NATIONAL RESEARCH UNIVERSITY
HIGHER SCHOOL OF ECONOMICS

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THE EFFECTIVENESS OF DIFFERENT TRADING STRATEGIES FOR PRICE-TAKERS

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: FINANCIAL ECONOMICS
WP BRP 29/FE/2014

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THE EFFECTIVENESS OF DIFFERENT TRADING STRATEGIES FOR PRICE-TAKERS[2]

Simulation models of the stock exchange are developed to explore the dependence between a trader's ability to predict future price movements and her wealth and probability of bankruptcy, to analyze the consequences of margin trading with different leverage rates and to compare different investment strategies for small traders. We show that in the absence of margin trading the rate of successful predictions should be slightly higher than 50% to guarantee with high probability that the final wealth is greater than the initial and to assure very little probability of bankruptcy, and such a small value explains why so many people try to trade on the stock exchange. However if trader uses margin trading, this rate should be much higher and high rate leads to the risk of excessive losses.

JEL Classification: G020, G170.

Keywords: agent-based system, simulation, stock exchange, trading strategies.

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[2] This study was undertaken in the framework of the Programme of Fundamental Studies of the Higher School of Economics in 2014. The author expresses sincere gratitude to Professor F. Aleskerov and Dr. H. Penikas for their valuable comments and help, and to the International Laboratory of Decision Choice and Analysis and Laboratory of Algorithms and Technologies for Network Analysis of HSE for partial financial support.

1. Introduction

The stock market is an organized financial market where securities can be bought and sold at prices formed by supply and demand. Stock market participants include individual investors and institutional investors such as banks, insurance companies, mutual, retirement and hedge funds. Traders have different trading goals, some use stocks as an investment to receive interest or dividends, others speculate to profit from short and medium term price movements.

In this paper we consider only price-takers, who are traders with relatively small capital and negligible market power. We suppose that they are speculators, because for a trader with a long-term investment goal it is more reasonable to invest in a mutual or hedge fund with professional investment management. Informed investment decisions should be based on the available economic and statistical information about the assets, analytical reports, economic and political news and can take into account even rumors concerning a security and its issuer. However, individual investors generally do not have sufficient time, experience, ability, or access to the inside information, for fully informed decision making. According to estimates from [Brown et al. 1999], the proportion of informed orders is less than 10% and, as was shown in [Barber 2008], individual investors tend to buy attention-grabbing stocks, which are in the news, experiencing abnormally high trading volume, or extreme one-day returns, while professional investors have more time and resources to continuously monitor a wider range of stocks. [Barber and Odean 2008] showed that the average trader turns over 75 percent of its portfolio annually and more than 250 percent for the traders who trade the most. These papers confirm the speculative nature of individual traders.

Speculators hope to make money in the stock market by betting on price moves. If such trader expects a price increase in the near future, she will buy securities now in order to sell them later and in the reverse situation (if she expects that prices will move down), the speculator will sell now and buy back later at a lower price. A speculator can, therefore, earn money on the stock exchange if she can correctly predict the future price of securities. The financial results of such a trader and the likelihood of bankruptcy depend on the accuracy of her predictions.

On average the stocks individual investors buy subsequently underperform those they sell [Odean 1999], which indicates their inability to predict the future price movements of securities. In [Barber and Odean 2008] 66,465 trader accounts at a large discount broker between 1991 and 1996 were analyzed, and the average trader earned an annual return of 16.4 percent, while the market returns were 17.9 percent, and those who trade most earn less (11.4 percent). It is also known that even economic analysts often fail to achieve even a 50% rate of correct forecasts

[Kahneman 2011]. In [Proskurin and Penikas 2013] the rate of successful expert recommendations on selling and buying stocks of Russian companies was calculated and it was showed that only 56.8% of expert recommendations were profitable.

That is why in the first model we consider an agent with one characteristic denoting her probability of correctly predicting asset price movements. In [Aleskerov and Egorova 2012] it was shown that if this probability is slightly higher than 50%, it allows the trader to receive a positive average gain from trading.

Another consequence of the time and ability restrictions of the small trader decision making process is a propensity to herd or imitate, when traders mimic the actions (rational or irrational) of another trader or group of traders. In the first case there are traders in the market (a so called ‘guru’), which other traders think to be more experienced, or lucky, or to have access to restricted information. The investment decisions of these players are of great interest to other agents and for some they can be a benchmark for decision making. The second case refers to a situation when the trader follows the mood of the market, that is, if everyone buys (a 'bull' market), then the trader will buy, and if everybody sells and the price falls (a 'bear' market), then the trader will also sell. Even professional players are tempted to follow the euphoria of bullish or bearish trends, although the propensity to herd is lower for professional investors than for the amateur [Venezia et al. 2011].

It was shown in [Tedeschi et al. 2012] that traders have incentives to imitate and for some it turns out to be profitable. In [Rothig and Chiarella 2010] the activities of small traders on currency futures markets were investigated and it was shown that small traders follow large speculators, indicating that they believe themselves to be less well informed than the large speculators.

We also consider more exotic strategies, based on the expectation of a crisis. Taleb, the author of [Taleb 2008], is the founder of the hedge fund "Empirica Capital", which bought derivatives with extreme values of strike price, because they are cheaper as extreme events are less probable. Thus Empirica was waiting for an opportunity to earn a lot of money in the rare event of a financial crisis (a financial crisis is called a ‘black swan’ in the Taleb’s terminology) and was losing a little money each day because of the premiums paid for purchased options. This strategy is reasonable in ‘a world of low levels of predictability’ [Makridakis and Taleb 2009], but is not used by the majority of traders.

The aim of this study is (1) to investigate the connection between a trader’s ability to predict future price movements, and her wealth and the probability of bankruptcy; (2) to analyze the consequences of margin trading with different leverage rates (margin trading is buying/selling of securities with cash/stocks borrowed from a broker and leverage rate is a ratio

between the collateral and the loan); (3) to compare different investment strategies of small traders using an agent-based simulation of a stock exchange.

The paper is organized as follows. Section 2 describes the general properties of the models and data used, Section 3 provides a detailed description of the models, Section 4 presents the results, and Section 5 concludes.

2. The models

2.1. The description of the market and agents

In our model there is a market of one asset and a population of N agents. All agents initially have \$10 000 in cash and no securities. In the basic model the agents differ only in p which models the probability of the agent to correctly predict the direction of the price movement the next day. It is assumed that p does not change during the life of the agent, and at the same time the basis of the agent decisions is not considered. In the model 'leader-follower' only leaders have p, and followers simply repeat leader's decisions with a 1 day delay. An agent does not know her own p in the basic model or the leader's p in the model 'leader-follower'.

There are two different approaches to modeling the stock exchange. In the first approach agents are price-takers, and in the second some agents are price-makers who have enough market power to impact to the price of an asset by their trading decisions. In the first case the price must be given exogenously, as it does not depend on the actions of price-takers, and in the second case the price is determined by aggregate supply and demand, and is therefore endogenously formed on the basis of orders submitted by the agents. In this paper we choose the first approach, as we explore small and medium-sized agents and assume that these agents do not have an impact on the asset price. We use a data of 6 stock indices to model the price of the asset, the time interval is 10 years.

We also assume that the market is sufficiently liquid and all the orders can be executed completely. All agents are speculators, so they are interested in the asset not as a long-term investment, but want to profit from price differences. Therefore, all orders are designed for the short-term and agents trade on the daily price fluctuations. In addition, the agents in our model submit only market orders. Such orders include the volume and do not specify a price, so it is immediately executed at the current market price. The agents in our model do not submit more complex orders.

We will also explore the impact of short selling and margin buying on wealth and the possibility of bankruptcy. Short selling denotes the selling of borrowed securities with a subsequent repurchase of the same securities. The trader opens a short position expecting a fall

in the asset price, which allows him to buy it later for less and return to the lender. Margin buying refers to the buying of securities with cash borrowed from a broker. The leverage rate denotes the ratio between the collateral and the loan. For example, a leverage rate of 1:5 means that the trader can borrow 5 times the collateral. Margin buying and short selling can increase trader's returns but it can also magnify losses. Margin trading will be prohibited on crisis days in our experiment. The prohibition can be caused by lenders as they see an increased risk of trader bankruptcy.

To assess the success of agents at the end of a day we evaluate their wealth as the sum of their cash and the number of their shares multiplied by the market price of the next day. The next day market price is used because agents will only be able to sell stocks the next day. If the wealth of the agent is less than a critical level (we use half of the initial wealth), then the agent will leave the market and we denote her as bankrupt. The usual definition of bankrupt as a person whose debt is greater than her assets cannot be applied in the models without leverage as in this case agent cannot become a debtor and can only lose all her money. This threshold was chosen as an estimation of an agent's success over 10 years of trading. The longer the interval of trading the lower such a threshold can be. This threshold is considered a stop-loss order which is designed to limit an investor's loss on a position.

As criteria for optimal strategy selection in the model when an agent does not know her leader's value of p we have chosen three simple criteria: the expected wealth, the probability of positive gain from trading and the probability of bankruptcy. The last criterion is related to a version of the Safety-First criterion [Roy 1952], when the agent should minimize the probability of the portfolio's return falling below a minimum desired threshold (here the minimum acceptable return is 0.5). Therefore for all models we consider three parameters: 1) the average wealth of agents on the final date, 2) the fraction of agents from the whole population whose wealth on the final date was higher than the initial wealth, 3) the fraction of bankruptcies in the whole population.

2.2 Description of data

As the agents have no impact on the price of the asset, the price of the asset must be specified exogenously and we have used here daily data of the different indices for the period 01.01.2000-31.12.2009. The time series consists of the closing prices. We used the S&P500 (Fig. 1, 2514 observations), the CAC 40 (2552 observations), the DAX (2542 observations), the FTSE (2525 observations), the Nikkei 225 (2453 observations) and Hang Seng (2488 observations)

indices. Different number of observations is caused by the presence of holidays in different countries.

To separate days potentially suitable for restrictive measures on short selling and margin buying, we have used the volatility index, calculated with a sliding interval of 20 days, and apply the same threshold rule as in [Aleskerov and Egorova 2012]: if the volatility value does not exceed the corresponding index value multiplied by a predetermined threshold value, then we will consider that day to be regular, and in the case of excess we assume that the market is in a crisis and impose a prohibition on the opening of short positions and the use of margin buying.

