Mathematical Modeling of the Individual Traders' Behavior on the Stock Exchange

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Introduction

- The classical models of portfolio allocation like Markowitz's 'mean-variance' model [Markowitz 1952], CAPM model [Sharpe 1964], APT model [Ross 1976] and their extensions have certain limitations:
  - assumption of trader's rationality,
  - need to assess the probability distribution of random variable (i.e. future price or return of securities),
  - computational complexity in case of many securities taken into account

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- The methods for probability distribution assessment like regression models, stochastic differential equations, neural networks, etc. also have limitations:
  - sophisticated procedures of parameter estimation for many models,
  - need of long and stable time series to estimate parameters of that probability distribution.

Introduction

- Classical investment models cannot help individual traders and investment funds to reach their goals:
  - the stocks individual investors buy subsequently underperform those they sell [Odean 1999],
  - Individual traders mostly cannot outperform the market portfolio [Barber and Odean 2008],
  - Substantial proportion of hedge funds failed to survive, the survived mostly they have returns lower then market return and for those funds, who succeed in beating the market portfolio, their success is not persistent [Malkiel and Saha 2005],
  - only 56.8% of expert recommendations on selling or buying stocks of Russian companies were profitable [Proskurin and Penikas 2013],
  - the forecasts of economic analysts perform worse than naive forecasts (the historical mean) [Soderlind 2008]

Introduction

- There are a lot of facts showing the irrationality of trader’s behavior:
  - Psychological factors and emotions (like fear and greed) influence to trader’s decisions [Lo & Repin 2002, Lo et al. 2005, Stracca 2004]
  - Disposition effect, i.e. the tendency to sell winners quicker than losers [Shapira & Venezia 2001, Talpsepp et al. 2014]
  - Overconfidence [Odean 1999, Barber & Odean 2000, Kuo & Lin 2013]
  - Overreaction to unexpected and dramatic events [De Bondt & Thaler 1985]
  - Herding behavior [Rothig & Chiarella 2010, Tedeschi et al. 2012, Venezia et al. 2011]

Introduction

- The key property of trader that should be taken into account is the probability of his correct prediction of future price changes of the securities.

- We propose a new approach : to estimate what financial results can be achieved by the trader with known probability of making a successful prediction of future price changes

Estimation of the trader’s probability to correctly predict the future price changes

Estimation of probability of correct prediction

- We propose a scheme to estimate the probability of trader's correct prediction of future price changes for the certain security based on the Bernoulli scheme, i.e. a set of experiments with the same conditions:
  - The time series of this security prices from the historical data are given to the trader,
  - He makes his prediction of price changes on the next moment according to his trading strategy,
  - His decision is compared with the actual movement of security's price.

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Estimation of probability of correct prediction

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$$\lim_{n\to\infty} P\left(\left|\frac{m}{n} - p\right| \le \varepsilon\right) = 1.$$

- where m is a number of correct trader's predictions in the series of n experiments, p is an probability of correct prediction

- The accuracy of this estimate can be evaluated and the needed number of experiments to achieve this accuracy can be calculated via Chebyshev's inequality:

$$P\left(\left|\frac{m}{n} - p\right| \le \varepsilon\right) \ge 1 - \frac{1}{4n\varepsilon^2}$$

The effectiveness of different trading strategies: analysis via computational simulation

Description of models

- Market description and price formation
  - Stock indices as a securities prices (S&P500, CAC, DAX, FTSE, Nikkei 225, Hang Seng, all for 2000-2010)
  - Traders use only market orders
  - Equal initial wealth allocation
  - Capital minimum level – a half of the initial wealth
  - Margin trading with leverage rate 1:2, 1:5, 1:10

