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FISCAL LIMITS AND MONETARY POLICY: DEFAULT VS. INFLATION[2]

In times of fiscal stress, governments fail to adjust fiscal policy in line with the requirements for debt sustainability. Under these circumstances, monetary policy impacts the probability of sovereign default alongside inflation dynamics. Uribe (2006) studies the relationship between inflation and sovereign defaults with a model in which the central bank controls a risky interest rate. He concludes that low inflation can only be maintained if the government sometimes defaults. This paper follows Uribe (2006) by examining monetary policy that controls a risky interest rate. However, it differs by the baseline assumption about the objectives of the central bank. In this paper, monetary policy is not pure inflation targeting: it is assumed that the central bank minimizes the probability of default under the upper restriction on inflation. An advantage of this framework is that it avoids the issue of zero risk premium, which exists in Uribe (2006), while at the same time allowing a study of the relationship between the constraints on monetary pol icy, the equilibrium default rate, and the risk premium. We show that monetary policy that controls the risky interest rate can mitigate default risks only when the upper limit on inflation is sufficiently high. The higher the agents believe the upper limit on inflation to be, the lower the equilibrium risk premium and probability of default are. Under a low default rate, constraints on inflation can only be fulfilled when fiscal shocks are either positive or small.

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1. Introduction

In the aftermath of the crisis of 2007-2008, some economies of the European Monetary Union (EMU) have found themselves in a complex situation. On the one hand, there is a pressing need to increase budget surpluses to mitigate default risks. Yet on the other hand, the scope of raising extra revenues through fiscal austerity is limited because such policy may lead to further recession and cause political crises. In the presence of fiscal stress, fiscal policy by itself may fail to insure the sustainability of government debt. In this environment, it is crucial to understand what the monetary policy controlling the cost of borrowing can do to mitigate the debt crisis.

Sovereign defaults are associated with devastating consequences for the financial system. Ensuring the stability of the financial system is one of the key functions of a central bank. When government debt is nominated in a national currency, the central bank is capable of resolving issues with debt sustainability by reducing the costs of servicing the debt. Uribe (2006) shows that in the presence of sovereign default risks, two fundamental functions of the central bank are in conflict, namely ensuring debt sustainability (stability of the financial system) and maintaining low inflation. In the literature studying default risks and monetary policy, authors often presuppose that one of the two aims of the central bank is dominant; the results concerning the dynamics of inflation and the risk premium are contingent on the underlying assumption about the central bank's priorities. Specifically, in Sargent and Wallace (1981), as well as in the papers on Fiscal Theory of Price Level (FTPL),[3] the authors presuppose that the primary goal of the central bank is to avoid sovereign defaults, regardless of the costs in terms of inflation. Rational agents are aware of the central bank's preferences and thus believe that the probability of default is zero. It follows that in those models there is no risk premium on government bonds.

By contrast, Uribe (2006) and Guillard and Kempf (2012) study the case when maintaining low inflation is a primary objective of the central bank – when monetary policy is conducted in a way that excludes deviations of inflation from the target. In these models, defaults emerge whenever debt becomes unsustainable under the target level of inflation.

In this paper, the baseline assumption is that although the central bank is eager to minimize the probability of default arising from fiscal stress, it is constrained by formal requirements concerning inflation – a maximum level of inflation that the central bank may allow to avoid sovereign default. This specification of the central bank's problem can be viewed as a compromise between baseline assumptions of FTPL models and models in which the central bank does not allow any deviations of inflation from the target, such as that of Uribe (2006), and Guillard and Kempf (2012). An advantage of this specification is that it avoids the issue of zero

[3] Leeper (1991), Woodford (1995, 1998), Cochrane (2001), and others.

risk premium that exists in Uribe (2006), while at the same time allowing for a study of the capabilities and limitations of monetary policy aimed at mitigating default risks.

In the model it is assumed that the central bank has two control variables: an interest rate on government bonds and a risk-free interest rate. When the central bank sets a risky rate, it affects the costs of debt servicing and the probability of default. By setting a risk-free rate, the central bank can pin down inflation, provided that the switch to risk-free interest rate control is permanent. We show that the switch to the Taylor rule risk-free interest rate control is not compatible with equilibrium on the financial market. We determine the threshold value of real debt that triggers sovereign default and show that this threshold is an increasing function of the upper limit on inflation.

We show that under this specification of monetary policy, the equilibrium risk premium and probability of default depend on the upper limit of inflation: The higher the limit, the lower the risk premium and the probability of default. When the upper limit on inflation is high enough, monetary policy that controls the risky interest rate can ensure a zero probability of default in equilibrium. Furthermore, if agents do not possess exact information concerning inflation constraint, the central bank has incentives to create inaccurate beliefs, suggesting the upper limit on inflation to be higher than the actual value in order to lower the risk premium on government bonds and reduce the probability of default.

Fiscal stress in the EMU

Our specification of the central bank's problem seems particularly relevant for the analysis of monetary policy within a monetary union. When the central bank of a monetary union implements accommodative policy intended to stabilize the debt of one of the member regions, the costs in terms of inflation are spread across all member regions. Fiscally prudent governments may be unwilling to share these costs and thus may have an incentive to collectively impose an upper limit on inflation, restricting the central bank's policy choices.[4] Alternatively, the central bank may determine the upper limit on inflation by comparing the costs associated with an increase in inflation with the costs arising from a sovereign default of one of the member states.[5] Finally, the upper limit on inflation may be treated as a formal commitment of the central bank.

[4] This outcome seems reasonable if fiscal policy differs across regions. For instance, if the probability of default is rather small in the majority of regions, costs associated with an increase in inflation for these regions exceed benefits from reduction of the probability of default resulting from an increase in the upper limit of inflation.

[5] Cooper, Kempf, and Peled (2010) show that in a monetary union the decision of the central bank on whether to bailout a member state depends on the allocation of risky bond holdings across regions. Since monetization leads to inflation growth, allocation of risky bonds might as well influence the maximum value of inflation that the central bank can tolerate to avoid defaults.

A study of monetary policy that controls the costs of borrowing appears to be urgent in light of the recently launched OMT program (Outright Monetary Transactions), a program presupposing that the European Central Bank would buy bonds of troubled governments to mitigate default risks, given that they implement fiscal austerity.

In the EMU, the ability of governments to flexibly adjust fiscal policy in line with the sustainability criteria is an issue. Trabandt and Uhlig (2009) show that over the past 20 years, European economies have drawn closer to the peaks of their respective Laffer curves: The scope of raising extra tax revenues via increases in tax rates is limited since further increases in the tax rate would cause only a minor gain in a government's earnings. Cochrane (2011a) asserts that even if an economy is supposed to operate well below the Laffer curve peak, a small rise in the tax rate may cause a prominent slowdown of economic growth, thereby reducing future taxable income. Bi, Leeper, and Leith (2012) show that expectations of increases in fiscal surpluses may have a different impact on output growth depending on the composition of fiscal consolidation. Particularly, expectations of an increase in the labor tax rate lead to a slowdown of output growth, whereas a decrease in government expenditures promotes it. Even if tax collection capacities are to be neglected, it is plausible that a government facing a debt sustainability constraint would rather default on its debt than perform fiscal contraction, even though such a move would facilitate debt service. Theoretical support for this view can be found in Eaton and Gersovitz (1981), who determine the "effective" tax rate – the highest rate it makes sense to impose before defaulting – which turns out to be lower than the rate corresponding to the Laffer curve peak.

Thus, austere tax policy has certain limitations. The scope of raising revenues through cutting transfers and government expenditures is limited as well. First, in a democratic environment it is difficult to implement such a policy without a substantial delay (see Alesina and Drazen, 1991). Second, due to adverse demographic trends on the one hand, and the government obligations to support future retirees with appropriate benefits on the other hand, expenditures related to aging are expected to rise substantially in the next 50 years. According to the IMF (2009), the net present value of these promised expenses is averaging 409% of GDP across advanced G20 countries, meaning that the transfers are not backed by tax revenues. These concerns show that fiscal stress is likely to remain a pressing issue in the long run.

