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LETTERS

Individual differences in non-verbal number acuity correlate with maths achievement

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Abstract hidden

Behavioural, neuropsychological and brain imaging techniques show that a signature of the approximate number system (ANS) is its imprecision[2–13]. Unlike exact verbal counting, the ANS produces numerical representations that grow increasingly imprecise as a linear function of the target array, with larger quantities represented less precisely than smaller quantities. This imprecision is expressed as a Weber fraction that indexes the amount of error in the underlying mental representation of any numerosity[3–5]. On average, the Weber fraction of adults is approximately 0.11, yielding successful non-verbal discrimination of arrays differing by as little as a 9:10 ratio[5,14]. Here we address whether there are significant individual differences in ANS acuity, and also whether these differences correlate with individual differences in symbolic maths achievement.

We examined 64 14-yr-old children with normal development whose performance in a variety of mathematical and more general cognitive tasks had been measured longitudinally, starting in kindergarten[15]. We tested for correlations between the current ANS acuity of the subjects and their past achievement in symbolic maths, while controlling for a wide range of other variables. Each subject's ANS acuity was assessed by psychophysical modelling of performance on a simple more/less judgement task similar to those used previously with infants and non-human animals. On each trial, subjects saw spatially intermixed blue and yellow dots presented on a computer screen too rapidly (200 ms) to serially count (Fig. 1a)[16]. Subjects indicated which colour was more numerous by key press and verbal response. The ratio between the two sets varied randomly among 1:2, 3:4, 5:6 and 7:8, with between 5 and 16 dots in each set. The colour of the more numerous set varied randomly, and half of the trials were area-controlled to ensure that responses were on the basis of the

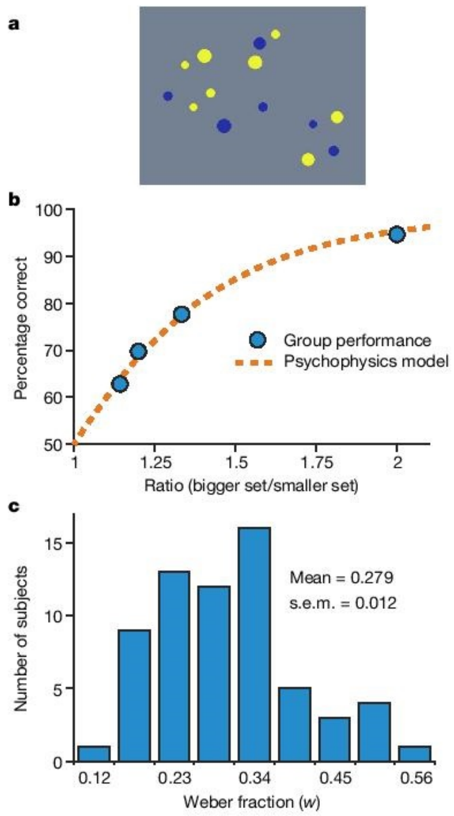

Figure 1 | Method and group performance. **a**, A representation of the trial from the numerical discrimination task. **b**, Group performance and modelled best-fit for all trials in the numerical discrimination task. **c**, Histogram of *w*, the acuity of the ANS, for the sample ($n = 64$), as determined by the psychophysical model for each subject.

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number of dots and not on the total dot area (see Supplementary Information). Subjects participated in two sessions of 10 practice trials and 40 test trials each, totalling 80 test trials (approximately 10 min of testing per subject).

Collapsing across subjects, numerical discrimination improved as the ratio between the presented numerosities increased, in accord with Weber's law and with previous investigations of the ANS[3–9] (Fig. 1b). This gradual improvement in performance as a function of ratio was modelled using classical psychophysical tools to determine the group Weber fraction (see Methods and Supplementary Information). This returned a value of 0.265 for the group Weber fraction (w) with an R^2 value of 0.995, suggesting that there is very high agreement between this psychophysical model of the ANS and the behavioural data (Fig. 1b). Next, we used this same method to fit each individual subject's data and thereby determine each subject's Weber fraction. This showed surprisingly large variation in the ANS acuity (w), ranging from 0.119 to 0.567 (Fig. 1c). The Weber fractions of subjects can also be translated into more intuitive whole numbers that show the ratio that would result in 75% correct performance. Using this translation, some subjects could discriminate numerical ratios as fine as 9:10 ($w = 0.11$) whereas others had difficulty with ratios finer than 2:3 ($w = 0.5$; mean subject $w \approx 4:5$).