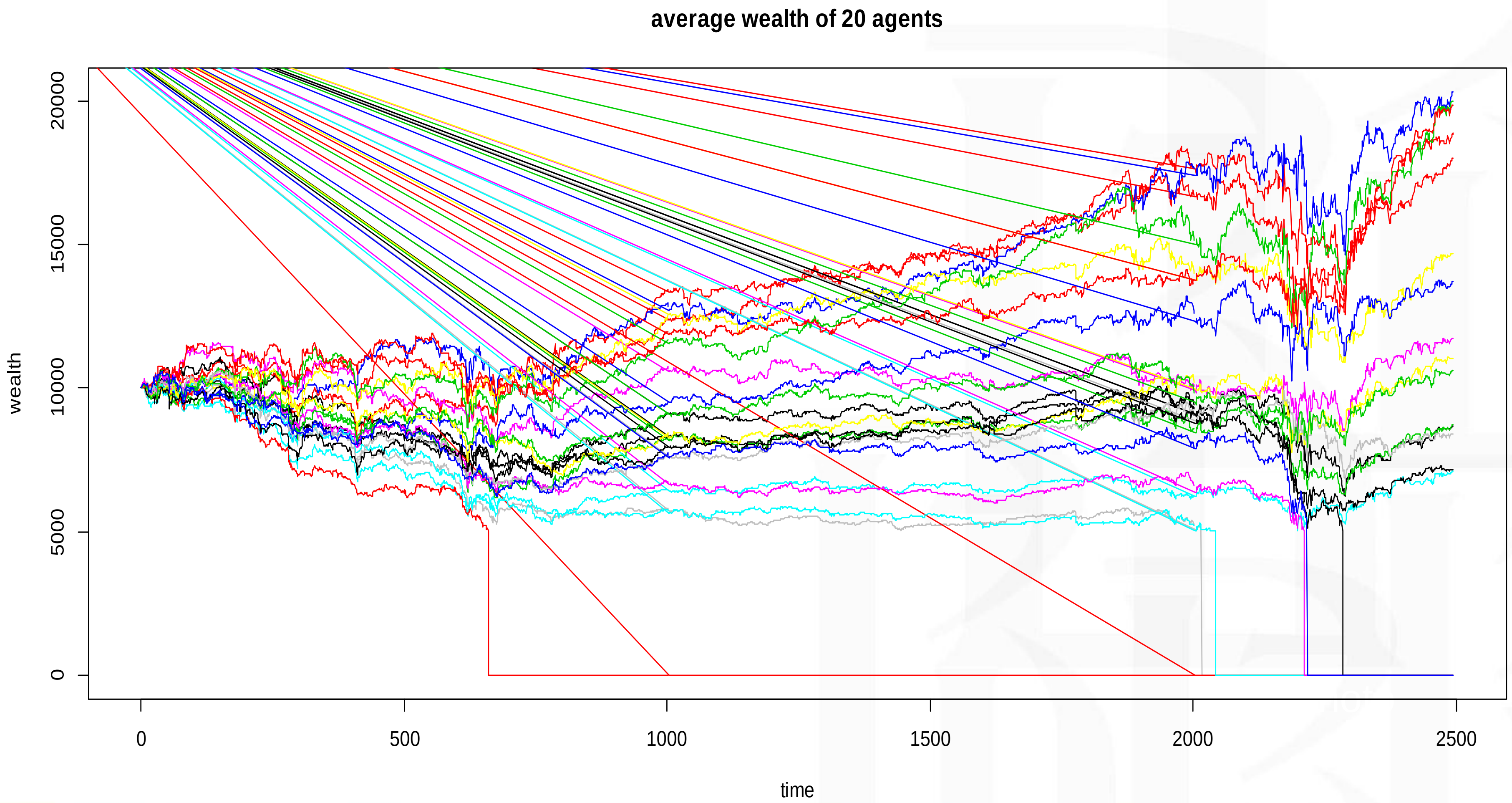
Description of models

- Traders and their strategies
  - Basic characteristic: p is a probability of correct prediction of the price movement on the next day;
  - Follower strategy: repeat leader’s decisions with one step delay;
  - ‘Black swan seeker’ strategy: trader tries to predict crisis events with higher possible gains

Description of models

- Financial results
  - The average wealth of not-bankrupts,
  - The fraction of agents with final wealth greater than initial wealth,
  - The fraction of bankrupts (at some step the wealth became lower then the half of the initial one).