Section 2 presents the model that lays out the design of fiscal policy and a household's problem. We determine conditions insuring that government debt can be sold to households and describe the central bank's problem. In Section 3 we define equilibrium, determine the conditions under which equilibrium exists, and express the default rate, the probability of default, and the risk premium as functions of the risky interest rate. In Section 4 we determine

conditions guaranteeing that the solution to the central bank's problem exists and characterize it, determining a risky interest rate. We explore equilibrium outcomes when households know the true value of the upper limit on inflation and when they do not know it, so the central bank can form beliefs about its value. Section 5 concludes the paper, while the Appendix presents a numerical example for the Greece's economy.

2. The model

2.1 Government

Consider an endowment economy, where the government collects lump sum taxes, pays transfers, and issues one-period bonds. The economy is subject to fiscal stress: Fiscal surpluses evolve exogenously and do not respond to changes in the real value of government debt, and as a result the government fails to insure debt sustainability when the inflation rate is particularly low. Using the terminology of Leeper (1991), fiscal policy is "active". We follow Uribe (2006) by assuming that fiscal surpluses (taxes minus transfers) follow an AR(1) process:

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \varepsilon_t, \tag{1}$$

where $\varepsilon_t \sim F(0,\sigma^2)$, $\varepsilon_t \in \left[-\varepsilon^{\max}, \varepsilon^{\max}\right]$, $\bar{s}$ is a steady state value of fiscal surplus. Government debt is risky: In period t the government defaults on a δ_t fraction of its debt. The dynamic budget constraint in period t is given by:

$$\frac{B_t}{P_t} = \frac{R_{t-1}B_{t-1}(1-\delta_t)}{P_t} - s_t \quad , \tag{2}$$

where B_t is the nominal debt in period t, P_t is the price level, and R_{t-1} is the gross nominal interest rate. Following Bi (2012), Bi and Traum (2012), and Guillard and Kempf (2012), we assume that default occurs when the real value of debt exceeds an upper limit, $\hat{b}_t$, in which case the default rate equals δ. Thus, the default rule is given by:

$$\delta_t = \begin{cases} \delta & if \quad b_t > \hat{b}_t \\ 0 & if \quad b_t \le \hat{b}_t \end{cases}. \tag{3}$$

We derive δ in section 3.1.

2.2 The household's problem

A representative household consumes c_t and purchases contingent claims, D_t. The value of contingent claims purchased in period t is given by $E_t r_{t+1} D_{t+1}$, where E_t represents rational expectations based on the full information set of period *t*, and r_{t+1} is a stochastic discount factor on contingent claims purchased in period *t*. Uncertainty arises due to fiscal stress: Because there is a possibility that the government might default on its debt, the value of assets in period *t+1* is unknown in period *t*. A household also receives endowment y_t and pays the government s_t lump-sum taxes minus transfers. Households may borrow from each other – we assume that private debt contracts are enforceable and private debt is risk-free.[6] Let R_t^f be the gross nominal risk-free interest rate.

Household maximizes utility from consumption over an infinite horizon, solving:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \to \max_{c_t} \tag{4}$$

subject to:

$$\textit{dynamic budget constraint: } c_t + \frac{E_t r_{t+1} D_{t+1}}{P_t} + s_t = y_t + \frac{D_t}{P_t}, \tag{5}$$

$$\textit{transversality condition: } \lim_{j \to \infty} E_t q_{t+j} D_{t+j} \geq 0, \tag{6}$$

where $q_{t+j} = r_t r_{t+1} \ldots r_{t+j}$.

The first-order condition for this problem is:

$$u'(c_t) = \beta \frac{1}{r_t} E_t \frac{P_t}{P_{t+1}} u'(c_{t+1}). \tag{7}$$

In the subsequent section we use equation (7) to derive Euler equations for the risky and risk-free interest rates.

[6] This assumption is necessary to characterize the behavior of the risk-free interest rate. It was also applied in Uribe (2006), Guillard and Kempf (2012), and others.

2.3 Sustainability of government debt and the relationship between inflation and the default rate

For simplicity, assume that endowment y_t is constant. As there is no production sector, the resource constraint is given by: $c_t = \bar{y}$.

In equilibrium, a no-arbitrage condition must be satisfied: Risky and risk-free assets must be equally attractive for consumers. Since households are identical, in equilibrium private borrowing must be equal to zero. The financial market equilibrium is given by:

$$B_t = D_t. \tag{8}$$

We derive equilibrium conditions for the risky and risk-free interest rates from first-order condition (7):

$$1 = \beta R_t E_t (1-\delta_{t+1}) \frac{P_t}{P_{t+1}}, \tag{9}$$

$$1 = \beta R_t^f E_t \frac{P_t}{P_{t+1}} \quad . \tag{10}$$

Analogously to transversality condition (6), there is a no-Ponzi game condition for the government:

$$\lim_{j\to\infty} \beta^j E_t \left[b_{t+j} \right] \leq 0, \tag{11}$$

where $b_{t+j} = B_{t+j}/P_{t+j}$ is the real value of government debt. When condition (11) is violated, the discounted infinite sum of expected government expenditures plus debt exceeds the discounted sum of expected revenues. Under these conditions rational households would not buy newly issued government bonds.

We iterate the dynamic budget constraint of the government (3) and apply Euler equation (9), obtaining:

$$\lim_{j\to\infty} E_t \left[\beta^j \frac{B_{t+j}}{P_{t+j}} \right] = R_{t-1} \frac{B_{t-1}}{P_t} (1-\delta_t) - E_t \sum_{h=0}^{\infty} \beta^h (s_{t+h}) \quad . \tag{12}$$

Combining (11) and (12), we derive a condition guaranteeing that the government does not engage in Ponzi schemes by choosing default rate δ_t:

$$b_{t-1}(1-\delta_t) \le \frac{\sum_{h=0}^{\infty} E_t \beta^h (s_{t+h})}{R_{t-1}} \pi_t \equiv \frac{(1-\beta)s_t + \beta(1-\rho)\bar{s}}{(1-\beta)(1-\rho\beta)R_{t-1}} \pi_t , \tag{13}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$.

When condition (13) is violated for a set of $\{s_t; b_{t-1}; \delta_t; \pi_t; R_{t-1}\}$, the debt b_t that finances the operational deficit in period *t* cannot be sold to households.[7]

Substituting $\delta_t = 0$ into (13), we obtain a condition that guarantees that the government debt is sustainable:

$$b_{t-1} \le \frac{\sum_{h=0}^{\infty} E_t \beta^h (s_{t+h})}{R_{t-1}} \pi_t = \frac{(1-\beta)s_t + \beta(1-\rho)\bar{s}}{(1-\beta)(1-\rho\beta)R_{t-1}} \pi_t . \tag{14}$$

As households are rational, the discounted sum of private income must not exceed the discounted sum of private consumption. Thus, in equilibrium, transversality condition (6) must hold as an equality. Using equilibrium conditions (8) and (9), we obtain:

$$\lim_{j\to\infty} \beta^j E_t \left[b_{t+j} \right] = 0 . \tag{15}$$

Equation (15) guarantees that, on the one hand, the dynamics of b_t satisfies the transversality condition and, on the other hand, the consumption choices of households are rational.

When equation (15) holds, the no-Ponzi game condition (11) holds as equality. Analogously to the derivation of condition (14), using (15) we obtain an equilibrium relationship between inflation and default rate:

$$\delta_t = 1 - \frac{\sum_{h=0}^{\infty} E_t \beta^h (s_{t+h})}{R_{t-1} b_{t-1}} \pi_t = 1 - \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t (1-\beta)}{(1-\beta)(1-\rho\beta)R_{t-1} b_{t-1}} \pi_t . \tag{16}$$

Qualitative interpretations of equation (16) may vary, depending on how monetary policy is conducted. For example, in Uribe (2006) the central bank sets the inflation rate at $\pi_t = \pi^*$ for all *t* and determines the risky interest rate, R_{t-1}. Since s_t is random, from equation (16) it follows that under such a policy rule, shocks to s_t result in non-zero default rates.

