NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

Igor G. Pospelov, Stanislav A. Radionov

MULTISECTOR MONOPOLISTIC COMPETITION MODEL

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: ECONOMICS
WP BRP 34/EC/2013

Igor G. Pospelov, Stanislav A. Radionov[1]

MULTISECTOR MONOPOLISTIC COMPETITION MODEL[2]

Abstract

We present a natural generalization of the Dixit-Stiglitz monopolistic competition model (DSM) — we assume that there is a continuum of industries, each of them described as in DSM, and each characterized with its own elasticity of substitution. Although firms in all industries share the same level of productivity and costs, exogenous technological progress leads to non-trivial reallocations of labor and production to industries with lower elasticities of substitution. Thus the model, despite is simplicity and absence of additional assumptions about industry structure, generates the structural changes described in the economic growth literature.

JEL classification: D43, J21, L13, O41.

Keywords: Dixit-Stiglitz model, monopolistic competition, economic growth, labor reallocations.

1 Introduction

In *Modern Economic Growth*, Simon Kuznets wrote: "We identify the economic growth of nations as a sustanined increase in per capita or per worker product, most often accompanied by an increase in population and usually by sweeping structutal changes. In modern times these were changes in the industrial structure within which product was turned out and resources employed — away from agriculture toward nonagricultural activities, the process of industrialization..." A number of economic models were developed to describe these structural changes and their relation to economic growth. For example, in [4] different economic sectors grow at

[1]National Research University Higher School of Economics. Research group on macro-structural modeling of Russian economy. Junior Researcher; E-mail: stradionov@gmail.com

[2]Authors are grateful to S.E. Pekarski and anonymous referee for their valuable comments. The reported study was partially supported by Russian Foundation for Basic Research (RFBR), research project Nos 12-01-00916a, 12-01-31333, 12-01-31189; Russian Foundation for Humanities (RFH), project No. 11-02-00241a; Basic research programs of Russian Academy of Sciences (RAS) No. 3 (project 3.14) and No. 14 (project 109). The study was implemented in the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE) in 2013.

different rates because they have different rates of technological progress. In [2] it was noted that similar effects may be obtained by assuming different capital intensities in the industries. In [10] authors assume that household consumption consists of agricultural, manufacturing and services goods. A special form of utility function generates reallocations of consumption: after the consumption of certain amount of agricultural good, the household starts to demand other goods, moreover, it consumes service goods only after some level of manufacturing consumption is reached. In [11], the structural change in the economy with agricultural and manufacturing industries is explained by assuming the absence of technological progress in agriculture and the learning-by-doing effect in industry. There are also several models incorporating, for example, financial development, demographic transition, urbanization, migration, production organization, see [1, ch. 21] for a survey. As we can see, all these models use rather artificial assumptions about industry structure.

We present a simple monopolistic competition model, based on the constant elasticity of substitution utility function, proposed in [6], which generates non-trivial market reallocations of the type described by Kuznets. The Dixit-Stiglitz model (DSM) is often used in economic growth models. In [17] the authors describe three ways of incorporating DSM in an economic growth framework. The first one is based on Romer's idea of knowledge spillovers (see [15], [8, ch. 3]), the second on the Schumpeterian creative destruction concept and quality improvement (see [3], [8, ch. 4]), and the third is based on the idea that firms undertake in-house R&D (see [18], [19], [14],[13]). The Melitz monopolistic competition model, described in [12], based on DSM, was also generalized to incorporate economic growth, see, for example [16]. But, to our knowledge, none of the existing models of these types generates the structural changes we are interested in.

The main difference between our model and the models described above is that we do not assume any special properties of different industries: the only difference between them are intra-industry elasticities of substitution. We think it is quite natural to assume that the "simpler" the commodity, the higher the elasticity of intra-industry substitution. This assumption is in line with empirical studies on product differentiation, see for example [9] for calculations and also [7] for discussion. Industries with high elasticities of substitution may be interpreted as agricultural, with lower as industrial, service and informational. Technological progress in our framework is modeled naturally as a decrease of firm variable costs (an increase of worker productivity) at the expense of an increase of fixed costs (which may be interpreted as investment). Although firms in all industries share the same levels of productivity and costs, labor and production flows from less differentiated (higher elasticity of substitution) to more differentiated (lower elasticity of substitution) goods. Thus our model, despite its simplicity and with no additional assumptions on industry structure, generates Kuznets structural changes.

We make some comments before we introduce our model. First, we restrict our analysis with comparative statics and do not develop a fully dynamic model because of computational difficulties. This simplification, however, does not lead to a loss of generality — one may think that we describe one step of model in discrete time. It is also possible to reformulate our model in continuous time, but we would like to avoid unnecessary technical complications. Second, an interpretation of industries with lower elasticities of substitution as "informational" in contrast to "material" industries, characterized with high elasticity of substitution is highly debatable, mainly because of the ambiguity of "informational good" concept, but, as we will show later, this may be justified in a CES framework. This interpretation can make our model correspond better to modern informational society. And third, the concept of constant elasticity of substitution may be considered as too restrictive. As was noted in [20], limitations of CES include independence of prices and markups from firm entry and market size, and the absence of the scale effect — the size of firms is independent of the number of consumers. This is, of course, true, and we will encounter some difficulties in the current setup of our model originating from these limitations, but CES remains the benchmark of monopolistic competition models, so we find it reasonable to start the analysis of new concepts in the CES framework.

