Exercises
Matchings and Markets
HSE
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Let (M, W, P) be a marriage game.

1. Let μ, μ' be two stable matchings. For every m in M, let $\mu^M(m)$ be $\mu(m)$ if $\mu(m) = \mu'(m)$ and be the "better" of $\mu(m), \mu\prime(m)$ for m otherwise. (In short notation, $\mu^M = \mu \vee_M \mu'$.) Show that μ^M is (i) a matching and (ii) stable. Define $\mu_M = \mu \wedge_M \mu'$ analogously by replacing "better" with "lesser". Show $\mu_W = \mu^M$.

(This is the *Stable Marriage Lattice Theorem*: The set of stable matchings for any marriage game is a lattice under the *group preferences* of all the individuals on one side. The male *supremum* of two stable matchings is at the same time their female *infimum*.)

2. Show that if a man is unmatched in one stable matching then he is unmatched in any other stable matching.

3. Show that if all men have the same preferences then there is a unique stable matching.

4. Consider the many-to-many matching game (M, W, P, Q) where Q_m is a number that denotes the *quota* of m (meaning m would like to be matched with as many as Q_m women) and Q_w denotes the *quota* of w. Describe the "generalized" *Deferred Acceptance Procedure* for this game.

Let $(B, S, (w_{bs}))$ be an assignment game.

5. Show that, if (p, μ) is a competitive equilibrium then (u, p), where $u_b = w_{b\mu(b)} - v_{\mu(b)}$ for $b \in \mu$ and $u_b = 0$ for $b \notin \mu$, is a core payoff.

6. Show that if (p, μ) is the minimum price equilibrium, then for any buyer $b \in B$, there is a μ - alternating path to either an object $s \in \mu$ with $p_s = 0$ ora buyer $b' \notin \mu$ with $u_{b'} = 0$. (Otherwise it would be possible to lower p_s to $p_s - \in$ for $\in$ sufficiently small for all s reachable from b by a μ - alternating path and still have an equilibrium.)

7. Consider the game where $B = \{1, ..., M\}, S = \{1, ..., N\}$ and $w_{mn} = mn$. Find the B-optimal competitive equilibrium when $M = N = 3$..

8. Consider the one-to-one house market where there are 2 buyers A, E and 2 sellers a, e. Let p_a, p_e be the prices of the houses. Let $U_{Aa}(p_a) = 2(6 - p_a)$

be the utility of A if she buys the house of a at price p_a, and let $U_{Ae}(p_e) = 18 - p_e, U_{Ea}(p_a) = 8 - p_a, U_{Ee}(p_e) = 2(12 - p_e)$. Find all competitive equilibria.

9. 3 students A, B, C have together rented a house with 3 rooms for a total rent of 1000. The rooms are different in quality. The maximum willingness-to-pay of A for the 3 rooms are respectively $500, 300, 300$. For B and C respectivelthese are $600, 500, 300$ and $400, 200, 100$. Who should get which room and how should they share the total rent if no student is to "envy" another (meaning no student will rather have the room another has gotten and will contribute the associated share.)