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COLOG ASSET PRICING, EVIDENCE FROM EMERGING MARKETS

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We introduce a new asset pricing model to account for risk asymmetrically in a very natural way. Assuming asymmetric investor behavior we develop a utility function similar to a quadratic utility but include a colog measure for capturing risk attitude. Asymmetry in investor preferences follows the asymmetric relationships between asset and market returns in equilibrium.

Moreover the local version of the model depends on the characteristics of domestic markets, which is reflected in the different relationship between asset and market returns. We test the model in the Russian and South African markets and show that market premium in the Russian market is higher than in the South African market.

Key words: asset pricing models, risk measures, asymmetric investor's preferences, market risk premium.

JEL codes: G12, G15

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1. Introduction

There are many different assumptions about asset pricing models in the literature. Some of these assumptions are difficult to interpret and others cannot explain the behavior of investors in financial markets. Asset pricing models themselves are often criticized for their lack of explanatory power of stock returns. That is why much research tries to improve the models by modifying the theoretical reasoning behind them.

One of the key assumptions for a large number of asset pricing models is that all investors are equally risk averse. That is why we only consider the preferences of a representative investor. In this case we develop an asset pricing model by maximizing for a representative investor the difference between the expected value and the risk measure.

Starting with the Markowitz (1952) variation many different risk measures were developed for financial models (see, for example Rachev et al. (2011)). During the last two decades researches agreed that risk measures should reflect the asymmetric attitude of investors toward risk and account for the strong aversion to large losses. Artzner et al. (1999) introduced coherent risk measures which assign different weights to return distribution quantiles. The expected shortfall is one of the well-known examples of coherent risk measurement. However coherent measures were not widely applied in asset pricing because of the properties of positive homogeneity and translation invariance. There were a number of attempts to apply risk measures to the development of asset pricing models. De Giorgi and Post (2008) used distortion risk measures (which also modify the probability space) for the development of a risk-reward pricing model. Choosing an appropriate distortion measure is difficult since it is not clear how to determine and interpret investor attitudes toward different probabilities of outcomes through a corresponding distortion function.

The drawbacks of these measures forced researches to apply other techniques for risk measurement. Rockafellar et al. (2006) introduce a class of deviation measures. Rachev et al. (2011) consider a broader class of dispersion measures which include dispersion and semi-variation. In this paper we consider a colog measure which also belongs to the class of dispersion measures (see Rachev et al. (2011)). The colog measure for a random variable X is defined as:

$$Colog(X,X) = E(X\log X) - E(X)E(\log X)$$

The main advantage of a colog measure of stock returns is that it accounts for asymmetric investor behavior toward risk along with a strong aversion to large losses and at the same time it preserves the easily interpretable properties of dispersion. Semi-deviation, semi-skewness and semi-kurtosis, as examples of lower partial moments of Bawa and Lindeberg (1977), have similar properties. But a colog measure explains simultaneously the asymmetric influence of different moments since the logarithm can be represented as a Taylor series. Moreover a colog measure does not exclude the positive price movements which are important for the investor and their risk attitude.

In financial models investor preferences can be defined by the appropriate choice of utility function. Developing the intertemporal asset pricing model, Merton (1973) applied the Bellman principle for the recursive definition of the utility of consumption. Since then it has become common to determine current utility through current consumption and expected utility which depends on future consumption and investor attitude to risk. Expected utility can be defined not only through future consumption but using a corresponding risk measure. For example, Tallarini (2000) adapted risk Epstein-Zin preferences to financial models and obtained an equation where utility depended on current consumption and the entropic risk measure of future consumption. Widely criticized for its symmetry, quadratic utility is another example of a representation involving risk measure. In the case of quadratic utility, the risk measure choice is the usual variation.

Similarly to these examples we apply a colog measure to determine investor preferences and the utility function, which in turn is used to obtain asset pricing equations in equilibrium.

The asset pricing models will be tested in the Russian and South African markets. We check if colog risk factors have any significant influence on the cost of equity and estimate the corresponding risk premium for each market.

