Bidimensional Sorting: Workers vs Entrepreneurs*

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1 Introduction

We know a lot on the behavior of firms. However, little is still known on the people standing behind the firms. In this paper, we revisit the underlying factors inducing individuals to become entrepreneurs. There are long-standing disputes around the term "entrepreneur". We share Lazear's (2005) point of view on this question: an entrepreneur is an individual who (i) has discretionary authority over her business; (ii) and this discretionary authority is large enough to cause significant variations in profits depending on the entrepreneur's performance. The use of the concept of profit maximization is plausible, since the majority of existing firms are small and run by their owners.

We consider agents facing a trade-off between launching their own firms or being hired as workers. The model is developed following the static Lucas (1978) model which compares the reservation wage of an agent as a worker with her expected salary as a manager. Yet, Lucas did not specify exogenous entrepreneurial ability, though it is common for researchers to think of it in terms of human capital, such as schooling and experience. Empirical evidence to date is inconclusive concerning the relationship between human capital and selection into entrepreneurship (Poschke, 2008). To accommodate this empirical fact, we introduce two exogenous dimensions of agents heterogeneity. While deciding on her occupation, an agent takes into account not only her productivity, as in Melitz (2003), but also her entrepreneurial ability. Consequently, the subset of agents indifferent between being entrepreneurs and workers is not reduced to a point and the model is able to generate fuzzy patterns of selection. We can think of the two abilities as being aggregates of a set of more general factors, which affect entrepreneurial performance and productivity differently. The characteristics considered may be determined by the ability to predict and adjust to idiosyncratic changes in consumer tastes, as in Takii (2008), human and social capital, family background, and attitude towards risk.

Lazear's (2005) model also excogitates bi-dimensional heterogeneity by using a joint

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distribution of two abilities that governs the choice between paid employment and entrepreneurship. Paid employment specialists earn the maximum of the drawn abilities, whereas solo entrepreneurs have to supply both skills themselves and receive a premium which is only as high as the weakest ability, multiplied by an exogenous value of entrepreneurial talent, λ. This setting leads to a reduced form of selection determined mostly by the value of λ, while our model delivers an endogenous cut-off curve.

Poschke (2008) approaches the occupational choice model as if both entrepreneurial performance and wages were affected by the random draw of firm productivity α, which in turn depends on an agent's productivity a. However, his model does not generate an explicit pattern of selection, since firm productivity is randomly assigned to individuals. Summarizing the discussion above, we infer that our model conveys: (i) an explicit form of the cut-off function describing the set of individuals indifferent between particular occupations; (ii) the cut-off function is a function of the given range of productivities and marginal cost of operating firms; and (iii) although individuals in the economy are driven by pecuniary incentives, they also compare themselves to the average individual.

The environment of the model is a two-sector monopolistic competition framework with uniformly distributed entrepreneurial ability and productivity. Variation in productivity leads to wage inequality among workers, and variation in entrepreneurial ability results in inequality of marginal cost among firms. We solve the model for a quasi-linear log upper-tier utility function and nested CES preferences. We use comparative statics to analyze the behavior of the cut-off function splitting individuals into two groups. When shifting the parameters of the productivity support, we obtain four patterns of employment structure. These patterns are able to explain the existence of both "out of opportunity" and "out of necessity" entrepreneurs, which are empirically observed by Poschke (2008). Comparative statics allow us to analyze changes in the employment structure when new individuals flow from outside into the economy. These shifts can be associated with migration and human capital reallocation due to trade liberalization. Thus, the model can be nicely extended to the open economy case, in which domestic entrepreneurs compete with emerging firms as if they were competing with an individual (endowed with marginal cost c and zero productivity). Under this assumption, the fraction of local entrepreneurs crowded out by foreign firms can be perceived as an alternative for Melitz's (2003) "probability of death" for small firms.

2 Model

As mentioned above, we consider an economy of unit population size. Nature randomly assigns entrepreneurial talent and productivity to each individual using a uniform distribution with support $[\underline{c}, \overline{c}]$ and $[\underline{\phi}, \overline{\phi}]$, respectively. Since entrepreneurial ability mimics marginal cost, the lower c, the more talented an entrepreneur is. On the contrary, the greater ϕ, the more productive a worker is. Each individual represents a point from a set $\Omega = [\underline{c}, \overline{c}] \times [\underline{\phi}, \overline{\phi}]$. In other words, the mass L of individuals in the economy is a measure of the ability set: $L = |\Omega| = (\overline{\phi} - \underline{\phi})(\overline{c} - \underline{c})$.

We consider a two-sector economy: monopolistic competition in the market for the differentiated good and perfect competition in the market for the homogenous good. The price of the latter is chosen as the numéraire.