2 Model

2.1 Model setup

There is a continuum of industries indexed with $\rho \in (0,1)$. For every industry, preferences of aggregate consumer are given by a CES function:

$$V(\rho) = \left(\sum_{i=1}^{n(\rho)} c_i(\rho)^{\rho} \right)^{\frac{1}{\rho}}, \tag{1}$$

where $n(\rho)$ is the number of firms working in this industry, $c_i(\rho)$ is the quantity of consumed good produced by i-th firm. Elasticity of substitution between goods in a given industry is $\sigma = 1/(1-\rho)$. So, if ρ is high then the elasticity of substitution in this industry is high, and goods in this industry are weakly differentiated and vice versa.

We assume that industries with low elasticities of substitution may be interpreted as "informational" and industries with high elasticities — as "material". The explanation is following: if all c_i's in the fixed industry are equal, than (1) may be rewritten in the following form, $V(\rho) = c_i(\rho)n(\rho)^{1/\rho} = (c_i(\rho)n(\rho))n(\rho)^{1/\rho-1}$, where expression in the parentheses is the material amount of goods, and the multiplier depends only on the variety of goods, not their physical amount. This multiplier is high for low ρ, which means variety matters more for these industries. This is exactly what we observe in modern informational industries, for example, cinema, because each movie is unique, in contrast to traditional material ones.

Preferences of the aggregate consumer over the goods in continuum of industries are given by the function

$$U = \frac{1}{\nu}\int_0^1 a\left(\rho\right) V\left(\rho\right)^{\nu} d\rho, \tag{2}$$

where $a(\rho) \geq 0$ may be interpreted as consumer's preference for the products of industry ρ, this is also needed for the dimension correctness, because goods in different industries can have different dimensions and units of measurement.

Firms are assumed to have both fixed and variable costs, so in order to produce $c_i(\rho)$ units of good, i-th firm has to use

$$l_i\left(\rho\right) = \alpha\, c_i\left(\rho\right) + f \tag{3}$$

units of labor. Productivity level $1/\alpha$ and fixed costs f are assumed to be equal across firms. The model becomes highly complicated without this assumption (see, for example [5], who deal with heterogeneous markets). Full stack of labor in the economy is denoted by L, the wage, the same for all workers, is w. So the aggregate consumer has wL units of income.

2.2 Market equilibrium and social welfare

Consider the aggregate consumer's problem of maximizing utility given budget constraint:

$$U=\frac{1}{\nu}\int_0^1 a(\rho)\left(\left(\sum_{i=1}^{n(\rho)} c_i(\rho)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho \to \max,$$

$$\int_0^1 \sum_{i=1}^{n(\rho)} c_i(\rho)\, p_i(\rho)\, d\rho \le wL.$$

To solve it, we form a Lagrange function in the following form:

$$\mathcal{L}=\frac{1}{\nu}\int_0^1 a(\rho)\left(\left(\sum_{i=1}^{n(\rho)} (c_i(\rho))^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu} d\rho + \xi^{\nu-1}\left(wL-\int_0^1 \sum_{i=1}^{n(\rho)} c_i(\rho)\, p_i(\rho)\, d\rho\right).$$

First of all, we should check if the problem has a solution. For this the integrand must be concave. Applying Sylvester's criterion, we conclude that the function is concave if the two following conditions are valid: $\nu < 1$ and $\nu < \rho$. So we have two possibilities: $\nu \in (0,1)$, $\nu < \rho$ or $\nu < 0$. We will consider only the second case, because, as we will see later, first case leads to some very unpleasant degeneracies at the point $\nu = \rho$. From an economic point of view, first case is problematic because we have to assume a relationship between parameters which do not have an economic meaning. So, from now on, ν is assumed to be negative.

Fixing some $j \in \{1 \ldots n\}$ and equalizing the partial derivative of Lagrange function with respect to $c_j(\rho)$ to zero, we get the optimum level of consumption of $c_j(\rho)$:

$$c_j(\rho)=\left(\frac{p_j(\rho)\,\xi^{\nu-1}}{a(\rho)\,V(\rho)^{\nu-\rho}}\right)^{\frac{1}{\rho-1}}. \tag{4}$$

We involute both sides of this equality to the power ρ and find a sum over $j = 1 \ldots n(\rho)$, and after some calculation we get

$$V(\rho)=\xi\, a(\rho)^{-\frac{1}{\nu-1}}\, P(\rho)^{\frac{1}{\nu-1}}, \tag{5}$$

where

$$P(\rho)^{\frac{\rho}{\rho-1}}=\sum_{i=1}^{n(\rho)} p_i(\rho)^{\frac{\rho}{\rho-1}} \tag{6}$$

is the price index associated with the goods index $V(\rho)$ as in [6] or [12]. Taking (5) into account, (4) may be rewritten in the following form:

$$c_j(\rho) = p_j(\rho)^{\frac{1}{\rho-1}} V(\rho) P(\rho)^{-\frac{1}{\rho-1}} . \tag{7}$$

Now consider the behavior of a firm. First of all, it is obvious that due to the fact that all firms have the same levels of fixed and variable costs, and the symmetry of consumer's preferences over the goods in a fixed industry, all firms in the industry face the same consumer's demand and set the same price. Taking (3) into account, the problem of firm in the industry ρ has the form

$$\pi(\rho) = p(\rho) c(\rho) - (c(\rho)\alpha + f) w \to \max . \tag{8}$$

Substituting (7) and finding maximum with respect to $p(\rho)$, we find the price set by firm:

$$p(\rho) = \frac{\alpha w}{\rho}, \tag{9}$$

like, again, in [12].