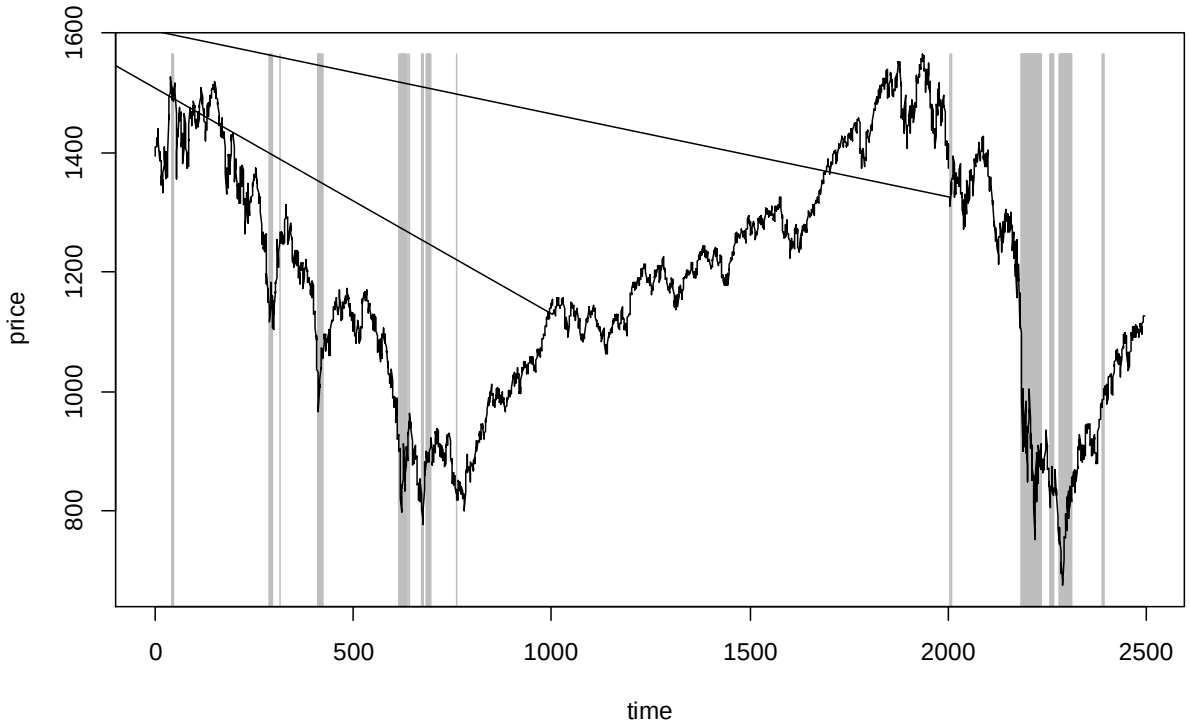

Fig.1. The time series of values for S&P500. The grey lines show the periods of crisis calculated with a threshold of 4%. There are 164 crisis days (6.6% of the total number of days).

3. Experiment description

3.1 Basic model

At the beginning of simulations all agents have $c_0 = \$10\,000$ of capital and $s_0 = 0$ of securities, and their initial wealth is $w_0 = c_0 + s_0 \cdot price_1 = \$10\,000$.

Agents are speculators and make decisions on the short-run. Agents differ only by the single characteristic p which models the ability of the agent to correctly identify the direction of the asset price $price_{t+1} - price_t$. The value p is random, and is distributed according to a uniform law on the interval $[p_{min}, p_{max}]$, it is selected at the beginning of the experiment and stays unchanged during the life of the agent on the market.

On each t-th iteration agent i make a decision to buy ($d_{i,t} = +1$) or sell ($d_{i,t} = -1$). If the agent expects the price to fall ($price_{t+1} \leq price_t$), it will be profitable to sell assets at the current price $price_t$ to prevent a decline in wealth. If at time t agent predicts a price increase ($price_{t+1} \geq price_t$), then the agent should buy shares at price $price_t$.

After making decisions all agents submit their orders to the stock exchange. Agents use market orders where they denote the volume of asset they want to buy or sell. If margin buying is forbidden and agent decision is $d_{i,t} = +1$ (buy), then the volume is set at $v_{i,t} = z \cdot \frac{c_{i,t}}{price_t}$, where z is a random value with uniform distribution R[0,1]. Otherwise the volume is set at $v_{i,t} = z \cdot \frac{w_{i,t} \cdot leverage}{price_t}$, where $leverage$ is the leverage rate. The presence of random variable z reflects the fact that the agent chooses how much she invests. This is caused by the cautious behavior of an agent because the rate of successful prediction is unknown. Agent-based models usually have such a feature, for example, in [Harras and Sornette 2011] the agent can only trade 2% of her capital per order.

If short selling is forbidden, $d_{i,t} = -1$ and the agent has some securities, then she will sell some of them and $v_{i,t} = z \cdot s_{i,t}$, $z \sim R[0,1]$. If short selling is allowed, then the agent sells more securities than she has. Securities can be borrowed and the trader has to buy the same volume the next day and return them to the lender. There are no other restrictions on the number of borrowed securities except the agent's wealth.

After the execution of all orders we recalculate the number of agent's securities $s_{i,t+1}$ and the agent's money $c_{i,t+1}$, and estimate agents total wealth as $w_{i,t} = c_{i,t+1} + s_{i,t+1} \cdot price_{t+1}$. If agent i's wealth at time t is less than a threshold $w_{i,t} \leq \frac{1}{2} w_0$ and she is declared bankrupt and leaves the market.

The goal of the experiment in the basic model is to determine the effect of p on the welfare of agents and the possibility of bankruptcy, and to determine the impact of short selling and margin buying.

3.2 The model with followers

In this model we divide the agents into two equal groups. The first group act according to the scheme from the section 2.3.1 and have a single characteristic p. Agents from the second group do not have a characteristic p, but observe the actions of agents from the first group and simply repeat their actions with a delay of one day, that is, an agent-follower j, observing the previous decision of her leader $d_{i,t-1}$, will repeat it the next day $d_{j,t} = d_{i,t-1}$.

For each leader there is only one follower, who does not know the value of her leader's p.

The goal is to evaluate the advantages and disadvantages of an imitation strategy and to determine which leader (that is, the leader with which value of p) it is more profitable or safe, from the viewpoint of the possibility of bankruptcy, to imitate.

3.3 The model with Black Swans

In this model the agents are divided into two groups of "ordinary traders" and "black swan seekers". The agents from the first group predict the movement of the asset price well in the period of economic stability, but not in a crisis, and agents of the second group, following Taleb's strategy, predict price movements well in a crisis, but not during stable periods.

The division of trading days into stable, Q-days, and crisis, R-days, is to forbid short positions and margin buying in crisis days, and is used to describe agent characteristics. The method for division was described in Section 2.2.

The difference in the characteristics is as follows. "Ordinary traders" on Q-days make the right decision with probability p^Q, which is assigned to the agent with a uniform distribution on the interval $[p_{min1}, p_{max1}]$, with $p_{min1} \geq 0.5$. On R-days this probability falls and $p^R = p^Q - \delta$. "Black swan seekers" on the contrary make the right decisions in the regular days with smaller probability than in a crisis: $p^Q \sim R[p_{min1}, p_{max1}]$, $p_{max1} \leq 0.5$ and $p^R = p^Q + \delta$. Thus "ordinary traders" should have better results in a stable economic situation and worse in a crisis, and "black swan seekers" are more profitable during a crisis and make losses otherwise.

The goal is to determine which strategy is most effective and whether any strategy has advantages in creating wealth or in lessening the probability of bankruptcy.

4. The results

4.1 The results for the basic model

For all models we consider three parameters: 1) the average wealth of remaining agents on final date, 2) the fraction of agents from the whole population whose wealth on the final date was higher than the initial wealth, 3) the fraction of bankruptcies in the whole population. Below we describe in details the results for the S&P500 index and give summary results for all indices at the end of this subsection. The complete results for all experiments are given in the Appendices.

Fig. 2 & 3 show the results without short selling and margin buying, that is the $leverage$ is 0. The parameter p is selected from the uniform distribution $R[0.4; 0.6]$. Vertical lines on Fig. 2 show the moment when the wealth of the corresponding agent reaches the threshold of 5000, and this agent leaves the market.

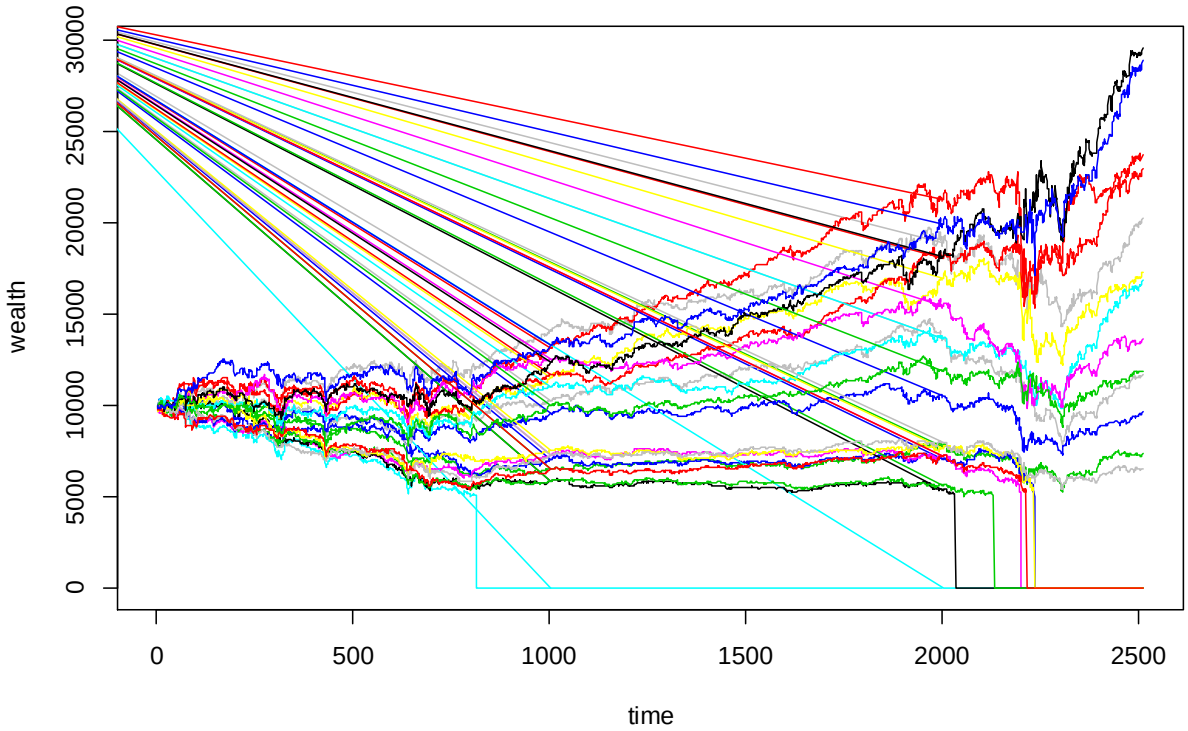

Fig. 2. Wealth of agents from the experiment with p ~R[0.4; 0.6]. For simplicity only 20 of 1000 trajectories of the wealth value $\boldsymbol{w_{i,t}}$ over time are given.

The left panel of Fig. 3 shows agent wealth on the final date, and the right panel shows the frequency of bankruptcies for p ~R[0.4; 0.6].

The average welfare of not-bankrupts on the final date is slightly lower than the initial wealth and equals to 9 934, and the highest possible level of wealth in this experiment is 47 610 (about 5 times higher than the initial level). The share of agents with the final wealth higher than the initial level is equal to 46.7% (Fig.3, left panel).

During the experiment 332 of 1000 traders were declared bankrupt. The results show that agents with a lower value of p have lower wealth on the final date and a higher probability of bankruptcy (Fig. 3, right panel).

