Monetary Policy and Quantitative Easing in an Open Economy: Prices, Exchange Rates and Risk Premia *

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Abstract

Under Quantitative Easing, Open Market Operations involve arbitrary portfolios of assets and not exclusively nominally risk free bonds held with a specific target composition. In a simple stochastic cash-in-advance model of a large open economy, quantitative easing inhibits the ability of the central bank to control the path of prices and exchange rates. This is the case even with non-Ricardian fiscal policy.

Alternative modes of conduct of monetary policy have measurable implications. A financial stability target, where the central bank trades only in nominally risk free bonds, implies that the risk premium is positively correlated with future interest rates. A price stability, or inflation, target induces the same correlation, while a monetary stability target reverses the sign of the correlation. Naïve estimations of aggregate risk premia may be misleading if monetary policy is not accounted for.

Key words: monetary policy; uncertainty; indeterminacy; fiscal policy; open economy.

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1 Introduction

How monetary policy transmits inflation expectations to other countries is a question of theoretical interest and practical importance. The failure to control inflation domestically can be the cause of suboptimal domestic fluctuations, if indeterminacy is real, and can de-stabilise trading partners via current account changes. Optimal fiscal-monetary policy supports an optimal allocation of resources; if such a policy is also consistent with other, suboptimal, equilibrium allocations, then, it does not "implement" the targeted allocation.[1] Under normal conditions, monetary policy sets a target for the short-term (here one period) interest rates, and conducts open market operations or repo transactions, using as collateral Treasury securities, with various maturities, but to conform to an ex-ante determined overall portfolio composition which has an exclusive focus on Treasuries of short maturity. Unconventional monetary policy expands the balance sheet by increasing the maturity range (and possibly range of assets) of the monetary authority portfolio. As under conventional monetary policy, under the recent US experience of Credit Easing it is the explicit target for the composition of the balance sheet that allows the monetary authority to target the stochastic path of inflation: the target for the composition of the portfolio guarantees the necessary restrictions to obtain determinacy. The absence of such restrictions under the UK and Japanese versions of QE manifests nominal (and possibly) real indeterminacy. Here we show that, non-traditional methods of conducting monetary policy such as quantitative easing affect the path of prices and furthermore, the interaction with interest rate rules generate specific risk premia associated with the correlation between interest rates and the martingale measure in an open economy.

To address these issues, we consider an open economy extension of Mcmahon et al (2012) and Nakajima and Polemarchakis (2005), similar in spirit to Lucas (1982) and Geanakoplos and Tsomocos (2002). Specifically, we consider large open stochastic cash-in-advance economies, and first show that indeterminacy is pervasive: monetary policy does not suffice to determine the stochastic path of inflation. This indeterminacy may affect real allocations even with flexible prices, depending on the conduct of monetary policy, the completeness of asset markets, and the timing of transactions in goods and asset markets.

[1] Chari and Kehoe (1999) and Bloise et al. (2005) survey the literature.

In an open economy, this indeterminacy proliferates. The stochastic distribution of prices is now independently indeterminate in each country. If all countries coordinate on to an interest rate monetary policy rule, the indeterminacy is purely nominal, while if even one country runs a money supply rule, then via current account changes, the indeterminacy becomes (globally) indeterminate. This result is beyond that of Dupor (2000), where like us, they explore exchange rate determination in a multi country/ currency model under a nominal interest rate peg. They too restrict the substitutability of currency as a method of payment across borders and maintained the possibility that the exchange rate is not unique for a conventional monetary/fiscal policy. Our result is stronger however. Their result resets on agents being indifferent as to the currency in which they hold their money balances, ours does not. Although the non-Ricardian fiscal policy pins down initial price levels and hence the initial exchange rate, the stochastic distribution of prices and exchange rates depends on asset demands. As the monetary authority is willing to supply state-contingent bonds, maintaining only the interest rate, individual asset prices are left undetermined. Furthermore, as agents are indifferent between purchasing assets in any country, the indeterminacy in one country proliferates globally.

The fact that the initial price level and the nominal equivalent martingale measure are indeterminate implies that monetary policy leaves indeterminacy of degree equal to the number of unique martingale probabilities in a finite-period model (1 less than the number of terminal nodes)[2].

[2] There is a vast and important literature on indeterminacy of monetary equilibria. Sargent and Wallace (1975) discussed the indeterminacy of the initial price level under interest rate policy; Lucas and Stokey (1987) derived the condition for the uniqueness of a recursive equilibrium with money supply policy; Woodford (1994) analyzed the dynamic paths of equilibria associated with the indeterminacy of the initial price level under money supply policy. In this paper, we give the exact characterization of the indeterminacy in stochastic economies in terms of the initial price level and the nominal equivalent martingale measure and extend the argument to the sticky-price case. Also, we show that there is a continuum of recursive equilibria with interest rate policy. In closely related models, Dubey and Geanakoplos (1992, 2003) considered non-Ricardian fiscal policy with no transfers and Geanakoplos and Tsomocos (2002) and Tsomocos (2008) extended their model to an open economy. Dreze and Polemarchakis (2000) and Bloise et al. (2005) studied the existence and indeterminacy of monetary equilibria with a particular Ricardian fiscal policy, seigniorage distributed contemporaneously as dividend to the private sector. The literature on incomplete markets shows the degree of real indeterminacy which proliferates when contracts are in nominal terms. Geanakoplos and Mas-Colell (1989) showed that there are generically $S-1$ degrees of indeterminacy, where S is the number of states. In an

The mainstream competitive model has locally unique equilibria with respect to the real side of the economy; however, it manifests nominal indeterminacy. Kareken and Wallace (1981) extend the O.L.G. indeterminacy result to a monetary model of the international economy. Tsomocos (2008) show that under non-ricardian fiscal policy, international monetary equilibria are locally unique[3].

The necessity of analysis of the determinacy of any model and specifically any monetary model is the question of money non-neutrality or lack thereof. In other words, a model as the traditional competitive model that produces real determinacy but nominal indeterminacy manifests neutrality of monetary policy. Changes of the money supply affect nominal variables without influencing the determination of the real allocations of an economy. Therefore, the study of the number of equilibria in an economic model lies at the heart of the neutrality debate in macroeconomics.

We then study determinate equilibria and argue that the correlation between monetary costs and real asset payoffs in monetary models creates risk-premia in expected exchange rates. Monetary costs generate a wedge between cash and credit goods, and consequently affect marginal utilities and equilibrium prices. This premium causes the term structure to lie above levels predicted by the pure expectation hypothesis. In equilibrium models where monetary policy is neutral, as in Lucas (1982), as risk premia are constant, interest rate differentials move one-for-one with the expected change in the exchange rate. Empirically, however, the expected change in the exchange rate is roughly constant and interest differentials move approximately one-for-one with risk premia. Furthermore, the forward premium anomaly, as documented by Fama (1984), Hodrick (1987), and Backus et al. (1995) among others, states that when a currencys interest rate is high, that currency is expected to appreciate. Here we show that not only does the stochastic distribution of prices and interest rates domestically matter, but also the correlation of monetary policy across countries, in determining risk premia. We do this by considering the general equilibrium model of Lucas (1982) who considered only "cash goods", to include "credit goods". The

abstract open economy, Polemarchakis (1988) allow A assets to be dominated in N distinct units of account or currencies. In addition to the purchasing power of one currency, the rates of exchange across currencies may now vary. In this setting Polemarchakis (1988) shows that, generically, the economy displays $NS - A(N-1) - N$ degrees of indeterminacy.

[3]This is in the model of Geanakoplos and Tsomocos (2002), which has qualitatively a similar structure to Lucas (1982)

International Finance models of Geanakoplos and Tsomocos (2002), Tsomocos (2008), Peiris and Tsomocos (2010) and Peiris (2010) study the effects of this and monetary policy becomes non-neutral since monetary changes affect nominal variables which in turn determine different real allocations. In a closed economy Espinoza et al. (2009) show that the risk-premia generated by the non-neutrality of a monetary policy exist in addition to the ones derived from the stochastic distribution of endowments as presented in Lucas (1978) and Breeden (1979). They provide a potential explanation for the Term Premium Puzzle. In such a setting there is a role for monetary policy to determine the equilibrium allocation, as presented in Tsomocos (2003) and Goodhart et al. (2006)[4].

2 Monetary World Economy

In this section, we describe the benchmark economy with flexible prices and characterize the set of equilibria . All markets are perfectly competitive. Money is valued through a cash-in-advance constraint, as in Lucas and Stokey (1987). We consider non-ricardian fiscal policy which determines the initial price level but leaves the probability measure associated with nominal state prices, which is referred to as the nominal equivalent martingale measure, indeterminate.

2.1 Households

Suppose that shocks follow a Markov chain with transition probabilities $f(s'|s) > 0$. The history of shocks up through date t is denoted by $s^t = (s_0, \ldots, s_t)$, and called a date-event. The initial shock, s_0, is given, and the initial date-event is denoted by 0. The probability of date-event s^t is $f(s^t)$. Successors of date-event s^t is $s^{t+i}|s^t$. For $s^{t+i}|s^t$, the probability that s^{t+i} occurs, conditional on s^t, is $f(s^{t+i}|s^t)$.

The world is inhabited by a continuum of individual producer-consumers, of unit mass, each of whom produces a single homogeneous good. There are

[4]In these models, the demand for money is supported by cash-in-advance constraints and financial frictions are explicitly introduced through endogenous default on nominal obligations. Shubik and Yao (1990), Shubik and Tsomocos (1992) and Shubik and Tsomocos (2002) present the importance of monetary transaction costs and nominal wealth within a strategic market game framework.

$N > 1$ countries, each of whom has a population of mass λ_n, $\sum_n \lambda_n = 1$. Agents of different nationalities differ only in that they must sell their produce in the currency of the country which they are native to. The first superscript $n \in N$ denotes the nationality of the agent while subscripts $(n, k, l \in N)$denote the country to which that transaction belongs. The second superscript, in the case of bond holdings, denotes the periods till maturity of that bond. The first element inside the brackets denotes the date and the second the state. That is for some variable x, $x_k^n(0)$ and $x_k^n(s^t)$ denotes an action by agent n in country k at time 0 and date-event s^t respectively. Similarly $b_k^{n,i}(s^t)$ denotes the bonds held by agent n in country k maturing in i periods at date-event s^t. Macro variables will follow the same notation, absent the first superscript.