Results for the basic model



Results for the basic model

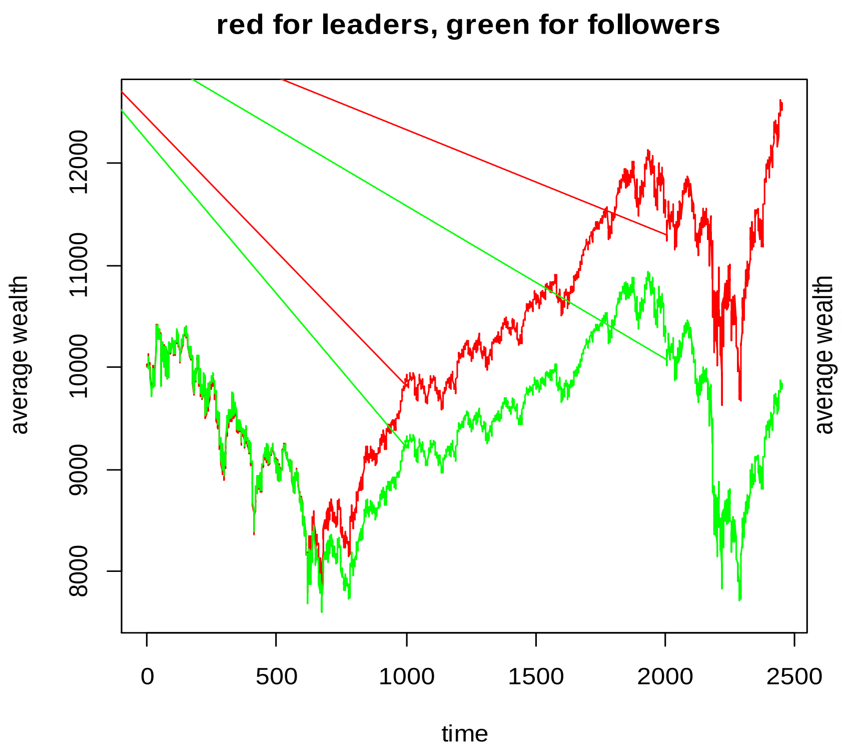
		The fraction of bankrupts,%			
	p	Leverage=0	Leverage=2	Leverage=5	Leverage=10
1	0.50	6.2	52.2	88.5	98.6
2	0.51	2.7	37.5	77.6	96.2
3	0.52	0.9	20.7	60.6	90.8
4	0.53	0.2	10.2	42.4	81.4
5	0.54	0.0	5.0	30.3	70.5
6	0.55	0.0	2.4	21.4	59.8
7	0.56	0.0	1.0	13.7	49.3
8	0.57	0.0	0.7	10.9	40.8
9	0.58	0.0	0.5	7.3	32.4
10	0.59	0.0	0.0	5.1	26.5
11	0.60	0.0	0.0	2.7	21.8
12	0.61	0.0	0.0	1.8	17.9
13	0.62	0.0	0.0	1.5	15.5
14	0.63	0.0	0.0	1.4	13.3
15	0.64	0.0	0.0	0.5	9.6
16	0.65	0.0	0.0	0.5	7.0