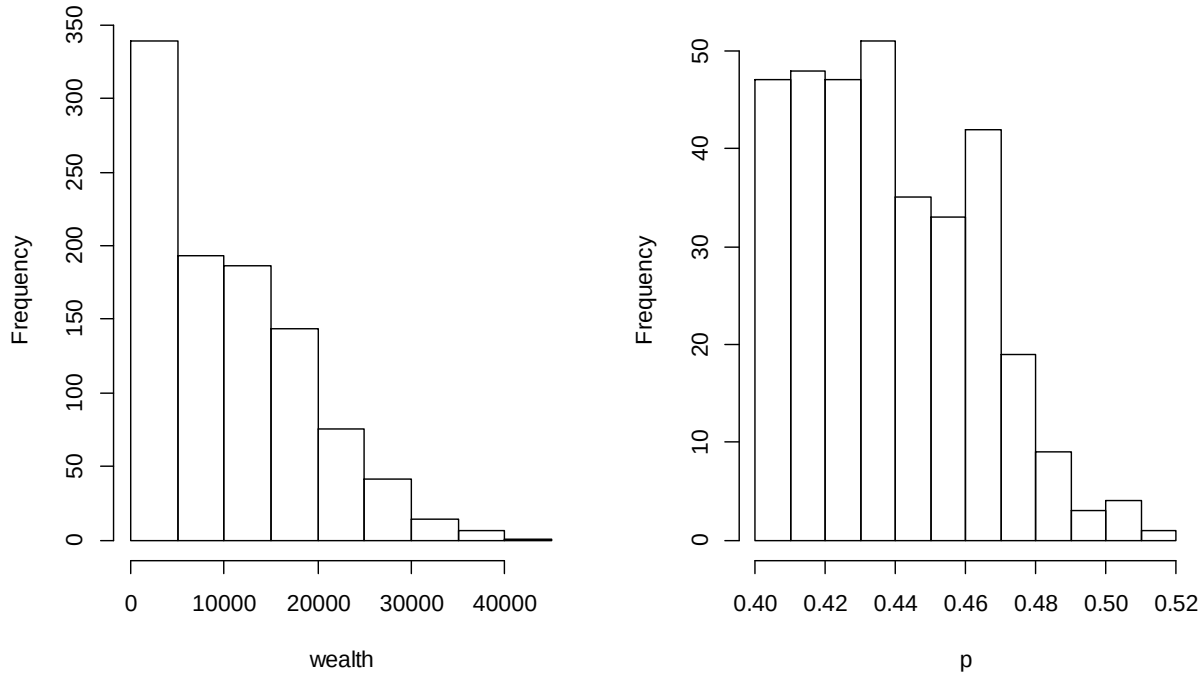

Fig. 3. The left panel - the histogram of the agent's wealth on the final date. The right panel - the histogram of the probability p among all bankrupts for the case p ~R[0.4; 0.6].

Further we investigate how the three main parameters (the average wealth of surviving agents, the fraction of agents whose the final wealth increased, and the fraction of bankruptcies in the whole population) depend on the value of p. We used a p not selected from a uniform distribution $R[p_{min}, p_{max}]$, but equal for all agents and repeated this 150 times. We then varied p from 0.3 to 0.7 with increments of 0.01, making a total of 6150 experiments for each level of leverage.

Fig. 4 and Table 1 show the results without margin trading and short selling for p from 0.47 to 0.57. On the left panel of Fig. 4 is the average final wealth (the value of p is on the horizontal axis and the average final wealth is on the vertical axis), on the central panel is the share of agents, whose wealth on the final date is higher than the initial level (the value of p is on the horizontal axis and the percentage of such agents in the whole population is on the vertical axis), and on the right panel is the share of bankruptcies in the whole population (the value of p is on the horizontal axis and the percentage of such agents in the whole population is on the vertical axis). The circles correspond to the results of experiments, the lines denote the average values of the three parameters for the whole set of experiments.

You can see that for agents with $p \geq 0.52$ the probability of bankruptcy is less than 1%, and without margin trading the agents with $p \geq 0.56$ never experienced a bankruptcy (remembering that a bankruptcy is here defined as a decline of agent's wealth below the threshold of 5000, that is, half of the initial wealth).

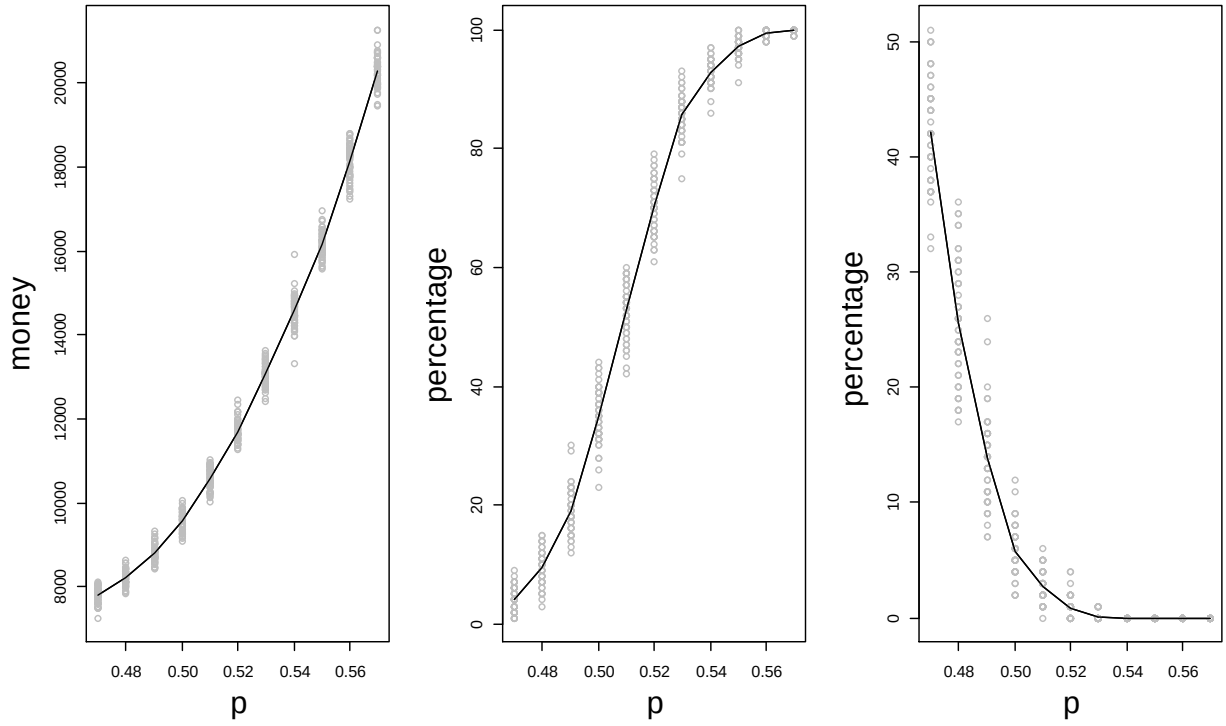

Fig. 4. The left panel - the average final wealth, the central panel - the share of agents, whose wealth on the final date is higher than the initial level, the right panel - the share of bankrupts in the whole population for the case of prohibited margin trading.

In Fig. 4 and Table 1, when p increases the wealth of the surviving agents grows, the fraction of bankrupts also grows and the fraction of agents with the final wealth greater than the initial reduces.

Tab. 1. Results of the experiments without margin trading
(*leverage*=0)

	p	№ of experiment							Average value
		№1	№2	№3	№4	№5	№6	…	
The average wealth of not-bankrupts on the final date, $									
1	0,51	10 396	10 511	11 076	10 796	10 542	10 657	…	10 594
2	0,52	12 213	11 854	11 993	11 720	12 424	11 669	…	11 703
3	0,53	13 117	13 187	13 184	13 035	12 915	12 776	…	13 068
4	0,54	14 735	14 920	14 744	14 390	15 156	14 288	…	14 588
5	0,55	16 292	16 112	16 079	16 569	16 078	15 852	…	16 162
6	0,56	17 763	18 337	18 783	17 755	18 424	18 620	…	18 140
The fraction of agents with the wealth on the final date greater than the initial level, %									
1	0,51	46	54	56	55	52	53	…	52.8
2	0,52	76	71	78	71	78	69	…	70.3
3	0,53	86	83	84	86	85	86	…	85.7
4	0,54	94	96	94	96	94	93	…	92.8
5	0,55	99	97	94	100	99	98	…	97.1
6	0,56	99	100	100	100	98	100	…	99.3
The fraction of bankrupts, %									
1	0,51	1	2	4	3	2	1	…	2.7
2	0,52	0	0	1	0	0	1	…	0.9
3	0,53	0	0	0	0	0	0	…	0.2
4	0,54	0	0	0	0	0	0	…	0.0
5	0,55	0	0	0	0	0	0	…	0.0
6	0,56	0	0	0	0	0	0	…	0.0

Of course, if we allow the holding of short positions and margin trading, then the frequency of bankruptcies significantly increases compared with the basic model. Note that welfare increases as well, especially for agents with high values of p (Table 2).

Tab. 2. Results of experiments with margin trading and short selling
(with different level of leverage)

		The fraction of agents with $wealth_{i,T} \geq wealth_0$, %			
	p	$leverage = 0$	$leverage = 2$	$leverage = 5$	$leverage = 10$
1	0.50	35.4	29.8	10.3	1.3
2	0.51	52.8	47.7	20.2	3.6
3	0.52	70.3	67.6	38.0	8.7
4	0.53	85.7	85.4	56.2	18.2
5	0.54	92.8	93.1	69.3	29.2
6	0.55	97.1	97.4	78.5	40.0
7	0.56	99.3	98.7	86.3	50.6
8	0.57	99.7	99.3	89.1	59.2
9	0.58	99.9	99.5	92.7	67.6
10	0.59	99.9	99.8	94.9	73.5
11	0.60	100.0	99.9	97.3	78.2
12	0.61	100.0	99.9	98.2	82.1
13	0.62	100.0	100.0	98.5	84.5
14	0.63	100.0	100.0	98.6	86.7
15	0.64	100.0	100.0	99.5	90.4
16	0.65	100.0	100.0	99.5	92.9
		The fraction of bankrupts,%			
1	0.50	6.2	52.2	88.5	98.6
2	0.51	2.7	37.5	77.6	96.2
3	0.52	0.9	20.7	60.6	90.8
4	0.53	0.2	10.2	42.4	81.4
5	0.54	0.0	5.0	30.3	70.5
6	0.55	0.0	2.4	21.4	59.8
7	0.56	0.0	1.0	13.7	49.3
8	0.57	0.0	0.7	10.9	40.8
9	0.58	0.0	0.5	7.3	32.4
10	0.59	0.0	0.0	5.1	26.5
11	0.60	0.0	0.0	2.7	21.8
12	0.61	0.0	0.0	1.8	17.9
13	0.62	0.0	0.0	1.5	15.5
14	0.63	0.0	0.0	1.4	13.3
15	0.64	0.0	0.0	0.5	9.6
16	0.65	0.0	0.0	0.5	7.0

Now we summarize the results for all 6 stock indices. The main conclusion is based on Table 3, and detailed results can be found in Appendix 1. Even without margin trading an agent needs to have a value of p higher than 0.5 to guarantee with high probability that the final wealth is greater than the initial, and to assure very little probability of bankruptcy. Margin trading requires higher values of p for the same results. For margin trading with a leverage rate of 1:10 the agent should have an extremely high p, which could be possible for insiders, for instance.