[7] See the formal proof in sections 3.1.

Such specification of a monetary policy has a disadvantage: In equilibrium, the default rate becomes negative whenever the value of fiscal shock exceeds zero. To see this, substitute the expression for $\delta_t = 0$ from equation (16) into the Euler equation for the risky interest rate (9):

$$\frac{\rho(1-\beta)s_t + (1-\rho)\bar{s}}{b_t(1-\rho\beta)(1-\beta)} = \frac{1}{\beta}. \tag{17}$$

Equation (17) states that the bigger fiscal surpluses are, the higher the real value of debt can be sustained in equilibrium. Applying (17) to (16) and assuming that $R_{t-1} = R^* = \pi^*/\beta$, $\pi_t = \pi^*$ as in Uribe (2006), we obtain the equilibrium default rate:

$$\delta_t = -\frac{\beta\varepsilon_t}{(1-\beta)(1-\rho\beta)} b_{t-1}. \tag{18}$$

Thus, when $\varepsilon_t > 0$, the equilibrium default rate is negative. Moreover, it follows from (10) that the implied risk premium is always zero since $R^f_{t-1} = \pi^*/\beta = R_{t-1}$.

Following Uribe (2006), this paper also focuses on monetary policy that controls the risky interest rate. However, unlike in Uribe (2006), our specification of monetary policy implies uncertainty over future inflation. Section 4 shows that the equilibrium risk premium and the probability of default are affected by an agent's beliefs about the upper limit on inflation: In equilibrium, the higher the upper limit on inflation, the lower the risk premium.

Now assume that the default rate is fixed. By substituting a fixed default rate into (16), we uniquely determine inflation in period *t*, because the discounted expected sum of surpluses is exogenous. This happens because under a fixed default rate and exogenous fiscal surpluses the transversality condition from the household's problem (11) holds as equality for only one value of π_t (and one value of P_t, since P_{t-1} is known in period *t*), and this particular level of π_t is realized as an equilibrium. This result is in line with FTPL, with the only difference being that in FTPL the value of δ_t is assumed to be zero. By substituting $\delta_t = 0$ we obtain the inflation rate and the price level, corresponding to the FTPL case:

$$\pi_t = \frac{R_{t-1}b_{t-1}}{\sum_{h=0}^{\infty}\beta^h E_t(s_{t+h})} \equiv \frac{R_{t-1}b_{t-1}(1-\beta)(1-\rho\beta)}{s_{t-1}(1-\beta)\rho + \bar{s}\,(1-\rho) + \varepsilon_t(1-\beta)}, \tag{19}$$

$$P_t = \frac{R_{t-1}B_{t-1}}{\sum_{h=0}^{\infty}\beta^h E_t(s_{t+h})} \equiv \frac{R_{t-1}B_{t-1}(1-\beta)(1-\rho\beta)}{s_{t-1}(1-\beta)\rho + \bar{s}\,(1-\rho) + \varepsilon_t(1-\beta)}. \tag{20}$$

The FTPL attributes this positive relation between inflation (or price level) and fiscal surpluses to the wealth effect. Suppose that in period *t* taxes are unexpectedly reduced (transfers are increased) and the reduction in taxes (increase in transfers) today are not associated with an expected increase in taxes (reduction in transfers) in the future. Thus, if the price level in period *t* does not change, then households in period *t* become wealthier because the expected discounted sum of net taxes falls. Since households are rational, an increase in wealth leads to an increase in aggregate demand and the price level (see Leeper, 1991; Woodford, 1995 and 1998; and Cochrane, 2001; among others). Similarly, when the government issues new bonds, household wealth rises because new bonds are not backed by corresponding increases in government surpluses. Note that when the default rate differs from zero the mechanism we have just outlined remains in place – the difference is quantitative, but not qualitative.

2.4 The central bank

We showed that in times of fiscal stress there is a negative relation between inflation and the default rate with equation (16). Thus, when the central bank allows increases in inflation, it mitigates default risks. This trade-off between low inflation and fiscal sustainability causes a controversy between two fundamental goals of the central bank: Suppressing inflation hikes and insuring stability of the financial market.

In this paper, we assume that, although the central bank seeks to minimize default risks arising from fiscal stress, it has to fulfill a formal requirement with respect to an upper limit on inflation. This assumption can be viewed as a compromise between FTPL (Leeper, 1991; Woodford, 1995 and 1998; and Cochrane, 2001; among others) and models where the central bank sets a risk-free interest rate and does not allow any deviations of inflation from the target, such as Guillard and Kempf (2012). Formally, the central bank minimizes the expected default rate under an exogenous restriction on inflation, which must not exceed the upper limit, $\pi^{\max}$. The central bank has two alternative policy variables – the interest rate on government bonds and the risk-free interest rate – and in period *t* can choose one of these two instruments.[8] When the

[8] When the central bank attempts to control the two instruments simultaneously, the actual risk premium does not coincide with the premium demanded by the financial market and there is no equilibrium with a non-negative demand for both assets.

central bank sets the risky interest rate R_{t-1} in period *t-1*, it influences the debt service: The lower the risky interest rate, R_{t-1} is, the cheaper it is to finance government debt in period *t*.

When the central bank sets a risk free interest rate, it may influence inflation dynamics. In particular, if in period *t* the central bank commits to setting the risk-free interest rate according to the Taylor rule in all subsequent periods, it pins down inflation in period *t* and in all subsequent periods. Formally, inflation in period t^A and in all subsequent periods will equal π^* if the risk-free interest rate for all $t \geq t^A$ is set according to:

$$R_t^f = R^* + \alpha(\pi_t - \pi^*), \qquad (21)$$

where $\alpha > 1/\beta$. Under this rule, inflation will always be equal to π^*, if equilibria with hyperinflation or a liquidity trap are excluded.[9] Thus, in period *t* the central bank chooses between switching to inflation targeting and maintaining control of the risky interest rate. To simplify this analysis, we assume that this decision is perceived as a binding commitment. If the switch to inflation targeting does not occur, then the central bank will choose the target value for the risky interest rate, R_t. Let us define the central bank's decision on whether to switch to inflation targeting in period *t* by $IT_t = \{0,1\}$, where $IT_{t^A} = 1$ means that in all $t \geq t^A$ the risk free interest rate will be set according to (21) and $IT_{t^A} = 0$ means that in period t^A the central bank decides to control the risky interest rate.

We make the following assumptions concerning the timing of events. At the beginning of period *t*, agents learn the realization of fiscal shock ε_t and s_t. To finance operational deficit, the government issues new bonds and sells them to households or defaults. Then the central bank sets the interest rate, R_t, or switches to inflation targeting and sets the risk-free interest rate, R_t^f. Thus, the solution to the central bank's problem is a function of fiscal shock s_t.

While it is rather common to assume that the central bank conducts monetary policy by setting the risk-free rate, it may seem somewhat unconventional to presuppose that the central bank may alternatively choose to govern the risky rate. The mechanism underlying the policy controlling the risky rate can be interpreted as follows: The central bank can alter the risky interest rate by trading government bonds, and the central bank may enter the market and change the risky rate at the end of period *t*, after the debt has been sold to households. At the beginning of period *t*, when government initially sells newly issued bonds to households, the latter already

[9] See Cochrane (2011).

know the target value for the risky interest rate because they know s_t. Suppose now that at the beginning of period *t*, when the initial offer of bonds is made, the interest rate on those bonds deviates from the target value. This implies that at the end of period *t* the central bank will enter the market and alter the interest rate with certainty. Under these circumstances a rational household can benefit by opening a short or long position, suggesting that there is an arbitrage opportunity. The existence of an arbitrage opportunity implies that the initial allocation of assets is not an equilibrium. Thus, in equilibrium, the interest rate on government bonds that corresponds to the initial placement of bonds must match the target value.