At each date-event s^t there are J nominally risk-free bonds available for trade in each country, each bond maturing at periods $j \in J$ from that date-event. We assume that $J = S$: markets are complete. Let the *annualized* interest rate at date-event s^t of a country k bond maturing in $j \in J$ periods be $r_k^j(s^t)$. The term-structure of interest rates in country k at s^t is then given by the j dimensional vector $\left\{r_k^1(s^t), ..., r_k^J(s^t)\right\}$. Furthermore, let the price of a j-period bond, denominated in country k currency, at date-event s^t be $q_k^j(s^t) = \frac{1}{(1+r_k^j(s^t))^j}$. Finally let the price of a bond maturing at date-event s^t be $q_k^0(s^t) = 1$ with interest rate $r_k^0(s^t) = 0$.

Let μ_k denote the nominal equivalent martingale measure in country k. It is a probability measure over the date-event tree with $\mu_k(s^t) > 0$, all s^t. Let $\psi_k(s^{t+1}|s^t)$ denote the corresponding no-arbitrage price at s^t of the (untraded) elementary security that pays off one unit of country k currency if and only if the date-event s^{t+1} is reached. Then,

$$\psi_k(s^{t+1}|s^t) = \frac{\mu_k(s^{t+1}|s^t)}{1 + r_k^1(s^t)},$$

where $r_k(s^t)$ is the one-period nominal interest rate at s^t.

More generally,

$$\psi_k(s^{t+i}|s^t) = \frac{1}{1 + r_k^1(s^t)} \cdots \frac{1}{1 + r_k^1(s^{t+i-1})} \mu_k(s^{t+i}|s^t).$$

As markets are complete in the sense of there being as many traded securities as states of nature, the no-arbitrage value of the elementary securities is determined entirely by the path of prices of the traded bonds. The no-arbitrage relationship between Arrow prices and interest rates is:

$$\sum_{s^{t+1}|s^t} \psi_k(s^{t+1}|s^t) = \frac{1}{1 + r_k^1(s^t)}. \tag{1}$$

The output produced by a domestic representative household of country n in period 0 is $y^n(0)$ and at date event s^t in period t, it is $y^n(s^t)$; consumption of products from country k are $c_k^n(0)$ and $c_k^n(s^t)$. As goods are identical across countries, we further define for representative agent n $c^n(0) = \sum_k c_k^n(0)$ and $c^n(s^t) = \sum_k c_k^n(s^t)$ where the superscript denotes the nationality of the agent, and the subscript the country of origin of the good consumed.

The preferences of the representative household are described by the lifetime expected utility

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u\big[c^n(s^t), \overline{y}^n(s_t) - y^n(s^t)\big] f(s^t), \tag{2}$$

Here, we interpret $\overline{y}$ as the endowment of time, and $\overline{y} - y$ as the consumption of leisure, l.[5] The flow utility function, $u(c, l)$, satisfies standard conditions:

We assume that a household cannot consume what it produces; instead, it has to purchase consumption goods with cash from other households.[6]

Concerning the timing of transactions we assume that at each date-event the asset market and currency market in each country open simultaneously and before the goods market. An important consequence of this assumption is that the cash the households obtain from sales of its output has to be carried over to the next period.

The representative household enters the initial period 0 with nominal assets in country n of $w_k(0)$. Then, the asset market opens, in which cash and a complete set of contingent claims are traded, as well as the currency market, in which cash denominated in one currency is traded for cash denominated in another currency. The price level in each country is given by $p_n(0)$ and $p_n(s^t)$.

Let $e_k(0)$ and $e_k(s^t)$ be the nominal exchange rate (rate at which a unit of country k money is worth in terms of country 1 money[7].

[5] In the terminology of Lucas and Stokey (1987), $\overline{y}$ and $\overline{y} - y$ are the endowment and consumption of "credit goods", and c is consumption of "cash goods".

[6] In this, we follow Lucas and Stokey (1987).

[7] $e_1(0) = 1$ and $e_1(s^t) = 1$.

The budget constraint for the representative household of country n in the asset and currency market is

$$\sum_{k \in N} e_k(s^t) \left[\hat{m}_k^n(s^t) + \sum_{j \in J} q_k^j(s^t) b_k^{n,j}(s^t) \right] \leq \sum_{k \in N} e_k(s^t) w_k^n(s^t), \tag{3}$$

where $\hat{m}_k^n(s^t)$ is the amount of country k cash obtained by the household and $b_k^{n,j}(s^t)$ the portfolio of country k bonds maturing in j periods and $w_k(s^t)$ is the wealth of the householder in country k, entering that date-event.

The market for goods opens next. The purchase of consumption goods in each country is subject to the cash-in-advance constraint

$$p_k(s^t) c_k^n(s^t) \leq \hat{m}_k^n(s^t). \tag{4}$$

The household also receives cash by selling its product, $y^n(s^t)$. The sales of goods occurs only in the native country of each household. Hence, the amount of cash that it carries over to the next period, at home (country 1)is

$$m_1^n(s^t) = p_1(s^t) y^n(s^t) + \hat{m}_1^n(s^t) - p_1^n(s^t) c_1^n(s^t). \tag{5}$$

Abroad, this is (country $l \neq 1$)is

$$m_l^n(s^t) = \hat{m}_l^n(s^t) - P_l^n(s^t) c_l^n(s^t). \tag{6}$$

Given (5) and (6), the cash-in-advance constraint (4) at home is equivalent to the constraint

$$m_1^n(s^t) \geq p_1(s^t) y^n(s^t). \tag{7}$$

Abroad, the cash-in-advance constraint is equivalent to

$$m_l^n(s^t) \geq 0. \tag{8}$$

It turns out that (7) and (8) are more convenient than (4) to describe the cash constraint in our economy. This is due to the assumption that the asset market precedes the goods market.

In each currency, the household enters state $s^{t+1}|s^t$ in the next period with nominal wealth

$$w_k^n(s^{t+1}|s^t) = m_k^n(s^t) + \sum_{j \in J} q_k^{j-1}(s^{t+1}) b_k^{n,j}(s^t). \tag{9}$$

Each bond purchased at s^t now has one period less till maturity. The constraints the household faces are summarized by: (i) the flow budget constraint:

$$\sum_{k\in N} e_k(s^t)\left[p_k(s^t)c_k^n(s^t) + m_k^n(s^t) + \sum_{j\in J} q_k^j(s^t)b_k^{n,j}(s^t)\right] \\ \leq \sum_{k\in N} e_k(s^t)w_k^n(s^t) + P_1(s^t)y_1^n(s^t), \tag{10}$$

(ii) the cash constraints: at home

$$m_1^n(s^t) \geq p_1(s^t)y_1^n(s^t), \tag{11}$$

and abroad

$$m_l^n(s^t) \geq 0, \tag{12}$$

and (iii) the natural debt limit (Ljungqvist and Sargent, 2000):

$$\sum_{k\in N} e_k(s^t)w_k^n(s^t) \geq -\sum_{i=t}^{\infty}\sum_{s^i|s^t}\psi_1(s^i|s^t)p_1(s^i)\overline{y}^n(s_i). \tag{13}$$

The natural debt limit (13) says that the amount the agent can borrow at a given date-event, $-\sum_{k\in N} e_k(s^t)w_k^n(s^t)$, is bounded by the present discounted value of his future earnings, $p_1(s^i)\overline{y}^n(s_i)$. It is equivalent to

$$\lim_{i\to\infty}\sum_{s^i|s^t}\sum_{k\in N} e_k(s^i)\psi(s^i|s^t)w_k^n(s^i) \geq 0. \tag{14}$$

Given prices, $p_k(s^t)$, $r_k(s^t)$, $e_k(s^t)$ and $q_k^j(s^t)$, the household chooses $c_k(s^t)$ and $y^n(s^t)$ so as to maximize utility (2) subject to the budget constraints (10). When interest rates are positive, money balances incur the cost of lost interest and so the cash in advance constraints (11) and (12) will bind.

In equilibrium the Law of One Price must hold for goods,

$$p_1(s^t) = e_k(s^t)p_k(s^t) \tag{15}$$

and (redundant) bonds, via the relationship between Arrow prices across countries,

$$\psi_1(s^{t+1}|s^t) = \frac{e_k(s^t)\psi_k(s^{t+1}|s^t)}{e_k(s^{t+1}|s^t)}. \tag{16}$$

The uncovered interest parity condition can be derived by summing across states as follows:

$$\psi_k(s^{t+1}|s^t)e_k(s^t) = \psi_1(s^{t+1}|s^t)e_k(s^{t+1}|s^t) \tag{17}$$

$$e_k(s^t)\sum_{s^{t+1}|s^t}\psi_k(s^{t+1}|s^t) = \sum_{s^{t+1}|s^t}\psi_1(s^{t+1}|s^t)e_k(s^{t+1}|s^t) \tag{18}$$

$$e_k(s^t)\sum_{s^{t+1}|s^t}\frac{\mu_k(s^{t+1}|s^t)}{1+r_k(s^t)} = \sum_{s^{t+1}|s^t}\frac{\mu_1(s^{t+1}|s^t)}{1+r_1(s^t)}e_k(s^{t+1}|s^t) \tag{19}$$

$$e_k(s^t)\frac{1+r_1(s^t)}{1+r_k(s^t)} = \sum_{s^{t+1}|s^t}\mu_1(s^{t+1}|s^t)e_k(s^{t+1}|s^t) \tag{20}$$

$$\tag{21}$$

Substituting the law of one price relationships, the flow budget constraints (10) reduce to the single, lifetime budget constraint in terms of country n (home) currency:

$$p_n(0)c^n(0) + \sum_{s^t}\psi_1(s^t|0)p_1(s^t)c^n(s^t)$$
$$\leq w_1^n(0) + \frac{p_1(0)y^n(0)}{1+r_n(0)} + \sum_{s^t|0}\psi_1(s^t|0)\frac{p_1(s^t)y^n(s^t)}{1+r_1(s^t)}. \tag{22}$$

The life time budget constraint 22 should bind at an optimum which is equivalent to the transversality condition; that is,

$$\lim_{i\to\infty}\sum_{s^i|s^t}e_k(s^t)\psi_k(s^i|s^t)w_k^n(s^i) = 0. \tag{23}$$

2.2 The monetary-fiscal authority

We consider a non-Ricardian fiscal policy: the initial liability of the monetary-fiscal authority is not taxed back[8]. As we will show in the proposition this determines the initial money supply and consequently, given the allocation, the initial price level.