Tab. 3. Parameters for 6 stock indices

		S&P 500	CAC 40	DAX	FTSE	Nikkei 225	Hang Seng
				$leverage = 0$			
1	The value of p, such that a surviving agent with higher p increases their wealth	0.51	0.51	0.50	0.51	0.52	0.48
2	The value of p, such that the probability of increasing wealth is higher than 99%	0.56	0.55	0.56	0.56	0.57	0.54
3	The value of p, such that the probability of increasing wealth is higher than 99,9%	0.60	0.57	0.60	0.58	0.59	0.58
4	The value of p, such that the probability of bankruptcy is less than 1%	0.52	0.54	0.56	0.52	0.54	0.52
5	The value of p, such that the probability of bankruptcy is less than 0,1%	0.54	0.57	0.60	0.56	0.58	0.58
				$leverage = 2$			
1	The value of p, such that a surviving agent with higher p increases their wealth	0.48	0.47	0.47	0.48	0.48	0.47
2	The value of p, such that the probability of increasing wealth is higher than 99%	0.57	0.57	0.58	0.56	0.58	0.58
3	The value of p, such that the probability of increasing wealth is higher than 99,9%	0.63	0.64	0.61	0.63	>0.65	0.65
4	The value of p, such that the probability of bankruptcy is less than 1%	0.57	0.61	0.60	0.59	0.58	0.58
5	The value of p, such that the probability of bankruptcy is less than 0,1%	0.61	0.64	0.61	0.63	>0.65	0.65
				$leverage = 5$			
1	The value of p, such that the probability of increasing wealth is higher than 99%	0.64	0.62	0.63	0.61	0.63	0.71
2	The value of p, such that the probability of increasing wealth is higher than 99,9%	0.75	0.72	>0.75	0.71	0.72	>0.75
3	The value of p, such that the probability of bankruptcy is less than 1%	0.64	0.62	0.64	0.61	0.63	0.71
4	The value of p, such that the probability of bankruptcy is less than 0,1%	0.75	0.72	>0.75	0.71	0.71	>0.75
				$leverage = 10$			
1	The value of p, such that the probability of increasing wealth is higher than 99%	0.74	0.70	0.80	0.70	0.73	>0.85
2	The value of p, such that the probability of increasing wealth is higher than 99,9%	>0.85	>0.85	>0.85	>0.85	>0.85	>0.85
3	The value of p, such that the probability of bankruptcy is less than 1%	0.74	0.70	0.80	0.70	0.73	>0.85
4	The value of p, such that the probability of bankruptcy is less than 0,1%	>0.85	>0.85	>0.85	>0.85	>0.85	>0.85

In Table 4 we present the results for the agent, who tosses a coin, i.e. her p is equal to 0.5. Without margin trading such agents on average cannot make a profit on the stock exchange, but using leverage causes dramatic increase of bankruptcy probability. Not many people would pay $10 000 for a lottery ticket with a 1% probability of winning $130 000 and a 99% probability of losing everything.

Tab. 4. Parameters for agent with $p = 0.5$

				$leverage = 0$			
		S&P 500	CAC 40	DAX	FTSE	Nikkei 225	Hang Seng
1	The average wealth of survivors, ($)	9 584	9 464	11 098	9 667	8 929	12 719
2	The fraction of agents who increase their wealth, (%)	35.35	28.05	45.35	37.36	18.83	71.12
3	The fraction of bankrupts, (%)	6.17	19.58	23.17	3.95	24.09	5.16
				$leverage = 2$			
1	The average wealth of survivors, ($)	14 131	19201	17 348	14 110	15 637	19 606
2	The fraction of agents who increase their wealth, (%)	29.80	20.44	25.73	29.25	18.87	34.99
3	The fraction of bankrupts, (%)	52.21	72.06	65.20	54.40	71.95	55.87
				$leverage = 5$			
1	The average wealth of survivors, ($)	43 427	64 072	66 643	44 156	56 847	61 566
2	The fraction of agents who increase their wealth, (%)	10.31	5.47	7.67	10.35	5.51	9.06
3	The fraction of bankrupts, (%)	88.54	93.95	91.16	88.12	93.66	89.86
				$leverage = 10$			
1	The average wealth of survivors, ($)	2e+05	3e+05	2e+05	3e+05	5e+05	3e+05
2	The fraction of agents who increase their wealth, (%)	1.32	0.21	0.67	1.26	0.47	0.89
3	The fraction of bankrupts, (%)	98.62	99.79	99.27	98.63	99.51	99.09

The main results:

1. If an agent is random in her decisions ($p = 0.5$) and does not use margin trading, then after 10 years of trading her average wealth will be around 9500, she will have positive return from trading with probability 0.35 and she has a 6% probability of bankruptcy.
2. If an agent does not use margin trading, it is sufficient to have $p \geq 0.52$ to 'survive' in the market for 10 years with a probability more than 0.99. The probability to gain from trading overall in this case is 0.93. And if $p \geq 0.56$, the agent is very unlikely to become bankrupt and will get positive return with a probability of 0.7. Such small values of p seem very attractive for small traders as they only need to marginally outperform the random trader. This could be a reason why the stock market is so appealing.
3. The decision to use margin buying and short selling changes the situation dramatically. The random agent (p = 0.5) will survive 10 years of trading with a probability of 0.48 when leverage is 1:2, with a probability of 0.11 when leverage is 1:5 and with a probability of only 0.01 when leverage is 1:10. Although the 10-year return will be 50%, 300% and 2000% respectively, in these cases, the random strategy shows the excessive risk.
4. If an agent uses margin trading, she needs to be more sophisticated and to have $p \geq 0.7$ to survive with probability 0.99 and $p > 0.85$ to survive with 0.001 probability. For ordinary traders such rate of success is almost impossible.

4.2 The results for the model with followers

In this section we discuss the results for the ‘follower’ strategy first for S&P 500 and for all indices at the end of the subsection. In Table 5, *leverage*=0 leads to fairly good results, the probability of bankruptcy is relatively small (lower than 0.09 for S&P500 and 0.15 for other indices) and wealth increases with a non-zero probability. Compared to the basic model, where p strictly determines all three parameters and it is preferable to have high p, the follower can earn without respect to her leader’s p, although the average return is close to 0. The advantage of follower strategy is the relatively small probability of bankruptcy without respect to the value of leader’s p.

If an agent uses margin trading, the average wealth and probability of bankruptcy grow. So, the ‘follower’ strategy can be considered mostly more profitable in comparison with the strategy of ‘tossing a coin’ (Table 4), when an agent does not use short selling and margin trading.

The high standard deviation of the average final wealth when *leverage*=10 is caused by the large number of agents being declared bankrupt.

Tab. 5. Results for 'follower' strategy for S&P500 data

	$leverage = 0$			$leverage = 2$			$leverage = 5$			$leverage = 10$		
Value of leader's p	Average wealth of follower, $	Better wealth of follower, %	Bankrupts among followers, %	Average wealth of follower, $	Better wealth of follower, %	Bankrupts among followers, %	Average wealth of follower, $	Better wealth of follower, %	Bankrupts among followers, %	Average wealth of follower, $	Better wealth of follower, %	Bankrupts among followers, %
0.40	11 044	31.0	0.7	21 085	50.9	35.5	91 778	21.8	75.5	51 940	1.8	98.2
0.41	11 018	28.8	0.9	18 032	49.1	40.1	48 753	14.5	80.9	260 832	2.5	97.5
0.42	10 654	28.5	1.0	17 378	46.4	38.2	66 802	16.4	81.8	30 727	1	99
0.43	10 728	26.2	0.9	17 584	47.3	36.4	47 134	15.5	83.6	323 271	0.5	99.5
0.44	10 336	25.3	1.4	16 845	40.9	42.7	49 738	12.7	84.5	59 670	1.2	98.8
0.45	10 131	23.1	1.5	17 378	32.7	48.2	52 434	18.9	79.1	163 849	1.6	98.64
0.46	9 926	22.2	1.9	16 780	44.5	45.5	50 334	17.3	80.9	230 542	1.4	98 .6
0.47	9 970	21.2	2.1	15 188	38.2	44.5	85 579	8.3	89.1	18 023	1.3	98.7
0.48	9 946	19.8	2.3	15 383	28.2	59.1	37 389	14.8	83.6	59 015	0.7	99.3
0.49	9 603	19.3	2.5	13 030	27.3	60.9	65 388	19.1	79.1	29 822	1.9	98.1
0.50	9 590	17.5	3.2	13 006	31.8	50.8	39 833	10.6	87.3	28 308	2.8	97.2
0.51	9 559	17.0	3.1	12 995	28.2	48.2	52 156	16.1	82.7	18 291	1.5	98.5
0.52	9 408	15.8	3.6	13 141	25.5	54.5	39 057	8.5	87.3	-	0	100
0.53	9 443	13.9	4.4	12 949	21.8	58.2	87 119	4.5	95.5	14 227	0.1	99.9
0.54	9 050	12.9	4.7	12 185	16.4	67.3	36 900	10.9	88.2	-	0	100
0.55	8 839	11.7	5.1	12 551	20.9	64.5	28 826	7.3	90.9	639 577	0.9	99.1
0.56	9 012	11.7	6.0	12 314	14.5	66.4	18 045	2.4	96.4	-	0	100
0.57	9 069	10.0	6.8	11 164	13.6	73.6	16 397	6.4	90.9	55 243	1.4	98.6
0.58	8 592	9.2	7.5	10 527	10.0	70.0	95 561	5.9	93.6	18 908	2.1	97.9
0.59	8 414	8.1	8.1	10 961	8.2	81.8	28 376	1.8	97.3	-	0	100
0.60	8 590	7.4	9.0	11 061	12.7	76.4	16 725	2.7	95.5	75 147	2.2	97.8

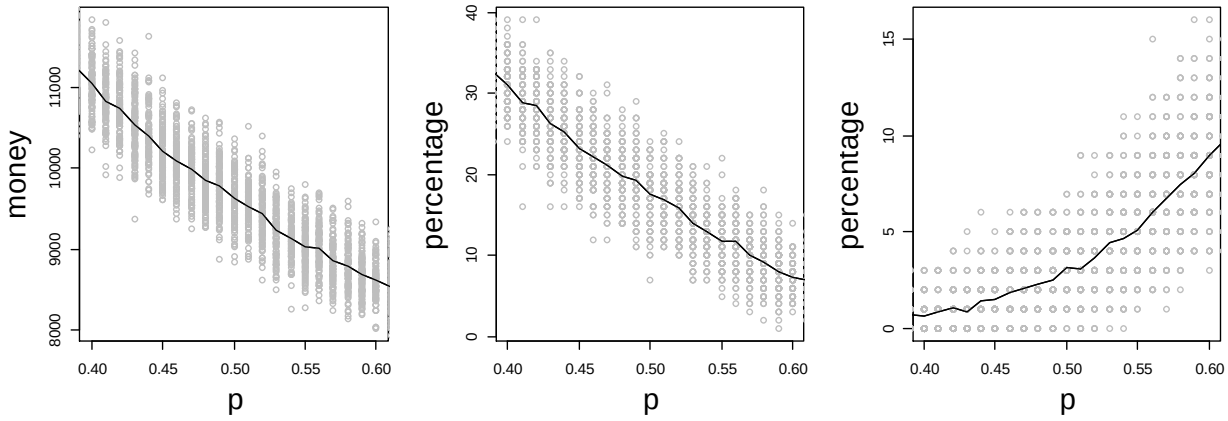

Fig. 5. The left panel - the average final wealth; the central panel - the share of agents, whose wealth on the final date is higher than the initial level; the right panel - the share of bankrupts in the whole population for the case of prohibited margin trading.