Consider an individual described by two characteristics $\omega \equiv (c, \phi)$ from the set Ω. While deciding on her occupation, she compares her expected payoffs:

$$I_\omega = \begin{cases} \pi_\omega & \text{if} \quad \omega \in \Omega_E \\ \phi & \text{if} \quad \omega \in \Omega_W, \end{cases} \tag{1}$$

where Ω_E and Ω_W are the sets of entrepreneurs and workers, respectively. Individuals indifferent between the two occupations are elements of a border set $M = \Omega_E \cap \Omega_W$.

The utility function is quasi-linear and defined by a two-tier utility function and a linear term denoting consumption of the homogenous good. Quasi-linearity is a property that allows us to get rid of wealth effects, which usually make the model solution more complicated. More formally, we assume that

$$U(x_\omega, A_\omega) = V\left(\int_{\Omega_E} u(x_\omega)\mathrm{d}\Omega_E\right) + A_\omega. \tag{1}$$

The consumer's budget constraint is given by

$$I_\omega = \int_{\Omega_E} p_\omega x_\omega \mathrm{d}\Omega_E + A_\omega, \tag{2}$$

which yields the following consumer problem:

$$\max_{x_\omega} V\left(\int_{\Omega_E} u(x_\omega)\mathrm{d}\Omega_E\right) + I_\omega - \int_{\Omega_E} p_\omega x_\omega \mathrm{d}\Omega_E \tag{3}$$

The budget constraint is always binding, since all income leftovers from purchasing the differentiated good are spent on the homogeneous good. The first-order conditions (FOCs) of the problem express price as

$$p(x_\omega) = \frac{u'(x_\omega)}{\mu}, \tag{4}$$

where $\mu \equiv \int_{\Omega_E} u(x_\omega)\mathrm{d}\Omega_E$. An entrepreneur $\omega \in \Omega_E$ maximizes her profit with respect to the sum of all individual demands $y_\omega = Lx_\omega$ for the variety produced:

$$\max_{y_\omega} \pi_\omega = [p(x_\omega) - c]y_\omega \qquad (5),$$

The FOCs of the producer problem relate the Lerner index to the 'relative love for variety' as follows:

$$\frac{p(x_\omega) - c}{p(x_\omega)} = -\frac{1}{\varepsilon_\omega} = \frac{1}{r_u(x_\omega)},$$

so that the price is given by:

$$p(x_\omega) = \frac{c}{1 - r_u(x_\omega)}.$$

An indifferent individual receives the same payoff under any occupational choice. Let the set of these individuals be described by an iso-curve $M(c(\phi), \phi) = 0$. The function $c(\phi)$ is a cut-off rule that returns a value of entrepreneurial talent yielding profit equal to productivity $\phi \, \forall \, \phi \in [\underline{\phi}, \overline{\phi}]$.

We distinguish the output and the price of a cut-off individual

$$\begin{aligned} y_\omega &\equiv y_{c,\phi} \\ y_\omega|_{M(c(\phi),\phi)=0} &\equiv y_{c(\phi)} \\ p(x_\omega|_{M(c(\phi),\phi)=0}) &\equiv p_{c(\phi)} \end{aligned}$$

The free entry condition follows from the definition of the cut-off function: there is no entry cost for the market of the differentiated good:

$$\pi_{c(\phi)} = \phi \tag{7}$$

Producer and consumer FOCs, the free entry condition, and the relationship between the outputs of the indifferrent and an ordinary entrepreneur are used to find an equilibrium bundle $\{\mathbf{x}_{c,\phi}, \mathbf{x}_{c(\phi)}, c(\phi), \mu\}$. We write down a system of equilibrium conditions considering a log upper-tier utility function and nested CES preferences:

$$\left\{ \begin{array}{rcl} \frac{c}{\rho} &=& \frac{\rho x_{c,\phi}^{\rho-1}}{\mu} \\ \frac{c(\phi)(1-\rho)}{\rho} x_{c(\phi)} L &=& \phi \\ \frac{c(\phi)}{c} &=& \left(\frac{x_{c(\phi)}}{x_{c,\phi}}\right)^{\rho-1} \\ \mu &=& \int_{\Omega_E} x_{c,\phi}^{\rho} \mathrm{d}\Omega_E \end{array} \right. \tag{9}$$

We solve the system step-by-step and get the following results:

$$\left\{ \begin{array}{rcl} c(\phi) &=& \frac{\rho^{\frac{\rho+1}{\rho}} \phi^{\frac{\rho-1}{\rho}}}{(1-\rho)^{\frac{\rho-1}{\rho}} \mu_\Omega^{\frac{1}{\rho}}} \\ x_{c(\phi)} &=& \phi^{\frac{1}{\rho}} \rho^{-\frac{1}{\rho}} (1-\rho)^{-\frac{1}{\rho}} \mu_\Omega^{\frac{1}{\rho}} \\ x_{c,\phi} &=& \rho^{\frac{2}{1-\rho}} c^{\frac{1}{\rho-1}} \mu_\Omega^{\frac{1}{\rho-1}} \\ \mu &=& \int_{\Omega_E} x_{c,\phi}^{\rho} \mathrm{d}\Omega_E \end{array} \right. \tag{11}$$