Results for the basic model

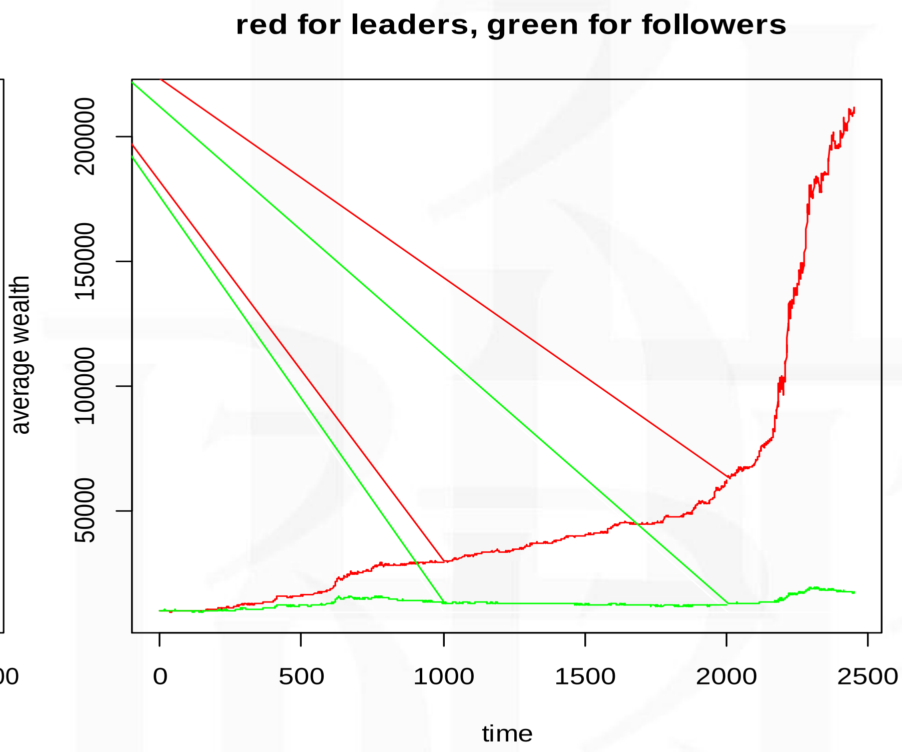
		Leverage=0					
		S&P	CAC	DAX	FTSE	Nikkei	HS
1	Average wealth of not-bankrupts	9 584	9 464	11 098	9 667	8 929	12 719
2	The fraction of agents with final wealth greater than initial wealth,%	35.35	28.05	45.35	37.36	18.83	71.12
3	The fraction of bankrupts,%	6.17	19.58	23.17	3.95	24.09	5.16
		Leverage=2					
1	Average wealth of not-bankrupts	14 131	19201	17 348	14 110	15 637	19 606
2	The fraction of agents with final wealth greater than initial wealth,%	29.80	20.44	25.73	29.25	18.87	34.99
3	The fraction of bankrupts,%	52.21	72.06	65.20	54.40	71.95	55.87
		Leverage=5					
1	Average wealth of not-bankrupts	43 427	64 072	66 643	44 156	56 847	61 566
2	The fraction of agents with final wealth greater than initial wealth,%	10.31	5.47	7.67	10.35	5.51	9.06
3	The fraction of bankrupts,%	88.54	93.95	91.16	88.12	93.66	89.86
		Leverage=10					
1	Average wealth of not-bankrupts	2e+05	3e+05	2e+05	3e+05	5e+05	3e+05
2	The fraction of agents with final wealth greater than initial wealth,%	1.32	0.21	0.67	1.26	0.47	0.89
3	The fraction of bankrupts,%	98.62	99.79	99.27	98.63	99.51	99.09