A question to address is whether these individual differences in ANS acuity (w) predict individual differences in symbolic maths achievement. Each of our subjects was tested annually from kindergarten to sixth grade (ages 5–11) on a battery of standardized and investigator-designed measures. This longitudinal assessment of mathematical, verbal and other cognitive abilities provides a unique opportunity to detect any enduring correlations between ANS acuity and symbolic maths ability while controlling for other factors. Each year (ages 5–11), symbolic maths ability was assessed using the 'test of early mathematical ability, second edition' (TEMA-2)[17] and/or the 'Woodcock–Johnson revised calculation subtest' (WJ-Rcalc)[18], yielding an age-referenced standardized score for each subject. We found that the ANS acuity (w) of subjects correlated with symbolic maths performance in every year tested (from kindergarten to sixth grade) for both of the standardized maths tests, as summarized in Table 1. ANS acuity in ninth grade retrospectively predicted the symbolic maths performance of individual students from as early as kindergarten, a 9-yr time span. The linear correlations of ANS acuity (w) with symbolic maths achievement (TEMA-2 and WJ-Rcalc) for the third grade are shown in Fig. 2a, b.

A further question to address was whether the correlation between ANS acuity and symbolic maths achievement was due to individual differences in more general cognitive or performance factors. In the third grade (when subjects were approximately aged 8) we administered several non-numerical standardized tests including measures of rapid lexical access for colour names (rapid automatic naming, RAN-colour)[19] and full-scale IQ (Wechsler abbreviated scale of intelligence, WASI-full)[20]. The RAN-colour is an appropriate control

Table 1 | Correlation of ANS acuity (w) with symbolic maths achievement

Grade	TEMA-2 R^2	t d.f. = 62	P	WJ-Rcalc R^2	t d.f. = 62	P
Kindergarten	0.137	3.134	0.003	0.127	2.959	0.004
First	0.140	3.171	0.002	0.326	5.480	8×10^{-7}
Second	0.238	4.399	4×10^{-5}	–	–	–
Third	0.324	5.448	9×10^{-7}	0.282	4.933	6×10^{-6}
Fourth	–	–	–	0.248	4.518	3×10^{-5}
Fifth	–	–	–	0.117	2.866	0.006
Sixth	–	–	–	0.251	4.564	2×10^{-5}

ANS acuity (w) measured in ninth grade retroactively correlated with symbolic maths achievement. R^2 values represent the proportion of the variance in symbolic maths achievement that is explained by ANS acuity. R^2 values >0.25 are considered large in behavioural science and are generally viewed as having large practical significance. t values represent the distance, measured in units of standard error, between the obtained correlation and the null hypothesis of no correlation. P values represent the probability of obtaining the observed correlation in a sample of data by random chance when there is truly no relation in the population.