2. Colog investor preferences

We assume that the usual assumptions of consumption models are fulfilled (see, for example Cochrane (2005)). Similar to quadratic utility we will use the following form of utility of consumption:

$$u(c) = (c - c_0) - a(c - c_0)\log(c - c_0), \tag{1}$$

where c is consumption, which is bounded below by $c_0 > 0$,
$a > 0$ is a coefficient of risk aversion.

We define investor preferences by considering the simple case of one period utility as follows:

$$U(c_t, c_{t+1}) = u(c_t) + \beta E_t(u(c_{t+1})), \tag{2}$$

where c_t, c_{t+1} are investor's consumptions at times t and $t+1$;
E_t is the expectation conditional on information at t;
$\beta < 1$ is a subjective discount factor for future consumption.

Let w_t be the total wealth at t. We assume that the wealth after consumption $k_t = w_t - c_t > c_0$ will be invested at the market portfolio rate $r_{w,t+1}$ and will be completely consumed in the following period $t+1$. In other words we suggest that

$$(w_t - c_t)(1 + r_{w,t+1}) = c_{t+1} > c_0. \tag{3}$$

Here we suppose that minimal consumption is stable over time and equal to c_0 in the next period.

Now we would like to advocate our choice of utility function in the form (1).

If we plug (3) into equation (1) for expected utility we obtain:

$$E_t\big(u(c_{t+1})\big) = k_t E_t\big(1 + r_{w,t+1}\big) - aE_t\Big(\big(k_t\big(1 + r_{w,t+1}\big) - c_0\big)\log\big(k_t\big(1 + r_{w,t+1}\big) - c_0\big)\Big)$$

The expected utility can be expressed as a function of the expectation of market return and the colog measure of market return. Compared with variation the main advantage of a colog measure is that it is asymmetrical and assigns more weight to downside movements of a market portfolio.

If an investor is restricted to investment in the finite list of N risky assets with returns $r_{i,t+1}, i = 1, \dots, N$ and a risk free asset with return $r_{0,t+1}$, then the optimal choice for portfolio selection would be presented by the following first order conditions:

$$u_c(c_t) = E_t(\beta u_c(c_{t+1})(1 + r_{i,t+1})), i = 0, \dots, N, \tag{4}$$

where $u_c(\cdot)$ is a first order derivative of the utility function. We plug the expression for expected utility into (4) and obtain:

$$u_c(c_t) = \beta E_t\Big(\big(1 + r_{i,t+1}\big)\big(k_t - a - ak_t\log\big(k_t r_{w,t+1} - k_t c_0\big)\big)\Big)$$

Then the stochastic discount factor could be written as:

$$M_{t+1} = \beta \frac{u_c(c_{t+1}(r_{w,t+1}))}{u_c(c_t)} = a_0 - a_1 \log\left(\theta_t r_{w,t+1} + 1\right), \tag{5}$$

where a_0, a_1 are positive coefficients which depend on model parameters a, c_0, β, marginal utility $u_c(c_t)$ and the remaining wealth k_t at time *t*;

$\theta_t = \frac{w_t - c_t}{w_t - c_t - c_0} > 1$ is a coefficient showing how savings (all invested in our case) will cover the minimal consumption in the future period.

The value of θ_t should satisfy condition (3), in other words $\theta_t \leq \max\left(-\frac{1}{r_{w,t+1}}\right)$ where max stands for the maximum of a random variable. In other words θ_t should not exceed one over the value of the maximum decrease of the market portfolio for the period.

The stochastic discount factor (5) differs from the regularly used logarithmic version of the pricing kernel for CARA utility. Moreover M_{t+1} can assign different weights to the up and down movements of the market portfolio.