These assumptions about the timing of events allows us to skip explicit modeling of the central bank's balance sheet, because in equilibrium the central bank does not enter the financial market: An equilibrium is formed as a result of adjustment of demand for bonds, depending on a household's beliefs concerning the target level of the risky interest rate.[10] When the central bank controls the risk-free interest rate in accordance with (21), deviations of the risk-free rate from the target are not compatible with equilibrium either (see Section 3.1). Thus, there is no need to explicitly introduce the central bank's balance sheet.[11]

3. Equilibrium

We now turn to the definition of a competitive equilibrium for this economy.

Definition 1. A competitive equilibrium is a set of sequences $\left\{c_t, s_t, \delta_t, \pi_t, b_t \geq 0, R_t \geq 1, R_t^f \geq 1, \hat{b}_t, IT_t, P_t, B_t\right\}_{t=0}^{\infty}$, *if:*

1. *Sequences satisfy:*

   - *Equilibrium condition (16):*

$$\delta_t = 1 - \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{(1-\beta)(1-\rho\beta)R_{t-1}b_{t-1}}\pi_t;$$

   - *Default rule (3):*

[10] Uribe (2006) also studies monetary policy that controls the risky interest rate without explicitly introducing the central bank's balance sheet.

[11] Under rule (21), all deviations of inflation from the target result in either hyperinflation or a liquidity trap. By assumption, hyperinflation and the liquidity trap are not equilibrium outcomes. Discussion on whether this assumption is relevant can be found in Woodford (2003) and Cochrane (2011).

$$\delta_t = \begin{cases} \delta & if \quad b_t > \hat{b}_t \\ 0 & if \quad b_t \le \hat{b}_t \end{cases};$$

- *Resource constraint:*

  $c_t = \bar{y}$;

- *Euler equation for the risk-free interest rate (10):*

  $$1 = \beta R_t^f E_t \frac{1}{\pi_{t+1}};$$

- *Government budget constraint (2):*

  $$b_t = \frac{R_{t-1} b_{t-1}(1-\delta_t)}{\pi_t} - s_t;$$

2. $\tilde{R}_t(s_t)$ *and* $\tilde{IT}(s_t, \delta_t)$ *solve the central bank's problem, and:*

   $R_t^f = R^* + \alpha(\pi_t - \pi^*)$ *for all* $t + j$, $j \ge 0$, *if* $IT_t = 1$;

   $R_t = \tilde{R}_t(s_t)$ *If* $IT_t = 0$;

3. *Fiscal surpluses follow (1):*

   $s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \varepsilon_t$;

   for given B_0, P_0.

In Section 3.1 we express the equilibrium value of $\hat{b}_t$ as a function of $\left\{b_{t-1}, s_t, R_{t-1}, \pi^{\max}\right\}$. We show that $IT_t(s_t, \bullet) = 1$ is not an equilibrium solution and find all $\left\{b_{t-1}, s_t, R_{t-1}, \pi^{\max}\right\}$, such that there exists an equilibrium with $IT_t(s_t, \bullet) = 0$. Assuming further that an equilibrium exists, we express δ_t as a function of $\left\{b_{t-1}, s_t, R_{t-1}, \pi^{\max}\right\}$. In section 3.2 we use the equilibrium definition of $\hat{b}_t$ to derive the risk premium in period *t-1* and the probability of default in period *t*

and express them as functions of $\{b_{t-1}, s_{t-1}, R_{t-1}, \pi^{\max}\}$. In Section 4 we solve the central bank's optimization problem and study the relation between $\tilde{R}_{t-1}$ and $\{b_{t-1}, s_{t-1}, \pi^{\max}\}$.

Before proceeding, we should verify that all variables from Definition 1 can be determined uniquely and expressed as functions of $\{b_{t-1}, s_{t-1}, R_{t-1}, \pi^{\max}\}$, provided that for a given set of $\{b_{t-1}, s_{t-1}, R_{t-1}, \pi^{\max}\}$ an equilibrium exists. Knowing s_{t-1}, we determine s_t from equation (1). If for $\{b_{t-1}, s_t, R_{t-1}, \pi^{\max}\}$ an equilibrium exists, then $IT_t(s_t, \bullet) = 0$, thereby $\hat{b}_t$ and δ_t can be uniquely determined (see Section 3.1). Knowing δ_t, s_t, b_{t-1}, we determine π_t from (16). For a given δ_t, s_t, π_t, b_{t-1}, we determine b_t from the government budget constraint (2). The solution to the central bank's problem can be expressed as a function of b_t, s_t: Knowing b_t, s_t, we determine the risky rate for newly issued bonds R_t (see Section 4). For a given R_t, s_t, b_t the risk premium and R_t^f can be uniquely determined (see Section 3.2). Knowing P_{t-1}, π_t, b_t we derive B_t, P_t. Therefore, when an equilibrium exists, all variables from Definition 1 are uniquely determined.

3.1 The equilibrium default rate

In this subsection we show that when control over the risky rate fails to insure that $\pi_t \le \pi^{\max}$ is fulfilled, the central bank has no other option but to abandon controlling the risky rate and switch to setting a risk-free interest rate to suppress inflation (see Proposition 1 below). Rational agents are aware of the central bank's objectives; when government debt is only sustainable under $\pi_t > \pi^{\max}$, newly issued bonds cannot be sold to private agents, who know that the central bank will not let inflation exceed $\pi^{\max}$ (see Proposition 2 below). It follows that if a given default rate can only be sustained when inflation exceeds $\pi^{\max}$, this default rate is not compatible with equilibrium (see Proposition 3 below). Based on these results, we then derive the threshold value of debt $\hat{b}_t$, such that whenever the real value of debt exceeds $\hat{b}_t$, default occurs. We also determine the conditions ensuring that an equilibrium exists.

Assumption 1. Hyperinflation and the liquidity trap are not possible in equilibrium.

To prove Proposition 1, we assume that Assumption 1 holds. From equation (16) it follows that under $IT_{t-1} = 0$ inflation in period *t* satisfies $\pi_t \le \pi^{\max}$ if:

$$b_{t-1} \le \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{(1-\beta)(1-\rho\beta)R_{t-1}(1-\delta_t)}\pi^{\max} \quad . \tag{22}$$

For other values of b_{t-1} the inflation constraint is violated. Suppose that the default rate in period *t* equals $\bar{\delta}_t$.

Proposition 1. *If for a given* b_{t-1}, R_{t-1}, ε_t, s_{t-1} *and* $\bar{\delta}_t$ *condition (22) is violated, then* $IT(s_t,0)=1$ *is a solution to the central bank's problem (the central bank switches to inflation targeting and sets the risk-free interest rate in line with (21) where* $\pi^* \le \pi^{\max}$

Proof. In this setup, if the central bank decides to refrain from switching to inflation targeting, $IT_t(s_t,0)=0$, then inflation exceeds $\pi^{\max}$. On the other hand, under $IT_t(s_t,0)=1$, inflation remains within $\pi_t \le \pi^* \le \pi^{\max}$, which can be shown by combining rule (21) with the Euler equation for the risk-free interest rate (9). Linearizing in the neighborhood of $\pi=\pi^*$ for small values of inflation, we obtain:

$$E_t\pi_{t+1} - \pi^* = \alpha\beta(\pi_t - \pi^*) \, . \tag{23}$$

The relationship between current and expected future inflation is depicted in Figure 1. When $\alpha\beta > 1$, the $\pi=\pi^*$ steady state is unstable. Thus, whenever inflation diverges from the target, rational agents expect either hyperinflation or a liquidity trap to emerge in the long run (on Figure 1, π^l corresponds to a zero interest rate). By assumption, expectations of hyperinflation or the liquidity trap are not possible in equilibrium. Thus, with $\alpha\beta > 1$ the only available equilibrium solution is $\pi_t = \pi^*$, since all other values of π_t lead to non-equilibrium expectations. Therefore, when the central bank switches to the Taylor rule inflation targeting and sets the risk-free interest rate according to (21), inflation equals $\pi_t = \pi^*$.[12] Thus, if for a given value of $\bar{\delta}_t$ condition (22) is violated in period *t*, the central bank switches to controlling the risk-free interest rate in period *t*, because otherwise inflation would exceed $\pi^{\max}$. □

[12] To prove Propositions 1-3, it is sufficient to assume that hyperinflation is not an equilibrium, in which case the equilibrium inflation under the risk-free interest rate control satisfies $\pi_t \le \pi^* \le \pi^{\max}$. If the liquidity trap is not excluded from the set of plausible equilibrium outcomes, whereas hyperinflation is, then equilibrium inflation lies within the interval $\left[\pi^l, \pi^*\right]$, and under $\pi^* \le \pi^{\max}$ Proposition 1 still holds.