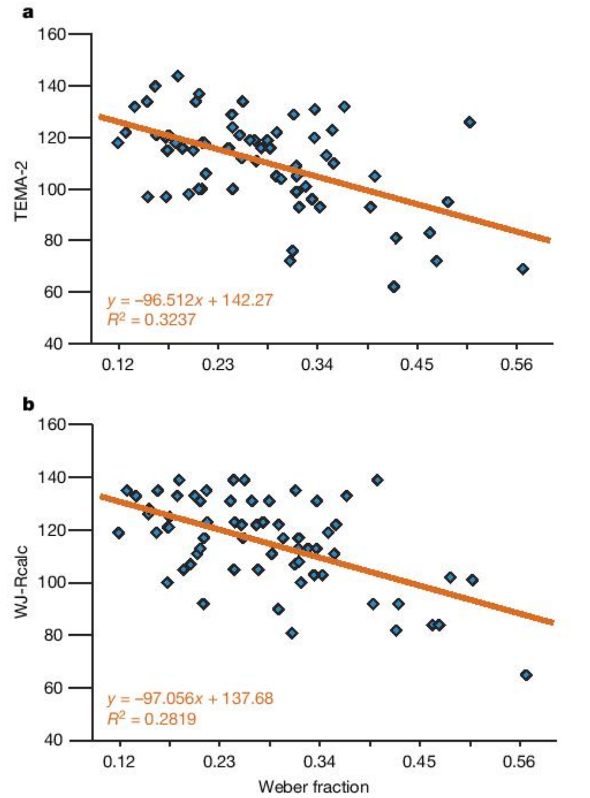

Figure 2 | Regressions. a, b, Linear regression of the standard score for each subject on the TEMA-2 test (**a**) or on the WJ-Rcalc test (**b**) of symbolic maths achievement and the acuity of the ANS (w). For TEMA-2 and WJ-Rcalc, higher numbers indicate better performance, whereas for the Weber fraction, lower numbers indicate better performance.

for our task because it measures the reaction time to identify the colours of 50 stimuli quickly; rapid colour naming is precisely the behaviour required by our ANS acuity assessment. The WASI-full IQ test acts as a control for general intelligence. WASI-full and RAN-colour did not correlate with one another in our sample ($P = 0.699$), making them largely orthogonal for purposes of linear regressions with ANS acuity. To examine the relationship of ANS acuity and symbolic maths achievement while controlling for other variables, two separate linear regressions were performed with ANS acuity (w) as the dependent variable and performance on either the TEMA-2 or the WJ-Rcalc test, and WASI-full and RAN-colour as independent variables. These showed that ANS acuity (w) correlated with symbolic maths achievement in the third grade even with rapid lexical access and general intelligence controlled for (Table 2).

To assess the strength of the correlation between ANS acuity (w) and symbolic maths achievement further, we performed extra linear regressions between w (measured at age 14) and an even broader range of standardized test scores obtained when subjects were in the third grade. These 16 measures controlled for the widest possible range of behavioural, cognitive and intelligence factors in our sample including many factors promoted as predictors of mathematical ability (for example, visual–spatial reasoning, working memory)[21–25]. ANS acuity (w) significantly correlated with symbolic maths achievement (measured in the third grade) for both TEMA-2 and WJ-Rcalc performance, with all 16 measures controlled for ($r_p^2 = 0.167$ and 0.200, respectively, where p represents partial correlation). In contrast, no other measure correlated with ANS acuity when symbolic maths performance and other variables were controlled for (Table 3). This means that success on tests of symbolic mathematics throughout the school years

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Table 2 | Correlations controlled for cognitive and performance factors

Measure (task)	r_p^2	t d.f. = 60	P
With TEMA-2			
Symbolic maths (TEMA-2)	0.146	3.205	0.002
Intelligence (WASI-full)	0.013	0.887	0.379
Task demands (RAN-colour)	0.004	0.492	0.625
With WJ-Rcalc			
Symbolic maths (WJ-Rcalc)	0.155	3.325	0.003
Intelligence (WASI-full)	0.070	2.124	0.038
Task demands (RAN-colour)	0.017	1.023	0.310

ANS acuity (w) measured in ninth grade retroactively correlated with third grade symbolic maths achievement and other measures. r_p^2 values represent the proportion of the variance in ANS acuity accounted for by the listed variable when controlling for the two remaining variables in each analysis (TEMA-2 or WJ-Rcalc). t values represent the distance, measured in units of standard error, between the obtained correlation and the null hypothesis of no correlation. P values represent the probability of obtaining the observed correlation in a sample of data by random chance when there is truly no relation in the population.

can be retrospectively predicted by a subject's ANS acuity in young adulthood, as measured by the simple task of determining which of two quickly flashed arrays has more dots, even with extensive controls for other cognitive and performance factors.

Our results are consistent with at least two interpretations. Given that it is functional in infancy[7], long before the onset of symbolic mathematics instruction, the ANS may have a causal role in determining individual maths achievement. Indeed, neuropsychological evidence suggests that the ANS is activated during symbolic mathematical reasoning across the lifespan[13]; therefore individual differences in ANS acuity might give rise to individual differences in maths ability.