We rewrite (4) using stochastic discount factor (5) as Euler equations for *N* asset returns $i = 0, \dots, N$ for equilibrium:

$$1 = E_t\left(M_{t+1}\left(1 + r_{i,t+1}\right)\right) \tag{6}$$

Equation (6) must hold for the risk free asset $r_{0,t+1}$and for the market portfolio $r_{w,t+1}$ assuming the investor has the ability to purchase them. For these two assets we obtain two linear equations with two unknown parameters a_0, a_1:

$$\begin{cases} a_0 - a_1 E_t \log\left(\theta_t r_{w,t+1} + 1\right) = \frac{1}{1 + r_{0,t+1}}; \\ a_0 E_t\left(1 + r_{w,t+1}\right) - a_1 E_t\left(1 + r_{w,t+1}\right)\log\left(\theta_t r_{w,t+1} + 1\right) = 1 \end{cases}$$

Recalling that:

$$E_t\left(1 + r_{w,t+1}\right)\log\left(\theta_t r_{w,t+1} + 1\right) - E_t\left(1 + r_{w,t+1}\right)E_t\log\left(\theta_t r_{w,t+1} + 1\right) = COV\left(r_{w,t+1}, \log\left(1 + \theta_t r_{w,t+1}\right)\right)$$

we solve for parameters a_0, a_1 and plug their values into (5):

$$M_{t+1} = \frac{E_t((r_{w,t+1} - r_{0,t+1})\log(1 + \theta_t r_{w,t+1}))}{(1 + r_{0,t+1})COV_t(r_{w,t+1}, \log(1 + \theta_t r_{w,t+1}))} - \frac{E_t(r_{w,t+1} - r_{0,t+1})}{(1 + r_{0,t+1})COV_t(r_{w,t+1}, \log(1 + \theta_t r_{w,t+1}))} \log(1 + \theta_t r_{w,t+1})$$

a_0, a_1 are determined as functions of a, β and other current parameters such as consumption c_t, wealth w_t and marginal utility $u'(c_t)$ at time *t*. Solutions of (6) for a_0, a_1 tie all these variables with the risk free return and the market return.

We rewrite (6) in the following way:

$$1 = E_t(M_{t+1})E_t\left(1 + r_{i,t+1}\right) + COV_t(M_{t+1}, r_{i,t+1})$$

Now we can use equation for SDF M_{t+1} and obtain asset pricing formula:

$$E_t\left(r_{i,t+1} - r_{0,t+1}\right) = \frac{COV_t\left(r_{i,t+1}, \log\left(1 + \theta_t r_{w,t+1}\right)\right)}{COV_t\left(r_{w,t+1}, \log\left(1 + \theta_t r_{w,t+1}\right)\right)} E_t(r_{w,t+1} - r_{0,t+1}) \tag{7}$$

Equation (7) is similar to the traditional form of asset pricing models such as CAPM. Because of the covariation of the asset return with $\log(1 + \theta_t r_{w,t+1})$, model (7) assigns greater values for negative returns on the market portfolio than models where covariation with the market return is considered. Moreover, the higher value of coefficient θ_t, which measures how investments exceed minimal consumption, the greater the asymmetry of the model since the value of $\log(\theta_t r_{w,t+1} + 1)$ becomes significantly different from $r_{w,t+1}$. The introduction of coefficient θ_t distinguishes the colog variation of return $COV_t(r_{w,t+1}, \log(1 + \theta_t r_{w,t+1}))$ from the colog measure mentioned in Rachev et al. (2011).

One of the assumptions of model (7) is that investors can purchase an asset with the return equal to return of the market portfolio. This is not always true in practice. Instead investors can usually purchase an asset with returns close to the market return such as the index exchange traded fund, where return fluctuations could asymmetrically differ from the fluctuations of the market portfolio because of the inaccurate imitation of the index structure, transaction and management costs. Moreover the index itself may not follow the market portfolio since it only includes tradable stocks. We hence assume that an investor can purchase only an asset with return $\log(1 + \theta_t r_{w,t+1})$, whose fluctuations depend on the level of market development which we measure with coefficient θ_t. In this case we can use Euler equations (6) and follow the procedures described above to obtain nonlinear pricing model of the form:

$$E_t(r_{i,t+1} - r_{0,t+1}) = \frac{COV_t(r_{i,t+1}, \log(1+\theta_t r_{w,t+1})}{VAR_t(\log(1+\theta_t r_{w,t+1})} E_t(\log(1 + \theta_t r_{w,t+1}) - r_{0,t+1}) \qquad (8)$$

Model (8) is similar to the nonlinear model developed by Rubinstein (1976). It differs though in right hand side where instead of the logarithm of return there is the asset return itself. Features of model (8) take into account the nonlinear relationship between asset returns and market returns.

