4th Workshop on combinatorics of moduli spaces, cluster algebras, and topological recursion

This meeting benefited from the support of the Higher School of Economics, Dynasty Foundation, Russian Foundation for Basic Research, grant PARCECO, and Branch of Mathematics of the Russian Academy of Sciences.

Program

Monday, **May 26** (at the Independent University of Moscow)

9:50-10:00	Opening
10:00-11:00	J.Andersen (QGM Aarhus, Denmark) TBA
11:10-12:10	B. Eynard (IPhT, Saclay, France) An ABC of mirror symmetry
coffee break	
12:40-13:30	A. Sergeev (Steklov Math. Inst., Moscow) On the moduli space of Yang-Mills fields on $\mathbb{R}^4$
lunch	
15:30-16:30	T. Nakanishi (Nagoya Univ., Japan) Exact WKB analysis and cluster algebras I
coffee break	
16:50-17:40	V. Roubtsov (Univ. Angers, France, ITEP, Moscow) TBA
17:40-18:10	K. Iwaki (RIMS, Kyoto, Japan) Exact WKB analysis and cluster algebras II
18:45	Welcome party at the IUM.

Tuesday,	**May 27** (at the Independent University of Moscow)
10:00-11:00	V. Fock (Strasbourg Univ., France) Combinatorics and solution of cluster integrable systems
11:10-12:10	Don Zagier (College de France, Paris, MPIM, Bonn) Partitions and quasimodular forms
coffee break	
12:40-13:30	P. Sulkowski (Caltech, USA, and Warsaw Univ., Poland) Moduli spaces, 3d gauge theories and homological knot invariants
lunch	
15:00-15:50	G. Borot (MPIM, Bonn) Blobbed topological recursion
coffee break	
16:10-17:00	P. Norbury (Melbourne Univ., Australia) Discretised moduli space and cohomological field theory
17:00-17:40	Siye Wu (Hong Kong Univ.) Hitchin's equations over a non-orientable manifold
tea break	
18:00-18:30	H. Williams (Berkeley, USA) Integrable Systems, Canonical Bases, and N=2 Gauge Theory
18:35-19:05	A. Basalaev (HSE and Hannover Univ.) Primitive form for the orbifolded LG B-model
Wednesday,	**May 28** (at the Steklov Mathematical Institute)
10:00-11:00	M. Shapiro (Michigan State Univ., USA) Cluster structures on Poisson-Lie groups
tea break	
11:20-12:30	S. Fomin (Univ. Michigan, Ann Arbor, USA) Subtraction-free complexity
12:35-13:35	A.Zorich (Univ. Paris VII, France) TBA
	¡afternoon and evening free¿

Thursday, May 29 (at the Steklov Mathematical Institute)

10:00-11:00	I. Krichever (Columbia Univ., USA, and HSE) Degenerations of real normalized differentials
11:10-12:10	A. Marshakov (Lebedev Physical Inst., ITEP, and HSE) TBA
coffee break	
12:30-13:10	R. Kashaev (Geneva Univ.) Vertex changing in Teichmller TQFT
13:10-13:40	A. Felikson (Durham Univ., UK) Bases for cluster algebras arising from triangulated orbifolds
13:40-14:00	G. Mutafyan (HSE) Characters of representations of quantum toroidal algebra $\widehat{\widehat{\mathfrak{gl}_1}}$
lunch	
16:00-17:00	A. Mironov (Lebedev Phys. Inst. and ITEP) Hypergeometric Hurwitz partition functions
tea break	
17:30-18:00	Y. Zenkevich (Inst. Nucl.Res., Moscow) Spectral duality between Heisenberg chain and Gaudin model
18:10-18:40	P. Dunin-Barkovsky Quantum spectral curve for the Gromov-Witten theory of the complex projective line
18:40-19:10	A. Popolitov (Univ. of Amsterdam and ITEP) Cluster variety face of quantum groups
19:30	workshop dinner at the Steklov Mathematical Institute