Results for the 'follower' strategy

Leverage = 0
$p \sim R[0.4; 0.6]$



Leverage = 5
$p \sim R[0.4; 0.6]$

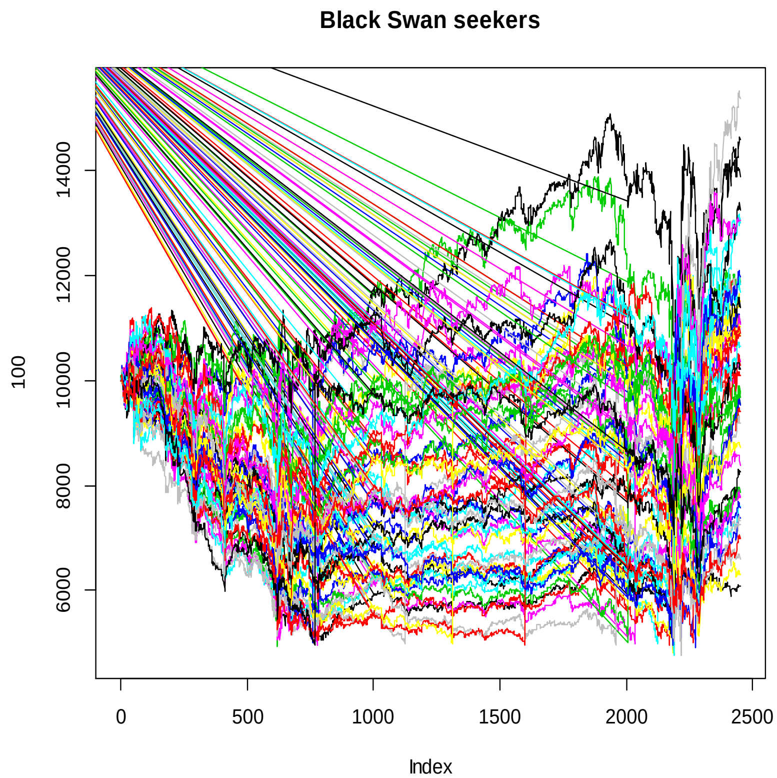


Value of leader's p	*leverage=0*			*leverage=2*			*leverage=5*			*leverage=10*		
	Average wealth of follower	Better wealth of follower	Bankrupts among followers	Average wealth of follower	Better wealth of follower	Bankrupts among followers	Average wealth of follower	Better wealth of follower	Bankrupts among followers	Average wealth of follower	Better wealth of follower	Bankrupts among followers
0.44	10 336	25.3	1.4	16 845	40.9	42.7	49 738	12.7	84.5	59 670	1.2	98.8
0.45	10 131	23.1	1.5	17 378	32.7	48.2	52 434	18.9	79.1	163 849	1.6	98.64
0.46	9 926	22.2	1.9	16 780	44.5	45.5	50 334	17.3	80.9	230 542	1.4	98 .6
0.47	9 970	21.2	2.1	15 188	38.2	44.5	85 579	8.3	89.1	18 023	1.3	98.7
0.48	9 946	19.8	2.3	15 383	28.2	59.1	37 389	14.8	83.6	59 015	0.7	99.3
0.49	9 603	19.3	2.5	13 030	27.3	60.9	65 388	19.1	79.1	29 822	1.9	98.1
0.50	9 590	17.5	3.2	13 006	31.8	50.8	39 833	10.6	87.3	28 308	2.8	97.2
0.51	9 559	17.0	3.1	12 995	28.2	48.2	52 156	16.1	82.7	18 291	1.5	98.5
0.52	9 408	15.8	3.6	13 141	25.5	54.5	39 057	8.5	87.3	-	0	100
0.53	9 443	13.9	4.4	12 949	21.8	58.2	87 119	4.5	95.5	14 227	0.1	99.9
0.54	9 050	12.9	4.7	12 185	16.4	67.3	36 900	10.9	88.2	-	0	100
0.55	8 839	11.7	5.1	12 551	20.9	64.5	28 826	7.3	90.9	639 577	0.9	99.1
0.56	9 012	11.7	6.0	12 314	14.5	66.4	18 045	2.4	96.4	-	0	100
0.57	9 069	10.0	6.8	11 164	13.6	73.6	16 397	6.4	90.9	55 243	1.4	98.6
0.58	8 592	9.2	7.5	10 527	10.0	70.0	95 561	5.9	93.6	18 908	2.1	97.9
0.59	8 414	8.1	8.1	10 961	8.2	81.8	28 376	1.8	97.3	-	0	100
0.60	8 590	7.4	9.0	11 061	12.7	76.4	16 725	2.7	95.5	75 147	2.2	97.8