In the next section we describe the methodology of empirical tests of models (7) and (8).

3. Pricing colog risk factor

For testing purposes conditional models are often replaced with unconditional versions with the introduction of additional instrumental variables (see, for example, Jannatan & Wang (1996)). A factor mimicking portfolios as instrumental variables in unconditional models can be used (see Fama & French (1993)). The methodology of empirical tests usually follows the procedures of Fama and Macbeth (1973) or a generalized method of moments.

However modeling portfolio construction in emerging markets faces difficulties because of the lack of publicly traded companies and the short history of the stock markets. That is why for checking the adequacy of the models we use an idea of Ang et al. (2006), who followed the Fama-Macbeth procedure but did not use factor modeling portfolios.

We consider local pricing models in this study. For each country we substitute the market return with the return of a domestic equity index and consider domestic short term government bonds valued in local currency as a proxy for the risk free rate. During the first stage we estimate sensitivity to market risk according to models (7) and (8). We made a comparison with beta coefficients obtained according to other models such as CAPM of Sharp (1964), Linter (1965) and Downside CAPM of Hogan and Warren (1974).

For the estimation of coefficients we considered adjusted for dividends weekly returns $r_{i,t+1,s}$ during the two year periods such that:

$$r_{i,t+1,s} = \frac{P_{i,s} + D_{i,s}}{P_{i,s-1}} - 1,$$

where $r_{i,t+1,s}$ is a return of *i*-th asset during the *s*-th week of $(t + 1)$-th two year period, $P_{i,t}$ is the closing price of *i*-th asset on the last day of week *s*, $P_{i,s-1}$ is the closing price of *i*-th asset on the last day of week $(s - 1)$, and $D_{i,s}$ is a dividend of *i*-th asset, if the ex-dividend date belongs to week *s*.

The value of θ_t differs from country to country and depends on the maximum fall of a domestic equity index during the considered period. We pick the value of θ_t according to the following equation:

$$\theta_t = 1 + \delta\left(\max\left(-\frac{1}{r_{w,t+1}}\right) - 1\right), \text{ for some } \delta\text{: } 0<\delta < 1. \tag{9}$$

Recall that the coefficient θ_t according to models (7) and (8) must indicate how savings exceed the minimum consumption in a given country. We suggest that for developed countries the level of savings is higher and the maximum fall of the stock market is smaller than for emerging countries. That is why a value of θ_t should be higher for developed countries.

We calculated the sensitivity coefficients in two different ways. The first was straightforward. Beta coefficients were estimated for the period $(t, t+1)$ according to different asset pricing models:

$$\beta_{i,t+1}^{CLOG1} = \frac{COV_t(r_{i,t+1}, \log(1+\theta_t r_{w,t+1}))}{COV_t(r_{w,t+1}, \log(1+\theta_t r_{w,t+1}))},$$

$$\beta_{i,t+1}^{CAPM1} = \frac{COV_t(r_{i,t+1}, r_{w,t+1})}{VAR_t\,(r_{w,t+1})} \tag{10}$$

$$\beta_{i,t+1}^{DCAPM} = \frac{E_t(\min[(r_{i,t+1} - E_t(r_{i,t+1}), 0]\cdot\min[(r_{w,t+1} - E_t(r_{w,t+1}), 0])}{E_t\min[(r_{w,t+1} - E(r_{w,t+1}), 0]^2}$$

In (10) variances and covariances mean their standard unbiased estimators from weekly data.

The second calculation of beta coefficients uses an adjustment for liquidity and volatility clustering. Liquidity is an important risk factor that affects the returns on assets especially in emerging markets. To adjust for liquidity we used the coefficient of illiquidity proposed by Amihud (2002):

$$ILLIQ_{i,s} = \frac{1}{Days_{i,s}}\sum_{d=1}^{Days_{i,s}}\frac{|r_{i,s,d}|}{Vol_{i,s,d}},$$

where $(s, s+1)$ is the period for which a value of the coefficient is estimated;

$Days_{i,s}$ is the number of days in $(s, s+1)$;

$r_{i,s,d}$ is the return of *i*-th asset at day *d* of period $(s, s+1)$;

$Vol_{i,s,d}$ is the trade volume of *i*-th asset on day *d* of period $(s, s+1)$.

