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Bulat N. Gafarov

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Bulat N. Gafarov[1]

DO UNOBSERVED COMPONENTS MODELS FORECAST INFLATION IN RUSSIA?

I apply the model with unobserved components and stochastic volatility (UC-SV) to forecast the Russian consumer price index. I extend the model which was previously suggested as a model for inflation forecasting in the USA to take into account a possible difference in model parameters and seasonal factor. Comparison of the out-of-sample forecasting performance of the linear AR model and the UC-SV model by mean squared error of prediction shows better results for the latter model. Relatively small absolute value of the standard error of the forecasts calculated by the UC-SV model makes it a reasonable candidate for a real time forecasting method for the Russian CPI.

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[1]Higher School of Economics (Moscow, Russia). Laboratory for Inflation Problems and Economic Growth Research. E-mail: gafarovb@gmail.com.

1 Introduction

After the liberalization of prices in 1990 inflation became one of the major concerns of the Russian government. Russia passed through the period of hyperinflation in the early 1990-s and experienced two rapid rises in prices during the crises of 1996 and 1998. Such high instability in domestic prices led to negative consequences for the economy. Fortunately, after the exchange rate depreciation in 1998 and subsequent changes in economic policy inflation tended toward stabilization. As a result, prices became more predictable, which stimulated interest for precise short term inflation forecasting methods in Russia. But despite this interest to the best of our knowledge in the late 2000-s any non-naive methods were absent in Russian academic literature. In this situation, application of the best foreign inflation forecasting practices to Russian economy becomes an interesting topic for research.

One good example of recent research in the area of inflation forecasting is the paper (Stock and Watson, 2007) *(SW hereafter).* In their study the authors present a new model with unobserved components and stochastic volatility (UC-SV hereafter) and show that their method outperforms all univariate and even multivaritate US inflation forecasts in terms of mean squared out-of-sample forecasting error in period after the Great Moderation. The Purpose of the present paper is to adopt UC-SV to Russia and compare its forecasting performance with the "textbook" benchmark, an autoregression model (AR hereafter).

In the present research I propose a Bayesian approach to calibrate the UC-SV model for the Russian inflation and add a seasonal component to take into account differences between methodologies of publication of the data by the Russian and the US statistical agencies. According to SW the UC-SV model outperforms the competitors in periods of relatively stable inflation. So as a measure of inflation I choose the official Consumer price index because it seems to be less volatile than other two main price indices, the GDP deflator and the Producer Price Index. The estimated parameters of the model turned out to be close to the parameters for the U.S. GDP deflator. I compute out-of-sample forecasts of the CPI based on the UC-SV model and compare them with "textbook" univariate recursive autoregessional (AR) forecasts. The Results of the comparisons of the forecast resemble qualitatively the results of SW. The UC-SV model allows to forecast Russian quarterly CPI better than the AR model on horizons from one to four quarters if I include an additional seasonal factor in the model. So the UC-SV model can be used as a better benchmark model for inflation in Russia.

The structure of the paper is the following. In the second part I describe a benchmark AR model and modifications of UC-SV. In the third part I present results of estimations and comparisons. The fourth part concludes.

2 Competitive Models

2.1 Pseudo out-of-sample methodology and a benchmark model

In contemporary literature it is acknowledged that in-sample prediction errors there is a poor measure of real-time prediction performance. Indeed, highly nonlinear models with a large number of parameters could fit data well, but due to misspecification or large errors in estimates of the parameters characterizing the model could provide poor real-time forecasts. Pseudo out-of-sample approach was established to avoid this problem. Pseudo out-of-sample forecasting is forecasting of a part of sample based on the model, estimated on another part of the sample. Unlike in-sample predictions, this approach is robust to over-fitting. The method is called ”Pseudo” to distinguish it from real-time forecasting, which is often called out-of-sample forecasting. The difference is the following. In contrast to ”real” out-of-sample forecasting, pseudo out-of-sample forecasting exercises are made after the entire sample became available. It the present paper I use a recursive scheme for forecasts comparison. Following (Stock,Watson, 2007) I choose mean squared prediction error () as a measure of forecasting performance. One can find a comprehensive review of this methodology in (Stock and Watson,2003).