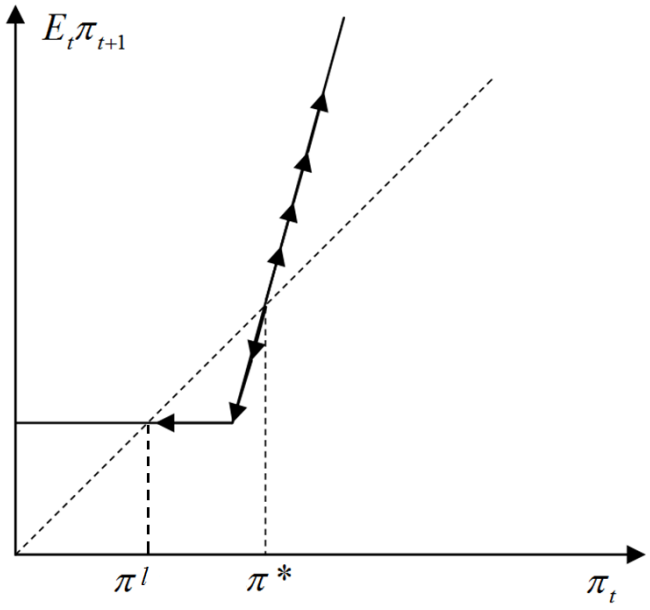


Figure 1. Inflation dynamics under the Taylor rule

Therefore, whenever under $\delta_t = \bar{\delta}_t$ the debt issued in period *t-1* exceeds the upper bound given by (22), the central bank is compelled to abandon control of the risky rate and switch to inflation targeting to fix inflation at π^* — otherwise the constraint on inflation will not be fulfilled.

Having determined the response of the central bank to violation of (22), we proceed by studying whether a violation of (22) can in fact occur in equilibrium, provided that households know the central bank's objective function. For this purpose, we check whether the default rate $\bar{\delta}_t$ is compatible with equilibrium, if condition (22) is violated for $\left\{b_{t-1}; s_t; R_{t-1}; \bar{\delta}_t\right\}$. We first show that if under $\delta_t = \bar{\delta}_t$ debt b_{t-1} is unsustainable, then $\delta_t = \bar{\delta}_t$ is not compatible with equilibrium.

Proposition 2. *If for a given* b_{t-1}, R_{t-1}, ε_t, s_{t-1} *and* $\bar{\delta}_t$ *condition (13) is violated, then the debt in period t cannot be sold to households.*

Proof. Suppose condition (13) is violated:

$$b_{t-1} > \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{(1-\beta)(1-\rho\beta)R_{t-1}(1-\bar{\delta}_t)}\pi_t .$$

From the government budget constrain (2) we obtain the real value of bonds that the government must issue in period *t* to finance the operational deficit:

$$b_t > \frac{\beta\left[\rho(1-\beta)s_t + (1-\rho)\bar{s}\right]}{(1-\rho\beta)(1-\beta)} .$$

From the first order condition (9), households will be eager to purchase these bonds, when their expected yield satisfies:

$$\frac{1}{\beta} \le R_t E_t\left(\frac{1-\delta_{t+1}}{\pi_{t+1}}\right).$$

Applying equilibrium condition (16), we conclude that for the bonds to be sold to households, the following must hold:

$$b_t \le \frac{\beta\left[\rho(1-\beta)s_t + (1-\rho)\bar{s}\right]}{(1-\rho\beta)(1-\beta)}\pi_t .$$

We reach a contradiction. □

We now show that if for $\delta_t = \bar{\delta}_t$ the inflation-insuring sustainability of post-default debt in period *t* exceeds $\pi^{\max}$, $\delta_t = \bar{\delta}_t$ is not an equilibrium solution for the default rate.

Proposition 3. *If for a given* b_{t-1}, R_{t-1}, ε_t, s_{t-1} *and* $\bar{\delta}_t$ *condition (22) is violated,* $\delta_t = \bar{\delta}_t$ *is not an equilibrium solution.*

Proof. Suppose that under given conditions δ_t equals $\bar{\delta}_t$ in equilibrium. From Proposition 1 it then follows that in period *t* the central bank is forced to fix inflation at $\pi^* \le \pi^{\max}$. In this case, under $\pi_t = \pi^*$ condition (13) is violated for given b_{t-1}, R_{t-1}, ε_t, s_{t-1} and $\bar{\delta}_t$. Then, from Proposition 2 it follows that the government cannot finance the operational deficit in period *t* through issuing new bonds because private agents are not interested in buying them. Hence, an equilibrium with $\delta_t = \bar{\delta}_t$ does not exist. □

Following Guillard and Kempf (2012), Bi and Traum (2012), and Bi (2012), we assume that when default emerges, the default rate equals a known constant value $0 < \delta \le 1$.[13] Note that if condition (22) is violated for $\bar{\delta}_t = \delta$, it is violated for $\bar{\delta}_t = 0$ as well. If this is the case, then from Proposition 3 it follows that there is no equilibrium either under $\bar{\delta}_t = 0$ or under $\bar{\delta}_t = \delta$. We can conclude that:

Corollary 1. *If for a given* b_{t-1}, R_{t-1}, ε_t, s_{t-1} *and* $\bar{\delta}_t = \delta$ *condition (22) is violated, then equilibrium does not exist.*

[13] In Arellano (2008) the default rate is also constant and equals 1. In Bi (2012) the value of the default rate depends on the properties of distribution for the fiscal limit.

Now suppose that, although condition (22) is violated under $\bar{\delta}_t = 0$, it is satisfied under $\delta_t = \delta$. Then, from Proposition 3 it follows that equilibrium only exists with default.

Corollary 2. *If for a given* b_{t-1}, R_{t-1}, ε_t, s_{t-1} *condition (22) is violated under* $\bar{\delta}_t = 0$ *and satisfied under* $\bar{\delta}_t = \delta$, *the equilibrium default rate equals* δ.

Now we can identify the threshold value for real debt from rule (3) by substituting $\bar{\delta}_t = 0$ into condition (22):

$$\hat{b}_t = \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{(1-\beta)(1-\rho\beta)R_{t-1}}\pi^{\max}. \qquad (24)$$

Debt that exceeds threshold $\hat{b}_t$ is unsustainable when inflation is less than $\pi^{\max}$. Whenever the real value of debt exceeds $\hat{b}_t$, a default occurs.

Finally, suppose that condition (22) is fulfilled under $\bar{\delta}_t = 0$ (and, consequently, under $\bar{\delta}_t = \delta$). In this case, inflation will remain below $\pi^{\max}$ regardless of $\bar{\delta}_t$, meaning that the central bank has no incentive to switch to inflation targeting and the central bank's response to both $\bar{\delta}_t = \delta$ and $\bar{\delta}_t = 0$ would be $IT_t(s_t) = 0$. In this case, debt b_{t-1} is sustainable and new bonds issued in period *t* to finance the operational deficit can be sold to households.