The monetary-fiscal-currency authority of country $k \in N$ enters each date-event s^t with nominal wealth $W_k(s^t|s^{t-1})$ and faces the following budget

[8]We also abstract from subsequent fiscal transfers as they do not affect the results.

constraint

$$M_k(s^t) + \sum_{j \in J} q_k^j(s^t) B_k^j(s^t) = W_k(s^t | s^{t-1}). \tag{24}$$

The wealth it carries forward to date event $s^{t+1}|s^t$ is given by

$$W_k(s^{t+1}|s^t) = M_k(s^t) + \sum_{j \in J} q_k^{j-1}(s^{t+1}) B_k^{j-1}(s^{t+1}). \tag{25}$$

where at date-event s^t, $M_k(s^t)$ is the money supply and $B_k^j(s^t)$ is the holding of bonds maturing in j periods and are the domestic liabilities of the monetary-fiscal authority. The initial wealth, $W_k(0)$, is given.

Monetary policy Monetary policy sets nominal interest rates, $r_k(s^t) \geq 0$ and a portfolio of bonds such that $B_k^{j-1}(s^{t+1}) > 0$.

Note that equilibrium will only require that the one period bonds be traded by the monetary-fiscal authority. Under *conventional* monetary policy the monetary-fiscal authority may, in addition, purchase longer-term bonds and hold them to maturity. As we will see, and presented in McMahon et al (2012), this additional set of restrictions on the monetary authority portfolio guarantees determinacy of the *domestic* price level, but not necessarily the exchange rate.

2.3 Equilibrium conditions

Since households are identical, the market clearing conditions for each $k \in N$ are

$$\sum_{n \in N} \lambda_n m_k^n(s^t) = M_k(s^t), \quad \sum_{n \in N} \lambda_n b_k^n(s^t) = B_k(s^t).$$

Also, consistency requires that

$$\sum_{n \in N} \lambda_n w_k^n(0) = W_k(0).$$

As only total quantities are relevant, we can summarise the world demand as

$$C^W(s^t) = \sum_{n \in N} \lambda_n c^n(s^t).$$

Similarly, world supply is

$$Y^W(s^t) = \sum_{n \in N} \lambda_n y^n(s^t).$$

The world resource constraints are therefore $C^W(s^t) = Y^W(s^t)$.

A competitive equilibrium with interest rate policy is defined as follows:

Definition 1. *Given initial nominal wealth, $\lambda_n w^n(0) = W_n(0)$ and interest rate policy, $\{r_k(s^t)\}$ a competitive equilibrium consists of an allocation, $\{c_k(s^t),\ y^n(s^t)\}$, a portfolio of households, $\{m_k^n(s^t), b_k^{n,j}(s^t)\}$, a portfolio of the monetary-fiscal authority, $\{M_k(s^t), B_k^j(s^t)\}$, spot-market prices, $\{p_k(s^t)\}$, exchange rates, $\{e_k(s^t)\}$ and a nominal equivalent martingale measure, $\mu_n(s^{t+1}|s^t)$, $\forall n \in N$, $s \in S$ and $t \in \{0, ..., \infty\}$ such that*

1. *the monetary-fiscal authority accommodates the money demand, $M_k(s^t) = \sum_{n \in N} \lambda_n m_k^n(s^t)$;*
2. *given interest rates, $r_k(s^t)$, spot-market prices, $p_k(s^t)$, exchange rates. $e_k(s^t)$, nominal equivalent martingale measure, $\mu_n(s^{t+1}|s^t)$, the household's problem is solved $\forall n, k \in N$ by $c_k^n(s^t)$, $y^n(s^t)$, $m_k^n(s^t)$, $b_k^{n,j}(s^t)$;*
3. *all markets clear.*

The existence of equilibrium, given a distribution of interest rates, requires further restrictions on the flow utility function.

Assumption 1. *The flow utility function, $u : \mathbb{R}{+}{+}^2 \to \mathbb{R}$, is continuously differentiable, strictly increasing, and strictly concave. Both goods are normal*[9]*:*

$$u_{11}u_2 - u_{12}u_1 < 0, \quad \text{and} \quad u_{22}u_1 - u_{12}u_2 < 0.$$

The Inada conditions hold:

$$\lim_{c \to 0} u_1 = \lim_{l \to 0} u_2 = \infty.$$

[9] In the notation, the subscripts denote the derivative of the corresponding arguments of the utility function.

The flow utility function, u, satisfies

$$\lim_{c \to i} \frac{u_1(c, y-c)}{u_2(c, y-c)} = \infty,$$

for each $y > 0$.

In particular, this guarantees that $u_1(c, y-c)/u_2(c, y-c)$ is strictly decreasing in c.

2.4 Equilibria with interest rate policy

We first show that in the equilibrium under consideration, with non-Ricardian fiscal policy, that the equilibrium exists and that the initial price level and exchange rates are determined by the allocation.

Proposition 1. *Given initial nominal wealth, $\lambda_n w_n^n(0) = W_n(0)$ and interest rate policy, $\{r_n(s^t)\}$, the initial price, $p_n(s^t)$ in each country and exchange rate $e_n(s^t)$ is unique;*

Proof Given the allocation $\{c^n(s^t), y^n(s^t)\}$, $\forall n \in N$, the initial price level in each country is determined by the interest rate policy, the allocation and the initial nominal wealth. To see this take the monetary-fiscal authority present-value budget constraint for some country n

$$\frac{r_n(0)}{1+r_n(0)} M_n(0) + \sum_{s^t} \psi_n(s^t|0) \frac{r_n(s^t)}{1+r_n(s^t)} M_n(s^t) = W_n(0),$$

and using the first order condition for assets of the home/domestic agent

$$\begin{aligned} W_n(0) = {} & \frac{r_n(0)}{1+r_n(0)} M_n(0) \\ & + \sum_{s^t} \frac{\beta u_1[c_1^n(s^t), \overline{y}^n(s^t) - y^n(s^t)] f(s^t|0)}{u_1[c^n(0), \overline{y}^n(0) - y^n(0)]} \frac{p_n(0)}{p_n(s^t)} \frac{r_n(s^t)}{1+r_n(s^t)} M_n(s^t). \end{aligned} \tag{26}$$

Note that the cash-in-advance constraint corresponds to $p_n(s^t)\lambda_n y^n(s^t) = M_n(s^t)$. Substituting, we can solve for the initial price level in each country.

$$p_n(0) = \frac{W_n(0)}{\frac{r_n(0)}{1+r_n(0)} \lambda_n y^n(0) + \sum_{s^t} \frac{\beta u_1[c^n 1(s^t), \overline{y}^n(s^t) - y^n(s^t)] f(s^t|0)}{u_1[c(0), \overline{y}^n(0) - y^n(0)]} \frac{r_n(s^t)}{1+r_n(s^t)} \lambda_n y^n(s^t)}.$$

The law of one price gives us the exchange rates $e_n(0) = \frac{P_1(0)}{p_n(0)}$.

$\square$

Traditional Monetary Policy and Credit Easing In McMahon et al (2012), we distinguish formally the differences between traditional monetary policy and the US conduct of credit easing. Here will will give a general formulation which encompasses both practices. Under traditional monetary policy, monetary authorities hold bonds of shorter duration, but pertinently, in an ex-ante specified target composition. Under Credit Easing, the duration of the portfolio may increase (operations twist) or they may purchase risky assets (such as mortgage backed securities), but with am explicit target composition which is committed to. Both these practices can be summarised in the definition below, which states that the monetary authority chooses a portfolio to target the stochastic path of the value of its portfolio. With this definition, we then show now that under traditional monetary policy and credit easing a unique vector of prices can be determined.

Definition 2. *Traditional Monetary Policy and Credit Easing is defined as an explicit target for the value, relative to other states, of the marketed wealth the monetary-fiscal authority carries into the subsequent period. Formally,* $W_n(s^{t+1}|s^t) = \xi_{s_{t+1}|s^t} \sum_{s^{t+1}|s^t} W_n(s^{t+1}|s^t)$ *where* $\sum_{s^{t+1}|s^t} \xi_{s^{t+1}|s^t} = 1$ *is chosen by the monetary-fiscal authority.*

Proposition 2. *Given initial nominal wealth,* $\lambda_n w_n^n(0) = W_n(0)$ *and traditional monetary policy under interest rate policy,* $\{r_n(s^t)\}$*, for all* $n, k \in N$*, the nominal equivalent martingale measure,* $\mu_n(s^{t+1}|s^t)$*, are determinate in each country: there exists a unique strictly positive probability measure* $\mu_n(s^{t+1}|s^t)$*, prices and portfolio* $\{p_n(s^t), M_n(s^t), B_n(s^{t+j}|s^t)\}$ *satisfying*

$$\frac{p_n(s^{t+1}|s^t)}{p_n(s^t)} = \frac{\beta u_1[c^n(s^{t+1}|s^t), \overline{y}^n(s^{t+1}|s^t) - y^n(s^{t+1}|s^t)] f(s^{t+1}|s^t)}{u_1[c^n(s^t), \overline{y}^n(s^t) - y^n(s^t)]} \frac{1 + r_n(s^t)}{\mu_n(s^{t+1}|s^t)},$$

$$M_n(s^t) \geq p_n(s^t)\lambda_n y^n(s^t)$$

(equality if $r_n(s^t) > 0$ *), support the allocation* $\{c^n(s^t), y^n(s^t)\}$*,* $\forall n \in N$*.*

