabilities) or debt relief, changes in investor confidence that affect the interest rate, the appearance or disappearance of the indexation of government spending, are all important in understanding the causes of recurrent hyperinflations and their stabilization.

Hysteresis. The explanation of why chronically high inflation countries sometimes fall into the hyperinflation regime involves the important hysteresis effect. Consider the low inflation branch AB and the extremely high (hyper-) inflation branch DC in Fig. 4.1. Assume that the economy is initially in point A in Fig. 4.1. A small increase in d_p initially leads to a gradual increase in the inflation rate and does not produce an abrupt shift unless the economy reaches point B . In point B , which is actually on the wrong side of the Laffer curve (where the inflation rate is indeed very high by international standards), there is a discrete jump in inflation that moves the economy to point C at the extremely high (hyper-) inflation branch. However, when the economy is in point C , a small decrease in the inflation-proof deficit cannot produce a downward jump in inflation to the low inflation branch (say, back to point B). It can produce a relatively small decrease in inflation along the branch DC . Only when the inflation-proof deficit d_p is significantly reduced, so that the economy has moved to point D , will there be a downward jump in inflation (a jump from point D to point A). Again, when the economy is in point A , it will not jump to the extremely high (hyper-) branch, unless d_p increases by too much.

The intuition behind the hysteresis effect is again based on the operation of the Patinkin effect and the arithmetic of the wrong side of ITLC. As long as stable inflation rate is an increasing function of the inflation-proof deficit d_p , an increase in the latter has two effects upon the total budget deficit $D(\pi) = d_E(1 - \beta\pi(d_p)) + d_p$. The first is a direct effect of an increase in $D(\pi)$ following an increase in d_p . The second effect is the reduction of $d_E(1 - \beta\pi(d_p))$ due to the Patinkin effect. Note also that when the economy is on the efficient side of the ITLC, an increase in d_p (which leads to an increase in inflation) is associated with an increase in inflation tax revenue, which is used to finance the rising $D(\pi)$. However, when the economy eventually slips into the wrong side, a further increase in d_p and inflation leads to a smaller inflation tax revenue. This can

be observed in a steady state only for the smaller values of $D(\pi)$ that resemble the arithmetic of the wrong side. The question is whether the situation of increasing d_p and decreasing $D(\pi)$ (which is financed by steady state seigniorage) is feasible along the whole wrong side of ITLC. The answer is that this is indeed feasible if the Patinkin effect is strong enough; this is explained in the next paragraph. However, when the Patinkin effect is weak, this situation is feasible only for the upper and lower segments of the wrong side. For the middle segment (which is associated with a relatively sharp decline in the inflation tax following an increase in the inflation rate) it is not feasible. This makes the corresponding steady states unstable. Indeed, for the sharp decline in $D(\pi)$ that is needed to meet rapidly falling inflation tax revenues, there needs to be a strong Patinkin effect to override the increase in the inflation-proof deficit. Otherwise, a decline in $D(\pi)$ can be met by inflation tax finance only if the latter is a sufficiently steep function of inflation (and this is the case for the upper and the lower segments of the wrong side).

The case shown in Fig. 4.2 does not demonstrate the hysteresis effect, but it resembles this nonlinearity. Here the steady state inflation rate is a monotonic function of d_p that changes its convexity (it increases more rapidly over a certain interval). The reason why a relatively large exposed deficit and a strong Patinkin effect preclude the hysteresis effect associated with changes in the inflation-proof deficit is straightforward. On one hand, when the exposed budget deficit is high enough, it becomes a more important part of the total budget deficit that is financed by inflation tax. Thus, it outweighs the inflation-proof deficit in determining the steady state inflation rate. On the other hand, when the Patinkin effect is relatively strong, it can turn the exposed deficit into a surplus even at relatively low inflation rates. Given the size of the exposed deficit, an increase in the steady state inflation rate leads to an increase in the level of the resulting surplus. This balances the increase in the size of the inflation-proof deficit and makes its nonlinear effect weaker. However, in general, as long as a very strong Patinkin effect seems to be an unrealistic assumption and in practice governments do not design the zero-inflation budget deficit to be “extraordinarily” high even in the face of a deep budget crisis, the case shown in Fig. 4.2 seems to provide a less realistic outcome than the hysteresis effect described in Fig. 4.1.

An important lesson that we learn in comparing the consequences of changes in the exposed deficit and of changes in the inflation-proof deficit is that the latter provides a more efficient fiscal instrument that can be used to stop extremely high (hyper-) inflation. Indeed, as shown in the previous subsection, a decrease in the exposed deficit leads to even higher rates of inflation once the economy is on the extremely high (hyper-) inflation branch. In contrast, a reduction in the inflation-proof deficit leads to a gradual reduction in inflation that is likely to be followed by an abrupt

downward jump in inflation after some critical point (or at least a rapid decrease in inflation if bifurcation does not occur).

