The Teachers Assignment Problem

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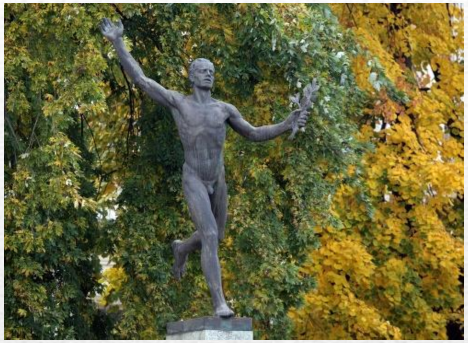
First some geography



Some history

- **1657: Universitas Cassoviensis**
- **1776: Academia Regia Cassoviensis**
- **1850-1921 Law Academy**
- **1959: Pavol Jozef Šafárik University**

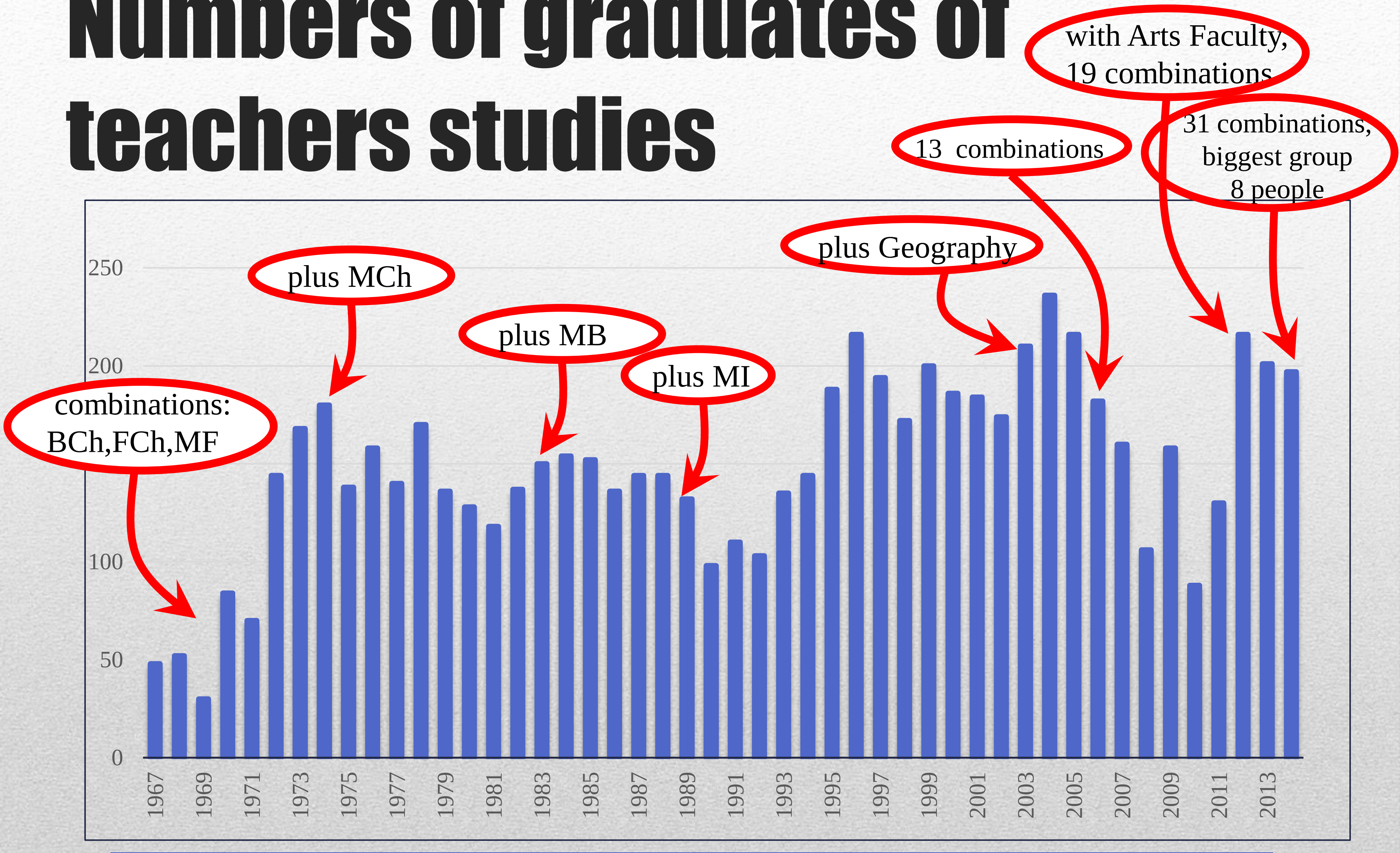
  Faculties: Medicine, Law, Public administration, Science, Arts
  1501 employees, 8138 students (2013)