Proof The previous proposition gives us unique values of $p_n(0)$ and $M_n(0)$. Now, iterating forward, the wealth the monetary-fiscal authority takes into date-event s^1 is $W_n(s^1|0) = M_n(0) + \sum_{j\in J} q_n(s^{1+j-1}|s^1)B_n(s^{1+j-1}|s^1)$. The present-value budget constraint at this date event is again

$$\frac{r_n(s^1)}{1+r_n(s^1)}M_n(s^1) + \sum_s^t \psi_n(s^t|s^1)\frac{r_n(s^t)}{1+r_n(s^t)}M_n(s^t) = W_n(s^1|0),$$

and using the first order condition for assets

$$\begin{aligned} W_n(s^1|0) = &\frac{r_n(s^1)}{1+r_n(s^1)}M_n(s^1) \\ &+ \sum_s^t \frac{\beta u_1[c1(s^t), \overline{y}(s^t) - y(s^t)]f(s^t|s^1)}{u_1[c(s^1), \overline{y}(s^1) - y(s^1)]}\frac{p_n(s^1)}{p_n(s^t)}\frac{r_n(s^t)}{1+r_n(s^t)}M_n(s^t). \end{aligned} \tag{27}$$

Using the cash-in-advance constraints and market clearing $p_n(s^t)\lambda_n y^n(s^t) = M_n(s^t)$ we can solve for the price level in each country at date-event s^1 as a function of the wealth carried into that date-event:

$$p_n(s^1) = \frac{W_n(s^1|0)}{\frac{r_n(s^1)}{1+r_n(s^1)}\lambda_n y^n(s^1) + \sum_s^t \frac{\beta u_1[c^n 1(s^t), \overline{y}^n(s^t) - y^n(s^t)]f(s^t|s^1)}{u_1[c(s^1), \overline{y}^n(s^1) - y^n(s^1)]}\frac{r_n(s^t)}{1+r_n(s^t)}\lambda_n y^n(s^t)}.$$

Now consider a restriction on the value of the wealth taken into this date-event of the form: $W_n(s^1|0) = \xi_{s_1} W_n(s_1|0)$ where $\sum_{s_1}\xi_{s_1} = 1$. This allows us to give the price level to be

$$p_n(s^1) = \frac{\xi_{s^1|0}\sum_{s^1|0} W_n(s^1|0)}{\frac{r_n(s^1)}{1+r_n(s^1)}\lambda_n y^n(s^1) + \sum_s^t \frac{\beta u_1[c^n 1(s^t), \overline{y}^n(s^t) - y^n(s^t)]f(s^t|s^1)}{u_1[c(s^1), \overline{y}^n(s^1) - y^n(s^1)]}\frac{r_n(s^t)}{1+r_n(s^t)}\lambda_n y^n(s^t)}.$$

Substituting this into the Fisher equation,

$$\frac{1}{1+r_n(0)} = \sum_s \psi_s(0) = \sum_{s^1} \frac{\beta u_1[c1(s^1), \overline{y}(s^1) - y(s^1)]f(s^1)}{u_1[c(0), \overline{y}(0) - y(0)]}\frac{p_n(0)}{p_n(s^1)}$$

gives us the value $\sum_{s^1|0} W_n(s^1|0)$ and hence the price level and exchange rate (via the law of one price) at date event s^1. Following in this fashion we can determine all prices in this economy. As we have used the Fisher equation to determine the price level it follows trivially that a valid and unique Martingale measure exists □

One may ask what sort of restriction we have imposed to obtain determinacy. Clearly it is requiring the monetary-fiscal authority to have a state contingent plan for the value of its assets for each period. However this is not as restrictive as it may seem. Consider monetary policy as it has traditionally been practised. Central Banks typically conduct transactions on most maturities of the term structure, but either target a specific composition of their portfolio and/or adhere to a *buy-and-hold* strategy. Targetting a specific composition of their portfolio introduces an aditional $S-1$ restrictions which pins down the stochastic path of prices. Another way to consider this is that the $S-1$ restrictions which determine the composition of the central bank portfolio must correspond to a target stochastic path for central bank assets. Within our notation, the buy and hold strategy would result in the *marketed* wealth of the monetary-fiscal authority to be *state-independent*, giving a natural value to the portfolio restriction to be $\xi_{s^{t+1}|s^t} = \frac{1}{s}$. Note that at each date-event, we require $S-1 \times N$ restrictions to obtain determinacy: $S-1$ in each country. As we will show in the next proposition, relaxing this constraint creates a multiplicity of equilibria which may occur under a naïve conduct of Quantitative Easing.

Quantitative Easing Under the UK and Japanese conduct of Quantitative Easing, the monetary authority committed to purchase assets to a pre-sepcified total value, but not to a target composition of its portfolio, as contrasted with traditional monetary policy or Credit Easing. We now show in the following proposition that in each country μ is not determined, or equivalently, the unique exchange rates at the terminal node, in addition to μ in any given country are not determined, and hence, there is $S-1 \times N$-dimensional indeterminacy.

Proposition 3. *Given initial nominal wealth, $\lambda_n w_n(0) = W_n(0)$ and naïve quantitative easing under interest rate policy, $\{r_n(s^t)\}$,*

1. *the nominal equivalent martingale measure, $\mu_n(s^{t+1}|s^t)$, are indeterminate in each country: for any strictly positive probability measure $\mu_n(s^{t+1}|s^t)$, any prices and portfolio $\{p_n(s^t), M_n(s^t), B_n(s^{t+j}|s^t)\}$ satisfying*

$$\frac{p_n(s^{t+1}|s^t)}{p_n(s^t)} = \frac{\beta u_1[c^n(s^{t+1}|s^t), \overline{y}^n(s^{t+1}|s^t) - y^n(s^{t+1}|s^t)] f(s^{t+1}|s^t)}{u_1[c^n(s^t), \overline{y}^n(s^t) - y^n(s^t)]} \frac{1 + r_n(s^t)}{\mu_n(s^{t+1}|s^t)},$$

$$M_n(s^t) \geq p_n(s^t) \lambda_n y^n(s^t)$$

(equality if $r_n(s^t) > 0$), support the allocation $\{c^n(s^t), y^n(s^t)\}$, $\forall n \in N$.

2. *Equivalently, any given $\nu_n(s^{t+1}|s^t) > 0$, $\sum_{s^{t+1}|s^t} \nu_n(s^{t+1}|s^t) = 1$ such that $e_n(s^{t+1}|s^t) = \nu_n(s^{t+1}|s^t) \sum_{s^{t+1}|s^t} e_n(s^{t+1}|s^t)$, support the allocation $\{c^n(s^t), y^n(s^t)\} \forall n, k \in N$ and a valid probability measure μ_n,.*

Proof Using the proof of Proposition 1 it is straightforward to see that given any μ, the prices and portfolio constructed as in the proposition support the equilibrium allocation. It is straightforward that these prices support exchange rates which are consistent with uncovered interest parity. To see this, for each agent $n \in N$,

$$\frac{p_n(s^1)}{p_n(0)} = \frac{\beta u_1[c^n 1(s^1), \overline{y}^n(s^1) - y^n(s^1)]f(s^1)}{u_1[c^n(0), \overline{y}^n(0) - y^n(0)]} \frac{1 + r_n(0)}{\mu_n(s^1|0)},$$

$$\frac{p_1(s^1)}{p_1(0)} = \frac{\beta u_1[c^n 1(1, s), \overline{y}^n(1, s) - y^n(1, s)]f(s^1)}{u_1[c^n(0), \overline{y}^n(0) - y^n(0)]} \frac{1 + r_1(0)}{\mu_1(s^1)},$$

using the law of one price

$$\frac{p_n(s^1)}{e_n(s^1)p_n(s^1)} \frac{e_n(0)p_n(0)}{p_n(0)} = \frac{\mu_1(s^1)}{\mu_n(s^1)} \frac{1 + r_n(0)}{1 + r_1(0)},$$

rearranging

$$e_n(0)\mu_n(s^1) \frac{1 + r_1(0)}{1 + r_n(0)} = \mu_1(s^1)e_n(s^1),$$

which is the law of one price for assets (taking interest rates to be identical globally). Summing over all states gives us the uncovered interest parity condition

$$e_n(0) \frac{1 + r_1(0)}{1 + r_n(0)} = \sum_{s^1} \mu_1(s^1)e_n(s^1).$$

Finally, choose an arbitrary $\mu_1(s^1) > 0$ and $\nu_n(s^2|s^1) > 0$ such that $e_n(s^2|s^1) = \nu_n(s^2|s^1) \sum_{s^2|s^1} e_n(s^2|s^1)$. The uncovered interest parity condition in Period 1 gives the exchange rates $e_n(s^1) = \nu_n(s^2|s^1) \sum_{s^2|s^1} e_n(s^2|s^1) \frac{1+r_1(s^1)}{1+r_n(s^1)} = \nu_n(s^2|s^1) \sum_{s^2|s^1} e_n(s^2|s^1)$. Finally, the uncovered interest parity condition in Period 0 gives $\overline{e}_n(s^2|s^1) = \frac{1+r_n(0)}{1+r_1(0)} \frac{e_n(0)}{\sum_{s^1} \nu_n(s^2|s^1)\mu_1(s^1)} = \frac{e_n(0)}{\sum_{s^1} \nu_n(s^2|s^1)\mu_1(s^1)}$. Now the law of one price for goods gives prices in all countries $p_1(0) = e_n(0)p_n(0)$ and

$p_1(s^1) = e_n(s^1)p_n(s^1)$. What remains is to verify that these prices support valid martingale measures.

$$\mu_n(s^1) = (1 + r_n(0))\frac{\beta u_1[c^n 1(s^1), \overline{y}^n(s^1) - y^n(s^1)]f(s^1)}{u_1[c^n(0), \overline{y}^n(0) - y^n(0)]}\frac{e_n(s^1)p_1(0)}{e_n(0)p_1(s^1)}.$$

$$\mu_n(s^1) = (1 + r_n(0))\psi(s^1)\frac{e_n(s^1)}{e_n(0)},$$

Using the no arbitrage condition in currency n

$$\sum_{s^1} \mu_n(s^1) = (1 + r_n(0))\sum_{s^1} \psi_n(s^1) = 1.$$

Hence μ_n is a valid martingale measure. It is straightforward that as the law of one price for goods and assets holds, a valid martingale in one country and $\overline{e}_n(s^2|s^1)$ support a valid martingale measure in every country. $\square$

The indeterminacy of μ implies that the inflation rate, $\tilde{\pi}_n(s^1) \equiv p_n(s^1)/p_n(0)$, is indeterminate. Thus, interest rate policy does not determine the stochastic path of inflation. Also shocks could be purely extrinsic. If $r_n(s^1)$ and $\overline{y}^n(s^1)$ are identical for all s^1, there is no uncertainty in "fundamentals;" nevertheless, there are equilibria in which the inflation rate, $p_n(s^1)/p_n(0)$, varies across states.