3.3. An implication: shifts in money demand and hysteresis

Unaccounted shifts in money demand seem to be an important part of the explanation for the weak correlation between the budget deficit and inflation. Consider, for example, a decrease in the demand for real money balances (which could result after financial liberalization, allowing the substitution of domestic currency in transactions, etc.). A downward shift in demand for real money balances shrinks the base of inflation tax. The ITLC moves down for any given budget deficit line. In fact, this produces qualitatively the same picture as an upward shift of the budget line given the Laffer curve. As in the case of changes in the inflation-proof deficit, in this case shifts in money demand may be associated with the substantial hysteresis effect described in Fig. 4.1. A decrease in money demand, associated initially with the gradual increase in the inflation rate along the low inflation branch, may produce an abrupt jump in inflation, moving the economy to the extremely high (hyper-) inflation branch. And once the economy is there, the demand for real money balances needs to rise substantially in order to return the economy to the low inflation branch.

This result corresponds to previous findings of the hysteresis effect in the money market in high inflation economies. Dornbusch, Sturzenegger and Wolf (1990) show that shifts in money demand in high inflation economies may represent an important hysteresis effect when money demand does not return to its low inflation level after stabilization. Arce (2006) provides a model of hyperinflation that incorporates this hysteresis effect. He shows that due to the hysteresis in money demand, fiscal-monetary reform that usually stops hyperinflation can not prevent its outburst from high inflation. However, unlike previous studies, the model presented here does not involve any specific mechanism that describes the adoption of new financial instruments. Instead, we provide an explanation based on the specific budgetary arithmetic of the Patinkin effect. In doing so, we do not call into question the importance of financial market mechanisms, but rather provide a complementary explanation of the hysteresis.

4. Concluding remarks

Two important problems which many developing countries face are inflation bias and deficit bias. There is reason to believe that the main source of inflation bias in developing countries is the significant financing of the budget deficit by seigniorage. This implies that inflation bias can be explained by deficit bias in many cases, especially in high inflation economies.

The stabilization of the economies in Latin America, Israel and of the transitional economies in Eastern Europe (economies with high inflation over the last decades) shows that it is important to

not only adopt the appropriate measures, but to do so in the correct order. One of the main conclusions that one comes to after the analysis of historical cases when inflation was stabilized is that of the impossibility of stopping high inflation even in the middle run by using only monetary policies that were not first supported by stabilization in the fiscal sphere.

This paper contributes to the literature on budget deficits and inflation in high inflation economies. The main finding is that recurrent outbursts of extreme inflation in these economies can be explained by the hysteresis effect associated with public finance. This interpretation meets the evidence that dramatic shifts between regimes of moderately high and extremely high (hyper-) inflation often occur without a visible deterioration in public finance or abrupt shifts in fiscal and monetary policies. The hysteresis effect can be explicitly explained by the action of two mechanisms: the arithmetic associated with the wrong side of the ITLC and the Patinkin effect. While the wrong side effect has been extensively discussed both theoretically and empirically in the literature, there is still much to be done in the empirical and theoretical research on the Patinkin effect. The variety of bifurcation scenarios described in the previous section can be seen as an outcome of introducing the Patinkin effect in a specific linear form. However, the most important implications of the analysis of the Patinkin effect, and the hysteresis effect in particular, seem to be rather robust to the choice of a particular specification.

Small changes in the inflation-proof part of the budget deficit (foreign or indexed debt service, among other items) and shifts in money demand (due to financial innovation, dollarization, changes in reserves requirements, etc.) are important factors that may trigger extremely high (hyper-) inflation in moderately high inflation economies. At the same time their reverse changes may act as stabilizers of hyperinflation, even if the changes are minor. This is the essence of the hysteresis effect.

Another important message of this paper is that it is necessary to clearly distinguish between attempted and realized fiscal policies. As the Patinkin effect deteriorates part of the budget deficit, an attempted fiscal expansion (an increase in the exposed deficit) may actually reduce the operational budget deficit by turning the exposed deficit into a surplus at a higher inflation rate. This theoretical evidence has implications for both the discussion of inflationary consequences of budget deficits and the proper design of stabilization policy.

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Appendix. Proof of the Proposition

Consider the vector field of the nonlinear equation $\dot{\pi} = F(\pi, d_E, d_P)$, where the function $F(\pi, d_E, d_P) = \theta((1-\alpha\theta)m)^{-1}(d_E(1-\beta\pi) + d_P - \pi m)$ is thrice continuously differentiable. To determine bifurcations it is necessary to evaluate the sign of the corresponding partial derivatives.²⁰

Changes in the exposed budget deficit d_E

For the bifurcation value $d_E^* = (m^* - \alpha d_P)(\alpha - \beta - \alpha\beta\pi^*)^{-1}$ the steady state π^* is singular, i.e., $\partial F(\pi^*, d_E^*, d_P)/\partial \pi = 0$. The point (π^*, d_E^*) can be a fold, transcritical, or pitchfork bifurcation.