Now we will impose a free entry condition and demand firm profit to be equal to zero. Substituting (5), (6), (9) into (8), after some calculations we get the following expression for the number of firms in the industry ρ:

$$n_M(\rho) = \left(\frac{(1-\rho)\alpha^{\frac{\nu}{\nu-1}} \xi \ w^{\frac{1}{\nu-1}}}{\rho^{\frac{\nu}{\nu-1}} f a(\rho)^{\frac{1}{\nu-1}}} \right)^{\frac{(\nu-1)\rho}{\nu-\rho}} \tag{10}$$

Substituting (10) to consumer's budget constraint, we get the equation defining ξ:

$$\int_0^1 a(x)^{-\frac{x}{\nu-x}} w^{\frac{x}{\nu-x}} f^{-\frac{\nu(x-1)}{\nu-x}} \alpha^{\frac{\nu x}{\nu-x}} \xi^{\frac{(\nu-1)x}{\nu-x}} \left(x^{-\frac{\nu}{\nu-1}} - x^{-\frac{1}{\nu-1}} \right)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu}{\nu-1}} dx = L. \tag{11}$$

Unfortunately, ξ can be found from (11) only numerically.

Substituting (9) into (8) and setting firm profit to zero, we find that firm output may be rewritten in the following form:

$$c_M(\rho) = \frac{f\rho}{\alpha(1-\rho)}, \tag{12}$$

which is again in line with [6].

Now consider the problem of the benevolent social planner, who maximizes consumer utility with respect to technological limitation.

$$U=\frac{1}{\nu}\int_0^1 a\left(\rho\right)\left(\left(\sum_{i=1}^{n(\rho)}c_i\left(\rho\right)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu}d\rho,$$

$$\alpha\int_0^1\sum_{i=1}^{n(\rho)}c_i\left(\rho\right)d\rho+f\int_0^1 n\left(\rho\right)d\rho\leq L.$$

Form the Lagrange function in the following form:

$$\mathcal{L}=\frac{1}{\nu}\int_0^1 a\left(\rho\right)\left(\left(\sum_{i=1}^{n(\rho)}c_i\left(\rho\right)^{\rho}\right)^{\frac{1}{\rho}}\right)^{\nu}d\rho+\left(L-\alpha\int_0^1\sum_{i=1}^{n(\rho)}c_i\left(\rho\right)d\rho+f\int_0^1 n\left(\rho\right)d\rho\right)$$

Differentiating with respect to $c(\rho)$ and $n(\rho)$, after some calculations we get

$$n_W(\rho)=\left(\frac{(1-\rho)\,\alpha^{\frac{\nu}{\nu-1}}\lambda}{\rho f a(\rho)^{\frac{1}{\nu-1}}}\right)^{\frac{(\nu-1)\rho}{\nu-\rho}},\tag{13}$$

$$c_W\left(\rho\right)=\frac{f\rho}{\alpha\,\left(1-\rho\right)}.$$

Substituting (12) into the planner's technological constraint, we get the formula defining λ:

$$\int_0^1\alpha^{\frac{\nu\,x}{\nu-x}}a\left(x\right)^{-\frac{x}{\nu-x}}\lambda^{\frac{(\nu-1)x}{\nu-x}}f^{-\frac{\nu\,(x-1)}{\nu-x}}x^{-\frac{(\nu-1)x}{\nu-x}}\left(1-x\right)^{\frac{\nu\,(x-1)}{\nu-x}}dx=L.\tag{14}$$

Similarly to (11), it can be solved only numerically.

As we can see, outputs in market equilibrium and in the social welfare problem are the same, but the numbers of firms are different. This proves the following

Proposition 1. For any $\nu\in(-\infty,0)$ and any parameters of the economy L,f,α,w market equilibrium is inefficient.

Thus, in market equilibrium theconsumer cannot optimally distribute her expenses across industries optimally, but can do it within an industry. Recall that in one-sector Dixit-Stiglitz model the equilibrium is efficient, but in the presence of second market of homogenous product (as it was proposed in the original paper), it is not. This result is disappointing, but rather expectable — market efficiency is rare thing in the presence of monopolists.

Figure 1 shows the distribution of the number of firms in the market equilibrium and in social welfare state for parameters of the economy $\alpha=0.01,f=0.5,L=30,a(\rho)=1,w=1$

and consumer's preferences characterized with $\nu = -1$. Figure 2 shows the same distribution of firms the same economy and for consumer's preferences characterized with $\nu = -5$.