The reason that μ is indeterminate is simple, and closely related to the well known fact that only relative prices are determined in equilibrium. As discussed in Introduction, in an economy where money only serves as a unit of account (without monetary policy), the nominal state prices $\psi_n(s^1)$ are indeterminate. Now consider our economy in which monetary policy sets nominal interest rates. Interest rate policy does two things: (i) it adds one restriction on the nominal state prices as shown in the no-arbitrage condition (1); (ii) it determines the relative prices of consumption goods and real balances: $r_n(0)/[1 + r_n(0)]$, $r_n(s^1)/[1 + r_n(s^1)]$, as shown in equation (22). The latter determines the equilibrium quantities of real money balances, but does not reduce the indeterminacy; the former determines the sum of the nominal state prices, $q_n(s^1)$, but their distribution, μ, remains indeterminate. This is why in an stochastic economy with active monetary policy indeterminacy is characterized by μ.

In an open economy, the indeterminacy proliferates for similar reasons. Even with perfect substitutes, as we have here, only the relative prices in countries are determined, here trivially 1. As the absolute price level is indeterminate, then so is the exchange rate. Put differently, fixing the exchange rate fixes only the ratio of prices in countries but not price levels globally.

3 Risk Premia in a Monetary Open Economy

The cash-in-advance constraints result in a positive demand for money and positive interest rates are a necessary condition such that the model exhibits nominal determinacy. However, this comes with a loss in efficiency by introducing a wedge between the marginal utilities of leisure and consumption.

Although positive interest rates are necessary for a determinate equilibrium, they also result in lower output. The monetary-fiscal authority may try to keep interest rates as low as possible, or accordingly to Friedman's rule consider the limiting case where they approach zero. Although efficient here, we abstract and consider positive interest rates: the introduction of additional frictions may mean that the Friedman rule may not be optimal.

Here we study the risk premia associated with the correlation between interest rates and the martingale measure in an open economy. We study determinate equilibria with non-Ricardian fiscal policy, which determines the initial price level, and also portfolio restrictions on the monetary-fiscal authority which also fix the distribution of prices across states. The restrictions on the monetary-fiscal authority determine the path of prices within each country. We consider three alternative objectives. The first is choosing a stable growth rate in inflation: we call this price stability. In a world of stochastic outputs, the portfolio choice alters the money supplies inversely with the output to maintain the same price across states of nature. Monetary stability results in money supplies to grow in a non-stochastic manner, and is consistent with the Friedman k% rule. Finally we consider financial stability which is the result of the monetary-fiscal authority holding a portfolio composed of a nominally riskless bond. Its implications are a combination of that under price and monetary stability and allows a positive role for interest rates to target the price level in order to maintain a stable growth rate in prices. All proofs in this section are in the Appendix. The section proceeds as follows...

3.1 Primitives

There are two periods. In the second period uncertainty is resolved. We fix a complete probability space $(\Omega, \mathcal{F}, P)$ for period 1. Here, Ω is a complete description of the exogenous uncertain environment at Period 1, the σ-algebra $\mathcal{F}$ is the collection of events distinguishable at period 1, and P is a probability measure over $(\Omega, \mathcal{F})$.

There are three periods: $t = \{0, 1, 2\}$. There is no uncertainty in the first or third period. In the second period, a single state $\omega \in \Omega$ realizes[10]. Each state occurs with a probability $\pi(\omega) > 0$. Production and consumption occur in the first two periods. The last period is added for an accounting purpose, where households and the fiscal authority redeem their debt.

3.2 Households

The world is inhabited by a continuum of individual producer-consumers in $N = 2$ countries, each of unit mass, and producing a single homogeneous good. Let the home country be Country 1 and the foreign country 2. Agents are denoted with superscripts while periods and, where appropriate, markets, by subscripts.

Individuals everywhere in the world have the same preferences, which are defined over consumption and effort expended in production. The preferences of the home agent is

$$\begin{aligned} &\frac{c^n(0)^{1-\rho} - 1}{1-\rho} + \frac{(\overline{y}(0) - y^n(0))^{1-\rho} - 1}{1-\rho} \\ &\qquad + \beta \int_\omega f(\omega) \left\{ \frac{c^n(1,\omega)^{1-\rho} - 1}{1-\rho} + \frac{(\overline{y} - y^n(1,\omega))^{1-\rho} - 1}{1-\rho} \right\} d\omega. \end{aligned} \tag{28}$$

Note that the endowment of leisure is state and agent independent while he probability measure and rate of discount factor is also agent independent for simplicity.

As before, there are no impediments or costs to trade between the countries. Let $e(0)$ be the nominal exchange rate, defined as the value of a unit of foreign currency in terms of home currency in Period 0, while in Period 1 and 2 it is $e(1, \omega)$ and $e(2, \omega)$ respectively.

[10]In the following, all the uncertainty will be due to the path of interest rates in one country.

The budget constraints for the representative home household is

$$
\begin{aligned}
p_1(0)c_1^1(0) + \int_\omega q_1(1,\omega)b_1^1(1,\omega)d\omega \\
+ e_{(0)}\left[p_2(0)c_2^1(0) + \int_\omega q_2(1,\omega)b_2^1(1,\omega)d\omega\right] + m_1^1(0) \\
\leq w_n^1(0) + p_1(0)y_1(0). \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
p_1(1,\omega)c_1(1,\omega) - b_1(1,\omega) + \frac{b_1^1(1,\omega)}{1+r_1(1,\omega)} \\
+ e(1,\omega)\left[p_2(1,\omega)c_2(1,\omega) - b_2^1(1,\omega) + \frac{b_2^1(1,\omega)}{1+r_2(1,\omega)}\right] + m_1^1(1,\omega) \\
\leq m_1^1(0) + p_n(1,\omega)y^n(1,\omega). \qquad (30)
\end{aligned}
$$

and in the final period

$$0 \leq m_1^1(1,\omega) + b_2(1,\omega) + e(2,\omega)b_2(2,\omega).$$

3.3 Individual Maximization

The first order conditions for the representative households in Period 1 gives us:

$$y^1(1,\omega) = \overline{y} - c^1(1,\omega)(1+r_1(1,\omega))^{1/\rho}, \qquad (31)$$

and

$$y^2(1,\omega) = \overline{y} - c^2(1,\omega)(1+r_2(1,\omega))^{1/\rho}. \qquad (32)$$

The marginal rates of substitution

$$q(1,\omega) = \beta f(\omega)\frac{p(0)}{p_1(1,\omega)}\left\{\frac{c^1(0)}{c^1(1,\omega)}\right\}^\rho. \qquad (33)$$

Equating 33 for the home and foreign agent gives

$$c^2(1,\omega) = \frac{c^2(0)}{c^1(0)}c^1(1,\omega). \qquad (34)$$

Market clearing condition is

$$c^1(1,\omega) + c^2(1,\omega) \quad = y^1(1,\omega) + y^2(1,\omega). \qquad (35)$$

3.4 Monetary-Fiscal Authority

Restricting the portfolio of the monetary-fiscal authority to be a fixed proportion of the gross value of liabilities provides the (when the state space is composed of a discrete number of S states) additional $S-1$ restrictions per currency which give determinacy. The reason is straightforward. As the equilibrium prices of the Arrow securities are determined solely by the demands of the households, altering the relative quantities of Arrow securities purchased by the monetary-fiscal authority will then change the relative prices of the Arrow securities which leads to nominal indeterminacy under an interest rate rule (see Nakajima and Polemarchakis (2005)). The Monetary authority portfolio restriction for Country 1:

$$B_k(0,\omega) \quad = \quad B_k(0)\pi_k(0,\omega) \tag{36}$$

where $\int_\omega \pi(\omega)d\omega = 1$. Hence

$$M_k(0) = B_k(0)\int_\omega q_k(0,\omega)\pi_k(0,\omega)d\omega + W_k(0) \tag{37}$$

$$B_k(0) = \frac{M_k(0) - W_k(0)}{\int_\omega q_k(0,\omega)\pi_k(0,\omega)d\omega}$$

$$B_k(0)\pi_k(0,\omega) = M_k(0) - \frac{r_k(1,\omega)}{1+r_k(1,\omega)}M_k(1,\omega) \tag{38}$$

where $B_k(0)$ is the gross value of debt purchased by the monetary-fiscal authority of country 1.

3.4.1 Monetary Policy Options

We now define the various monetary policy regimes available. As we are in a stochastic world, the monetary-fiscal authority is required to choose a path of interest rates and a choice of its portfolio to target a stable growth rate in prices or money supplies. Inn addition it can choose any arbitrary choice of asset holdings (in some ex-ante determined proportion). We will consider the simplest of these: a portfolio of state contingent bonds in equal proportion producing the payoff of a nominally riskless bond.