Corollary 3. *If for a given* b_{t-1}, R_{t-1}, ε_t, s_{t-1} *and* $\bar{\delta}_t = 0$ *condition (22) is satisfied, then the equilibrium default rate equals zero.*

3.2 The probability of default and the risk premium

In this section we study the relationship between the risky interest rate, the probability of default, and the risk premium. From equation (24), we obtain a threshold value of fiscal shock:

$$\hat{\varepsilon}_t = \left(\frac{R_{t-1}\beta}{\pi^{\max}} - 1\right)b_{t-1}\frac{1-\rho\beta}{\beta}. \qquad (25)$$

Default in period *t* emerges whenever the realization of fiscal shock turns out to be smaller than $\hat{\varepsilon}_t$. Note that the value of $\hat{\varepsilon}_t$ is known in period *t-1* and that it goes up as $R_{t-1}/\pi^{\max}$ (the gross real interest rate on government bonds in case inflation reaches $\pi^{\max}$) increases. It follows that the central bank can manipulate the threshold value, provided that it can alter the risky interest rate or expectations over the upper limit on inflation. When the risky interest rate

surpasses a certain level, namely $R_{t-1} > \beta\pi^{\max}$, even positive shocks to fiscal surpluses can trigger defaults.

In the succeeding analysis, we focus on cases in which fiscal disturbances are always relatively small in order to highlight that even when households believe that the range of fiscal shocks is narrow, they still demand a positive risk premium on government bonds, limiting the central bank's choices. Another motivation for this strategy is that when large negative shocks occur, an equilibrium cannot form for valid values of price level and the default rate. Suppose in period *t* a large fiscal shock occurs such that $\varepsilon_t < -\left[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)\right]$. Then, from equation (16) it follows that in period *t* a household's demand will be non-negative only under a negative price level or a default rate that exceeds unity. Thus, from now on we presuppose that $\varepsilon_t \in \left[-\varepsilon^{\max}; \varepsilon^{\max}\right]$ and that $\varepsilon^{\max} \le \left[\rho s_{t-1} + (1-\rho)\bar{s}/(1-\beta)\right]$ for all s_{t-1}. In the Appendix we provide quantitative estimates for Greece's economy, showing that this assumption is realistic.

Now we can write down an estimate for the probability of default in period *t*, calculated in period *t-1*:

$$\Pr(\varepsilon_t \le \hat{\varepsilon}_t) = \int_{-\varepsilon^{\max}}^{\hat{\varepsilon}_t} f(\varepsilon) d\varepsilon, \tag{26}$$

where $f(\varepsilon)$ is the density of the distribution of shock to fiscal surpluses. From equation (26) it follows that the probability of default depends on the relation between the risky interest rate and the upper limit on inflation, $R_{t-1}/\pi^{\max}$: The bigger the value of the gross real interest rate under maximum inflation, the higher the probability of default.

Now we can derive the risk premium on government bonds:

$$\begin{aligned}\frac{R_{t-1}}{R_{t-1}^f} - 1 = R_{t-1}\beta\Bigg[&\int_{-\varepsilon^{\max}}^{\hat{\varepsilon}_t} \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}b_{t-1}(1-\beta)(1-\rho\beta)(1-\delta)} dF(\varepsilon) + \\ &+ \int_{\hat{\varepsilon}_t}^{\varepsilon^{\max}} \frac{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) + \varepsilon_t(1-\beta)}{R_{t-1}b_{t-1}(1-\beta)(1-\rho\beta)} dF(\varepsilon)\Bigg] - 1 = \\ &= \frac{1}{1-\delta} - 1 + \left(1 - \frac{1}{1-\delta}\right)\int_{\hat{\varepsilon}_t}^{\varepsilon^{\max}} \left(1 + \frac{\varepsilon_t\beta}{b_{t-1}(1-\rho\beta)}\right) dF(\varepsilon).\end{aligned} \tag{27}$$

The value of the integral on the right-hand side of the last equation is positive. When $\hat{\varepsilon}_t = -\varepsilon^{\max}$, the probability of default equals zero and agents do not demand a risk premium. When $-\varepsilon^{\max} < \hat{\varepsilon}_t \le \varepsilon^{\max}$, the value of the integral is smaller than unity and the risk premium is

positive. An increase in $R_{t-1}/\pi^{\max}$ would lead to an increase in the threshold value of fiscal shock $\hat{\varepsilon}_t$, as well as the probability of default and the risk premium.

A qualitative interpretation is as follows. Default emerges when the gap between the real value of debt and the sum of fiscal surpluses could only be eliminated through inflation that exceeds the upper limit. Rational households are aware of this regularity. When the upper limit is high enough, the probability of default is relatively low. The higher the risky interest rate, the higher the costs of debt service, the bigger the expected gap between the real value of debt and its backing, and the higher the probability of default.

4. The central bank's problem: the choice of a risky interest rate

We have characterized the relationship between the probability of default and the costs of debt service, $R_{t-1}b_{t-1}$. Since the probability of default depends on R_{t-1}, the risk premium is determined uniquely for a given value of the risky interest rate. We have established that a switch to inflation targeting is not an equilibrium solution. Thus, in equilibrium, the central bank sets the risky interest rate R_{t-1}, whereas the risk-free rate R^f_{t-1} adjusts in accordance with equation (27), which specifies the risk premium.

In this section we study the choice of the central bank over the risky interest rate and examine the relation between the upper limit on inflation $\pi^{\max}$, and the probability of default in equilibrium.

Choosing R_{t-1} in period *t-1*, the central bank minimizes the expected value of the default rate in period t $E_{t-1}\delta_t$. Considering that in the event of default the default rate equals δ, this problem can be reduced to minimization of the probability of default in period *t*:

$$\Pr(\varepsilon_t \le \hat{\varepsilon}_t) = \int_{-\varepsilon^{\max}}^{\hat{\varepsilon}_t} f(\varepsilon)d\varepsilon \to \min_{R_{t-1}} \tag{28}$$

$$s.t.: \quad R_{t-1} \ge 1; \tag{29}$$

$$R_{t-1} \ge \frac{1}{1-\delta} + \left(1 - \frac{1}{1-\delta}\right)\int_{\hat{\varepsilon}_t}^{\varepsilon^{\max}} \left(1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)}\right)dF(\varepsilon) \equiv \psi(R_{t-1}), \tag{30}$$

where:

$$\delta_t = \begin{cases} \delta & \text{если} \quad \varepsilon_t \le \hat{\varepsilon}_t \\ 0 & \text{если} \quad \varepsilon_t > \hat{\varepsilon}_t \end{cases},$$

$$\hat{\varepsilon}_t = (\frac{R_{t-1}\beta}{\pi^{\max}} - 1) b_{t-1} \frac{1-\rho\beta}{\beta}.$$

When the central bank chooses the value of the risky interest rate, it takes into account the zero lower bound on the risk-free interest rate given by (30) – a condition obtained by substituting the risk premium from (27) into $R_{t-1}^{f} \geq 1$. Condition (30) ensures that for a given $\{R_{t-1}, s_{t-1}, b_{t-1}\}$ debt b_{t-1} can be sold to households in period *t-1*.

In the following sections we study for which values of $\pi^{\max}$ and default rate δ the central bank's problem has solutions. We then examine the equilibrium relationship between the probability of default, the default rate δ, and the upper limit on inflation $\pi^{\max}$.

4.1 The existence of a solution

Let $\tilde{R}_{t-1}$ be the solution to the central bank's problem. As shown in section 3.1, for certain sets of $\{\varepsilon_t, \pi^{\max}, \tilde{R}_{t-1}\}$ there is no equilibrium with $\pi_t \leq \pi^{\max}$.

Suppose in period *t* fiscal shock equals the largest negative value, $\varepsilon_t = -\varepsilon^{\max}$. From (16) it follows that the equilibrium inflation would satisfy $\pi_t \leq \pi^{\max}$ if:

$$\tilde{R}_{t-1} \leq \frac{\pi^{\max}\left[s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) - \varepsilon^{\max}(1-\beta)\right]}{(1-\delta)b_{t-1}(1-\beta)(1-\rho\beta)}. \qquad (31)$$

Thus, a solution to the central bank's problem satisfying $\pi_t \leq \pi^{\max}$ will exist for any $\varepsilon_t \in \left[-\varepsilon^{\max}; \varepsilon^{\max}\right]$, if the risky interest rate in period *t -1* complies with condition (31). If it does not comply, then, in the event of a large negative shock, inflation, which makes up for the gap between the real value of debt and the discounted sum of fiscal surpluses (corrected for a given δ), will exceed $\pi^{\max}$.