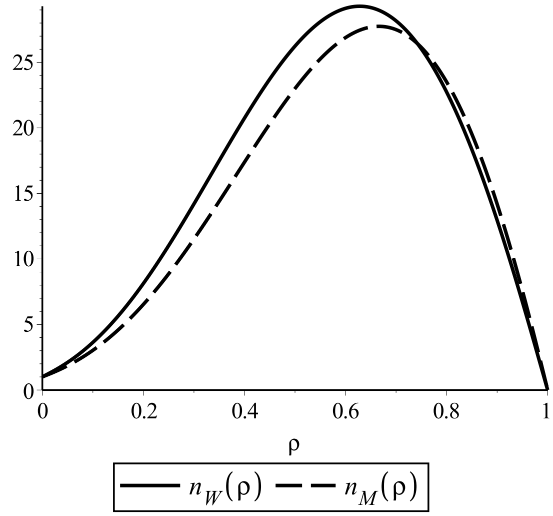

Figure 1: $\nu = -1$.

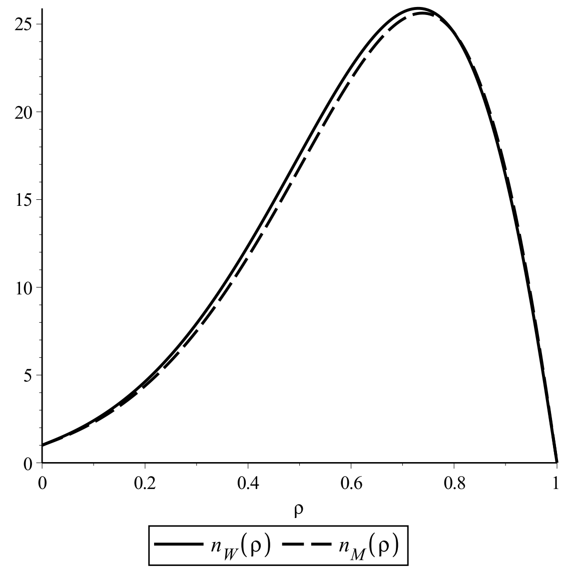

Figure 2: $\nu = -5$.

As we see, smaller ν, closer the market equilibrium to the social welfare state. Note that as ν tends to $-\infty$, utility function (2) converges to Leontieff-like minimum function:

$$U_{-\infty} = \min_{\rho \in (0,1)} V(\rho). \tag{15}$$

This observation leads us to

Proposition 2. For consumer's preferences defined by (15), market equilibrium is efficient.

Proof. As ν tends to $-\infty$ in (10) and (11), we get

$$n_M^*(\rho) = \left(\frac{f\rho}{\xi\,(1-\rho)\,\alpha}\right)^{-\rho}, n_W^*(\rho) = \left(\frac{f\rho}{\lambda\,(1-\rho)\,\alpha}\right)^{-\rho},$$
$$\int_0^1 \xi^x f^{-x+1} \alpha^x x^{-x} (-x+1)^{x-1}\, dx = L,$$
$$\int_0^1 \lambda^x f^{-x+1} \alpha^x x^{-x} (-x+1)^{x-1}\, dx = L.$$

Obviously, $\lambda = \xi$ and $n_M^*(\rho) = n_W^*(\rho)$. ■

2.3 Effects of technological progress and population growth

Let us consider the effects of technological progress in this model. We assume that in our framework technological progress means increase of workers' productivity $1/\alpha$. This, however, doesn't come without cost — we assume that fixed costs f also increase. Thus, progress is due to the increase in capital expenditures. Figures 3 – 6 show the effect of technological progress in economy with parameters $\alpha = 0.01, f = 0.5, L = 30, w = 1$ and consumer's preferences defined by parameters $a(\rho) \equiv 1, \nu = -\infty$, on the distribution of number of firms, output and labor. Technological progress is modeled by decreasing α 3 times and increasing f 3 times. Solid line is before progress, dashed — after.

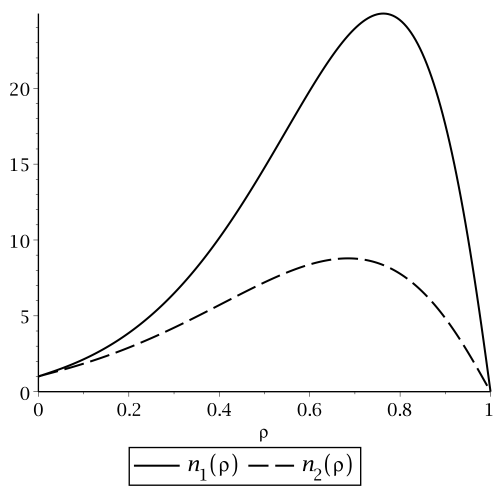

Figure 3: Number of firms.

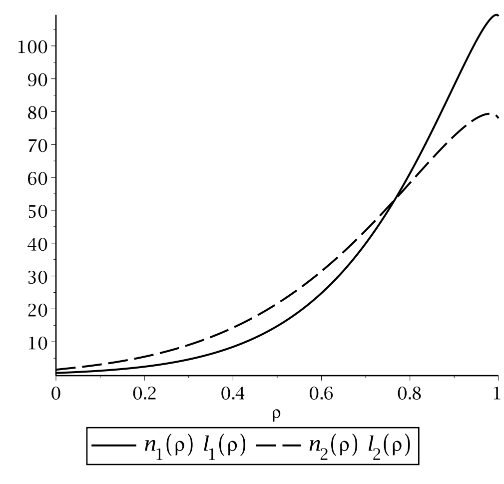

Figure 4: Number of workers.