To the best of our knowledge this is the first academic paper about inflation forecasting in Russia. So as a benchmark for comparison I use the same AR model with unit restriction as in SW,

$$\Delta\pi_t = \mu + \alpha(L)\Delta\pi_t + \nu_t \tag{1}$$

where μ is a constant term, $\alpha(L)$ is a lag polynomial,$\Delta\pi_t$is a first difference of inflation measure. I estimate the number of lags by the Akaike criterion (AIC).

2.2 Unobserved Components Model with Stochastic Volatility

In the paper SW the authors show the evidence in favor of changing parameters in linear time series models of the US inflation. They show that the best model from the ARIMA class for the US GDP inflation is IMA(1,1), but the coefficient in this model varies over time. To describe the entire US sample of 50 years of price observations the authors proposed a new model to allow for the variation in the coefficients. They suggested the following reasoning. One can easily check, that IMA(1,1) model,

$$\Delta\pi_t = \varepsilon_t - \theta\varepsilon_{t-1} \tag{2}$$

is equivalent to the local level model with noise (Harvey,2006), where a signal to noise ratio is constant. This ratio is a one-to-one function of the IMA(1,1) coefficient θ. Thus, one could model the time varying θ through variation in signal to noise ratio. This idea led the authors to the following specification, called Unobserved Components with Stochastic Volatility (UC-SV),

$$\pi_t = \tau_t + \eta_t \tag{3}$$

$$\tau_t = \tau_{t-1} + \varepsilon_t \tag{4}$$

$$\eta_t = \sigma_{\eta,t}\zeta_{\eta,t} \tag{5}$$

$$\varepsilon_t = \sigma_{\varepsilon,t}\zeta_{\varepsilon,t} \tag{6}$$

$$\ln\sigma^2_{\eta,t} = \ln\sigma^2_{\eta,t-1} + \nu_{\eta,t} \tag{7}$$

$$\ln\sigma^2_{\varepsilon,t} = \ln\sigma^2_{\varepsilon,t-1} + \nu_{\varepsilon,t} \tag{8}$$

where $\pi_t$is an inflation measure, $\zeta_t = (\zeta_{\eta,t}, \zeta_{\varepsilon,t})$ is i.i.d. $N(0,I)$, $\nu_t = (\nu_{\eta,t}, \nu_{\varepsilon,t})$ is i.i.d. $N(0,\gamma I)$,γ is a scalar parameter, which controls the smoothness of the stochastic volatility process. This specification allows variances of innovations η_t and ε_t to drift independently over time, thus allowing for variation in θ. Modeling the variation of coefficient in time as a random walk is very flexible and it is very common for Bayesian VAR literature.

SW shows that the UC-SV model either outperformed or performed as well as other univariate and some activity-based multivariate models in terms of mean squared prediction errors on the full the U.S. 50-year sample. The gains in forecasting performance were higher in stable period of the Great Moderation (1984-2004). This feature made the model attracting for application to the Russian CPI index, as it seems natural to expect structural breaks in it due to transition in pricing system and changes in monetary policy. Furthermore, approximately since 2002 the CPI inflation became relatively stable and in this aspect started to resemble the US inflation since 1990-s. These features make the UC-SV model an appropriate candidate for forecasting model for the Russian CPI.

2.2.1 Bayesian estimation

The authors of SW used Bayesian methods, namely Monte Carlo Markov Chain (MCMC) routine, to estimate the UC-SV model. These methods have some additional flexibility and computational advantages in comparison with classical methods, such as Maximum Likelihood (ML) and Generalized Method of Moments (GMM). The first advantage is that in Bayesian framework one can easily impose restrictions on parameters through proper specification of the priors. The second advantage is that one does not need to have a closed form density specification and to solve nonlinear multivariate optimization problems. Indeed, the MCMC method provides estimates of the parameters based on averages from simulated sample from a posterior distribution (Lancaster, 2004).

The Bayesian approach is not just another way to estimate statistical models. This is also new interpretation of statistical data and models. In this approach I assume some prior distributions and update them using information from the sample. In this framework I do not assume that the sampling will continue and that the estimated parameter of the models will converge to some limiting values. So the Bayesian framework is subjective and there are no identification concerns in this approach. Under the classical statistics framework I assume existence of some parameters which describe the whole population this feature may seem strange. But from the application perspective it does not bring a big difference in interpretation of model predictions and choice of the priors is up to the researcher.