Results for the Black Swan seekers' strategy

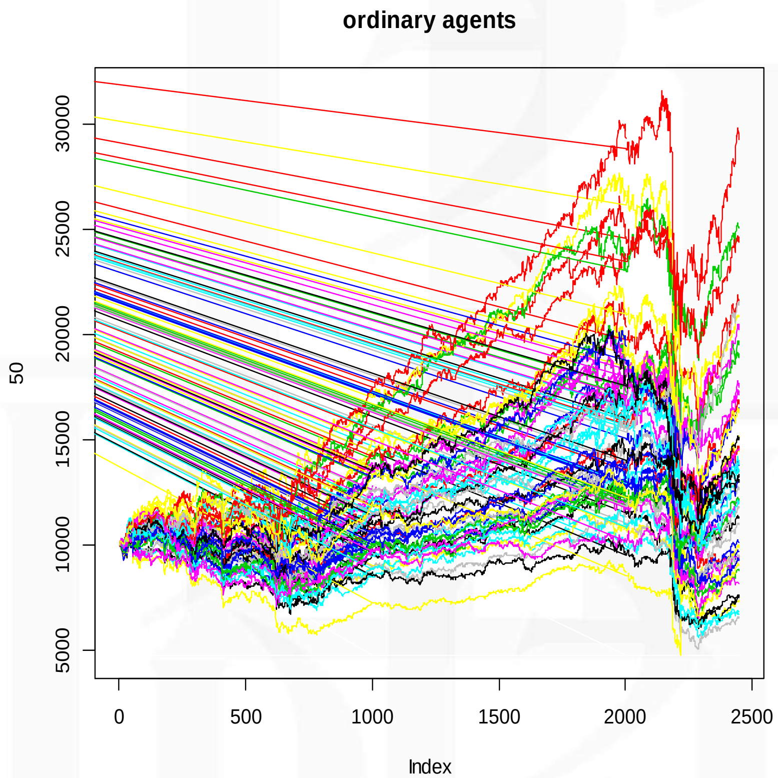
Black swan seekers

$p_{sign}^{Q} \sim R[0.4;0.5]$ $p_{sign}^{R} \sim R[0.8;0.9]$



Ordinary agents

$p_{sign}^{Q} \sim R[0.5;0.6]$ $p_{sign}^{R} \sim R[0.1;0.2]$



Results

- The results of such agent-based simulation can help trader with known probability of correct prediction of future price changes to decide:
  - whether it is worse trading on the stock exchange with this probability,
  - whether it is profitable to use certain trading strategy
  - to use or not to use margin trading and (in case of positive answer) what value of leverage to choose.

The optimal portfolio selection for a trader with known probability of future price changes

(joint work with Prof. Alexander S. Belenky)

First model

- Assumptions:
  1. At each moment of time t the trader possesses:
     - certain volume of securities $v_{i,t}, i = \overline{1,n}$,
     - amount of money m_t

First model

- Assumptions:
  2. At each moment of time *t* the trader divides the whole set *N* of available securities into 3 disjoint subsets:
     - I_t^+ is the subset of securities such that the trader predicts the increase of their price,
     - I_t^- is the subset of securities such that the trader predicts the decrease of their price,
     - I_t^0 is the subset of securities such that the trader cannot make a prediction about direction of price changes

First model

- Assumptions:
  3. the traders knows his probability p_i of making a correct prediction of future price changes
  4. his trading strategy allows to estimate the borders $s_{i,t+1}^{max}$ and $s_{i,t+1}^{min}$ of possible future price of i's security, $i = \overline{1,n}$

First model

- Let us denote:
  - $x_{i,t}^{+}$ is the volume of securities such that the trader intends to buy at the moment t,
  - $x_{i,t}^{-}$ is the volume of securities such that the trader intends to sell from his portfolio at the moment t,
  - $z_{i,t}^{-}$ is the volume of securities such that the trader intends to get from a broker to open a short position at the moment t,
  - $s_{i,t}$ and $s_{i,t+1}$ are the current and future price of i's security

First model

$s_{i,t+1}, i \in I_t^+$	$s_{i,t+1} > s_{i,t}$	$s_{i,t+1} \leq s_{i,t}$
P	p_i	$1 - p_i$

$s_{i,t+1}, i \in I_t^-$	$s_{i,t+1} \geq s_{i,t}$	$s_{i,t+1} < s_{i,t}$
P	$1 - p_i$	p_i

First model

$s_{i,t+1}, i \in I_t^+$	$s_{i,t+1} > s_{i,t}$	$s_{i,t+1} \leq s_{i,t}$
P	p_i	$1 - p_i$