Definition 3. *Monetary stability is the outcome of monetary policy that sets interest rates and money supplies in the second period which are state* *independent. Formally,* $r_k(0), r_k(\omega) \geq 0$ *and a choice of* $\pi_k(0,\omega)$ *such that* $\int_\omega \pi_k(0,\omega)d\omega = 1$ *and* $M_k(1,\omega) = M_k(1,\omega')$ $\forall \omega, \omega' \in \Omega$.

Definition 4. *Price stability is the outcome of monetary policy that sets interest rates and prices in the second period which are state independent. Formally,* $r_k(0), r_k(\omega) \geq 0$ *and a choice of* $\pi_k(0,\omega)$ *such that* $\int_\omega \pi_k(0,\omega)d\omega = 1$ *and* $p_k(1,\omega) = p_k(1,\omega')$ $\forall \omega, \omega' \in \Omega$.

Definition 5. *Financial Stability occurs when the Central Bank purchases equal quantities of state-contingent bonds. Formally,* $r_k(0), r_k(\omega) \geq 0$ *and* $B_k(0,\omega) = B_k(0,\omega') = \overline{B}_k$ $\forall \omega, \omega' \in \Omega$*, where* $\overline{B}_k$ *is the gross value of debt.*

3.5 Monetary Policy and the Aggregate Risk Premium

The state space is continuous, and will be indexed by the interest rates of country 1, between two bounds. Formally $\Omega = [\omega_0, ..., \omega_1]$, where interest rates in country 1 are $r_1(1,\omega_0) = \underline{r}_1$ and $r_1(1,\omega_1) = \overline{r}_1$, while for country 2, $r_2(1,\omega_i) = r_2(1)$ $\forall i \in [0,1]$. That is, the only uncertainty is the date 1 interest rate in country 1.

3.5.1 Monetary Policy and the Real Risk Premium

The real risk premium caused by monetary policy is determined by change in $\frac{u_1(c^1(1,\omega))}{u_1(c^1(0))}$. In the following we characterise how the direction of the risk premia in response to higher interest rates.

Proposition 4. *A higher expected spot interest rate in one country reduces consumption and production globally in that state.*

This shows that the path of interest rates in one country effects the allocation globally: the non-neutrality of monetary policy results in the global real risk premium being determined by the combination of interest rates globally.

3.5.2 Monetary Policy and the Nominal Risk Premium

The (stochastic) rate of inflation depends on the choice of nominal targets. Clearly a policy of price stability denies the presence of a nominal risk premium. However a policy of Monetary or Financial Stability has clear implications for the nominal risk premium.

Proposition 5. *A policy of Monetary Stability results in expected interest rates being positively related to the price level and hence nominal risk premium.*

Proposition 6. *A Monetary Policy of Financial Stability results in a negative correlation between money supplies and interest rates.*

Proposition 7. *A policy of Financial Stability results in expected interest rates being negatively related to the price level and hence nominal risk premium.*

3.5.3 Monetary Policy and the Martingale Measure

Asset prices are given by

$$q_1(1,\omega) = \beta f(\omega) \frac{p_1(0)}{p_1(1,\omega)} \left\{ \frac{c^1(0)}{c^1(1,\omega)} \right\}^{\rho}$$

Proposition 8. *A global monetary policy of Price Stability results in a positive risk premium. Equivalently, the martingale measure in each country is positively correlated to expected interest rate policy.*

Proposition 9. *A global monetary policy of Monetary Stability results in a negative risk premium. Equivalently, the martingale measure in each country is negatively correlated to expected interest rate policy.*

Proposition 10. *A Global Policy of Financial Stability results in a positive risk premium. Equivalently, the martingale measure in each country is positively correlated to expected interest rate policy.*

3.6 Monetary Policy and the Term Structure of Interest Rates

Here we examine the implications of the choice of monetary policy on the risk premium in the interest rate market. Under a policy of price stability we find the same result as in Espinoza et al. (2009) and Espinoza and Tsomocos (2008)the forward interest rate is an upwardly biased indicator of future interest rates.. However, under monetary stability we get the opposite result reflecting the importance of considering the choice of monetary policy in determining the informational content in observed risk premia in the market.

Proposition 11. *Given a distribution of future interest rates, price stability and financial stability result in the forward interest rate being an upwardly biased indicator of expected interest rates.*

Proposition 12. *Given a distribution of future interest rates, monetary stability result in the forward interest rate being an downwardly biased indicator of expected interest rates.*

3.7 Monetary Policy and the Path of Exchange Rates

Here we characterise the path of exchange rates under alternative monetary policy regimes globally. The exchange rate is given by

$$e(1,\omega) = \frac{P_1(1,\omega)}{P_2(1,\omega)} \tag{39}$$

$$= \frac{M_1(1,\omega)}{M_2(1,\omega)} \frac{y^2 1(1,\omega)}{y^1 1(1,\omega)}. \tag{40}$$

We can also show that output in country 1 falls relative to the output in country 2.

Proposition 13. *Output in the country with the higher interest rate will be relatively lower.*

The money supplies will depend on the monetary policy choice. Under Price stability, the exchange rate is unchanged across states by definition.

3.7.1 Exchange Rates under Financial Stability

Proposition 14. *Under a global Policy of Financial Stability, the exchange rate in the country with the relatively higher interest rate across states, will be relatively more appreciated across countries.*

We are now ready to prove the relative values of exchange rates.

Proposition 15. *Under a global Policy of Financial Stability, the exchange rate in the country with the relatively higher interest rate across states, will be relatively lower across countries.*

3.7.2 Exchange Rates under Monetary Stability

Proposition 16. *Under a global monetary policy of Monetary Stability, the exchange rate in the country with the relatively higher interest rate across states, will be relatively more depreciated across countries.*

3.7.3 Monetary Policy and Forward Exchange Rate Premium

Proposition 17. *Monetary Policy results in the Forward Exchange Rate being*

1. *downwardly biased under Financial Stability,*
2. *unbiased under Price Stability,*
3. *downwardly biased under Monetary Stability.*

If home interest rates are higher and more volatile, then it may seem economically profitable for foreign investors to take advantage of this difference.

3.8 Numerical Analysis

The lifetime budget constraint for household n in currency n becomes

$$p_n(0)\left[c^n(0) - \frac{y_n(0)}{1+r_n(0)}\right] + \int_\omega q_n(1,\omega)p_n(1,\omega)\left[c^n(1,\omega) - \frac{y^n(1,\omega)}{1+r_n(1,\omega)}\right]d\omega \\ \leq w_n^n(0). \tag{41}$$

Recall that the state price gives us $q_n(1,\omega) = \beta f(\omega)\frac{p_n(0)}{p_n(1,\omega)}\left\{\frac{c^n(0)}{c^n(1,\omega)}\right\}^\rho$. Substituting this in and rearranging gives

$$c^n(0)^{-\rho}\left[c^n(0) - \frac{y_n(0)}{1+r_n(0)}\right] + \int_\omega f(\omega)c^n(1,\omega)^{-\rho}\left[c^n(1,\omega) - \frac{y^n(1,\omega)}{1+r_n(1,\omega)}\right]d\omega \\ \leq w_n^n(0)\frac{c^n(0)^{-\rho}}{p_n(0)}. \tag{42}$$

From the first order conditions and market clearing, we get[11] $c^1(1,\omega)$ and $c^2(1,\omega)$ as functions of constants, endogenous variables $\{c^1(0), c^2(0)\}$ and

[11] see proof for proposition 4 in Appendix.

state variables $\{r_1(1,\omega), r_2(1,\omega)\}$. Using this, and the first order equations 31 and 32 we get expressions for $y^1(1,\omega)$ and $y^2(1,\omega)$ also as functions of constants, endogenous variables $\{c^1(0), c^2(0)\}$ and state variables $\{r_1(1,\omega), r_2(1,\omega)\}$.

What remains is to determine the initial price level in each country. This can be obtained using the present value budget constraint for each country by combining equations 37 and 38.

$$M_k(0)\frac{r_k(0)}{1+r_k(0)} + \int_\omega q_k(0,\omega)\frac{r_k(1,\omega)}{1+r_k(1,\omega)}M_k(1,\omega) = W_k(0).$$

Using the definition of the state price, and the cash-in-advance constraint: $p_n(1,\omega)y^n(1,\omega) = M_k(1,\omega)$,

$$M_k(0)\frac{r_k(0)}{1+r_k(0)} + \int_\omega p_n(0)y_n(1,\omega)\left\{\frac{c^n(0)}{c^n(1,\omega)}\right\}^\rho \frac{r_k(1,\omega)}{1+r_k(1,\omega)} = W_k(0).$$

Finally using the cash-in-advance constraint: $p_n(0)y^n(0) = M_k(0)$, and rearranging

$$p_n(0) = \frac{W_k(0)}{y_n(0)\frac{r_n(0)}{1+r_n(0)} + \int_\omega y_n(1,\omega)\left\{\frac{c^n(0)}{c^n(1,\omega)}\right\}^\rho \frac{r_k(1,\omega)}{1+r_k(1,\omega)}} \qquad (43)$$

which, using the arguments used earlier is a function of constants, endogenous variables $\{c^1(0), c^2(0)\}$ and state variables $\{r_1(1,\omega), r_2(1,\omega)\}$.

Substituting equation 43 into the two budget constraints represented by equation 42, we have a system of two equations that solve $\{c^1(0), c^2(0)\}$ as a function of state variables $\{r_1(1,\omega), r_2(1,\omega)\}$.

3.8.1 Simulation

The parameters of the initial allocation are given as follows.

In the second period the interest rates in the two countries follow a bivariate log-normal distribution. The mean of the interest rates in each country are given by $e^{\mu_n+\sigma_n^2/2}$, the variance is given by $(e^{\sigma_n^2}-1)(e^{2\mu_n+\sigma_n^2})$ and the correlation by ρ where μ_n, σ_n and ρ are the parameters from bivariate normal distribution. We fix the Country 2 parameters to be $\mu_2 = -4.5$ and $\sigma_2 = 1.5$ which translate into a mean of 0.0514 and a standard deviation of 0.2319 in the log normal distribution. For country 1 we assume the same mean but

	$Country1$	$Country2$
Initial Wealth, $w(h,1,i)$	1.0	1.0
Endowment of Leisure $\bar{l}$	1.0	1.0
Risk Aversion, $\rho(h)$	0.9	0.9
Preference for Leisure, $\kappa(i)$	1.0	1.0
Period 1 Interest Rate 0, $r(1,i)$	3.0%	3.0%
Discount Factor, β	1.0	1.0

Table 1: Parameters of Initial Equilibrium

solve the economy for 100 values of σ_1 between 1.25 and 1.75 and 100 values of ρ between .059 and .095. The plot of these are presented below.