The authors of SW *a priori* impose lower bounds on $\ln\sigma^2_{\varepsilon,t}$and $\ln\sigma^2_{\eta,t}$ to avoid over-fitting and imposed the particular value for the only parameter $\gamma = 0.04$. These restrictions, according to the authors, lead to good results for the US GDP deflator. It seems that the calibration $\gamma = 0.04$ was based on the behavior of the model on the entire US sample. This approach could lead to two problems. The first problem is that the forecasts of inflation, based on the model with such calibration are not pseudo out-of-sample as they use information from the entire sample. Thus it made the comparison of UC-SV forecasts with other models incorrect. The second problem is the following. Even if such a calibration performs well for the US GDP deflator, no one can guarantee that such priors will perform well for other inflation series, in particular, for the Russian CPI. These two problems can be solved if by using non-degenerate prior distribution for γ.

Also, for the sake of generality I omit the restriction on the variance of $\nu_t = (\nu_{\eta,t}, \nu_{\varepsilon,t})$, namely, make different variances for the components possible,

$$Var(\nu_{it}) = \gamma_i \tag{9}$$

where i represents eitherε, orη. To resample the parameters of interest in the Gibbs sampler framework one need to specify a posterior distribution for γ_i conditional on the other parameters, namely τ_t,$\ln\sigma^2_{\varepsilon,t}$and $\ln\sigma^2_{\eta,t}$. One obtains a likelihood function for γ_i conditional on the series$\ln\sigma^2_{\varepsilon,t}$ and $\ln\sigma^2_{\eta,t}$by using equations (7) and (8) ,

$$L(\gamma_i) = p(\gamma|\tau, \ln\sigma^2_{\varepsilon}, \ln\sigma^2_{\eta}) = (\frac{1}{\sqrt{2\pi\gamma_i}})^T \exp\left[-\frac{\sum_{t=2}^{T+1}(\ln\sigma^2_{i,t} - \ln\sigma^2_{i,t-1})^2}{2\gamma_i}\right] \tag{10}$$

where T is a size of a sample. For the proper specification of the posterior distribution one need to specify a prior distribution for γ_i. One way to do it is to use an improper non-informative prior, either Jeffrey's prior or a constant prior. Another way is to use some informative prior which has an expectation $E\gamma = 0.04$. A good candidate for this purpose is a conjugate prior for the likelihood function (10), as it corresponds to uninformative priors updated through the same Bayesian procedure by previous researchers. It is an inverse-gamma distribution,

$$f(\gamma;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}(\gamma)^{-\alpha-1}\exp(-\beta/\gamma) \tag{11}$$

which has an expectation $E\gamma = \frac{\beta}{\alpha-1}$. Note that the case$\alpha = 0$ and $\beta = 0$ corresponds to Jeffrey's non-informative prior. The posterior distribution for (10) and (11) also has an inverse-gamma distribution, but with parameters

$$\hat{\alpha} = \alpha + \frac{T}{2}, \hat{\beta} = \beta + \frac{\sum_{t=2}^{T+1}(\ln\sigma^2_{i,t} - \ln\sigma^2_{i,t-1})^2}{2}. \tag{12}$$

I use both informative and non-informative priors in this paper. I choose the parameters for the informative prior to approximately match posterior results based on the USA data from (Stock, Watson, 2007). I assume that the value $E\gamma = 0,04$ was based on the likelihood function with non-informative prior with sample size T=200, the sample size of the USA GDP deflator in (Stock, Watson, 2007). It means, that $\alpha = 200/2 = 100$ and $\beta = 0.04(100-1) = 3.96$. To guarantee a stability of the simulation algorithm for the non-informative prior I add an additional maximum boundary $\gamma < 1$. The results of estimation can be found in the third part. One can find the estimation scheme, based on the original code from (Stock, Watson 2007), in the appendix at the end of the paper.