Fold bifurcation. In the bifurcation point $\partial^2 F(\pi^*, d_E^*, d_P)/\partial \pi^2 = \theta((1-\alpha\theta)m^*)^{-1}(\alpha m^* - \alpha\beta d_E^*)$ and $\partial F(\pi^*, d_E^*, d_P)/\partial d_E = \theta((1-\alpha\theta)m^*)^{-1}(1-\beta\pi^*)$. If in equilibrium the budget deficit line $D(\pi) = d_E(1-\beta\pi) + d_P$ is tangent to ITLC, then $-\beta d_E = m^* + m'_\pi \pi^*$. Taking into account that $m'_\pi = -\alpha m^*$, and thus $m^* - \beta d_E^* = m^*(2 - \alpha\pi^*)$, the sign of $\partial^2 F(\pi^*, d_E^*, d_P)/\partial \pi^2$ is positive if $\pi^* < 2/\alpha$, and negative if $\pi^* > 2/\alpha$. Note that $\pi^* = 2/\alpha$ is the inflection point of the ITLC on its wrong side, and that the low inflation steady state is always to the left of an inflection point, while the extremely high (hyper-) inflation steady state is always to the right. If $\pi^* > 1/\beta$, then $\partial F(\pi^*, d_E^*, d_P)/\partial d_E < 0$. In this case there are two branches of equilibria with stable extremely high inflation and unstable high inflation to the left of the fold bifurcation point (see Fig. 3.1-3.4, 3.6,

²⁰ See e.g. Lorenz (1989).

3.7, 3.9) or there are two branches of equilibria with unstable extremely high inflation and stable high inflation to the right of the fold bifurcation point (Fig. 3.4 and 3.7 demonstrate the coexistence of two bifurcation points that constitute a hysteresis). If $\pi^* < 1/\beta$, then $\partial F(\pi^*, d_E^*, d_p)/\partial d_E > 0$. In this case there are either two branches of equilibria with unstable extremely high inflation and stable high inflation to the right of the fold bifurcation point (see Fig. 3.7, 3.10, 3.11) or there are two branches of equilibria with unstable extremely high inflation and stable high inflation to the left of the fold bifurcation point (Fig. 3.9 demonstrates two fold bifurcation points that constitute a hysteresis).

Transcritical bifurcation. If in equilibrium the budget deficit line is tangent to ITLC, then $\partial^2 F(\pi^*, d_E^*, d_p)/\partial \pi^2 > 0$ if $\pi^* < 2/\alpha$ and $\partial^2 F(\pi^*, d_E^*, d_p)/\partial \pi^2 < 0$ if $\pi^* > 2/\alpha$. The equality $\pi^* = 1/\beta$ is possible only when point E lies on the ITLC, and then $\partial F(\pi^*, d_E^*, d_p)/\partial d_E = 0$ and $\partial^2 F(\pi^*, d_E^*, d_p)/\partial \pi \partial d_E = -\beta\theta((1-\alpha\theta)m^*)^{-1} < 0$. In this case (π^*, d_E^*) is a transcritical bifurcation point where there is a cross of two branches of equilibria, thus changing their stability property (see Fig. 3.6 and 3.10).

Pitchfork bifurcation. If point E coincides with the inflection point of the ITLC, i.e. $\pi^* = 1/\beta = 2/\alpha$ and $d_p = \pi^* m^* = m^*/\beta$, then $\partial F(\pi^*, d_E^*, d_p)/\partial d_E = 0$, $\partial^2 F(\pi^*, d_E^*, d_p)/\partial \pi^2 = 0$, $\partial^2 F(\pi^*, d_E^*, d_p)/\partial \pi \partial d_E = -\beta\theta((1-\alpha\theta)m^*)^{-1} < 0$, and $\partial^3 F(\pi^*, d_E^*, d_p)/\partial \pi^3 = -\alpha^2\theta(1-\alpha\theta)^{-1} < 0$. Thus, (π^*, d_E^*) is a pitchfork bifurcation point (see Fig. 3.8).

Changes in the inflation-proof budget deficit d_p

Point (π^*, d_p^*) is a fold bifurcation point for the value of $d_p^* = \alpha^{-1}(m^* - d_E(\alpha - \beta - \alpha\beta\pi^*))$. Indeed, $\partial F(\pi^*, d_E, d_p^*)/\partial \pi = 0$, $\partial F(\pi^*, d_E, d_p^*)/\partial d_p = \theta((1-\alpha\theta)m^*)^{-1} > 0$, and if in low inflation and extremely high (hyper-) inflation equilibria the budget deficit line is tangent to ITLC (which could be the case only when the budget deficit line is flatter than the slope of ITLC at the inflection point), then the sign of $\partial^2 F(\pi^*, d_E, d_p^*)/\partial \pi^2 = \theta((1-\alpha\theta)m^*)^{-1}(\alpha m^* - \alpha\beta d_E)$ is positive if $\pi^* < 2/\alpha$, and negative if $\pi^* > 2/\alpha$. Fig. 4.1 demonstrates that the two corresponding fold bifurcation points constitute a hysteresis.

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