The results for $leverage = 0$ are shown in Fig.5. On the left panel is the average final wealth (the value of p is on the horizontal axis and the size of average final wealth is on the vertical axis), on the central panel is the share of agents whose wealth on the final date is higher

than the initial level (the value of p is on the horizontal axis and the percentage of such agents in the whole population is on the vertical axis), and on the right panel is the share of bankrupts in the whole population (the value of p is on the horizontal axis and the percentage of such agents in the whole population is on the vertical axis). For all these experiments the value of p was from 0.4 to 0.6 in increments of 0.01. The circles correspond to individual results, the lines denote the average values of the three parameters for the whole set of experiments.

The evident slope of all parameters, giving a small advantage for lower values of leader's p, can be explained as a particular property of chosen simplest strategy of imitation. Since our followers simply repeat the decision of her leader with a one step delay, the key factor of her success is the frequency of trend changes. If the right decision for Day 1 (see Fig.6) was 'to buy' and for Day 2 is 'to sell' and the leader was qualified enough to make these proper decisions (i.e. leader has high p), then the follower will make mistake on Day 2 choosing 'to buy' after her leader, because price trend changes on Day 2 from bullish to bearish. And follower's decision to sell on Day 3 will be correct as price trend continues to decline. Conversely, if the leader more often make wrong decisions (her p is low), then she will be likely to choose 'to sell' on Day 1 and 'to buy' on Day 2, hence, the follower will gain on Day 2 and lose money on Day 3. And the same reasons can be given for the days when trend was switched from bearish to bullish (see Day 4 on Fig.5).

The S&P500 data has 2514 observation days and 54,5% are trend-switching, so it is slightly more profitable to follow weak leaders, the CAC 40 52,7%, the DAX 51,5%, the FTSE 52,5%, the Nikkei 225, 51,5% and the Hang Seng 51,35%.

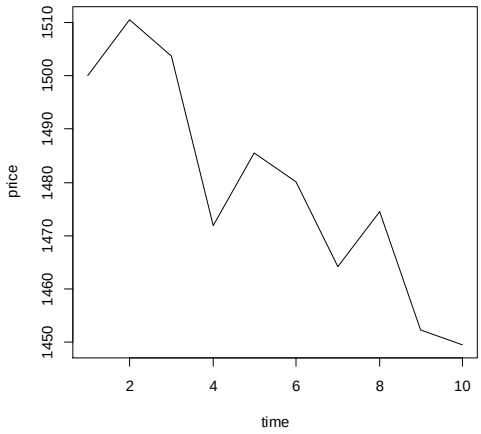

Fig. 6. The price data

For all 6 stock indices the results are very similar. Without margin trading, the probability of the bankruptcy is low and the final wealth of the followers is close to the initial value, even for the Nikkei (always slightly lower) and DAX and Hang Seng (always slightly higher) indices. And for margin trading with a leverage rate of 1:10 the probability of bankruptcy was extremely

high, although a successful agent can increase her wealth by a factor from 10-100. The detailed results can be found in Appendix 2.

The main results:

1. The follower strategy is almost independent of the quality of the leader. The strategy of simply repeating the leader's actions and the features of the data leads to a small advantage in having poorly qualified leader.
2. Even if the leader is unsuccessful and has a low p, the follower has a return close to 0 when $leverage = 0$, but the advantage of a 'follower' strategy is the relatively small probability of bankruptcy.
3. If an agent uses short selling and margin buying, the probability of bankruptcy increases and the probability having a positive return decreases.
4. In general, three parameters for all leverage rates for the 'follower' strategy are very close to the parameters of random strategy. Thus, the imitation strategy in general cannot help stock exchange earnings.
5. If the trader does not know her value of p (a newcomer, for example), the 'follower' strategy can be preferable from the viewpoint of safety, as the probability of bankruptcy is low.

4.3 Results for the model with black swans

In our model $N, = 1000$ and $\delta = 0.3$. Half of the agents use Taleb's strategy, that is, they earn less on a regular day and a lot in the period of crisis, so for them p^Q was selected from $R[0.4; 0.5]$ and p^R from $R[0.7; 0.8]$. The others use traditional strategy and for them p^Q was selected from $R[0.5; 0.6]$ and p^R from $R[0.2; 0.3]$. The results are shown on Fig. 6.

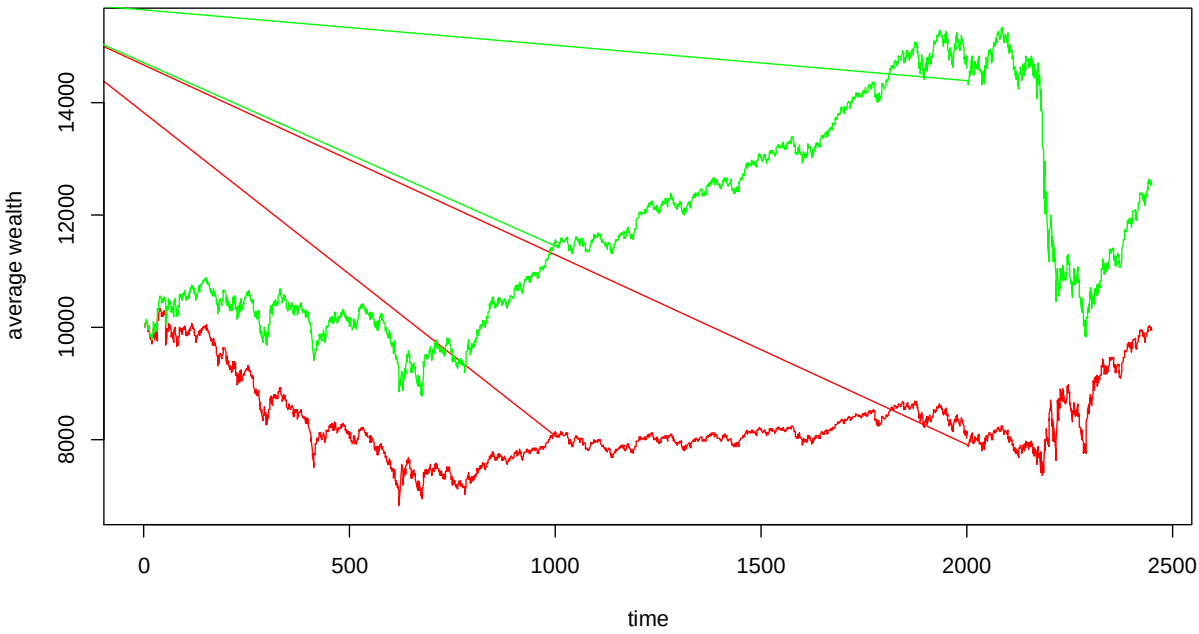

Fig.6. The average wealth of ordinary agents (green line) and the "black swan seekers" (red line).

The wealth of ordinary agents is higher on average than the wealth of Taleb's followers, although it is greatly reduced during the crisis. Histograms of the wealth distribution are shown in Fig. 7, where on the left panel there is the histogram of the agent's wealth on the final date for the sample of the 'black swan seekers', and on the left panel there is the histogram of the agent's wealth on the final date for the sample of ordinary agents (the size of the final wealth is on the horizontal axis and the frequency of the values is on the vertical axis).

Perhaps the low efficiency of the 'black swan seekers' in a period of stability cannot be counterbalanced for many of them by their gains during crisis days.

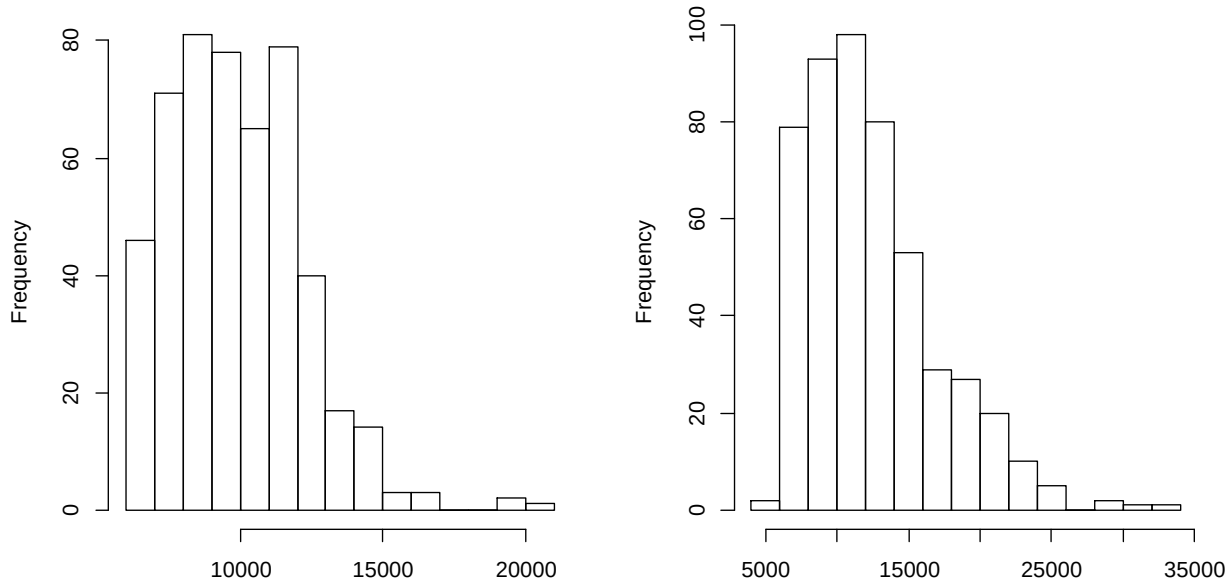

Fig.7. The left panel shows agent wealth on the final date for the sample of the ‘black swan seekers’, the left panel shows agent wealth on the final date for the sample of ordinary agents.

"Black swan seekers" are much more exposed to the risk of bankruptcy (Fig. 8, the value of *p* is on the horizontal axis and the frequency of the values is on the vertical axis) compared with ordinary agents. In this model there were 239 bankruptcies in the group of ‘black swan seekers’ and only 12 bankruptcies in the group of ordinary traders.