One fundamental question is to determine the value of μ. That value is found by solving the following integral equation:

$$\mu = \rho^{\frac{2\rho}{1-\rho}} \mu^{\frac{\rho}{\rho-1}} \int_{\Omega_E} c^{\frac{\rho}{\rho-1}} \mathrm{d}\Omega_E$$

The solution depends on a specific set of entrepreneurs that is observed in the economy. Remember that each individual can be mapped to a location in the Descartes coordinate system. The set of entrepreneurs is the one obtained by the intersection of the rectangular and the cut-off curve. Table 1 represents four possible forms of the sets of entrepreneurs. The cut-off curve is supposed to be monotonically decreasing because of the model set up: lower marginal cost and greater productivity imply greater probability of an individual to become an entrepreneur.

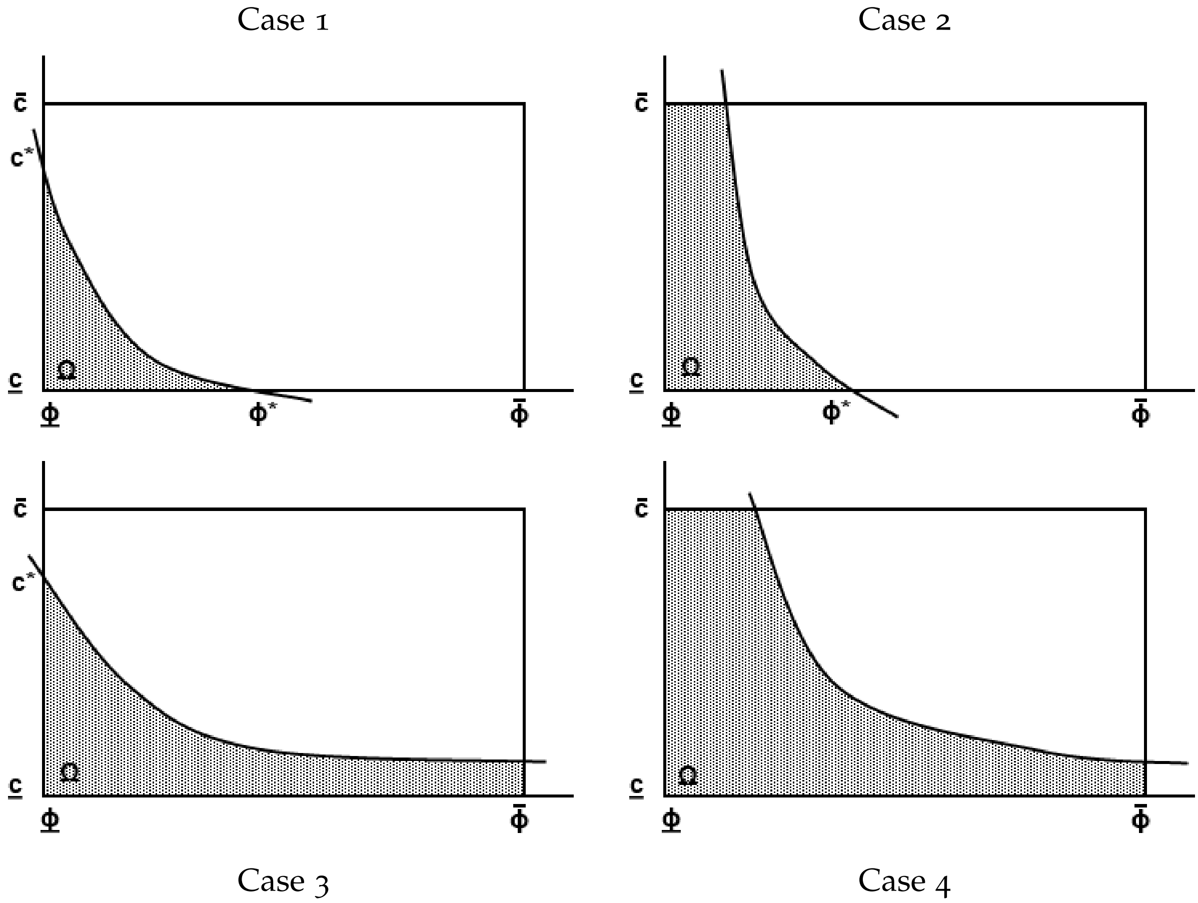

Table 1: Cut-off curve and possible sets of entrepreneurs

The mathematical description of the set of entrepreneurs depends on a particular case.