Much research (e.g. Ang et al. (2006)) mentions that the idiosyncratic volatility of stock returns depends on time. We decided to take into account the autocorrelation of volatility using the GARCH(1,1) model. We considered two time series models for the estimation of adjusted market sensitivity coefficients. One of these models corresponds to a traditional market model:

$$r_{i,t+1,s} - r_{0,t+1,s} = \gamma_{0,i} + \beta_{i,t+1}^{CAPM2}(r_{w,t+1,s} - r_{0,t+1,s}) + \gamma_{1,i}\cdot ILLIQ_{i,s} + \sigma_{i,s}^{\varepsilon} z_{i,s}\,, \tag{11}$$

where $r_{i,t+1,s}, r_{w,t+1,s}, r_{0,t+1,s}$ are the returns of *i*-th stock, market portfolio and a risk free asset for the time interval $(s, s+1) \in (t, t+1)$;

$\sigma_{i,s}^{\varepsilon}$ is the volatility of error terms in model GARCH(1,1);

$\beta_{i,t+1}^{CAPM2}$ is the coefficient of sensitivity to market portfolio movements.

The second time series regression is obtained from the pricing model (8):

$$r_{i,t+1,s} - r_{0,t+1,s} = \gamma_{0,i} + \beta_{i,t+1}^{CLOG2}(\log(1+\theta_t r_{w,t+1,s}) - r_{0,t+1,s}) + \gamma_{1,i} \cdot ILLIQ_{i,s} + \sigma_{i,s}^{\varepsilon} z_{i,s} \quad (12)$$

Running regressions (11) and (12) we estimate $\beta_{i,t+1}^{CAPM2}$, $\beta_{i,t+1}^{CLOG2}$ adjusted for liquidity and the clustering of volatility which allows us to determine more precisely the sensitivity to market risk.

Beside beta coefficients which measure market risk through the co-movement of asset and market returns we considered the influence of other risk factors in the second stage of the Fama-Macbeth procedure. Particularly we studied the impact of colog deviation, which can be defined for each asset return $r_{i,t+1}$ by the following:

$$\sqrt{COV_t(r_{i,t+1}, \log(1+\theta_t r_{i,t+1})}$$

Colog deviation differs from standard deviation since it is an asymmetric risk measure. Moreover colog deviation takes into account positive return movements which differentiates it from semi-deviation.

We avoid overloading our model by including pair of risk measures from each pricing model only. More precisely we included in the model beta coefficients $\beta_{i,t+1}^{CLOG1}$, $\beta_{i,t+1}^{CLOG2}$, $\beta_{i,t+1}^{CAPM1}$, $\beta_{i,t+1}^{CAPM2}$, $\beta_{i,t+1}^{DCAPM}$ along with the corresponding risk measures: standard deviation (SD), standard semi-deviation (SDD), which is defined as the square root of semi-variance in the sense of Hogan and Warren (1974), and the colog deviation (SDC) of returns. Following Ang et al. (2006), we added variables in each model to control for asset specific characteristics which allowed us to underline the impact of the risk factors. We obtained the following model explaining the average of excess returns $r_{i,t+1,s} - r_{0,t+1,s}$ of i-th asset on each time interval $(t, t+1)$:

$$mr_{i,t+1} = \lambda_{0i} + \lambda_{1i} beta_{i,t+1} + \lambda_{2i} devms_{i,t+1} + \lambda_{3i} \overline{Size}_{i,t} + \alpha\lambda_{4i} \overline{GO}_{i,t} + \varepsilon_{i,t}, \quad (13)$$

where $mr_{i,t+1}$ is an average weekly excess return of i-th asset on $(t, t+1)$;

$\lambda_{ji}, j = 0, \dots, 4$ are constant sensitivities to the risk factors and control variables respectively;

$beta_{i,t+1}$ are different beta coefficients obtained at the first stage of the Fama-Mcbeth procedure for $(t, t+1)$;

$devms_{i,t+1}$ are different deviation measures estimated on $(t, t+1)$;

$\overline{Size}_{i,t}$ is the proxy for size of a firm measured as the logarithm of capitalization at the beginning of interval t;

$\overline{GO}_{i,t}$ is the proxy for firm growth which is measured as B/M at the beginning of interval t;

$\varepsilon_{i,t}$ is an error term.