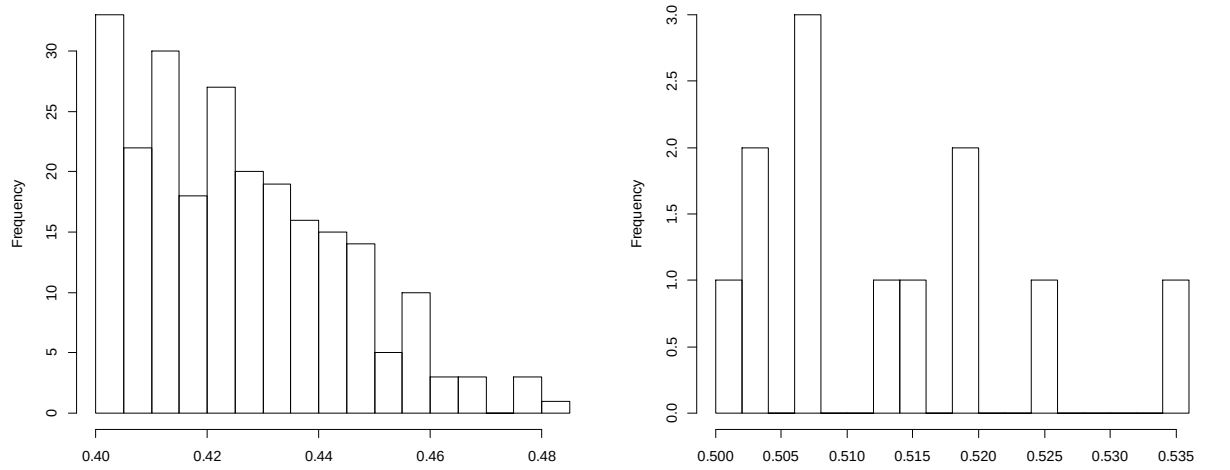

Fig.8. The left panel shows the probability *p* among all bankruptcies for ‘black swan seekers’, the right panel shows the probability *p* among all bankruptcies for ordinary agents.

The distribution of parameter *p* among the bankruptcies in both groups are consistent with the results of the basic model.

The main results:

1. The average 10-year wealth of the ‘black swan seeker’ strategy is comparable with the average wealth of the traditional strategy, but the probability of failing is much higher.

Thus, the traditional strategy of traders is not worse than Taleb's strategy. Black swans are too rare an event to be the basis of investment decisions.

5. Conclusion

We have analyzed different trading strategies of small and medium-size investors on the stock exchange, and the consequences of using short selling and margin buying.

The results show that there is a critical level of agent experience such that agents with this or higher level are almost sure to survive in the market over the long run. This critical level is just slightly higher than ½ for cautious behavior (i.e. the absence of margin trading) [Aleskerov and Egorova 2012], and such a small value could explain why so many people try to trade on the stock exchange. But if trader uses margin trading, the critical level is much higher as is the risk of excessive losses.

When the agent randomly chooses to buy or to sell assets (i.e. her probability of making the right decision is 0.5) it is disadvantageous without margin trading and dangerous with it, because the probability of bankruptcy is very high. For a small trader without trading experience or access to insider information (i.e. without a high p) it can be more reasonable to follow another trader's decision. In contradiction to the seemingly obvious suggestion that following the agent with high prediction rate would be more profitable, we found the reverse, although the difference between 'following a good predictor' and 'following a bad predictor' strategies is not significant. The follower strategy can be rational and appealing for traders as an attempt to receive more information, but the reverse is a herding effect that can lead to the large losses, chaos and speculative bubbles in the market (see, for example [Corcos et al. 2002], [Cont and Bouchaud 2000], [Harras and Sornette 2011]).

Another result shows the danger of a "black swan strategy". There may be so few crisis days that a trader can go bankrupt while waiting for the opportunity to catch a 'black swan', and make up for her losses, although we cannot exclude the case of success with this strategy.

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Appendix A.

$leverage = 0$

		Average wealth of not-bankrupts						Fraction of agents with $wealth_{i,T} \geq wealth_0$, %						Fraction of bankrupts,%					
	p	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng
1	0.45	7 017	6 756	7 484	6 959	6 704	8 171	0.35	0.05	0.61	0.32	0.02	3.33	77.35	95.47	88.76	71.32	95.71	72.24
2	0.46	7 358	6 957	7 811	7 235	6 970	8 671	1.52	0.21	1.80	1.03	0.17	8.43	60.26	90.11	77.98	53.52	89.43	55.81
3	0.47	7 686	7 249	8 462	7 646	7 290	9 303	3.13	1.31	6.19	3.54	0.92	19.33	42.48	74.76	65.27	34.99	77.05	36.90
4	0.48	8 178	7 796	9 101	8 157	7 696	10 184	9.70	4.36	13.72	9.59	2.77	35.00	25.00	54.70	50.15	19.75	61.36	22.36
5	0.49	8 838	8 528	9 984	8 870	8 223	11 264	20.00	13.00	27.70	21.17	8.45	53.47	14.04	35.18	35.54	9.91	41.30	10.99
6	0.50	9 584	9 464	11 098	9 667	8 929	12 719	35.35	28.05	45.35	37.36	18.83	71.12	6.17	19.58	23.17	3.95	24.09	5.16
7	0.51	10 594	10 941	12 473	10 741	9 809	14 564	52.80	50.65	64.13	57.52	35.13	85.49	2.68	9.94	14.92	1.35	11.67	1.78
8	0.52	11 703	12 770	14 125	11 895	11 004	16 788	70.27	71.76	79.57	74.16	54.92	94.38	0.90	4.23	8.93	0.47	5.03	0.62
9	0.53	13 068	15 179	16 162	13 198	12 529	19 241	85.74	87.18	90.67	86.91	73.90	97.85	0.24	1.33	4.39	0.07	1.86	0.15
10	0.54	14 588	18 121	18 514	14 770	14 409	22 278	92.83	96.06	95.87	94.54	87.49	99.32	0.00	0.25	2.27	0.01	0.49	0.03
11	0.55	16 162	21 752	21 241	16 512	16 594	25 691	97.08	99.00	98.11	98.19	95.38	99.90	0.00	0.29	1.37	0.02	0.07	0.00
12	0.56	18 140	25 765	24 337	18 405	19 026	29 689	99.32	99.82	99.34	99.46	98.58	99.99	0.00	0.06	0.52	0.00	0.07	0.00
13	0.57	20 349	31 098	28 258	20 509	21 829	34 357	99.74	100.00	99.79	99.94	99.50	99.99	0.00	0.00	0.21	0.00	0.02	0.01
14	0.58	22 527	37 127	32 614	22 891	25 356	39 630	99.86	100.00	99.86	100.00	99.91	100.00	0.00	0.00	0.13	0.00	0.00	0.00
15	0.59	25 109	43 663	37 631	25 605	29 164	45 622	99.94	100.00	99.98	100.00	100.00	100.00	0.00	0.00	0.02	0.00	0.00	0.00
16	0.60	28 412	52 552	43 410	28 599	33 500	53 151	100.0	100.00	100.00	100.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00

leverage $= 2$

		Average wealth of not-bankrupts						Fraction of agents with $wealth_{i,T} \geq wealth_0$, %						Fraction of bankrupts,%					
	p	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng
1	0.45	7 903	7 015	8 623	7 504	7 463	8 538	0.05	0.00	0.07	0.01	0.00	0.02	99.70	100.00	99.79	99.80	99.95	99.82
2	0.46	7 945	7926	8 551	8 554	9 396	9 902	0.31	0.06	0.16	0.33	0.11	0.53	98.50	99.83	99.23	98.64	99.67	98.47
3	0.47	9 220	10656	10 170	9 182	8 985	10 798	1.34	0.61	1.17	1.37	0.40	2.26	95.35	98.89	96.92	95.17	98.60	95.06
4	0.48	10 252	12136	11 415	10 217	10 964	12 603	4.87	2.27	3.86	5.01	2.34	7.48	86.25	95.92	91.75	87.10	94.77	86.93
5	0.49	11 684	14348	13 718	11 663	12 682	15 471	13.56	6.72	11.71	14.19	8.11	17.89	73.55	88.72	80.97	72.41	85.73	74.02
6	0.50	14 131	19201	17 348	14 110	15 637	19 606	29.80	20.44	25.73	29.25	18.87	34.99	52.21	72.06	65.20	54.40	71.95	55.87
7	0.51	17 681	25554	23 421	17 836	20 816	26 529	47.67	42.55	44.25	50.72	36.61	55.35	37.53	49.83	48.30	35.17	53.97	37.91
8	0.52	22 691	38547	33 017	23 773	28 440	37 473	67.63	65.16	62.97	70.25	57.20	72.12	20.66	29.50	32.38	20.77	35.66	24.12
9	0.53	31 491	66399	48 469	32 886	41 916	55 670	85.43	78.94	79.00	84.77	74.45	84.93	10.18	19.22	19.14	10.93	21.77	13.53
10	0.54	46 256	1e+05	73 823	47 445	63 143	86 725	93.10	91.03	88.45	93.32	86.23	91.65	5.01	8.11	10.93	5.24	12.26	7.76
11	0.55	68 028	2e+05	1e+05	68 872	1e+05	1e+05	97.42	94.74	93.96	97.20	93.44	95.29	2.43	5.16	5.90	2.56	6.21	4.61
12	0.56	1e+05	4e+05	2e+05	1e+05	2e+05	2e+05	98.69	98.00	97.53	99.01	96.42	97.72	1.01	1.95	2.44	0.96	3.49	2.27
13	0.57	1e+05	7e+05	3e+05	1e+05	3e+05	3e+05	99.31	99.37	98.79	99.59	98.09	98.75	0.69	0.63	1.21	0.40	1.90	1.25
14	0.58	2e+05	1e+06	4e+05	2e+05	4e+05	5e+05	99.48	99.63	99.55	99.88	99.15	99.43	0.54	0.37	0.45	0.12	0.85	0.57
15	0.59	3e+05	2e+06	7e+05	3e+05	7e+05	8e+05	99.83	99.79	99. 85	99.94	99.70	99.77	0.04	0.21	0.15	0.06	0.30	0.23
16	0.60	5e+05	4e+06	1e+06	5e+05	1e+06	1e+06	99.87	99.89	99.98	99.95	99.87	99.86	0.02	0.11	0.02	0.05	0.13	0.14
17	0.61	7e+05	6e+06	2e+06	7e+05	2e+06	2e+06	99.92	99.93	100.00	100.00	99.92	99.95	0.00	0.07	0.00	0.00	0.08	0.05
18	0.62	1e+06	1e+07	3e+06	1e+06	3e+06	3e+06	99.99	99.98	100.00	99.99	99.97	99.96	0.00	0.02	0.00	0.01	0.03	0.04
19	0.63	1e+06	2e+07	5e+06	2e+06	5e+06	5e+06	100.00	99.99	100.00	100.00	100.00	99.98	0.00	0.01	0.00	0.00	0.00	0.02

20	0.64	2e+06	3e+07	7e+06	2e+06	8e+06	9e+06	100.00	100.00	100.00	100.00	100.00	99.99	0.00	0.00	0.00	0.00	0.00	0.01
21	0.65	3e+06	5e+07	1e+07	3e+06	1e+07	1e+07	100.00	100.00	100.00	100.00	99.99	100.00	0.00	0.00	0.00	0.00	0.01	0.00