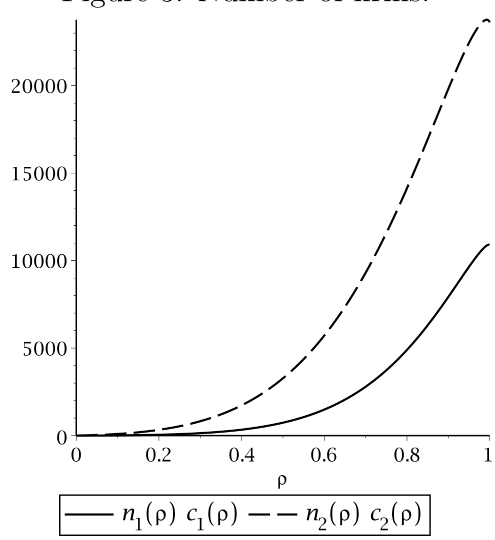

Figure 5: Output.

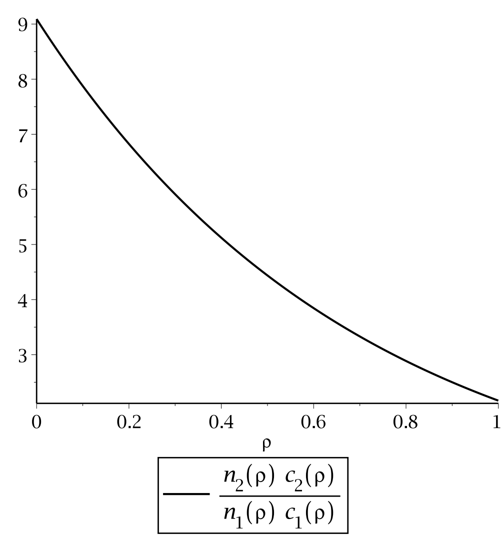

Figure 6: Ratio of outputs.

Conclusions are summed up in the following

Proposition 3. With decrease of variable costs and increase of fixed costs: $\alpha' = \frac{1}{k}\alpha, f' = k^{\mu} f, k > 1$, for any parameters of economy and consumer's preferences of the type (1)-(2) with $a(\rho) = 1$ for any ρ,

1. for $\mu \leq 1$ output increases in all industries $\rho \in (0, 1)$,

2. labor increases in industries with $\rho \in (0, \rho^*)$ and decreases for industries with $\rho \in (\rho^*, 1)$ for some $\rho^* \in (0, 1)$.

3. if $\mu + \nu \leq 0$, consumer's utility increases.

Proof. 1. Obviously, with the change of costs, ξ will change as well: $\xi' = \theta\xi$, where θ can depend on μ. We need to prove that $c(\rho)\, n(\rho) < c'(\rho)\, n'(\rho)$. Substituting expressions from (11) and (12), we get

$$\frac{\rho f'}{\alpha' (1-\rho) w} \left(\frac{(1-\rho)\alpha^{\frac{\nu}{\nu-1}} \xi\; w^{\frac{1}{\nu-1}}}{\rho^{\frac{\nu}{\nu-1}} f a(\rho)^{\frac{1}{\nu-1}}} \right)^{\frac{(\nu-1)\rho}{\nu-\rho}} = k^{-\frac{\nu(\mu+1)(\rho-1)+\rho}{\nu-\rho}} \theta^{\frac{(\nu-1)\rho}{\nu-\rho}} \times \\ \times\, c(\rho)\, n(\rho) > c(\rho)\, n(\rho)\,. \tag{16}$$

Note that $-\frac{\nu(\mu+1)(\rho-1)+\rho}{\nu-\rho} > 0$ and $\frac{(\nu-1)\rho}{\nu-\rho} > 0$ for all $\nu < 0$ and $\rho \in (0, 1)$. So if we prove that $\theta > 1$, inequality (16) will be proven. In order to prove $\theta > 1$, consider budget constraint (11) of economy after technological progress, which may be rewritten in the following form:

$$\int_0^1 k^{\frac{-\nu(\mu x-\mu+x)}{\nu-x}} \theta^{\frac{(\nu-1)\rho}{\nu-\rho}} (1-x)^{\frac{\nu\,(x-1)}{\nu-x}} x^{-\frac{\nu\, x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = L, \tag{17}$$

where A is a combination of parameters of the model. Budget constraint for economy before technological progress is then

$$\int_0^1 (1-x)^{\frac{\nu\,(x-1)}{\nu-x}} x^{-\frac{\nu\, x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = L. \tag{18}$$

Note that if we prove that

$$\int_0^1 k^{\frac{-\nu(\mu x-\mu+x)}{\nu-x}} (1-x)^{\frac{\nu\,(x-1)}{\nu-x}} x^{-\frac{\nu\, x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx < L, \tag{19}$$

that will mean exactly that for (17) to be true, θ must be greater than 1. Due to (18), inequality (19) may be rewritten in the following form:

$$\int_0^1 \left(1 - k^{-\frac{\nu(\mu x - \mu + x)}{\nu - x}}\right) (1-x)^{\frac{\nu\,(x-1)}{\nu - x}}\, x^{-\frac{\nu\,x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu - x}} dx > 0. \tag{20}$$

Denote the integrand in (20) as $g(x)$. Figure 6 shows the graph of $g(x)$. This function is positive for $\rho > \frac{\mu}{1+\mu}$ and negative otherwise.