Restriction $\pi_t \leq \pi^{\max}$ is violated when:

$$\pi^{\max} < \frac{b_{t-1}(1-\beta)(1-\rho\beta)}{s_{t-1}(1-\beta)\rho + \bar{s}(1-\rho) - \varepsilon^{\max}(1-\beta)}\left[1 - \delta \int_{e_t}^{\varepsilon^{\max}} 1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)}\right] \equiv \pi^{c}, \qquad (32)$$

where $e_t = \frac{1}{1-\delta}\left[1 - \frac{\varepsilon^{\max}\beta}{b_{t-1}(1-\beta)(1-\rho\beta)}\right]$.

At this point, we conclude that an equilibrium with a low default rate and inflation below $\pi^{\max}$ is feasible only for positive or relatively small negative shocks to fiscal surpluses.

4.2 Solution

In this section we solve the central bank's problem. The objective function given by (28) decreases in R_{t-1}. Hence, the solution to (28) is a minimum interest rate satisfying both (29) and (30). Figure 2 illustrates the two constraints, where the shaded area depicts a case in which both constraints are fulfilled. Note that these solutions exist for all realizations of a fiscal shock if $\pi^{\max} \geq \pi^{c}$.[14]

The function $\psi(R_{t-1})$ from the right-hand side of condition (30) is increasing in R_{t-1}:

$$\psi'_{R_{t-1}} = \frac{\delta}{1-\delta} R_{t-1} \frac{\beta(1-\rho\beta)b_{t-1}}{(\pi^{\max})^2} f(\hat{\varepsilon}_t) > 0 \ .$$

This function is convex for all R_{t-1}, such that $\hat{\varepsilon}_t \leq 0$:

$$\psi''_{R_{t-1}} = \frac{\delta}{1-\delta} \frac{\beta(1-\rho\beta)b_{t-1}}{(\pi^{\max})^2} \left[f'_{\hat{\varepsilon}} \frac{(1-\rho\beta)b_{t-1}R_{t-1}}{\pi^{\max}} + f(\hat{\varepsilon}) \right] > 0.$$

In the following analysis we only study equilibria with $\hat{\varepsilon}_t \leq 0$ — which is to say equilibria in which default can only be caused by negative fiscal shocks, but not positive ones. From (25) we obtain that $\hat{\varepsilon}_t \leq 0$ when $\tilde{R}_{t-1} \geq \pi^{\max} / \beta$. In equilibrium, the central bank sets $\tilde{R}_{t-1} \geq \pi^{\max} / \beta$ when constraint (30) is fulfilled at $\tilde{R}_{t-1} = \pi^{\max} / \beta$.[15] From (30) it follows that $\pi^{\max} / \beta \geq \psi(\pi^{\max} / \beta)$ if:

$$\pi^{\max} \geq \frac{\beta}{1-\delta} \left[1 - \frac{\delta}{2} - \frac{\beta\delta}{b_{t-1}(1-\rho\beta)} \int_0^{\varepsilon^{\max}} \varepsilon dF(\varepsilon) \right] \equiv \pi^{L}. \qquad (33)$$

Thus, we presuppose that condition (33) is fulfilled.[16]

We now determine the values of $\pi^{\max}$ from (33), under which the probability of default is zero. Solutions with a zero probability of default are available if condition (30) is fulfilled for

[14] See (32).

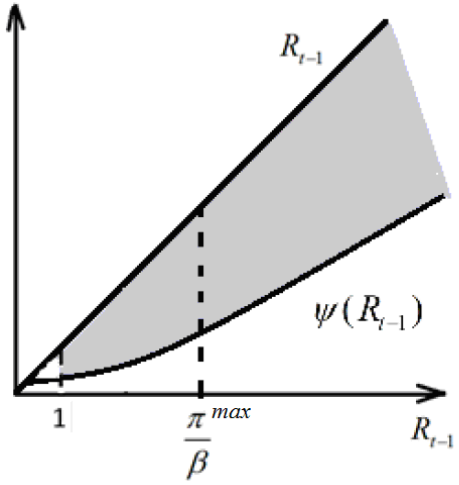
[15] This is true, because $\psi(R_{t-1})$ is an increasing convex function under $\hat{\varepsilon}_t \leq 0$.

[16] When constraint (33) is not fulfilled, the qualitative results do not change, whereas the algebra becomes substantially more complicated.

$R_{t-1}=1$: Since the objective function decreases in R_{t-1} when $R_{t-1}=1$ is possible, it is an equilibrium solution for which the probability of default is zero, which is the case if :

$$\pi^{\max} \geq \frac{\beta\left[\rho s_{t-1}+(1-\rho)\bar{s}/(1-\beta)\right]}{\left[\rho s_{t-1}+(1-\rho)\bar{s}/(1-\beta)\right]-\varepsilon^{\max}} \equiv \pi^{H}, \qquad (34)$$

so that $\psi(1)\leq 1$ (see Figure 2a). Thus, the probability of default is zero when the upper limit on inflation is sufficiently high. By contrast, when $\pi^{\max}<\pi^{H}$, $\psi(1)>1$ (see Figure 2b). In this case an equilibrium with a zero probability of default does not exist.



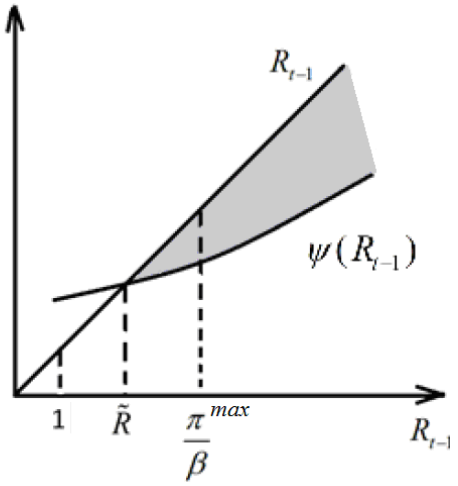


Figure 1a. Zero probability of default Figure 2a. Non-zero probability of default

When inequality (34) holds and $R_{t-1}=1$, households know that even if $\varepsilon_t=-\varepsilon^{\max}$, default would not emerge, because inflation that ensures debt sustainability is below $\pi^{\max}$. Thus, when $R_{t-1}=1$, the risk premium is zero and the zero lower bound constraint on the risk-free interest rate is fulfilled. At the same time, the central bank does not have incentives to set $R_{t-1}>1$ because the corresponding probability of default is higher. Therefore, under (34) $R_{t-1}=1$ is an equilibrium solution.

When the upper limit on inflation is low ($\pi^{L}\leq\pi^{\max}<\pi^{H}$), an equilibrium with a zero probability of default is not feasible. Even if the central bank sets $R_{t-1}=1$ in period *t-1*, there is a non-zero probability that inflation that ensures debt sustainability would exceed $\pi^{\max}$ in period *t* (and, thus, default would emerge). Hence, under $R_{t-1}=1$ in period *t-1*, households demand a positive risk premium on government bonds, which means that the zero lower bound constraint for a risk-free interest is violated. Thus, solution $R_{t-1}=1$ is not feasible.

To illustrate these theoretical results, the Appendix provides a numerical example for Greece's economy.