1795- 1861
poet, historian, first scientific Slavist

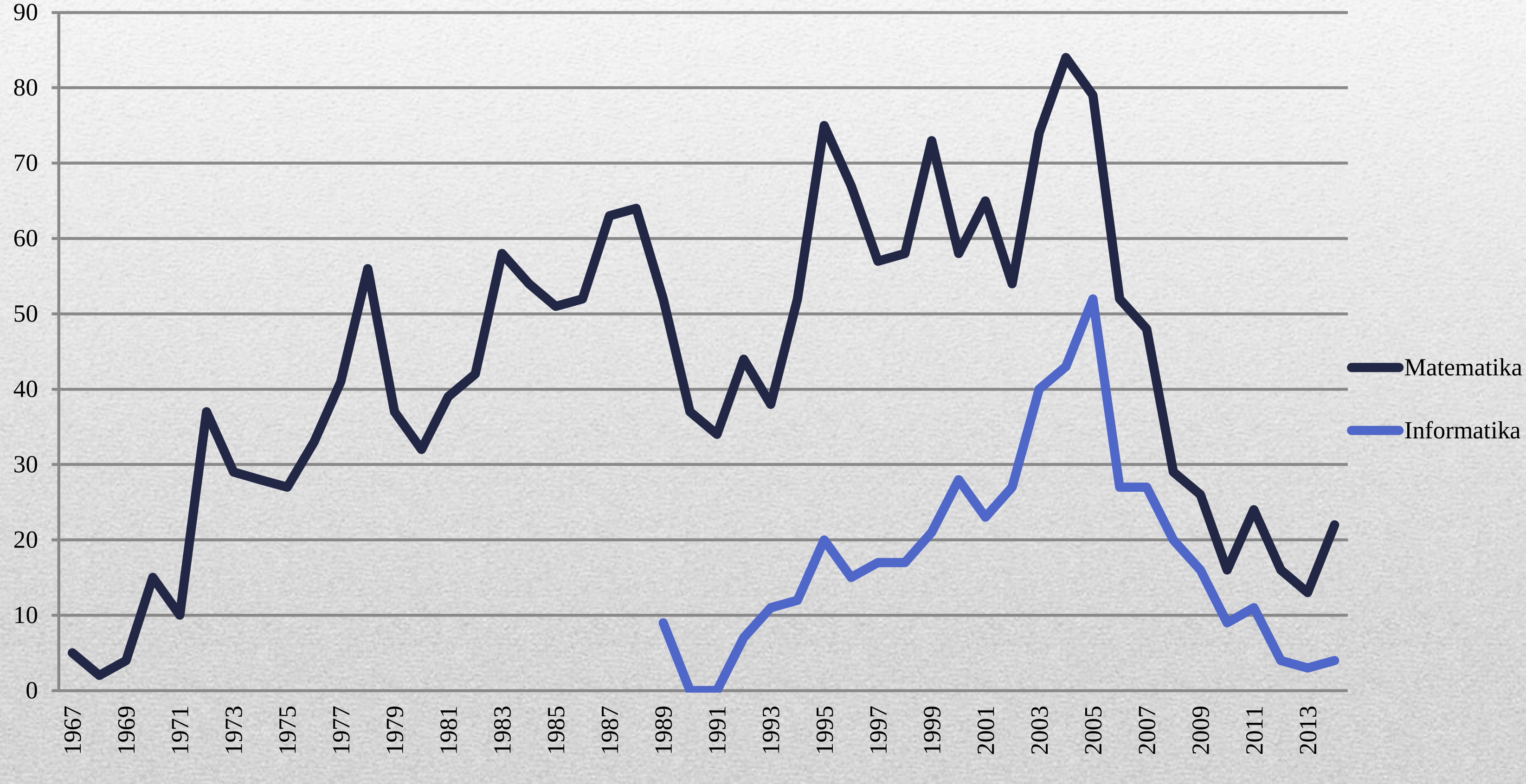
- **1963: Science faculty**

  326 employees, 1283 students (2013)
  originally: only teachers study, first graduates 1967
  combination of 2 subjects
  Mathematics, Physics, Chemistry, Biology
  Informatics (since 1989), Geography (since 2003)
  study programmes with Arts faculty (since 2011)

Numbers of graduates of teachers studies



Graduates in combinations with Mathematics and Informatics



Practical placement

Teachers study for upper elementary and lower secondary schools

- **specialization in two subjects (MF, IB, SjG,...)**
- practical placements at schools several times during their study

- ideally at different types of schools
- each student needs an approved supervising teacher for each subject
- university/faculty provides a list of teaching schools + teachers
- binary preferences: some schools are unacceptable for a student (e.g. because of commuting)

Two types of placements:

A: students divided into groups (4-6) with the same subject

groups visit classes and observe lessons, then analyze with the teacher;

one subject in period 1, second subject in period 2

B: student teaches pupils herself: both subjects simultaneously at the same school

Assignments are made by hand: several days needed

Assignment procedures elsewhere

Assignment procedures elsewhere

Matching in Practice

European network for research on matching practices in education and related markets

Home | About | People | Events | Matching Practices in Europe | Research

Welcome

"Matching in Practice" was created in September 2010 to bring together the growing community of researchers in Europe working on the various aspects of assignment and matching in education and related labour markets, with a view to actively foster the interactions between the different strands of approaches used by these researchers (theory, experiments, analysis of field data, policy/market design) and aggregate expertise about the actual functioning of these markets in Europe.