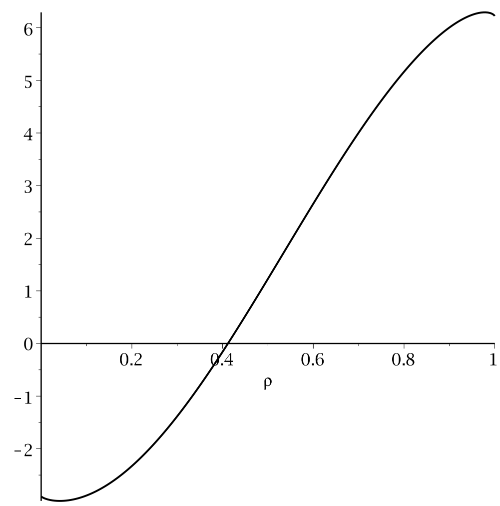

Figure 7: $g(x)$ for $\mu = 0.7$.

We will prove that if $\mu \leq 1$, than

$$g(x) > -g(1-x) \text{ for all } x \in (\frac{\mu}{1+\mu}, 2/frac\mu 1 + \mu), \tag{21}$$

which proves (20). Note that for $\mu > 1$ it doesn't have to be true. Consider the function $h(x) = -\frac{g(x)}{g(1-x)}$. Analysis of this function shows that $\lim_{x \to \frac{\mu}{1+\mu}} h(x) = 1$ and it is monotonically increasing provided that k is not too big compared to A. So $h(x) > 1$ for $\rho \in (0.5, 1)$ and hence (21) holds, hence (20) holds, hence $\theta > 1$, which concludes the proof.

2. We will prove that $l'(1) < l(1), l'(0) < l(0)$ and l is a monotonic function of ρ. $\lim_{\rho \to 1} l(\rho) = \alpha^{\frac{\nu}{\nu-1}} \xi\, w^{\frac{1}{\nu-1}}$, so $\frac{l'(\rho)}{l(\rho)} = k^{-\frac{\nu}{\nu-1}} \theta$. We need to prove that

$$\theta < k^{\frac{\nu}{\nu-1}}. \tag{22}$$

Assume otherwise: $\theta \geq k^{(\nu/(\nu-1))}$ and substitute it to the budget constraint in the form

$$L = \int_0^1 k^{-\frac{\nu(\mu x - \mu + x)}{\nu - x}} \theta^{\frac{(\nu-1)\rho}{\nu-\rho}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx \geq$$
$$\geq \int_0^1 k^{\frac{\nu\mu(1-x)}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx > \int_0^1 (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = L$$

This contradiction proves (21) and the fact that $l'(1) < l(1)$.

$\lim_{\rho \to 0} l(\rho) = f$, so obviously $l'(0) > l(0)$. Finally, analysis of the derivative of l shows that is indeed monotonically increasing.

3. Consumer's utility may be written in the following form:

$$U' = \frac{1}{\nu} \int_0^1 \left(c'(\rho)\, n'(\rho)^{\frac{1}{\rho}} \right)^\nu d\rho = \frac{1}{\nu} \int_0^1 \left(k^{-\frac{\mu\rho - \mu + \rho}{\nu - \rho}} \theta^{\frac{\nu-1}{\nu-\rho}} c(\rho)\, n(\rho)^{\frac{1}{\rho}} \right)^\nu d\rho >$$
$$> \frac{1}{\nu} \int_0^1 k^{\frac{\nu(\mu\rho - \mu - \nu + \rho)}{\nu - \rho}} \left(c(\rho)\, n(\rho)^{\frac{1}{\rho}} \right)^\nu d\rho > \frac{1}{\nu} \int_0^1 \left(c(\rho)\, n(\rho)^{\frac{1}{\rho}} \right)^\nu d\rho = U.$$

Here we used (21) in the first inequality and assumption $\nu \leq -1$ in the second one ($\mu\rho - \mu - \nu + \rho > -\mu - \nu > 0$). ■

There are few things regarding these results we should point out. First, the decrease of the number of firms in all industries, which can be seen in figure 3, looks counterintuitive. This is one of pitfalls of our model, originating from the lack of flexibility of the original CES function. Considering another utility function in (1) may overcome this difficulty. We can also point to a possible workaround: in the presence of population growth this effect will vanish. Other results, however, are more promising. The second part of proposition 3 is the most important to us: it means that because of technological progress, labor flows from less differentiated to more differentiated industries. Consumer utility increase, proven, with some restrictions, in third part of Proposition 3, indicates that this reallocation leads to economic growth. Figure 6 indicates another good property of output: the rate of output growth is higher in more differentiated industries. We do not include a proof because it can be done trivially by analyzing the derivative of the ratio. Note that another important indicator, output per worker, does not show non-trivial dynamics in our framework: it can be easily calculated that $c(\rho)/l(\rho) = \rho/\alpha$, so with technological progress this ratio increases in all industries equally.

Let us now consider the effect of population growth. Figures 7, 8, 9 show the effects of population growth of 150% times in the same economy as in the previous figures, but everything is now per capita.