Friday,	**May 30** (at the Independent Univ. of Moscow)
10:00-11:00	P. Zograf (St.Petersburg Department of Math. Inst. RAS) Enumeration of Grothendieck's dessins 1: Virasoro constraints and KP hierarchy
11:10-12:10	M.Kazaryan (Steklov and HSE, Moscow) Enumeration of Grothendieck's dessins 2: Topological recursion
coffee break	
12:40-13:20	J. Harnad (Concordia Univ., Montreal) 2D Toda tau functions as combinatorial generating functions
lunch	
15:30-16:20	P.Zinn-Justin (Univ. Paris VI, France) The geometry of loop models
16:30-17:20	A. Zabrodin (Semenov Chem. Physics Inst.. ITEP, and HSE) Dispersionless DKP hierarchy and elliptic Loewner equation
tea break	
17:40-18:20	D. Korotkin (Concordia Univ. and CRM, Montréal, Canada) TBA
Saturday,	**May 31** (at the Higher School of Economics)
10:00-10:50	O. Marchal (Univ. St.Etienne, France) Painlev 5 equation and the topological recursion
11:00-12:00	S. Natanzon (HSE, and ITEP, Moscow) Symmetric solutions to dispersionless 2D Toda hierarchy, Hurwitz numbers and conformal dynamics
coffee break	
12:30-13:30	D.Orlov (Steklov, Moscow) TBA
13:30	closing

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Titles and abstracts

Full-length research talks

Joergen Elegaard Andersen (QGM Aarhus, Denmark)

Gaëtan Borot (MPIM, Bonn)

Blobbed topological recursion

The topological recursion of Eynard, Orantin and Chekhov is an algorithm that computes, starting from initial data $W_{0,1}$ and $W_{0,2}$, a set of meromorphic differentials $W_{g,n}$ by recursion on $2g-2+n$. They have many nice properties, and are relevant to compute asymptotic expansion in random matrix theory and integrable systems, to enumerate of surfaces of genus g with n boundaries in various contexts, etc. I will describe the blobbed topological recursion, which provides the general solution of loop equations. It extends the topological recursion by adding extra initial condition for each (g,n), and allows to include all previously found solutions of the loop equations in the same formalism. I will explain its relevance in multi-trace matrix models and map enumeration, and describe some of its properties (the latter is part of a joint, ongoing work with Shadrin).

Bertrand Eynard (IPHT CEA Saclay, France, CRM, Montreal)

An ABC of mirror symmetry

we recall how A-models of enumerative geometry are related to B-model spectral curves, and to matrix models (C-model) as well as to integrable systems. The whole can be beta deformed as in the AGT picture. We shall present general features of those approaches and illustrate them on many examples, knots, discrete surfaces, intersection numbers, Liouville CFT, etc.

Volodya Fock (Strasbourg Univ., France, and ITEP, Moscow)

Combinatorics and solution of cluster integrable systems

Recently A.B.Goncharov and R.Kenyon suggested a class of integrable systems admitting cluster description and enumerated by Newton polygons - convex polygons on the plane with integer vertices. Though it turned out to be isomorphic to known integrable systems on Poisson-Lie groups $\widehat{PGL}(N)$, the new approach being rather simple allows to improve our understanding of such systems. In particular it allows to study discrete flows commuting with continuous ones and given by cluster transformations.

Solutions for such system can be easily expressed in terms of Riemann theta functions using a combinatorial analogue of the Abel map.

Particular cases of these systems are relativistic Toda systems, pentagram map, Somos (at least 4 and 5) recurrences, Their quantisation gives lattice models and Hofstadter model.

In the talk we will explain the main construction using a few simple examples.

Sergey Fomin (University of Michigan, USA)

Subtraction-free complexity

I will discuss the problem of dependence of computational complexity on the set of allowed arithmetic operations. An important role in this context is played by the notion of subtraction-free complexity, the version that allows addition, multiplication, and division, but not subtraction.

The talk is aimed at a general mathematical audience, and assumes no special background in complexity theory. Cluster algebras make a brief appearance in the algorithm of subtraction-free computation of Schur functions. This is joint work with D. Grigoriev and G. Koshevoy (arXiv:1307.8425).

John Harnad (Concordia Univ., and CRM, Montréal, Quebec, Canada)

2D Toda τ-functions as combinatorial generating functions

Two ways of constructing generating functions for certain geometrical invariants of a combinatorial nature are introduced and the resulting functions shown to be 2D Toda τ-functions of hypergeometric type. The first method involves generation of paths in the Cayley graph of the symmetric group S_n by multiplication of the sums C_λ in the group algebra $\mathbf{C}[S_n]$ of the symmetric group over elements in a given conjugacy class conjugacy class by elements of an abelian group within the center. Applying the characteristic map extended to the tensor product $\mathbf{C}[S_n] \otimes \mathbf{C}[S_n]$ leads to double expansions summed over products of monomial symmetric functions, in which the coefficients count the number of such paths. Applications include the generating functions for double Hurwitz numbers introduced by Okounkov, which count branched coverings of the Riemann sphere with nonminimal branching at two points, and various generalizations thereof. The second method is the standard construction of hypergeometric τ-functions as vacuum state matrix elements of products of vertex operators in a fermionic Fock space with elements of the abelian group of *convolution symmetries*. A homomorphism between these two group actions is derived and shown to be intertwined by

the characteristic map composed with fermionization, showing the equivalence of the two methods.