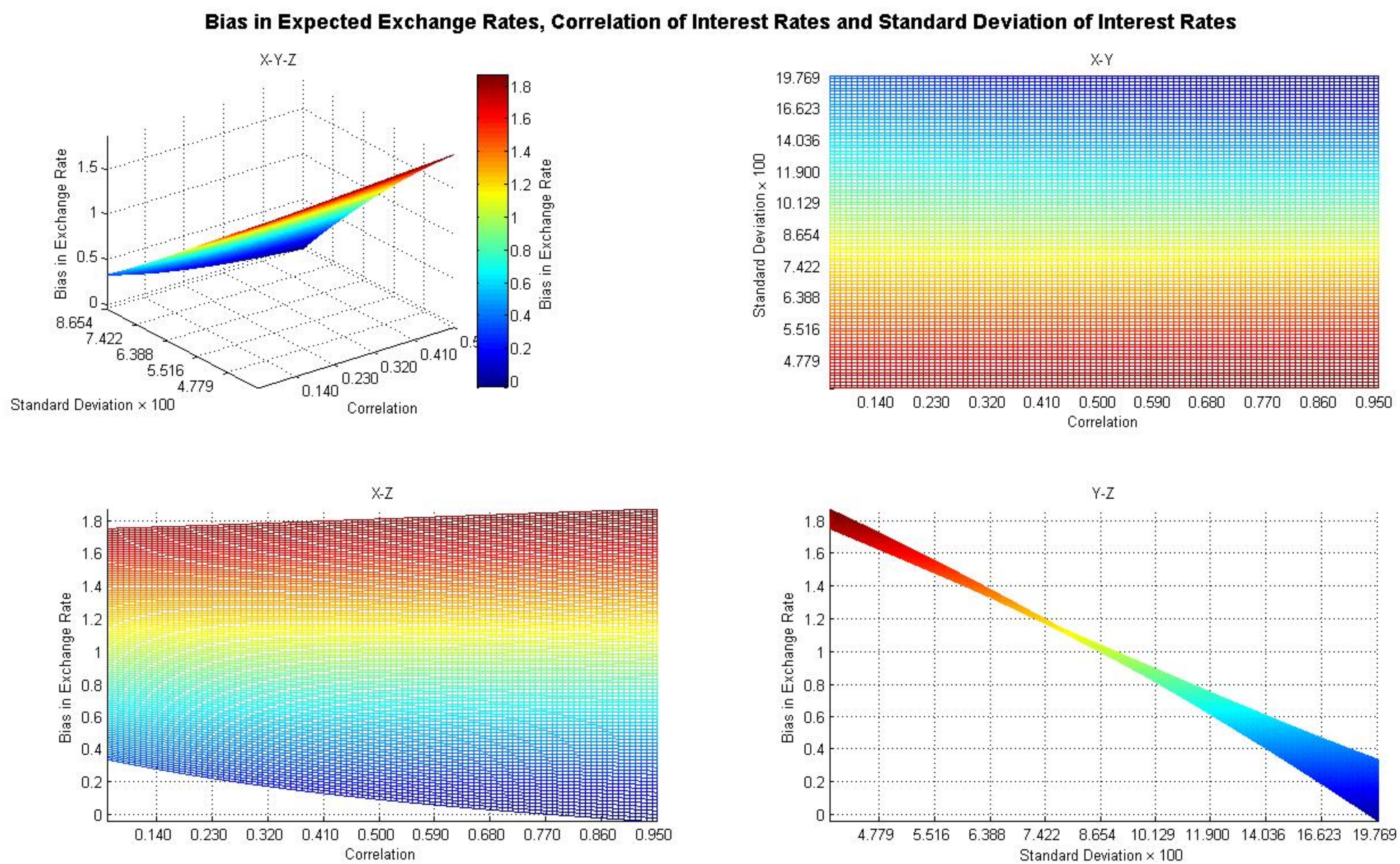

Figure 1: Log Difference between Objective and Risk Neutral Expected Exchange Rate

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4 Appendix

Proof of Proposition 4

Proof. **Consumption** Substituting 31 and 32 into market clearing equation 35

$$c^2(1,\omega) \;=\; \frac{2\overline{y}-c^1(1,\omega)(1+(1+r_1(1,\omega))^{1/\rho})}{(1+(1+r_2(1,\omega))^{1/\rho})}. \tag{44}$$

Finally substitute 34 into 44

$$c^1(1,\omega) \;=\; \frac{2\overline{y}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+(1+(1+r_1(1,\omega))^{1/\rho})}. \tag{45}$$

Now two states ω and ω' where monetary policy sets interest rates such that $r_1(1,\omega) > r_1(1,\omega')$ but $r_2(1,\omega) = r_2(1,\omega')$ we get

$$c^n(1,\omega) < c_1^n(\omega').$$

From 34

$$c^2(1,\omega) < c_1^2(\omega').$$

Production From 31 and 32

$$y^1(1,\omega) < y_1^1(\omega')$$

and

$$y^2(1,\omega) < y_1^2(\omega').$$

□

Proof of Proposition 5

Proof. By construction, $M_1(1,\omega) = M_1(1,\omega')$. In section 3.5.1 we showed that $y_1(1,\omega) < y_1(1,\omega')$ whenever $r_1(1,\omega) > r_1(1,\omega')$. As the cash-in-advance holds, then it must be that $p_1(1,\omega) > p_1(1,\omega')$. □

Proof of Proposition 6

Proof. The period 0 budget constraint

$$\frac{r_k(0)}{1+r_k(0)}M_k(0)+\overline{B}_k\sum_{\omega}q_k(1,\omega)=W_k(0) \tag{46}$$

$$\frac{r_k(0)}{1+r_k(0)}M_k(0)+\overline{B}_k\frac{1}{1+r_k(0)}=W_k(0) \tag{47}$$

The period 1 money supply is then

$$\frac{r_k(1,\omega)}{1+r_k(1,\omega)}M_k(1,\omega)+\overline{B}_k=M_k(0) \tag{48}$$

hence

$$M_k(1,\omega)=\frac{1+r_k(1,\omega)}{r_k(1,\omega)}\left[M_k(0)-\overline{B}_k\right]. \tag{49}$$

This gives us that taking two states ω and ω' where monetary policy sets interest rates such that $r(1,\omega)>r_k(\omega')$, $M(1,\omega)<M_k(\omega')$. □

Proof of Proposition 7

Proof. Take states $\omega,\omega'\in S$ such that $r_1(1,\omega)>r_1(1,\omega')$ and $r_2(1,\omega)=r_2(1,\omega')$. From the cash-in-advance constraint

$$\begin{aligned}p_1(1,\omega)&=\frac{M_1(1,\omega)}{y_1(1,\omega)}\\&=\frac{\left(M_1(0)-\overline{B}_1(0)\right)\frac{1+r_1(1,\omega)}{r_1(1,\omega)}}{\overline{y}-\frac{2\overline{y}(1+r_1(1,\omega))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+(1+(1+r_1(1,\omega))^{1/\rho})}}.\end{aligned}$$

The relative price levels are

$$\frac{p_1(1,\omega)}{p_1(1,\omega')}=\frac{\frac{r_1(\omega')}{1+r_1(\omega')}}{\frac{r_1(\omega)}{1+r_1(\omega)}}\frac{1-\frac{2(1+r_1(1,\omega))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+(1+(1+r_1(1,\omega))^{1/\rho})}}{1-\frac{2(1+r_1(1,\omega'))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega'))^{1/\rho})+(1+(1+r_1(1,\omega'))^{1/\rho})}}.$$

Given our assumptions about interest rates, the first part of the expression, $\frac{\frac{r_1(\omega')}{1+r_1(\omega')}}{\frac{r_1(\omega)}{1+r_1(\omega)}}$, is less than 1, as is the second, $\frac{1-\frac{2(1+r_1(1,\omega))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+(1+(1+r_1(1,\omega))^{1/\rho})}}{1-\frac{2(1+r_1(1,\omega'))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega'))^{1/\rho})+(1+(1+r_1(1,\omega'))^{1/\rho})}}$. Hence $p_1(1,\omega) < p_1(1,\omega')$. □

Proof of Proposition 8

Proof. As prices are non-stochastic under this policy regime, all the variation in the state price is derived from how consumption changes. In Proposition 4 we showed that $c_1(1,\omega) < c_1(1,\omega')$ whenever $r_1(1,\omega) > r_1(1,\omega')$, hence it must be that

$$\begin{aligned} r_1(1,\omega) > r_1(1,\omega') &\Leftrightarrow q_1(1,\omega) > q_1(1,\omega') \\ &\Leftrightarrow q_2(1,\omega) > q_2(1,\omega') \\ &\Leftrightarrow \mu_1(1,\omega) > \mu_1(1,\omega') \\ &\Leftrightarrow \mu_2(1,\omega) > \mu_2(1,\omega'). \end{aligned}$$

□

Proof of Proposition 9

Proof.

$$\begin{aligned} q_1(1,\omega) &= \beta f(\omega) \frac{p_1(0)}{p_1(1,\omega)} \left\{ \frac{c^1(0)}{c^1(1,\omega)} \right\}^{\rho} \\ &= \beta f(\omega) \frac{p_1(0) c^1(0)^{\rho}}{M_1(1,\omega)} \left\{ \frac{y_1(1,\omega)}{c^1(1,\omega)} \right\}^{\rho} y_1(1,\omega)^{1-\rho}. \end{aligned}$$