2.2.2 Treatment of seasonality

In the paper SW the authors study seasonally adjusted GDP deflator and they do not include the seasonal factors in the model. Rosstat does not provide the adjusted time series, so in the present paper I use unadjusted time series for Russian CPI with explicit seasonal component in the model. This approach allows forecasting explicitly both seasonally adjusted and unadjusted series.

I use seasonal dummies to recursively filter the Russian CPI time series from seasonality. However, the dummies could leave some residual seasonality due to potential variability in the seasonal pattern. So I propose another modification of the UC-SV which incorporates an additional unobservable seasonal factor to allow for such seasonal noise. which incorporates an additional unobservable seasonal factor. Following (Harvey, 2006) I change equation (3) and add another equation,

$$\pi_t = \tau_t + s_t + \eta_t \tag{13}$$

$$s_t = -s_{t-1} - s_{t-2} - s_{t-3} + \psi_t \tag{14}$$

where s_t is an unobservable seasonal factor, and ψ_t is the innovation for the factor. This model referred to as the extended model in the rest of the paper. The variance of ψ_t, in principle, could be modeled in a manner similar to $\ln\sigma^2_{\varepsilon,t}$ and $\ln\sigma^2_{\eta,t}$. However, in this paper I study only the constant variance of ψ_t to keep things simple. Values for $Var\psi$ could be estimated, but this task is beyond the scope of the study. So in this work I use only some non-zero values as part of sensitivity analysis.

The original MCMC algorithm from SW was changed to estimate this extended model. At the stage when series of τ are re-sampled, expectations of τ are estimated on the basis of the two-sided Kalman smoother, not the conventional formula for conditional expectations of components of a normal vector. These changes were necessary to simplify computations.

Table 1: Bayesian estimates of γ.

Model	Prior	$\gamma_{\eta,5\%}$	$E\gamma_\eta$	$\gamma_{\eta,95\%}$	$\gamma_{\varepsilon,5\%}$	$E\gamma_\varepsilon$	$\gamma_{\varepsilon,95\%}$
Original	informative	0.034	0.043	0.052	0.034	0.043	0.052
Extended	non-informative	0.821	0.979	1.00	0.776	0.972	1.000
Extended	informative	0.035	0.042	0.050	0.035	0.041	0.049

Remark: $\gamma_{\eta,5\%},\gamma_{\eta,95\%},\gamma_{\varepsilon,5\%},\gamma_{\varepsilon,95\%}$ *correspond respectively to 5% and 95% quartiles of re-sampled posterior distributions of* γ_η *and* γ_ε*. Values* $E\gamma_\eta$ *and* $E\gamma_\varepsilon$ *correspond to posterior expectations of the parameters.*

3 Results

3.1 Data

Consumer price index (CPI) was chosen as a measure of Russian inflation. On the basis of the monthly data the following measure was obtained,

$$\pi_t = 400 \ln(p_t/p_{t-1}) \tag{15}$$

where p_t is a CPI at the end of the quarter t. This measure corresponds to the annual rate of inflation.

I omit the data before the year 2000 due to the large crisis outliers in 1998-1999 years as the method is sensitive to such problems. So the sample was from the first quarter of the year 2000 to the fourth quarter of the year 2010, 44 points altogether.

3.2 Estimation of parameters

The only parameters in the model are variances of innovations in $\ln \sigma^2_{\varepsilon,t}$and $\ln \sigma^2_{\eta,t}$. These parameters were estimated with use of both the original model (3)-(8) and extended model (4)-(8) and (13)-(14). The simulation sample for the forecasting was 100 burn in iterations and 5000 informative iterations.

It can be seen from the table that the estimates based on the non-informative prior differ strikingly from the estimates based on the informative prior. The latter estimates differ only slightly from the prior expectations.

Smoothed estimates for series τ, σ_η, σ_ε are presented below. These estimates are based on the original model with the informative priors. Other variants are not presented here to save space.

In Figure 1 one can see some time variation in σ_η,σ_ε which corresponds to changes in coefficients of AR approximations. However, these changes are not very prominent, so linear models could forecast inflation as good as the UC-SV during the sample. Estimates of τ_t have an interpretation as an indicator of long-run inflationary expectations (Mishkin,2007) and could be used, for example, for VAR modeling of Russian economy.