$leverage = 5$

		Average wealth of not-bankrupts						Fraction of agents with $wealth_{i,T} \geq wealth_0$, %						Fraction of bankrupts,%					
	p	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng
1	0.45	-	-	-	8 409	9.461	-	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	100.00	99.99	99.99	100.00
2	0.46	46 652	-	17 356	14 622	-	28 153	0.03	0.00	0.03	0.03	0.00	0.04	99.96	100.00	99.97	99.95	100.00	99.95
3	0.47	10 248	21 345	17 147	14 042	13 374	20 353	0.08	0.01	0.09	0.11	0.06	0.22	99.84	99.99	99.87	99.85	99.92	99.67
4	0.48	29 051	38 195	22 942	21 101	26 838	30 908	1.32	0.21	0.58	1.06	0.28	0.85	98.17	99.74	99.19	98.57	99.63	98.86
5	0.49	27 169	59 652	45 005	26 693	41 355	42 835	3.27	0.95	2.47	3.59	1.47	3.15	96.01	98.79	97.13	95.38	98.14	96.20
6	0.50	43 427	64 072	66 643	44 156	56 847	61 566	10.31	5.47	7.67	10.35	5.51	9.06	88.54	93.95	91.16	88.12	93.66	89.86
7	0.51	66 700	2e+05	1e+05	72 086	1e+05	1e+05	20.24	14.68	17.50	21.71	13.70	19.74	77.63	84.16	81.25	75.55	85.24	78.63
8	0.52	1e+05	3e+05	3e+05	1e+05	2e+05	2e+05	37.96	29.47	31.46	38.82	26.79	32.99	60.60	68.95	67.33	59.18	71.69	65.66
9	0.53	3e+05	1e+06	5e+05	3e+05	5e+05	6e+05	56.24	47.84	47.33	55.75	41.80	48.61	42.41	51.63	52.01	43.06	57.37	50.67
10	0.54	6e+05	6e+06	1e+06	6e+05	2e+06	1e+06	69.26	66.84	62.37	70.67	57.26	59.80	30.34	33.00	37.29	28.84	42.42	39.90
11	0.55	1e+06	2e+07	3e+06	1e+06	4e+06	4e+06	78.49	76.89	74.52	81.54	69.85	70.72	21.41	22.89	25.38	18.32	30.00	29.21
12	0.56	4e+06	8e+07	1e+07	4e+06	1e+07	1e+07	86.33	84.74	83.01	88.59	78.80	76.59	13.66	15.33	16.98	11.37	21.19	23.40
13	0.57	8e+06	6e+08	3e+07	9e+06	4e+07	3e+07	89.12	91.06	89.96	93.04	85.54	82.09	10.88	8.94	10.04	6.96	14.46	17.91
14	0.58	2e+07	1e+09	9e+07	2e+07	1e+08	1e+08	92.67	94.44	93.66	96.16	90.73	86.43	7.33	5.56	6.34	3.84	9.27	13.57
15	0.59	6e+07	7e+09	3e+08	6e+07	4e+08	3e+08	94.90	96.94	95.91	97.79	94.00	88.84	5.10	3.06	4.09	2.21	6.00	11.16
16	0.60	2e+08	3e+10	8e+08	2e+08	1e+09	8e+08	97.32	98.39	97.00	98.75	96.23	91.92	2.67	1.61	3.00	1.25	3.77	8.08
17	0.61	4e+08	8e+10	3e+09	4e+08	5e+09	2e+09	98.16	98.86	98.32	99.13	97.83	93.39	1.84	1.14	1.68	0.87	2.17	6.61
18	0.62	1e+09	5e+11	7e+09	1e+09	1e+10	7e+09	98.48	99.05	98.88	99.60	98.56	94.39	1.52	0.95	1.12	0.40	1.44	5.61
19	0.63	3e+09	9e+11	2e+10	3e+09	5e+10	2e+10	98.61	99.48	99.00	99.72	99.11	95.26	1.39	0.52	1.00	0.28	0.89	4.74
20	0.64	7e+09	2e+13	7e+10	8e+09	1e+11	6e+10	99.49	99.70	99.33	99.81	99.50	96.56	0.51	0.30	0.67	0.19	0.50	3.44
21	0.65	2e+10	6e+13	2e+11	2e+10	5e+11	2e+11	99.54	99.81	99.57	99.86	99.82	96.82	0.46	0.19	0.43	0.14	0.18	3.18
22	0.66	4e+10	1e+14	7e+11	5e+10	2e+12	5e+11	99.65	99.92	99.78	99.94	99.83	97.46	0.35	0.08	0.22	0.06	0.17	2.54
23	0.67	1e+11	3e+14	2e+12	1e+11	5e+12	1e+12	99.91	99.95	99.86	99.96	99.90	97.68	0.09	0.05	0.14	0.04	0.10	2.32
24	0.68	3e+11	7e+14	6e+12	4e+11	2e+13	5e+12	99.94	99.98	99.88	99.99	99.98	98.36	0.06	0.02	0.12	0.01	0.02	1.64
25	0.69	8e+11	4e+15	2e+13	1e+12	5e+13	1e+13	99.96	99.99	99.89	99.98	99.97	98.60	0.04	0.01	0.11	0.02	0.03	1.40
26	0.70	2e+12	9e+15	6e+13	3e+12	2e+14	4e+13	99.95	99.99	99.95	99.96	99.97	98.87	0.05	0.01	0.05	0.04	0.03	1.13
27	0.71	4e+12	2e+16	2e+14	6e+12	6e+14	1e+14	99.96	99.99	99.95	100.00	99.98	99.09	0.04	0.01	0.05	0.00	0.02	0.91
28	0.72	8e+12	5e+16	6e+14	2e+13	2e+15	3e+14	99.98	100.00	99.99	100.00	100.00	99.25	0.02	0.00	0.01	0.00	0.00	0.75
29	0.73	2e+13	1e+17	2e+15	4e+13	6e+15	1e+15	99.99	100.00	99.98	100.00	100.00	99.27	0.01	0.00	0.02	0.00	0.00	0.73
30	0.74	5e+13	4e+17	5e+15	1e+14	2e+16	3e+15	99.99	100.00	99.98	100.00	100.00	99.31	0.01	0.00	0.02	0.00	0.00	0.69
31	0.75	1e+14	7e+17	2e+16	3e+14	7e+16	9e+15	100.00	100.00	99.99	100.00	100.00	99.47	0.00	0.00	0.01	0.00	0.00	0.53

$leverage = 10$

		Average wealth of not-bankrupts						Fraction of agents with $wealth_{i,T} \geq wealth_0$, %						Fraction of bankrupts,%					
	p	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng
1	0.45	-	-	-	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00	100.00	100.00
2	0.46	-	-	-	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00	100.00	100.00
3	0.47	-	-	2e+05	4e+04	-	-	0.00	0.00	0.01	0.01	0.00	0.00	100.00	100.00	99.99	99.99	100.00	100.00

4	0.48	2e+04	9e+04	3e+04	3e+04	1e+04	1e+04	0.04	0.01	0.01	0.03	0.01	0.01	99.96	99.95	99.99	99.93	99.99	99.98
5	0.49	2e+04	4e+05	2e+05	2e+05	1e+05	1e+05	0.25	0.11	0.15	0.29	0.08	0.22	99.57	99.89	99.80	99.65	99.92	99.75
6	0.50	2e+05	3e+05	2e+05	3e+05	5e+05	3e+05	1.32	0.21	0.67	1.26	0.47	0.89	98.62	99.79	99.27	98.63	99.51	99.09
7	0.51	3e+05	5e+05	7e+05	4e+05	3e+06	2e+06	3.64	1.05	2.14	3.80	1.74	2.62	96.23	98.89	97.75	95.91	98.19	97.24
8	0.52	3e+06	1e+07	8e+06	2e+06	7e+06	9e+06	8.68	4.32	6.33	9.97	5.12	7.03	90.81	95.58	93.44	89.56	94.66	92.74
9	0.53	9e+06	7e+07	4e+07	5e+06	2e+07	8e+07	18.17	12.58	13.55	19.80	11.29	13.97	81.42	87.37	86.21	79.86	88.42	85.76
10	0.54	5e+07	1e+08	1e+08	4e+07	2e+08	1e+08	29.22	23.89	22.69	32.47	21.38	22.45	70.51	75.95	77.19	67.29	78.44	77.33
11	0.55	2e+08	1e+09	1e+09	6e+08	1e+09	6e+08	40.04	37.74	34.54	45.40	31.93	31.12	59.80	62.16	65.39	54.50	67.92	68.76
12	0.56	6e+08	8e+09	2e+10	1e+09	1e+10	9e+09	50.55	49.21	45.66	57.62	44.83	38.92	49.44	50.79	54.33	42.36	55.13	61.04
13	0.57	1e+10	3e+10	9e+10	9e+09	1e+11	1e+11	59.21	60.42	55.33	66.67	53.47	46.01	40.79	39.58	44.65	33.33	46.51	53.98
14	0.58	6e+10	1e+11	7e+11	5e+10	7e+11	4e+11	67.63	70.68	63.64	74. 88	63.83	52.22	32.37	29.32	36.36	25.12	36.17	47.78
15	0.59	3e+11	2e+12	7e+12	3e+11	2e+13	3e+12	73.45	77.26	71.94	80.90	71.88	57.70	26.55	22.74	28.06	19.10	28.12	42.30
16	0.60	2e+12	2e+13	4e+13	2e+12	1e+14	2e+13	78.19	82.53	76.24	85.38	77.49	61.69	21.81	17.47	23.76	14.62	22.51	38.31
17	0.61	1e+13	1e+14	2e+14	2e+13	6e+14	2e+14	82.06	87.54	80.87	88.43	82.90	64.60	17.94	12.46	19.13	11.57	17.10	35.40
18	0.62	8e+13	4e+15	3e+15	9e+13	2e+16	2e+15	84.49	90.95	84.22	91.83	86.76	69.06	15.51	15.78	15.78	8.17	13.24	30.94
19	0.63	5e+14	3e+16	2e+16	8e+14	1e+17	1e+16	86.72	93.26	86.02	93.48	90.15	71.57	13.28	13.98	13.98	6.52	9.85	28.43
20	0.64	2e+15	6e+17	4e+17	6e+15	7e+17	1e+17	90.43	94.48	88.46	94.82	92.21	74.27	9.57	11.54	11.54	5.18	7.79	25.73
21	0.65	2e+16	3e+18	3e+18	3e+16	7e+18	3e+18	92.89	96.14	90.23	96.02	93.97	76.54	7.11	9.77	9.77	3.98	6.03	23.46
22	0.66	2e+17	2e+19	2e+19	3e+17	2e+20	6e+18	93.41	96.89	91.35	97.31	95.14	78.82	6.59	3.11	8.65	2.69	4.86	21.18
23	0.67	1e+18	2e+20	3e+20	1e+18	2e+21	6e+19	95.33	97.53	92.94	97.71	96.08	80.09	4.77	2.47	7.06	2.29	3.92	19.91
24	0.68	4e+18	1e+21	2e+21	1e+19	2e+22	3e+20	95.97	98.32	94.34	98.45	96.94	81.62	4.03	1.78	5.66	1.55	3.06	18.38
25	0.69	3e+19	2e+22	1e+22	5e+19	9e+22	3e+21	96.34	98.91	94.85	98.67	97.47	82.93	3.66	1.09	5.15	1.33	2.53	17.07
26	0.70	5e+20	1e+23	9e+22	5e+20	8e+23	2e+22	97.02	99.05	95.61	99.12	98.56	84.00	2.98	0.95	4.39	0.88	1.44	16.00
27	0.71	4e+21	9e+23	1e+24	3e+21	8e+24	2e+23	97.35	99.39	95.90	99.30	98.42	85.50	2.65	0.61	4.10	0.70	1.58	14.50
28	0.72	3e+22	8e+24	7e+24	2e+22	8e+25	2e+24	97.98	99.53	96.78	99.50	98.93	85.91	2.02	0.47	3.22	0.50	1.07	14.09
29	0.73	1e+23	4e+25	5e+25	1e+23	2e+27	1e+25	98.84	99.86	97.59	99.63	99.11	87.50	1.16	0.14	2.41	0.37	0.89	12.50
30	0.74	8e+23	7e+26	3e+27	9e+23	2e+28	9e+25	99.32	99.87	97.58	99.58	99.42	87.89	0.68.	0.13	2.42	0.42	0.58	12.11
31	0.75	4e+24	4e+27	5e+27	5e+24	9e+28	9e+26	99.54	99.71	97.86	99.75	99.45	89.48	0.46	0.29	2.14	0.25	0.55	10.52
32	0.76	2e+25	1e+28	3e+28	3e+25	3e+30	7e+27	99.67	99.86	98.16	99.80	99.64	90.14	0.33	0.14	1.84	0.20	0.36	9.86
33	0.77	1e+26	8e+28	6e+29	2e+26	2e+31	5e+28	99.71	99.89	98.61	99.87	99.77	90.87	0.29	0.11	1.39	0.13	0.23	9.13
34	0.78	9e+26	2e+29	4e+30	1e+27	9e+31	4e+29	99.82	99.90	98.88	99.90	99.71	91.03	0.18	0.10	1.12	0.10	0.29	8.97
35	0.79	7e+27	1e+30	1e+32	9e+27	1e+33	3e+30	99.86	99.93	98.90	99.94	99.88	91.58	0.14	0.07	1.10	0.06	0.12	8.42
36	0.80	8e+28	1e+31	5e+32	9e+28	1e+34	3e+31	99.91	99.95	99.34	99.97	99.88	92.66	0.09	0.05	0.66	0.03	0.12	7.34
37	0.81	3e+29	2e+32	2e+33	4e+29	7e+34	3e+32	99.92	99.97	99.43	100.00	99.90	93.42	0.08	0.03	0.57	0.00	0.10	6.58
38	0.82	1e+30	1e+33	4e+34	3e+30	2e+36	2e+33	99.94	99.98	99.42	99.98	99.89	93.48	0.06	0.02	0.58	0.02	0.11	6.52
39	0.83	1e+31	9e+33	2e+35	2e+31	1e+37	1e+34	99.95	99.99	99.49	99.99	99.94	94.34	0.05	0.01	0.51	0.01	0.06	5.66
40	0.84	9e+31	7e+34	1e+36	1e+32	1e+38	1e+35	99.95	99.99	99.60	99.99	99.92	94.76	0.05	0.01	0.40	0.01	0.08	5.24
41	0.85	8e+32	8e+35	9e+36	7e+32	1e+39	7e+35	99.97	99.99	99.69	99.99	99.97	94.47	0.03	0.01	0.31	0.01	0.03	5.53