(* Based on joint work with Mathieu Guay-Paquet)

Rinat Kashaev (Geneva Univ., Switzerland)

Vertex changing in Teichmller TQFT

I will talk about the behavior of the partitions functions of Teichmller TQFT under adding or removing vertices in shaped triangulations.

Maxim Kazarian (Steklov and HSE)

Enumeration of Grothendieck's dessins 2: Topological recursion

(this is the second part of the joint talk with P.Zograf)

These two talks are based on a joint work of the speakers. We enumerate the branched covers of the complex projective line with prescribed ramification over ∞ and fixed numbers of preimages of 0 and 1 with no other critical values. We show that the corresponding generating function has several remarkable properties – in particular, it obeys Virasoro constraints, an evolution equation, the KP hierarchy and a topological recursion.

Dmitrii Korotkin (Concordia Univ. and CRM, Montréal, Canada)

Igor Krichever (Columbia Univ., USA, and HSE, Moscow)

Degenerations of real normalized differentials

Olivier Marchal (Univ. J.Monnet, St.Etienne, France)

Painlev 5 equation and the topological recursion

The purpose of the talk is to recover gap probability results of hermitian matrix models and in particular their connection with the Painlev 5 integrable system directly from the topological recursion. This provides an alternative and more algebraic proof of these results that could be adapted for other kinds of integrable systems.

Andrei Marshakov (Lebedev Physical Inst., ITEP, and HSE, Moscow)

Andrei Mironov (Lebedev Physical Inst., and ITEP, Moscow)

Hypergeometric Hurwitz partition functions

Tomoki Nakanishi (Nagoya Univ., Japan)

Exact WKB analysis and cluster algebras I

We develop the mutation theory in the exact WKB analysis using the framework of cluster algebras. Under a continuous deformation of the potential of the Schrödinger equation on a compact Riemann surface, the Stokes graph may change the topology. We call this phenomenon the mutation of Stokes graphs. Along the mutation of Stokes graphs, the Voros symbols, which are monodromy data of the equation, also mutate due to the Stokes phenomenon. We show that the Voros symbols mutate as variables of a cluster algebra with surface realization.

Sergei Natanzon (HSE, and ITEP, Moscow)

Symmetric solutions to dispersionless 2D Toda hierarchy, Hurwitz numbers and conformal dynamics

We explicitly construct the series expansion for a certain class of solutions to the 2D Toda hierarchy in the zero dispersion limit, which we call symmetric solutions. We express the Taylor coefficients through some universal combinatorial constants and find recurrence relations for them. These results are used to obtain new formulas for the genus 0 double Hurwitz numbers. They can also serve as a starting point for a constructive approach to the Riemann mapping problem and the inverse potential problem in 2D. The talk is based on joint work with Anton Zabrodin

Paul Norbury (Melbourne Univ., Australia)

Discretised moduli space and cohomological field theory

Recently Dunin-Barkowski, Orantin, Shadrin and Spitz proved an equivalence between semi-simple cohomological field theories and topological recursion on local curves. Given a semi-simple cohomological field theory it may be fruitful to find its spectral curve and study the consequences of topological recursion. We do this for a fundamental example - a Hurwitz space parametrising genus zero covers of the sphere with Frobenius manifold structure due to Dubrovin. We find a relation with an enumerative problem which counts the number of integer points inside rational convex polytopes that naturally make up a cell decomposition of the moduli space of curves. One immediate consequence is the appearance of the Euler characteristic of the moduli space of curves in the Hurwitz space.

Dmitrii Orlov (Steklov Math, Inst., Moscow)

Volodya Roubtsov (Univ. Angers, France, ITEP, Moscow)

Armen Sergeev (Steklov Math. Inst., Moscow)

On the moduli space of Yang-Mills fields on $\mathbb{R}^4$

There is a well-known description of the moduli space of instantons on $\mathbb{R}^4$ given by Atiyah–Drinfeld–Hitchin–Manin. In contrast to that the structure of Yang–Mills gauge fields on $\mathbb{R}^4$ remains to a large extent unknown. In particular, there is no good twistor decription of this space. In our talk we shall discuss the known facts and conjectures concerning the structure of the Yang–Mills moduli space.