Figure 1: Estimates of the standard deviations of the (a) transitory and (b) permanent innovations, (c) estimates of the τ_t and seasonally adjusted π_t(dotted line), using the original UC-SV(.04) model. The dashed lines are the 5% and 95% quantiles of the posterior distributions of $\sigma_{\eta,t}$, $\sigma_{\varepsilon,t}$ and τ_t,

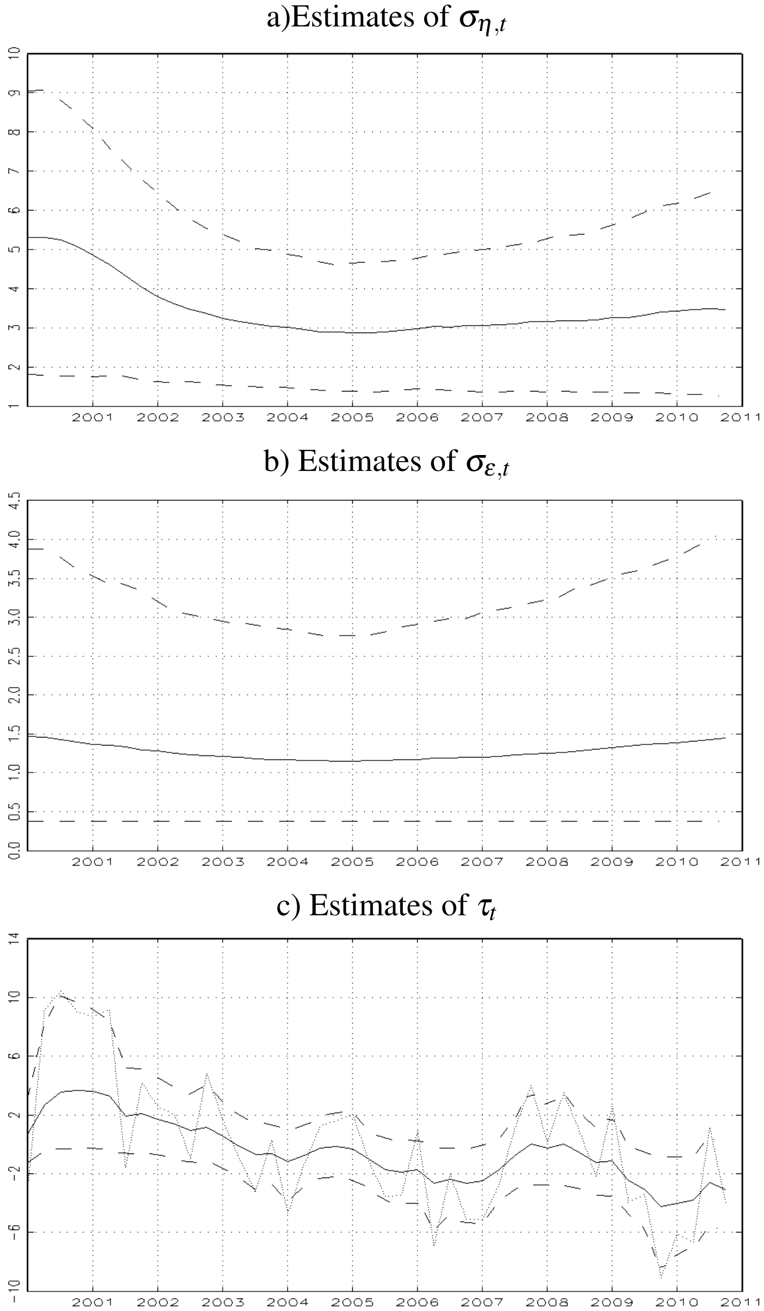

Table 2: Mean squared forecast errors of different models

Model	Priors	H=1	H=2	H=3	H=4
AR(AIC)	-	4.01	4.49	5.45	5.89
UC-SV, original	*degenerative*	4.16	**4.37**	**5.00**	**5.20**
UC-SV, original	*informative*	4.17	**4.38**	**5.03**	**5.21**
UC-SV, extended,$Var\psi_t = 0$	*non-informative*	4.27	4.50	5.55	**5.63**
UC-SV, extended,$Var\psi_t = 0$	*informative*	4.06	**4.37**	**5.33**	**5.47**
UC-SV, extended, $Var\psi_t = 0.03$	*informative*	**3.92**	**4.36**	**5.23**	**5.46**
UC-SV, extended, $Var\psi_t = 0.1$	*informative*	**3.80**	**4.33**	**5.24**	**5.52**
UC-SV, extended, $Var\psi_t = 1$	*informative*	**3.78**	**4.39**	**5.54**	**5.72**