Appendix B.

$leverage = 0$

Value of leader's p	Average wealth of follower (not-bankrupts)						Better wealth of follower						Bankrupts among followers					
	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng
0.40	11 044	10 346	12 199	10 982	9 075	12 708	30.99	22.43	30.50	30.96	10.66	36.07	0.66	3.90	8.97	0.42	11.56	2.26
0.41	11 018	10 255	12 145	10 851	9 088	12 748	28.75	21.70	29.65	29.67	10.61	35.95	0.85	4.62	9.57	0.55	11.22	2.61
0.42	10 654	10 219	11 975	10 729	9 072	12 692	28.47	21.57	28.71	28.85	10.35	36.00	1.04	4.34	9.63	0.45	11.59	2.37
0.43	10 728	10 148	11 890	10 551	8 983	12 729	26.21	20.53	28.10	27.03	10.20	36.24	0.89	4.97	10.19	0.71	11.45	2.44
0.44	10 336	9 995	11 795	10 451	9 021	12 744	25.28	19.62	26.85	26.11	10.09	36.18	1.40	5.13	10.00	0.78	11.94	2.25
0.45	10 131	9 907	11 784	10 345	9 026	12 775	23.12	18.34	27.21	25.23	10.13	36.23	1.54	5.62	10.08	0.75	11.90	2.32
0.46	9 926	9 837	11 650	10 252	8 930	12 905	22.21	17.94	26.50	24.15	9.75	37.02	1.90	5.17	10.69	1.09	11.98	2.30
0.47	9 970	9 746	11 617	10 104	8 917	12 742	21.19	16.50	25.62	22.69	9.87	36.43	2.05	6.48	11.30	1.33	11.61	2.09
0.48	9 946	9 645	11 395	9 941	8 978	12 908	19.78	16.34	25.03	21.21	10.12	37.16	2.32	6.64	10.86	1.47	11.40	1.99
0.49	9 603	9 547	11 223	9 847	8 944	12 855	19.28	15.41	23.89	20.48	9.95	36.88	2.50	6.46	10.99	1.61	11.25	2.18
0.50	9 590	9 524	11 214	9 782	8 867	12 911	17.49	15.22	23.25	19.78	9.29	37.19	3.18	6.62	11.75	1.92	11.54	1.82
0.51	9 559	9 456	11 154	9 685	8 888	12 943	16.95	14.66	22.57	18.93	9.57	37.39	3.08	7.49	12.06	2.14	11.62	1.83
0.52	9 408	9 385	10 937	9 492	8 826	12 842	15.83	14.22	21.49	16.93	9.07	36.97	3.64	7.73	12.40	2.07	11.28	2.04
0.53	9 443	9 258	10 871	9 409	8 950	12 942	13.92	12.79	20.76	16.06	9.91	37.21	4.44	8.22	13.06	2.52	11.89	1.85
0.54	9 050	9 186	10 753	9 306	8 874	12 897	12.90	12.37	20.34	15.15	9.36	37.19	4.70	8.25	12.93	3.18	11.32	1.73
0.55	8 839	9 217	10 679	9 209	8 894	12 898	11.73	12.57	19.89	13.85	9.31	37.12	5.07	8.71	13.41	3.30	12.52	1.75
0.56	9 012	9 034	10 506	9 094	8 853	12 961	11.69	10.83	18.58	12.79	9.01	37.50	6.03	9.06	13.31	3.85	11.76	1.77
0.57	9 069	9 036	10 492	8 990	8 854	12 891	9.97	10.95	18.24	11.99	9.35	37.55	6.76	9.84	13.36	4.01	11.49	1.75
0.58	8 592	8 871	10 332	8 896	8 772	12 957	9.18	9.40	17.60	10.60	8.92	37.20	7.48	10.43	13.70	4.51	11.60	1.96
0.59	8 414	8 920	10 230	8 768	8 781	13 006	8.06	10.00	16.92	9.44	8.83	38.19	8.08	9.97	14.11	4.91	11.55	1.56
0.60	8 590	8 742	10 163	8 748	8 786	12 955	7.39	8.42	16.22	9.56	8.69	38.03	9.01	11.23	14.66	5.52	11.92	1.41

$leverage = 10$

Value of leader's p	Average wealth of follower (not-bankrupts)						Better wealth of follower						Bankrupts among followers					
	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng	S&P	CAC	DAX	FTSE	Nikkei	Hang Seng
0.40	4e+05	2e+05	3e+05	3e+05	2e+05	1e+05	3.28	0.78	0.92	2.72	0.30	0.18	96.6	99.14	99.04	97.06	99.68	99.82
0.41	7e+05	5e+05	6e+05	6e+05	1e+06	2e+05	2.62	0.82	1.02	2.40	0.26	0.28	97.3	99.12	98.90	97.36	99.74	99.70
0.42	4e+06	3e+05	2e+05	6e+05	1e+06	1e+05	2.06	1.06	0.64	2.32	0.30	0.34	97.8	98.90	99.30	97.58	99.70	99.64
0.43	1e+06	2e+05	2e+05	3e+05	6e+04	9e+04	2.46	0.72	1.06	2.12	0.14	0.20	97.4	99.26	98.88	97.82	99.84	99.78
0.44	2e+05	1e+06	3e+05	3e+05	2e+06	1e+05	1.98	0.66	0.80	2.16	0.28	0.52	97.8	99.28	99.12	97.68	99.72	99.46
0.45	4e+05	2e+05	4e+05	4e+05	2e+05	1e+06	1.92	0.76	1.00	2.00	0.32	0.60	97.9	99.24	98.98	97.82	99.68	99.36
0.46	3e+05	4e+05	1e+06	5e+05	9e+05	1e+05	1.82	0.86	0.88	2.44	0.40	0.34	98.0	99.06	99.02	97.38	99.56	99.62
0.47	4e+05	3e+05	5e+05	3e+05	7e+04	4e+06	1.46	0.74	0.84	1.68	0.42	0.42	98.4	99.20	99.12	98.26	99.54	99.54
0.48	5e+05	2e+05	6e+05	3e+05	2e+05	2e+06	1.28	0.60	0.92	1.68	0.36	0.56	98.6	99.30	99.00	98.12	99.60	99.32
0.49	2e+05	2e+05	2e+05	3e+05	2e+05	2e+05	1.26	0.54	0.74	1.28	0.48	0.88	98.6	99.42	99.18	98.62	99.52	99.02
0.50	2e+05	3e+05	4e+05	7e+05	2e+05	1e+06	1.32	0.68	0.62	1.40	0.40	0.94	98.5	99.24	99.34	98.46	99.52	99.04
0.51	3e+05	2e+06	2e+05	7e+05	1e+05	2e+05	0.96	0.72	0.58	0.92	0.30	0.84	99.0	99.20	99.32	98.98	99.66	99.10
0.52	2e+05	1e+05	2e+05	2e+05	8e+05	3e+06	0.88	0.46	0.54	1.22	0.46	0.94	99.1	99.52	99.38	98.58	99.52	98.96
0.53	2e+05	1e+05	3e+05	9e+04	1e+06	4e+06	0.84	0.36	0.40	0.74	0.34	1.12	99.0	99.58	99.58	99.08	99.66	98.74
0.54	1e+05	1e+05	5e+05	3e+05	5e+05	4e+05	0.32	0.46	0.54	0.98	0.34	1.40	99.6	99.48	99.42	98.98	99.66	98.44
0.55	9e+04	6e+04	9e+04	3e+05	4e+05	3e+05	0.56	0.48	0.58	0.80	0.50	1.30	99.3	99.46	99.42	99.10	99.44	98.58

0.56	2e+05	2e+05	4e+05	1e+05	5e+05	4e+05	0.58	0.56	0.28	0.64	0.52	1.66	99.3	99.40	99.64	99.30	99.40	98.26
0.57	8e+04	2e+05	7e+05	1e+05	2e+05	3e+05	0.38	0.50	0.60	0.72	0.52	1.70	99.6	99.44	99.36	99.12	99.44	98.18
0.58	5e+04	1e+05	3e+05	4e+05	1e+05	4e+05	0.36	0.48	0.34	0.42	0.28	1.94	99.6	99.52	99.66	99.56	99.70	97.94
0.59	7e+04	1e+05	6e+04	2e+05	2e+05	1e+06	0.26	0.46	0.42	0.40	0.58	1.96	99.7	99.48	99.52	99.52	99.34	97.96
0.60	1e+05	2e+05	1e+05	1e+05	1e+05	6e+05	0.22	0.46	0.30	0.28	0.50	2.20	99.8	99.52	99.58	99.66	99.38	97.56

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