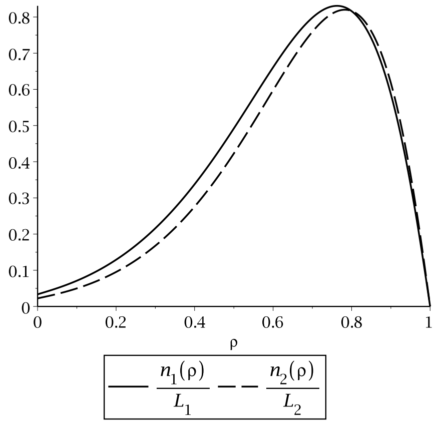

Figure 8: Number of firms per capita.

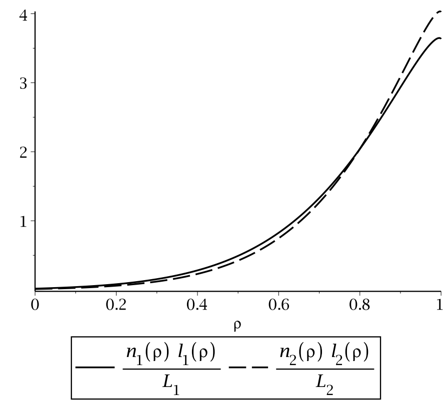

Figure 9: Output per capita.

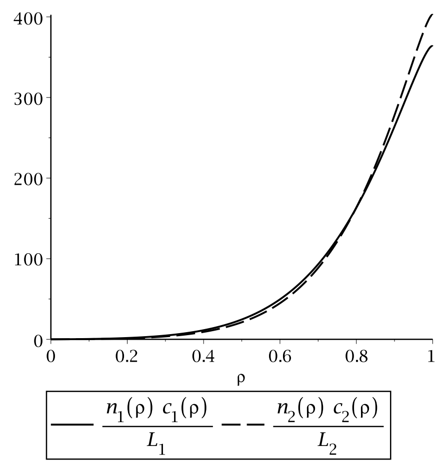

Figure 10: Number of workers in industry per capita.

Proposition 4. With the population growth, for any parameters of economy and consumer's preferences of the type (1)-(2) with $a(\rho) = 1$ for any $\rho \in (0, 1)$,

1. output per capita decreases in industries with $\rho \in (0, \rho_1)$ and increases in industries with $\rho \in (\rho_1, 1)$ for some $\rho_1 \in (0, 1)$.

2. labor per capita decreases in industries with $\rho \in (0, \rho_2)$ and increases in industries with $\rho \in (\rho_2, 1)$ for some $\rho_2 \in (0, 1)$.

3. consumer's utility per capita decreases.

Proof. 1. Denote $L' = kL, k > 1, \xi' = \delta\xi$. First, we shall prove that $\delta > 1$. Assume otherwise, $\delta \leq 1$, then

$$L' = kL = \int_0^1 \delta^{\frac{(\nu-1)x}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx \leq$$
$$\leq \int_0^1 (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = L,$$

which contradicts to the assumption $k > 1$. Next, we prove that $\delta > k$. To do so, consider budget constraint in the form (17):

$$L' = kL = \int_0^1 \delta^{\frac{(\nu-1)x}{\nu-x}} (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx <$$
$$< \delta \int_0^1 (1-x)^{\frac{\nu(x-1)}{\nu-x}} x^{-\frac{\nu x}{\nu-1}} A^{\frac{(\nu-1)x}{\nu-x}} dx = \delta L.$$

$\lim_{\rho\to 0} \frac{c(\rho)n(\rho)}{L} = 0$, so we have to analyze derivatives: $\lim_{\rho\to 0} \left(\frac{c(\rho)n(\rho)}{L}\right)' = \frac{f}{L}$ and $\frac{f}{L'} < \frac{f}{L}$, hence $\frac{c(\rho)n'(\rho)}{L'(x)} < \frac{c(\rho)n(\rho)}{L(x)}$ for small ρ's.

$\lim_{\rho\to 1} \frac{c(\rho)n(\rho)}{L} = \frac{\alpha^{\frac{1}{\nu-1}} \xi w^{\frac{1}{\nu-1}}}{L}$ and $\frac{xi'}{L'} = \frac{\delta\xi}{kL} > \frac{\xi}{L}$, hence $\frac{c(\rho)n'(\rho)}{L'(x)} > \frac{c(\rho)n(\rho)}{L(x)}$ for big ρ's. Analysis of derivative of the function $\frac{c(\rho)n(\rho)}{L(x)}$ shows that it is monotonically increasing.

2. Similarly to 1: $\lim_{\rho\to 0} \frac{c(\rho)n(\rho)}{L} = \frac{f}{L}$, $\lim_{\rho\to 1} \frac{c(\rho)n(\rho)}{L} = \frac{\alpha^{\frac{\nu}{\nu-1}} \xi w^{\frac{1}{\nu-1}}}{L}$.