Remark: *Bold numbers denote forecasts with MSE less than corresponding MSE of the AR(AIC) forecast. H denotes forecast horizons. First date for* h=1 *was 2006:IV, for* h=2 *2007:I, for* h=3 *2007:II, and for* h=4 *2007:III. The last date for comparison for all forecasts is the same – 2010:IV.*

3.3 Forecasting performance

I have chosen three models for forecasts comparisons: recursive AR(AIC), original UC-SV and extended UC-SV. The first two models are the most competitive time series models in the case of the US inflation (Stock, Watson 2007), and the last one is an adaptation of the UC-SV model for the Russian data. Different priors and different parameters for seasonal noise were used as sensitivity check. Particular values for $Var\psi_t \in \{0, 0.03, 0.1, 1\}$ were chosen arbitrarily, but small enough not to distort the results too much. For the original model I take two priors: the degenerative one with $\gamma = 0.04$ and the inverse-gamma with $\alpha = 100$ and $\beta = 3.96$ which is referred to as an informative prior. For extended model I try two priors : non-informative Jeffrey's prior with $\alpha = 0$ and $\beta = 0$ with upper bound $\gamma < 1$, and informative prior with $\alpha = 100$ and $\beta = 3.96$. The simulation sample for the forecasting was 100 "burn-in" iterations and 1000 informative iterations.

As a measure of forecast performance I use Mean Squared Error (MSE hereafter). Let $\hat{\pi}_t$ be the value of forecast of the CPI ,π_t, the first and the last dates comparison to be T_1 is T_2 correspondingly. Then the measure is

$$MSE = \sqrt{\frac{1}{T_2 - T_1 - 1} \sum_{t=T_1}^{T_2} (\pi_t - \hat{\pi}_t)^2}.$$

I can say that some model performs better than the benchmark if its MSE is smaller than the MSE of the benchmark model.

I study four forecasting horizons (h): one, two, three and four quarters ahead. The first date for *h=1* is 2006:IV, for *h=2* 2007:I, for *h=3* is 2007:II, and for *h=4* is 2007:III. The last date for comparison of all forecasts is the same – 2010:IV. According to the pseudo out-of-sample methodology a forecast for every date is based only on information prior to that date. I also sequentially estimate the seasonal dummies for every date. The mean squared errors of the forecasts are presented in Table 2.textbook

The results in Table 2 demonstrate that all UC-SV forecasts with informative priors outperform notably the benchmark AR(AIC) on three and four quarters horizons. For the smaller horizons *h=1* and *h=2* differences in prediction errors between the AR(AIC) and the original UC-SV with informative priors are small. But the extended UC-SV model with $Var\psi_t \neq 0$ outperforms the forecasts based on the AR(AIC) on one quarter ahead outperforms all the modifications of UC-SV.

One can see that addition of the seasonal factor in the UC-SV model led to almost 5% improvement of MSE in comparison with the AR(AIC) for *h=1* and 9% improvement in comparison with the original UC-SV model. This improvement, however, lead to slight deterioration of MSE of forecasts for three and four quarters ahead in comparison to the original UC-SV model. These differences may be a result of additional filtering of the seasonal factor, but it may be also due to re-sampling variability. It is difficult to give a curtain answer as inference theory for comparison of such models is not well established. In principle, one can calculate standard errors for MSE in assumption that the forecasting model is true DGP model using Monte Carlo simulations. However this task seems too computer intensive as the used number of elementary iterations of resampling will be performed about 1000 times to simulate a sample with reasonable size. Another problem is that critical values for the MSE computed this way would be incorrect in case if the forecasting model would be misspecified. So I leave this question for the future research.