Case 1 & Case 3	$\Omega_E^1 : [\underline{c},\ c(\phi)] \times [\underline{\phi},\ \overline{\phi}]$
Case 2 & Case 4	$\Omega_E^2 : \Omega_E^1 \Delta \left([\overline{c},\ c(\phi)] \times [\underline{\phi},\ \overline{\phi}]\right)$

Table 2: Mathematical description of sets of entrepreneurs

For now, we solve only for Case 3, since all the results can be represented in a closed form. Making explicit the domain of integration in (12), we get an integral equation

$$\mu = \rho^{\frac{2\rho}{1-\rho}} \mu^{\frac{\rho}{\rho-1}} \int_{\underline{c}}^{c(\phi)} \int_{\underline{\phi}}^{\overline{\phi}} c^{\frac{\rho}{\rho-1}} \mathrm{d}\phi \mathrm{d}c \tag{13}$$

$$\mu^{\frac{1}{1-\rho}} = \frac{\rho^{\frac{2\rho}{1-\rho}}(1-\rho)}{1-2\rho}\left[c(\phi)^{\frac{1-2\rho}{1-\rho}} - \underline{c}^{\frac{1-2\rho}{1-\rho}}\right](\overline{\phi} - \underline{\phi}) \tag{14}$$

Without loss of generality, though significantly simplifying the subsequent analysis, we set $\underline{\phi} = 0$ and $\underline{c} = 0$. Hence

$$\mu^{\frac{1}{1-\rho}} = \frac{\rho^{\frac{2\rho}{1-\rho}}(1-\rho)}{1-2\rho}\left(c(\phi)^{\frac{1-2\rho}{1-\rho}}\overline{\phi}\right). \tag{15}$$

Omitting lengthy computations, we get

$$\mu^{\frac{1}{\rho}} = \frac{\rho^{\frac{-4\rho^2+\rho+1}{(1-\rho)^2}}(1-\rho)^{\frac{1-\rho}{\rho}}\underline{\phi}L^{\frac{1-2\rho}{\rho}}}{(1-2\rho)\phi^{\frac{1-2\rho}{\rho}}}. \tag{16}$$

Using (16), we rewrite the expression for the cut-off function as follows:

$$c(\phi) = \frac{\rho^{\frac{5\rho^3-2\rho^2-2\rho+1}{\rho(1-\rho)^2}}(1-2\rho)L}{\overline{\phi}\phi^{\rho}} \tag{17}$$

$$\rho^{\frac{5\rho^3-2\rho^2-2\rho+1}{\rho(1-\rho)^2}} \equiv \kappa > 0, \quad \forall \rho \in (0,1) \tag{18}$$

Finally, the equilibrium bundle is given by

$$\begin{cases} c(\phi) &= \frac{\kappa(1-2\rho)\overline{c}}{\phi^{\rho}} \\ x_{c(\phi)} &= \frac{\alpha\phi^2 L^{\frac{1-2\rho}{\rho}}}{(1-\rho)(1-2\rho)} \\ x_{c,\phi} &= \frac{\beta\underline{\phi}^{\frac{\rho}{\rho-1}}L^{\frac{1-2\rho}{1-\rho}}}{(1-\rho)(1-2\rho)^{\frac{\rho}{\rho-1}}\phi^{\frac{1-2\rho}{\rho-1}}} \\ \mu^{\frac{1}{\rho}} &= \frac{\gamma(1-\rho)^{\frac{1-\rho}{\rho}}\underline{\phi}L^{\frac{1-2\rho}{\rho}}}{(1-2\rho)\phi^{\frac{1-2\rho}{\rho}}}, \end{cases}, \tag{19}$$

where

$$\rho^{\frac{-4\rho^3+2\rho^2-\rho+1}{\rho(1-\rho)^2}} \equiv \alpha > 0, \quad \forall\, \rho \in \left(0, \frac{1}{3}\right); \quad \rho^{\frac{4\rho^3+\rho^2-5\rho+2}{(1-\rho)^3}} \equiv \beta > 0, \quad \forall\, \rho \in (0,1);$$

and

$$\rho^{\frac{-4\rho^2+\rho+1}{(1-\rho)^2}} \equiv \gamma > 0, \quad \forall\, \rho \in \left(0, \frac{1+\sqrt{17}}{8}\right).$$

3 Conclusions

The project has just been started, and it is therefore being updated on a frequent basis: one remiss mistake and signs of derivatives are different. However, there are several conclusions that can be drawn: (i) an endogenous cut-off function exists and it is unique since μ is defined unambiguously in the real number plane; (ii) there are two forces that tend to balance each other at $\rho = \frac{1}{2}$, because $(1-2\rho)$ enters every equilibrium parameter; (iii) further analysis should be conducted separately for the case of "close enough" ($\rho < \frac{1}{2}$) varieties and "not close enough" ones ($\rho > \frac{1}{2}$).

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