3. Consider consumer's utility per capita in economy after population growth:

$$\hat{U}' = \frac{1}{\nu}\int_0^1 \left(\frac{c'(\rho)\, n'(\rho)^{\frac{1}{\rho}}}{L'}\right)^\nu d\rho = \frac{1}{\nu}\int_0^1 \left(\frac{\delta^{\frac{(\nu-1)\rho}{\nu-\rho}}}{k}\right)^\nu \left(\frac{c(\rho)\, n(\rho)^{\frac{1}{\rho}}}{L}\right)^\nu d\rho <$$
$$< \frac{1}{\nu}\int_0^1 \left(k^{\frac{(\nu-1)\rho}{\nu-\rho}-1}\right)^\nu \left(\frac{c(\rho)\, n(\rho)^{\frac{1}{\rho}}}{L}\right)^\nu d\rho < \frac{1}{\nu}\int_0^1 \left(\frac{c(\rho)\, n(\rho)^{\frac{1}{\rho}}}{L}\right)^\nu d\rho = \hat{U}.$$

■

As we can see, and as it might be expected, population growth leads the economy to, in a sense, opposite direction compared to economic growth, consumer now cares more about "simple" homogenous goods. This behavior may be interpreted in the following way: as the population increases, it becomes harder and harder to "feed" them, so people start to care less about luxury (differentiated goods) and more about simple material goods.

3 Conclusion

We developed a simple and natural model, which generalizes the Dixit-Stiglitz model. Our model provides a natual way to describe technological growth, which leads to non-trivial labor and product reallocations. These reallocations may be interpreted as transfers from "simple" homogenous to "more complicated" differentiated goods (or from "material" to "informational" ones). Under some assumptions, it leads to an increase in consumer utility, which may be considered in our framework as economic growth. So, despite its simplicity, our model is able to reproduce essential features of modern economies, described in economic growth literature.

It is also true that the model generates a strange effect of the decrease of the number of firms as a result of technological growth. Perhaps this problem may be resolved by considering not CES, but some other consumer utility function, for example an arbitrary function from VES class studied in [20]. It would be interesting if the reallocations will persist under a broader class of utility functions. Non-efficiency is also disappointing, although we found one utility function which leads to efficient equilibrium. It may be possible to find another utility function such that market equilibrium is efficient. One more important thing is to analyze our interpretation of "informational" and "material" industries and provide empirical confirmations.

References

[1] D. Acemoglu. *Introduction to Modern Economic Growth*. Princeton University Press, 2007.

[2] D. Acemoglu and V. Guerrieri. Capital deepening and nonbalanced economic growth. *Journal of Political Economy*, 116(3):467–498, June 2008.

[3] P. Aghion and P. Howitt. A model of growth through creative destruction. *Econometrica*, 60(2):323–51, March 1992.

[4] W. J. Baumol. Macroeconomics of unbalanced growth: The anatomy of urban crisis. *Economic Journal*, 57(3):415–426, June 1967.

[5] S. Dhingra and J. Morrow. Monopolistic competition and optimum product diversity under firm heterogeneity. Technical report, London School of Economics, 2012.

[6] A. K. Dixit and J. E. Stiglitz. Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3):297–308, 1977.

[7] H. Gray and J. Martin. On the meaning and measurement of product differentiation in international trade: A reply. *Review of World Economics (Weltwirtschaftliches Archiv)*, 118(2):335–337, June 1982.

[8] G. M. Grossman and E. Helpman. *Innovation and Growth in the Global Economy*, volume 1 of *MIT Press Books*. The MIT Press, 1993.

[9] G. Hufbauer. The impact of national characteristics & technology on the commodity composition of trade in manufactured goods. In *The Technology Factor in International Trade*, NBER Chapters, pages 143–232. National Bureau of Economic Research, Inc, August 1970.

[10] P. Kongsamut, S. Rebelo, and D. Xie. Beyond balanced growth. *Review of Economic Studies*, 68(4):869–82, October 2001.

[11] K. Matsuyama. Agricultural productivity, comparative advantage, and economic growth. *Journal of Economic Theory*, 58(2):317–334, December 1992.

[12] M. J. Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.

[13] P. Peretto and S. Smulders. Technological distance, growth and scale effects. *Economic Journal*, 112(481):603–624, July 2002.

[14] P. F. Peretto. Sunk costs, market structure, and growth. *International Economic Review*, 37(4):895–923, November 1996.

[15] P. M. Romer. Endogenous technological change. *Journal of Political Economy*, 98(5):71–102, 1990.

[16] P. J. Schroder and A. Srensen. Firm exit, technological progress and trade. *European Economic Review*, 56(3):579–591, December 2012.

[17] J. Smulders and T. v. d. Klundert. Monopolistic competition and economic growth. Open Access publications from Tilburg University urn:nbn:nl:ui:12-123118, Tilburg University, 2004.

[18] P. Thompson and D. Waldo. Growth and trustified capitalism. *Journal of Monetary Economics*, 34(3):445–462, December 1994.

[19] T. van de Klundert and S. Smulders. Strategies for growth in a macroeconomic setting. *The Manchester School of Economic & Social Studies*, 63(4):388–411, December 1995.

[20] E. Zhelobodko, S. Kokovin, M. Parenti, and J.-F. Thisse. Monopolistic competition: Beyond the constant elasticity of substitution. *Econometrica*, 80(6):2765–2784, 2012.

Stanislav A. Radionov

National Research University Higher School of Economics (Moscow, Russia). Research group on macro-structural modeling of Russian economy. Junior Researcher;

E-mail: stradionov@gmail.com, Tel. +7 (965) 164-81-29