Surprisingly, the informative priors concentrated near the value $\gamma = 0.04$ gave as good results for Russia as for the U.S. At the same time, the non-informative prior gave the worst results. Perhaps, it happened because of the insufficient number of simulations.

Note that I scale quarterly inflation by 400. So the absolute values of the forecast errors value $MSE = 3.80$, for example, would approximately imply a standard error of the quarterly inflation forecast equal to 0.95%.

4 Conclusion

The first contribution of the paper is the suggested modification of the simulation algorithm for posterior distributions in the UC-SV model framework. This modification allows estimation of variances of innovations in stochastic volatilities and calibration of the UC-SV model for any time series, not only to the U.S. GDP deflator. I apply the new variant of the estimation algorithm to Russian CPI. The estimates for the Russian inflation exhibit little difference from the parameters used in the original paper SW.

Another contribution is introduction of a seasonal noise in the model. Such modification improved forecasts of the Russian CPI inflation for short horizons. These results suggest that further research in modeling of the seasonal component of the Russian CPI is required. In particular, addition of estimation stage for the variation of the seasonal noise could be a new topic for research.

The last contribution is purely practical. I show that the forecasts based on UC-SV model diminish mean squared out-of-sample prediction errors for all horizons from one quarter to four quarters in comparison with simple the AR models. Absolute value of standard errors is about 1% for one quarter ahead forecast and 1.3% for four quarter ahead forecast. So the UC-SV model can be used for forecasting the Russian CPI.

5 Literature

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6 Appendix

6.1 Resampling scheme

In this part the algorithm used for simulation of a sample is described. This algorithm is a realization of Gibbs sampler.

- Step 1. Compute and subtract means for every quarter are from the initial series of π_t.
- Step 2. Set initial values for $\gamma_i, \ln\sigma^2_{\eta,t}, \ln\sigma^2_{\varepsilon,t}$.
- Step 3. Estimate conditional expectation and variance of s_t, τ_t on the basis of values from the previous step. This can be done with the use of either the formula for conditional expectation and variance for a normal vector, or using Kalman filter. In the original paper (Stock, Watson 2007) the former approach was used. I use the latter approach in the present study for estimation of the extended UC-SV model due to its computational effectiveness.
- Step 4. Simulate new realization of s_t, τ_t on the basis of the parameter estimates from the previous step. [2]
- Step 5. Estimate conditional expectations and variances of $\ln\sigma^2_{\eta,t}, \ln\sigma^2_{\varepsilon,t}$, as in step 3, on the basis of values s_t, $\tau_t$and γ from the previous step.
- Step 6. Simulate new realization of $\ln\sigma^2_{\eta,t}, \ln\sigma^2_{\varepsilon,t}$, on the basis of conditional expectations and variances of $\ln\sigma^2_{\eta,t}, \ln\sigma^2_{\varepsilon,t}$ and γ from the previous step.
- Step 7. Simulate new realization of γ on the basis of likelihood function from the previous step and the prior distribution.
- Step 8. Save values of s_t, τ_t,$\ln\sigma^2_{\eta,t}, \ln\sigma^2_{\varepsilon,t}$ and γ. Go to step 3.

[2] I make simulation of the unobserved states for the extended model with use of the algorithm of Durbin and Koopman (Durbin, Harvey,Koopman, Shephard 2004)

This cycle generates a path for a Markov chain which has the posterior distribution for s_t, τ_t, $\ln \sigma^2_{\eta,t}, \ln \sigma^2_{\varepsilon,t}$ and γ as a limiting distribution. So when the chain is converged it starts generate a sequence on simulated values s_t, τ_t, $\ln \sigma^2_{\eta,t}, \ln \sigma^2_{\varepsilon,t}$ and γ. On the basis of this sample all Bayesian estimates are computed.

The contribution of the present paper to the algorithm is the following. I use Kalman filter to resample s_t, τ_t in steps 3 and 4, and add step 7 to simulate γ.

Bulat A. Gafarov

Higher School of Economics (Moscow, Russia). Laboratory for Inflation Problems and Economic Growth Research. Research Fellow.

E-mail: gafarovb@gmail.com

Phone: +1-814-777-8107