Note that from 44 and 31, the ratio $\frac{y_1(1,\omega)}{c^1(1,\omega)} = \frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho}) - \frac{1}{2}(1+(1+r_1(1,\omega))^{1/\rho})$ and comparing states, is negatively correlated with $r_1(1,\omega)$. The additional relevant ratio to determining the risk premium is $\frac{y_1(1,\omega)^{1-\rho}}{M_1(1,\omega)}$. As under Monetary Stability money supplies are non-stochastic, then this ratio will move in the same direction as output. As we have shown in Proposition 4 that output falls, then the state price must be negatively correlated with interest rates. It follows that:

$$
\begin{aligned}
r_1(1,\omega) > r_1(1,\omega') &\Leftrightarrow q_1(1,\omega) < q_1(1,\omega') \\
&\Leftrightarrow q_2(1,\omega) < q_2(1,\omega') \\
&\Leftrightarrow \mu_1(1,\omega) < \mu_1(1,\omega') \\
&\Leftrightarrow \mu_2(1,\omega) < \mu_2(1,\omega').
\end{aligned}
$$

This result shows that the effect of inflation outweighs the effect of the real allocation in determining the risk premium. □

Proof of Proposition 10

Proof. From Proposition 7, we found that under Financial Stability, the price level is negatively correlated with interest rates. From Proposition 4 we found that consumption is also negatively correlated with interest rates. Hence it follows that:

$$
\begin{aligned}
r_1(1,\omega) > r_1(1,\omega') &\Leftrightarrow q_1(1,\omega) > q_1(1,\omega') \\
&\Leftrightarrow q_2(1,\omega) > q_2(1,\omega') \\
&\Leftrightarrow \mu_1(1,\omega) > \mu_1(1,\omega') \\
&\Leftrightarrow \mu_2(1,\omega) > \mu_2(1,\omega').
\end{aligned}
$$

□

Proof of Proposition 11

Proof. We can calculate the price of a bond which pays a unit of currency at the end of the second period through no arbitrage and is given by $q_1(0:2) = \int_\omega \frac{q_1(1,\omega)}{1+r_1(1,\omega)} d\omega$ or equivalently $\frac{1}{1+r_1(0)} \int_\omega \frac{\mu_1(1,\omega)}{1+r_1(1,\omega)} d\omega$. The positive correlation between the martingale measure and the nominal interest rate implies that $\int_\omega \frac{\mu_1(1,\omega)}{1+r_1(1,\omega)} < \int_\omega \frac{f(\omega)}{1+r_1(1,\omega)} d\omega$ or in other words $q_1(0:2) < \frac{1}{1+r_1(0)} \int_\omega f(\omega) \frac{f(\omega)}{1+r_1(1,\omega)} d\omega$. Let the (per period) interest rate on the long term bond be $r_1(0:2)$ and the forward interest rate between period 1 and 2 be $r_1^f(1:2)$. Therefore the price of the two period bond is

$$
q_1(0:2) = \frac{1}{(1+r_1(0:2))^2} = \frac{1}{1+r_1(0)} \frac{1}{1+r_1^f(1:2)}.
$$

It follows that

$$\frac{1}{1+r_1^f(1:2)} < \int_\omega \frac{f(\omega)}{1+r_1(1,\omega)}d\omega$$

and $f(1,\omega) > \int_\omega f(\omega)r_1(1,\omega)d\omega$: the forward interest rate is an upwardly biased indicator of future interest rates. $\square$

Proof of Proposition 12

Proof. The proof is the same as above, with inequalities reversed. $\square$

Proof of Proposition 13

Proof.

$$\begin{aligned}
\frac{y^1(1,\omega)}{y^2(1,\omega)} &= \frac{\overline{y}-c^1(1,\omega)(1+r_1(1,\omega))^{1/\rho}}{\overline{y}-c^2(1,\omega)(1+r_2(1,\omega))^{1/\rho}} \\
&= \frac{\frac{\overline{y}}{c^1(1,\omega)}-(1+r_1(1,\omega))^{1/\rho}}{\frac{\overline{y}}{c^1(1,\omega)}-\frac{c^2(1,\omega)}{c^1(1,\omega)}(1+r_2(1,\omega))^{1/\rho}} \\
&= \frac{.5\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+.5(1+(1+r_1(1,\omega))^{1/\rho})-(1+r_1(1,\omega))^{1/\rho}}{.5\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+.5(1+(1+r_1(1,\omega))^{1/\rho})-\frac{c^2(0)}{c^1(0)}(1+r_2(1,\omega))^{1/\rho}} \\
&= \frac{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho})+1-(1+r_1(1,\omega))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1-(1+r_2(1,\omega))^{1/\rho})+1+(1+r_1(1,\omega))^{1/\rho}}.
\end{aligned}$$

Taking two states, such that $r_1(\omega^*) > r_1(\omega')$ and $r_2(\omega^*) = r_2(\omega')$, it must be the case that $\frac{y^1(1,\omega^*)}{y^2(1,\omega^*)} < \frac{y^1(1,\omega')}{y^2(1,\omega')}$. $\square$

Proof of Proposition 14

Proof.

$$\frac{M_1(1,\omega)}{M_2(1,\omega)} = \frac{M_1(0)-\overline{B}_1(0)}{M_2(0)-\overline{B}_2(0)}\left\{\frac{\frac{1+r_1(1,\omega)}{r_1(1,\omega)}}{\frac{1+r_2(1,\omega)}{r_2(1,\omega)}}\right\} \tag{50}$$

Taking two states, such that $r_1(\omega^*) > r_1(\omega')$ and $r_2(\omega^*) = r_2(\omega')$, it must be the case that $\frac{M_1(1,\omega^*)}{M_2(1,\omega^*)} < \frac{M_1(1,\omega')}{M_2(1,\omega')}$. $\square$

Proof of Proposition 15

Proof. Propositions 13 and 14 tell us that the nominal and real effects on the exchange rates move in opposite directions. Therefore we will determine the net effect by considering them jointly:

$$
\begin{aligned}
e(1,\omega) &= \frac{P_1(1,\omega)}{P_2(1,\omega)} \\
&= \frac{M_1(1,\omega)}{M_2(1,\omega)} \frac{y^2 1(1,\omega)}{y^1 1(1,\omega)} \\
&= \frac{M_1(0) - \overline{B}_1(0)}{M_2(0) - \overline{B}_2(0)} \left\{ \frac{\frac{1+r_1(1,\omega)}{r_1(1,\omega)}}{\frac{1+r_2(1,\omega)}{r_2(1,\omega)}} \right\} \frac{\frac{c^2(0)}{c^1(0)}(1-(1+r_2(1,\omega))^{1/\rho}) + 1 + (1+r_1(1,\omega))^{1/\rho}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega))^{1/\rho}) + 1 - (1+r_1(1,\omega))^{1/\rho}}
\end{aligned}
$$

This function is clearly non-monotonic. However taking two states such that $r_1(\omega^*) > r_1(\omega')$ and $r_2(\omega^*) = r_2(\omega')$. Define $g_1(\omega^*) = log(e_1(\omega^*))$. This gives us that

$$
\begin{aligned}
\left.\frac{\partial g_1(\omega^*)}{\partial s^*}\right|_{r_1(\omega^*)=0} &= \frac{1}{1+r_1(\omega^*)} - \frac{1}{r_1(\omega^*)^2} \\
&+ \frac{1/\rho(1+r_1(1,\omega^*))^{1/\rho-1}}{\frac{c^2(0)}{c^1(0)}(1-(1+r_2(1,\omega^*))^{1/\rho}) + 1 + (1+r_1(1,\omega^*))^{1/\rho}} \\
&+ \frac{1/\rho(1+r_1(1,\omega^*))^{1/\rho-1}}{\frac{c^2(0)}{c^1(0)}(1+(1+r_2(1,\omega^*))^{1/\rho}) + 1 - (1+r_1(1,\omega^*))^{1/\rho}} \\
&\to -\infty.
\end{aligned}
$$

Setting this to zero we can find the interest rate at which the inequality is reversed, though for equilibrium values of $\frac{c^2(0)}{c^1(0)} \approx 1$ and $r_2(1,\omega^*) \approx 0$, for reasonable values of $r_1(1,\omega^*)$, the effect on the exchange rate is negative (appreciation): exchange rates are stronger or more appreciated in states where interest rates are higher. That is $r_1(1,\omega^*) > r_1(1,\omega') \Leftrightarrow e(1,\omega^*) < e(1,\omega')$. □

Proof of Proposition 16

Proof. As $e(1,\omega) = \frac{M_1(1,\omega)}{M_2(1,\omega)} \frac{y^2 1(1,\omega)}{y^1 1(1,\omega)}$, and using Proposition 13, it follows that $r_1(1,\omega^*) > r_1(1,\omega') \Leftrightarrow e(1,\omega^*) > e(\omega')$. □

Proof of Proposition 17

Proof. The propositions above tell us that for Price Stability and Financial Stability, the risk premium or state price is negatively correlated with the exchange rate. This means that, given an objective probability measure $f(\omega)$,$\mu_1(\omega^*) < f(\omega^*)$ whenever $e_1(\omega^*) < \int_\omega f(\omega)e_1(\omega)d\omega$ Let the forward Exchange Rate be $F(1)$ or:

$$
\begin{aligned}
e_{(0)}\frac{1+r_1(0)}{1+r_2(0)} &= \int_\omega \mu_1(\omega)e(1,\omega)d\omega && (51)\\
&= e^f(1) && (52)\\
&< \int_\omega f(\omega)e(1,\omega)d\omega. && (53)
\end{aligned}
$$

The expected exchange rate under the subjective measure is $\int_\omega f(\omega)e(1,\omega)$. Now as we have shown that as Arrow prices, and hence the Martingale measure are positively correlated with the exchange rate, then the Forward exchange rate, or the expected exchange rate under the Martingale Measure, is biased upwards (more depreciated). That is, $e^f(1) < \int_\omega f(\omega)e(1,\omega)d\omega$. $\square$