$s_{i,t+1}, i \in I_t^-$	$s_{i,t+1} \geq s_{i,t}$	$s_{i,t+1} < s_{i,t}$
P	$1 - p_i$	p_i

$$f_1(u) = \begin{cases} \dfrac{1}{s_{i,t+1}^{max} - s_{i,t}}, if\ u \in [s_{i,t}, s_{i,t+1}^{max}] \\ 0, if\ u \notin [s_{i,t}, s_{i,t+1}^{max}] \end{cases}$$

$$f_1(v) = \begin{cases} \dfrac{1}{s_{i,t} - s_{i,t+1}^{min}}, if\ v \in [s_{i,t+1}^{min}, s_{i,t}] \\ 0, if\ v \notin \left[s_{i,t+1}^{min}, s_{i,t}\right]. \end{cases}$$

First model

$Ms_{i,t+1}, i \in I_t^+$	$\frac{s_{i,t} + s_{i,t+1}^{max}}{2}$	$\frac{s_{i,t+1}^{min} + s_{i,t}}{2}$	$s_{i,t}$
P	p_i	$\frac{1-p_i}{2}$	$\frac{1-p_i}{2}$

$Ms_{i,t+1}, i \in I_t^-$	$\frac{s_{i,t+1}^{min} + s_{i,t}}{2}$	$\frac{s_{i,t} + s_{i,t+1}^{max}}{2}$	$s_{i,t}$
P	p_i	$\frac{1-p_i}{2}$	$\frac{1-p_i}{2}$

$Ms_{i,t+1}, i \in I_t^0$	$s_{i,t}$	$\frac{s_{i,t+1}^{min} + s_{i,t}}{2}$	$\frac{s_{i,t} + s_{i,t+1}^{max}}{2}$
P	p_i	$\frac{1-p_i}{2}$	$\frac{1-p_i}{2}$

First model

$$M[\Delta W_{t+1}] = \sum_{i\in I_t^0} v_{i,t}\left(Ms_{i,t+1} - s_{i,t}\right) + \sum_{i\in I_t^+}\left(v_{i,t} + x_{i,t}^+\right)\left(Ms_{i,t+1} - s_{i,t}\right) +$$

$$+\sum_{i\in I_t^-}\left(v_{i,t} - x_{i,t}^-\right)\left(Ms_{i,t+1} - s_{i,t}\right) + \sum_{i\in I_t^-} z_{i,t}^-\left(Ms_{i,t+1} - s_{i,t}\right) \to max$$

$$\sum_{i\in I_t^0} v_{i,t}Ms_{i,t+1} + \sum_{i\in I_t^+}\left[v_{i,t} + x_{i,t}^+\right]Ms_{i,t+1} + \sum_{i\in I_t^-}\left[v_{i,t} - x_{i,t}^-\right]Ms_{i,t+1} +$$

$$+\left(m_t - \sum_{i\in I_t^+} x_{i,t}^+ Ms_{i,t} + \sum_{i\in I_t^-} x_{i,t}^- s_{i,t} + \sum_{i\in I_t^-} z_{i,t}^-\left[s_{i,t} - Ms_{i,t+1}\right]\right) \geq \frac{1}{2}\left[\sum_{i=1}^{n} v_{i,t}s_{i,t} + m_t\right],$$

$$k_t\left(m_t + \sum_{i=1}^{n} v_{i,t}s_{i,t}\right) \geq \sum_{i\in I_t^+} x_{i,t}^+ s_{i,t} + \sum_{i\in I_t^-} z_{i,t}^- s_{i,t} - \left(m_t + \sum_{i\in I_t^-} x_{i,t}^- s_{i,t}\right),$$

$$x_{i,t}^- \leq v_{i,t}, i \in I_t^-,$$

$$x_{i,t}^+ \geq 0, i \in I_t^+,$$

$$x_{i,t}^- \geq 0, i \in I_t^-,$$

$$z_{i,t} \geq 0, i \in I_t^-,$$

Thank you!

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