The main goal of running regression (13) is the estimation of the significance of the coefficients $beta_{i,t+1}$ which allows us to make a conclusion about the adequacy of models (7) and (8). Moreover the estimators of $\lambda_{1i,t}$ provide us with the market risk premium.

4. Empirical results

We tested our models on data from publicly traded stocks in Russia and South Africa obtained from the Bloomberg database. We gathered information for the ten year period from January 2003 to December 2012. The dataset included stock returns of 104 non-financial companies listed on the Moscow Interbank Currency Exchangeexchange and 160 companies listed on Johannesburg Stock Exchange.

We used adjusted for dividends the MICEX index as a proxy for the market portfolio in the Russian market and adjusted for dividends FTSE/JALSH index as a proxy for the market portfolio in the South African market. We took short term government OFZ bond yields from the Central Bank of the Russian Federation website and three month treasury bond yields from the South African Reserve Bank website as proxies for risk the free rates in Russian and South African markets respectively.

We suppose that the ratio of savings and minimal consumption remained stable over the ten year period in Russia and South Africa which resulted in a constant $\theta = \theta_t$ over the period. For the calculation of θ values we used the maximum weekly percentage fall in the Russian and South African markets from January 2003 to December 2012 which were 24% and 9% respectively. We picked several values for δ: 0.3, 0.5, 0.7, 0.9. After testing our model we concluded that the most significant results were obtained for higher values of δ. That is why we choose a δ value of 0.9. This can be explained by the level of asymmetry in each market which increased with the rise of δ. Hence, we obtained $\theta = 3.71$ for Russia and $\theta = 10$ for South Africa.

Stock returns and Amihud illiquidity were calculated each week for every company adjusted for dividends. For size and value variables we used log capitalization and B/M values at the beginning of each two year period.

The estimation of $\beta_{i,t+1}^{CAPM2}$, $\beta_{i,t+1}^{CLOG2}$ according to models (11) and (12) with an embedded GARCH(1,1) procedure showed that Amihud illiquidity is significant in 90% of regressions. But its influence was limited to cases when beta coefficients unadjusted for liquidity were high. In those cases illiquidity decreases sensitivity of stock returns to market fluctuations. We can conclude that the liquidity adjustment estimated covariation of market returns and illiquid stock returns correctly.

After running regressions (13) on panel data we concluded that OLS was the best method according to the F-test and Breusch-Pagan test probably because of missing data. In the Russian and South African markets we found that beta coefficients estimated using colog models were highly significant. In Table 1 we include the estimators of the regression coefficients of model (13) and their levels of significance.

Table 1. OLS regression on weekly data, $\delta = 0.9$.

$$mr_{i,t+1} = \lambda_{0i} + \lambda_{1i} beta_{i,t+1} + \lambda_{2i} devms_{i,t+1} + \lambda_{3i} \overline{Size}_{i,t} + \alpha\lambda_{4i}\overline{GO}_{i,t} + \varepsilon_{i,t}$$

