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On New Resonances and Normal Forms of Autonomous Systems with One Zero Eigenvalue

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Abstract—In a neighborhood of a singular point, we consider autonomous systems of ordinary differential equations such that the matrix of their linear part has one zero eigenvalue, while the other eigenvalues lie outside the imaginary axis. We study the reducibility of such systems to polynomial normal form.

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INTRODUCTION

This paper continues the studies started in [1], where we analyzed systems of ordinary differential equations whose linear part matrix has one eigenvalue, while the other eigenvalues lie outside the imaginary axis. For brevity, such systems are called *systems with one zero root*. In the present paper, we study the structure of normal form and reducibility of the above-mentioned real autonomous system of ordinary differential equations to normal form in a neighborhood of a singular point. The normal form of systems of ordinary differential equations has been studied sufficiently well. In particular, the problem of analytic reducibility to normal form has been investigated in detail (see, e.g., [2]). Most of the papers concerned with such problems deal with systems with a nondegenerate singular point (or an invariant manifold), but even weakly degenerate systems have been studied very poorly. For partially degenerate systems, see [3]. The problem of infinite-smooth equivalence between formally equivalent systems with one zero root or two pure imaginary roots of the matrix of the system linear part was studied in [4], [5]. Most of the studies deal with the problem of finite-smooth equivalence of systems with a nondegenerate linear part; these aspects were thoroughly reviewed in the book [6]. In the present paper, we consider the class of transformations with singularities and show that the use of such transformations allows one to obtain very useful information about the normal form.

We consider a real autonomous system

$$\dot{\xi} = \frac{d\xi}{dt} = Q(\xi), \tag{1}$$

where $\xi, Q(\xi) \in \mathbb{R}^{n+1}$, $n > 0$, and $Q(\xi)$ is a function of class C^∞ in a neighborhood of the origin, $Q(0) = 0$, and the matrix $\widetilde{A} = Q'(0)$ has n eigenvalues lying outside the imaginary axis and one zero eigenvalue.

Let $\lambda_1, \dots, \lambda_n$ be eigenvalues of the matrix $\widetilde{A}$ that lie outside the imaginary axis, and let $\lambda_0 = 0$. Using a standard linear transformation, we reduce system (1) to the following form, where the matrix $\widetilde{A}$ is written in Jordan form:

$$\begin{aligned} &\dot{x} = f(x, y), \\ &\dot{y}_i = \varepsilon_i y_{i+1} + \lambda_i y_i + g_i(x, y), \qquad i = 1, \dots, n. \end{aligned} \tag{2}$$

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