Michael Shapiro (Michigan State Univ., Lansing, USA)

Cluster structures on Poisson-Lie groups

(joint work with Michael Gekhtman, Alek Vainshtein) This talk can be viewed as a progress report on the conjecture formulated by the authors earlier, stating that each class in the Belavin-Drinfeld classification of Poisson-Lie structures on a complex simple group G corresponds to a cluster structure in $O(G)$.

This conjecture has been verified

— for any G in the case of the standard Poisson-Lie structure (trivial Belavin-Drinfeld data);

—- for all Belavin-Drinfeld classes in $SL(n)$, $n < 6$;

— for the Cremmer-Gervais Poisson-Lie structure on $SL(n)$.

To construct an initial cluster in the Cremmer-Gervais case, we needed an insight that comes from analyzing the corresponding Poisson-Lie structure in the Drinfeld double of G. Working with the Drinfeld double, one is able to obtain an initial cluster as a restriction to the diagonal subgroup of a certain family of regular log-canonical functions on the double. This also hints at a possibility of endowing the double itself with a cluster structure. However, our work in progress shows that while it is possible to construct a regular log-canonical coordinate system in the double in all the cases in which conjecture is established, in order to stay within the ring of regular functions, one is forced to replace one of the exchange relations with a generalized exchange relation in the sense proposed by L.Chekhov and M.S.

Conjecture 2. For any Belavin-Drinfeld triple there exists a generalized cluster structure on the Drinfeld double $D(G)$ compatible with the corresponding Poisson- Lie structure.

Theorem. Conjecture 2 is valid for the trivial and Cremmer-Gervais Belavin- Drinfeld data.

Piotr Sułkowski (Caltech, USA, and Warsaw University, Poland)

Moduli spaces, 3d gauge theories and homological knot invariants

Compactifications of M5-branes on non-trivial 3-manifolds lead to N=2 supersymmetric theories in 3 dimensions. If the 3-manifold in question is a knot complement, various Chern-Simons amplitudes - or corresponding knot invariants - for this knot determine the properties of the resulting 3d, N=2 theory. This relation between N=2 theories and Chern-Simons theory is referred to as the 3d-3d correspondence. In this talk, after reviewing basics of this correspondence, we stress the importance of proper identification of moduli spaces on both sides of the correspondence. We also argue that M-theory realization of this correspondence implies that N=2 theories obtained in this way posses one special flavor symmetry, which is related to certain deformation of Chern-Simons theory and homological knot invariants.

Siye Wu (Univ. Hong Kong)

Hitchin's equations over a non-orientable manifold

We study Hitchin's equations and Higgs bundles over a non-orientable manifold whose oriented cover is compact Kähler. Using the involution induced by the deck transformation, we show that Hitchin's moduli space is Langrangian/complex with respect to the hyper-Kähler structure on Hitchin's moduli space associated to the oriented cover. We then establish a Donaldson-Corlette type correspondence and relate Hitchin's moduli space to representation varieties. This is a joint work with N.-K. Ho and G. Wilkin.

Anton Zabrodin (Semenov Chem. Physics Inst.. ITEP, and HSE, Moscow)

Dispersionless DKP hierarchy and elliptic Loewner equation

We show that the dispersionless DKP hierarchy (the dispersionless limit of the Pfaff lattice) admits a suggestive reformulation through elliptic functions. We also consider one-variable reductions of the dispersionless DKP hierarchy and show that they are described by an elliptic version of the Loewner equation. With a particular choice of the driving function, the latter appears to be closely related to the Painlevé VI equation with special choice of parameters.

Don Zagier (College de France, Paris, and MPIM, Bonn, Germany)

Partitions and quasimodular forms

Paul Zinn-Justin (LPTHE, Paris VI, France)

The geometry of loop models

In this work in collaboration with A. Knutson, we investigate the correspondence between algebraic geometry and quantum integrable systems from the point of view of Grobner degenerations. The latter is very combinatorial in nature and at a technical level, applies equally well to cohomology and K-theory. Following Knutson and Miller, I shall describe the simplest framework in which this approach works, namely (matrix) Schubert varieties and Schubert polynomials. I shall next formulate several variations and extensions, which will lead us naturally to loop models on 2d lattices and to the Yang–Baxter equation: first noncrossing loops (Temperley–Lieb model), then crossing loops (Brauer model).