Country	Model specification	Regression coefficients				
		β-coeicient λ_{1i}	Deviation measure λ_{2i}	Log Mcap$_{t-1}$ λ_{3i}	Book/et$_{t-1}$ λ_{4i}	Constant λ_{0i}
Russia	$\beta_{i,t+1}^{CLOG1}$, SDC	0.00233*	0.01874	-0.00023	-0.00416***	0.00735**
	$\beta_{i,t+1}^{CLOG2}$, SDC	0.00437***	0.02454*	-0.00039	-0.00416***	0.00769*
	$\beta_{i,t+1}^{CAPM1}$, SD	0.00344**	0.02561*	-0.0003	-0.00415***	0.00679**
	$\beta_{i,t+1}^{CAPM2}$, SD	0.00173	0.03453**	-0.00015	-0.00408***	0.0059*
	$\beta_{i,t+1}^{DCAPM}$, SDD	0.0023	-0.07414***	-0.00042*	-0.00414***	0.01386***
South Africa	$\beta_{i,t+1}^{CLOG1}$, SDC	0.00128***	0.09757***	-0.00056***	-0.00345***	0.00414***
	$\beta_{i,t+1}^{CLOG2}$, SDC	0.00249***	0.09639***	-0.00066***	-0.00343***	0.00472***
	$\beta_{i,t+1}^{CAPM1}$, SD	0.00173**	0.08821***	-0.00061***	-0.00336***	0.00479***
	$\beta_{i,t+1}^{CAPM2}$, SD	0.00152***	0.09054***	-0.00058***	-0.00338***	0.00459***
	$\beta_{i,t+1}^{DCAPM}$, SDD	0.00112*	0.1452***	0.00094***	-0.00363***	0.00834***

*, **, *** is significance at 10%, 5% and 1% level

SD is standard deviation, SDC is colog deviation, SDD is semi-deviation.

The sensitivity of $\beta_{i,t+1}^{CLOG2}$ was significant at the 1% level in the Russian market. The market weekly premium for colog risk in this case was 0.00437 and the constant was less significant. If we rewrite this premium in terms of weekly returns we obtain the following:

$$\left(\log\left(1+\theta_t r_{w,t+1,s}\right)-r_{0,t+1,s}\right).$$

If we transform this premium to the regular form of $r_{w,t+1,s}-r_{0,t+1,s}$ and annualize we obtain a market premium of 4.6% in Russia. For the more developed market in South Africa with a higher value of θ estimators $\beta_{i,t+1}^{CLOG1}$ и $\beta_{i,t+1}^{CLOG2}$ were significant at the 1% level. But the market premium in South Africa after transformation is half of the Russian premium. Moreover in the South African market deviation measures and control variables were also significant. The high level of the significance of constants in the South African market can be explained by the lack of other important control variables.

As for the explanatory power of the models we obtained mixed results. The coefficient of determination was approximately 42% in the Russian market and approximately 58% in the South African market for the colog specifications of model (13). In Russia the explanatory power for the colog specification with adjustment for liquidity and volatility clustering was higher but in South Africa the coefficients of determination were close for different specifications.

5. Conclusion

We propose a new model of asset pricing under the assumption of colog investor preferences. The main advantage of this model is that it accounts for market risk asymmetrically without losing interpretability. The asymmetry is higher for developed countries where the markets are more stable and savings are much higher than minimal consumption. We also proposed a nonlinear version of the pricing model which responds to restrictions on investor abilities to purchase a market portfolio.

We tested our models in Russian and South African markets using the two stage procedures of Fama and Macbeth (1973). At the first stage we adjusted market sensitivity coefficients for liquidity and volatility clustering. At the second stage we included deviation measures and control variables which allowed us to deduce more precisely the estimators for sensitivity coefficients. The results supported the hypothesis about the strong influence of the sensitivity coefficients obtained according to the colog pricing models. These findings support the suggestion of the asymmetry of the covariation of market and asset returns. This asymmetry mainly arises from asymmetrical preferences of investors. The CARA utility function cannot explain the level of this asymmetry. Colog preferences show the stochastic discount factor that accounts for up and down market movements in different ways.

Market risk premiums for Russia and South Africa differ significantly which coincide with the findings of other authors (e.g. Bekaert G., (1995)). This can be explained partly by the different levels of market development in Russia and South Africa, which result in a more stable equity market for South Africa. Our model takes into account the level of market development in the country, which corresponds with higher asymmetry of asset and market movements. The higher premium in the Russian market then can be explained by the low level of diversification.

Future work might include taking into consideration a more sophisticated many period utility model and empirical tests in other developed and emerging markets.

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