Table 3 | Correlations controlled for all available factors

Measure (task)	r_p^2	t d.f. = 40	P
With TEMA-2			
Symbolic maths (TEMA-2)	0.167	2.831	0.007
Intelligence (WASI-full)	0.005	0.472	0.640
Task demands (RAN-colour)	0.023	0.981	0.332
Verbal IQ (WASI-verbal)	0.005	0.459	0.649
Performance IQ (WASI-performance)	0.006	0.482	0.632
Executive functions (CNT-B3)	0.021	0.918	0.364
Visual working memory (MemPuzl)	0.067	1.694	0.098
Visual segmentation (DTVPfg)	0.009	0.599	0.552
Object perception (DTVPfc)	0.001	0.172	0.864
Visual reasoning (DTVPvc)	0.025	1.004	0.321
Spatial reasoning (DTVPps)	0.012	0.701	0.488
Visual motor integration (VMI)	0.035	1.213	0.232
Word knowledge (WJ-RIwid)	0.012	0.706	0.484
Reading (WJ-Rwa)	0.001	0.225	0.823
Rapid lexical access (RAN-letter)	0.049	1.435	0.149
Rapid lexical access (RAN-number)	0.012	0.685	0.497
Gender	0.028	1.069	0.291
With WJ-Rcalc			
Symbolic maths (WJ-Rcalc)	0.200	3.149	0.003
Intelligence (WASI-full)	0.013	0.736	0.466
Task demands (RAN-colour)	0.004	0.391	0.698
Verbal IQ (WASI-verbal)	0.009	0.605	0.549
Performance IQ (WASI-perf)	0.013	0.727	0.472
Executive functions (CNT-B3)	0.035	1.208	0.234
Visual working memory (MemPuzl)	0.084	1.916	0.062
Visual segmentation (DTVPfg)	0.032	1.148	0.254
Object perception (DTVPfc)	0.001	0.201	0.842
Visual reasoning (DTVPvc)	0.008	0.578	0.566
Spatial reasoning (DTVPps)	0.018	0.869	0.390
Visual motor integration (VMI)	0.013	0.725	0.473
Word knowledge (WJ-RIwid)	0.014	0.757	0.454
Reading (WJ-Rwa)	0.000	0.014	0.988
Rapid lexical access (RAN-letter)	0.014	0.757	0.454
Rapid lexical access (RAN-number)	0.000	0.037	0.970
Gender	0.012	0.684	0.498

ANS acuity (w) measured in ninth grade retroactively correlated with third grade symbolic maths achievement and other measures. r_p^2 values represent the proportion of the variance in ANS acuity accounted for by the listed variable when controlling for all other variables in the list. t values represent the distance, measured in units of standard error, between the obtained correlation and the null hypothesis of no correlation. P values represent the probability of obtaining the observed correlation in a sample of data by random chance when there is truly no relation in the population.

Alternatively, individual differences in the quantity or quality of engagement in formal mathematics might increase ANS acuity. This latter possibility is hinted at by cross-cultural differences in Weber fractions, with maths-educated adults having better ANS acuity than adults from indigenous cultures lacking maths education[5,14]. These causal relationships, possible tertiary factors and the trainability of ANS acuity[26] remain to be explored. Further evidence will add to the present results, which suggest that our ability to reason over symbolic numbers is deeply entwined with an evolutionarily ancient system for numerical approximation.

METHODS SUMMARY

At age 14 (that is, ninth grade), ANS acuity was assessed for 64 subjects (see Methods). The percentage correct on the ANS task was modelled for each individual subject as 1 − error rate, where error rate is defined as:

$$\frac{1}{2}\mathrm{erfc}\left(\frac{n_1-n_2}{\sqrt{2}w\sqrt{n_1^2+n_2^2}}\right)$$

where erfc(x) is the complementary error function related to the integration of the normalized Gaussian distribution. This model fits percentage correct as a function of the Gaussian approximate number representations for the two sets displayed on a trial (n_1 and n_2, that is, blue dots and yellow dots) with a single free parameter, the Weber fraction (w; see Supplementary Information)[5]. Correlations presented were between this estimate of ANS acuity (w), measured at age 14, and scores on standardized cognitive and performance measures, from kindergarten to sixth grade.

Full Methods and any associated references are available in the online version of the paper at www.nature.com/nature.

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Supplementary Information is linked to the online version of the paper at www.nature.com/nature.

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