Peter Zograf (St. Petersburg branch of Steklov Math. Inst.)

Enumeration of Grothendieck's dessins 1: Virasoro constraints and KP hierarchy

(this is the first part of the joint talk with M.Kazarian)

These two talks are based on a joint work of the speakers. We enumerate the branched covers of the complex projective line with prescribed ramification over ∞ and fixed numbers of preimages of 0 and 1 with no other critical values. We show that the corresponding generating function has several remarkable properties – in particular, it obeys Virasoro constraints, an evolution equation, the KP hierarchy and a topological recursion.

Anton Zorich (Univ. Paris VII, France)

30-minutes communications

Aleksei Basalaev (HSE, Moscow, Leibniz universitt Hannover)

Primitive form for the orbifolded LG B-model

Motivated by the global mirror symmetry conjecture for the orbifolded Landau-Ginzburg models we define the action on the certain class of Frobenius manifolds that corresponds to the Saito's primitive form change. Using this action we give the classification of the rank 3 Frobenius manifolds defined over the field and also ?satisfying additional symmetry assumption.

Petr Dunin-Barkowski (UVA, The Netherlands, ITEP, Moscow)

Quantum spectral curve for the Gromov-Witten theory of the complex projective line

We construct the quantum spectral curve for the Gromov-Witten theory of the complex projective line. In order to construct the wave function, we

propose a unique mechanism of integrating meromorphic differentials $W_{g,n}$ appearing in the corresponding spectral curve topological recursion procedure into rational functions. After that we rewrite the wave function in terms of stationary Gromov-Witen invariants of CP^1, which allows us to express it in terms of a semi-infinite wedge product. Using this formalism, we reduce the quantum curve equation to a combinatorial problem, which we then solve using representation theory of S_n. The talk is based on joint work with M.Mulase, P.Norbury, A.Popolitov and S.Shadrin.

Anna Felikson (Durham Univ., UK)

Bases for cluster algebras arising from triangulated orbifolds

We construct bases of cluster algebras coming from unpunctured orbifolds. These are generalizations of bangles and bracelets bases constructed by Misiker, Schiffler and Williams for cluster algebras arising from unpunctured surfaces. This is a joint work with Pavel Tumarkin.

Kohei Iwaki (RIMS, Kyoto Univ., Japan))

Exact WKB analysis and cluster algebras II

This is the second part of our talk on "Exact WKB analysis and cluster algebras" which is joint work with Tomoki Nakanishi (Nagoya). In the relation between exact WKB analysis and cluster algebras, there is a key formula which describes the jump rule of Voros symbols when a saddle trajectory appears in Stokes graph. We call the formula Delabaere-Dillinger-Pham (DDP) formula. I explain a derivation of DDP formula, and relate the formula to cluster transformation. I will also explain how the periodicity of cluster algebras derives the identity of Stokes automorphisms.

Alexandr Popolitov (ITEP, Moscow and UVA, The Netherlands)

Cluster variety face of quantum groups

Using the well-known free-field formalism for quantum groups, we demonstrate in case of $A(n)_q$, that quantum group is naturally also a cluster variety. Widely used formulae for mutations are direct consequence of independence of group element on the order of simple roots. Usual formulae for 2n Poisson leaf emerge in classical limit, if all but few 2n coordinates vanish.

Harold Williams (Berkeley Univ., USA)

Integrable Systems, Canonical Bases, and N=2 Gauge Theory

We identify the Hamiltonians of certain integrable systems with generating functions of Euler characteristics of quiver Grassmannians. The phase

spaces of these systems (relativistic Toda chains) are cluster varieties, and the relevant generating functions are to be interpreted as generalized canonical basis elements (for example, all cluster variables are examples of such generating functions). For the open chain, a geometric perspective on this result is obtained by identifying this cluster variety with a space of irregular flat connections, which we can make precise through the spectral network formalism of Gaiotto-Moore-Neitzke. This in particular relates cluster charts with spectral curves of the corresponding Hitchin system, which here is simply the closed nonrelativistic Toda chain. The identification of this as the correct Hitchin system to associate with our cluster variety relies in turn on the role of the former as the Seiberg-Witten system of pure N=2 gauge theory and the latter as the cluster variety associated with its BPS quiver. This then leads to yet another interpretation of the open relativistic Toda Hamiltonians as expectation values of Wilson loops as functions of vacua.

Yegor Zenkevich (Inst. Nucl.Res., Moscow)

Spectral duality between Heisenberg chain and Gaudin model