4.3 Expectations over the upper limit on inflation and dynamic inconsistency

Now suppose that in period *t-1* households do not know the true value of $\pi^{\max}$ and the central bank can affect a household's expectations over $\pi^{\max}$ for period *t* by making a public statement in period *t-1*. Choosing $\tilde{\pi}_t^{\max}$, the central bank solves:

$$\Pr(\varepsilon_t \le \hat{\varepsilon}_t) = \int_{-\varepsilon^{\max}}^{\hat{\varepsilon}_t} (1 + \frac{\varepsilon_t \beta}{b_{t-1}(1-\rho\beta)}) dF(\varepsilon) \to \min_{\tilde{\pi}^{\max}, R_{t-1}} \tag{35}$$

$$R_{t-1} \ge 1;$$

$$R_{t-1} \ge \frac{1}{1-\delta} + \left(1 - \frac{1}{1-\delta}\right) \int_{\hat{\varepsilon}_t^e}^{\varepsilon^{\max}} (1 + \frac{\varepsilon\beta}{b_{t-1}(1-\rho\beta)})\, dF(\varepsilon),$$

where the expected value of fiscal shock depends on $\tilde{\pi}_t^{\max}$:

$$\hat{\varepsilon}_t^e = (\frac{R_{t-1}\beta}{\tilde{\pi}_t^{\max}} - 1) b_{t-1} \frac{1-\rho\beta}{\beta},$$

and the actual threshold value of shock depends on $\pi^{\max}$:

$$\hat{\varepsilon}_t = (\frac{R_{t-1}\beta}{\pi^{\max}} - 1) b_{t-1} \frac{1-\rho\beta}{\beta}.$$

As noted before, the bigger the risky interest rate R_{t-1}, the higher the expected default rate $E_{t-1}\delta_t$. The range of R_{t-1} that satisfies the zero lower bound restriction for the risk-free rate depends on the risk premium that households demand for government bonds: The lower the risk premiums, the lower the minimum value of R_{t-1} that the central bank can set. At the same time, regardless of the risk premium, the (gross) risky interest rate cannot be lower than 1. The solution $R_{t-1} = 1$ becomes feasible when the risk premium is zero, which would be the case if $\tilde{\pi}_t^{\max} \ge \pi^H$. Thus, the solution for problem (35) is $\tilde{R}_{t-1} = 1$, $\tilde{\pi}_t^{\max} \ge \pi^H$ because for $\tilde{R}_{t-1} > 1$ the probability of default is higher and under $\tilde{\pi}_t^{\max} < \pi^H$ the solution $\tilde{R}_{t-1} = 1$ is not feasible.

Therefore, when the central bank can influence a household's expectations over $\tilde{\pi}_t^{\max}$, the probability of default turns out to be smaller than under $\tilde{\pi}_t^{\max} = \pi^H$. This feature invokes the issue of dynamic inconsistency: The solution for $\tilde{\pi}_t^{\max}$ does not correspond to the true value of the upper limit on inflation, if $\tilde{\pi}_t^{\max} < \pi^H$.

At this point we can draw the following conclusion. When households do not know exactly what the restrictions on inflation that the central bank faces are, the latter has incentives to create inaccurate beliefs, by suggesting that the upper limit on inflation is higher than the true value, in order to lower the risk premium and the equilibrium probability of default. On the other hand, when households believe that the upper limit on inflation is lower than its actual value, the equilibrium probability of default is higher than in cases when the beliefs of households are accurate.

5. Conclusion

In recent years fiscal stress has become a matter of concern for some developed European countries. Governments facing fiscal limits are unable to flexibly adjust their fiscal policy in line with the requirements for debt sustainability. In these countries fiscal shocks may lead to an escalation of default risks.

In these circumstances the policy of the central bank affects the probability of default on government bonds, alongside having an impact on inflation. In this paper we studied the capabilities and limitations of monetary policy that controls the risky interest rate in an environment where the central bank strives to minimize the probability of sovereign default while facing restrictions on the upper limit of inflation.

We have arrived at the following conclusions. The higher the upper limit on inflation, the lower the equilibrium probability of default and the risk premium on government bonds demanded by the market. Equilibrium with a low default rate and inflation below the upper limit is only feasible when fiscal shocks are either positive or negative but small. An equilibrium with a zero probability of default is feasible when the upper limit on inflation is sufficiently high: The smaller current fiscal surpluses are, the higher the value of the upper limit on inflation that ensures a zero probability of default is.

Furthermore, an agent's beliefs about the restrictions on inflation have a prominent effect on equilibrium outcomes. When the upper limit on inflation is believed to be higher than its

actual value, the equilibrium probability of default is lower than in cases when the beliefs reflect the true value of the upper limit on inflation.

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Appendix

Table 1 reports annual data on the fiscal surplus as a percentage of GDP for Greece for 2000-2012. Suppose that $\rho \in [0,5;0,8]$ and $\beta = 0,99$.[17]

According to The Stability and Growth Pact, the debt to GDP ratio in European countries must not exceed 60 percent. It follows from (17) that to support this level $\bar{b} = 60\%$ in the long run, the steady-state surplus must equal $\bar{s} = 0.6\%$.

We estimate the value of fiscal shock according to $\varepsilon_t = s_t - \bar{s} - \rho(s_{t-1} - \bar{s})$. The minimum value of fiscal shock $-\varepsilon^{\max}$ for the period 2005-2012 belongs to the interval $\{-11; -2.95\}$. Define $[\rho s_{t-1} + (1-\rho)\bar{s} / (1-\beta)] \equiv \Delta_t$. The value of Δ_t is within the range $\{-28.15; -22.2\}$. Thus, the assumption that $\varepsilon^{\max}$ satisfies $\varepsilon^{\max} \le [\rho s_{t-1} + (1-\rho)\bar{s} / (1-\beta)]$ is realistic.

Under monetary policy that controls the risky interest rate, an equilibrium with a zero probability of default is only feasible when the upper limit on inflation is believed to be very high (higher than 21% for 2004-2007 and even higher after 2007). But this is only true if agents believe that the long-term value of surplus equals 0.6% of GDP. If the long-term surplus is believed to equal $\bar{s} = 0.7 (\bar{s} = 1)$, then the upper limit on inflation for 2004-2007 must exceed $\pi^{\max} = 15 (\pi^{\max} = 11)$ for an equilibrium with a zero probability of default to exist.

Year	s_t	Estimates under $\bar{s} = 0.6$			Estimates for $\pi^{\max}$		
		Δ_t	ε_t	$-\varepsilon^{\max}$	for $\bar{s} = 0.6$	for $\bar{s} = 0.7$	for $\bar{s} = 1$
2000	-3.7						
2001	-4.5	-28.15	-2.95	-2.95	1.1	1.09	1.05
2002	-4.8	-27.75	-2.85	-2.95	1.11	1.09	1.06
2003	-5.6	-27.6	-3.5	-3.5	1.13	1.11	1.07
2004	-7.5	-27.2	-5	-5	1.21	1.17	1.11
2005	-5.2	-26.25	-1.75	-5	1.22	1.18	1.11
2006	-5.7	-27.4	-3.4	-5	1.21	1.17	1.11
2007	-6.5	-27.15	-3.95	-5	1.21	1.17	1.11
2008	-9.8	-26.75	-6.85	-6.85	1.33	1.26	1.17
2009	-15.6	-25.1	-11	-11	1.76	1.56	1.31
2010	-10.7	-22.2	-3.2	-11	1.96	1.67	1.34
2011	-9.5	-24.65	-4.45	-11	1.78	1.57	1.32
2012	-10	-25.25	-5.55	-11	1.75	1.56	1.32

Table 1

* Data provided by Eurostat

[17] Bi and Traum (2012) calibrate the model with sovereign risks for Greece's economy. We follow them in assuming that $\beta = 0.99$ and rely on their results, suggesting that mean values of auto-regressive coefficients for taxes, transfers, and government purchases are 0.5, 0.5, and 0.8, respectively.

** For a calculation of $-\varepsilon^{max}$, we assume that it equals the minimum value of fiscal shock over the preceding period, starting from 2000 when Greece entered Eurozone. For instance, for 2005 with $\bar{s}=0.6$, we obtain $-e^{max}=\min\{-2.95;-2.85;-3.5;-5;-1.75\}=-5$. This analysis can be generalized by adding uncertainty over ε^{max}.

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