Upcoming Events

Meeting of COST Action IC1205 on Computational Social Choice
April 14 - April 16

MATCH-UP 2015: The Third International Workshop on Matching Under Preferences
April 16 - April 17

Not so successful stories

THE TIMES

UK News

News | Opinion | Business | Money | Sport | Life | Arts | Puzzles | Papers |

Welcome to your preview of The Times

Online selection of new doctors ‘grossly unfair’

By Nigel Hawkes Health Editor
Published at 12:00AM, March 4 2006

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RADICAL changes to medical training, introduced by the Department of Health, aim to train a new generation of doctors in the skills of communicating and working as a team.

Traditionally, junior doctors have served a form of apprenticeship, selected and mentored by senior figures who have guided their careers. Suspicious of what it saw as an “old boy” network, the department has introduced a scheme that aims to select and distribute medical students to their first posts by an entirely different system.

Not so successful stories

Not so successful stories

Na Feira, Crato ouviu dois coros: um afinado de alunos, outro de protesto de professores sem trabalho

SARA DIAS OLIVEIRA 26/09/2014 - 18:20

MULTIMÉDIA

Nesta sexta-feira, o ministro da Educação referiu-se a um "dia de festa" pela inauguração da nova EB2,3 Fernando Pessoa, em Santa Maria da Feira, escola com 1160 alunos, 41 turmas, 90 professores e 31 auxiliares. À sua espera, dentro da escola, estava um coro afinado de alunos do 9.º ano com várias canções preparadas e acompanhadas ao piano. Lá fora, rodeado por um cordão policial, um coro de protestos em alta voz, megafone em punho, mobilizado pelo movimento nacional de professores Boicote e Cerco, com "Crato rua, a escola não é tua" na ponta das línguas e um cartaz com uma fórmula matemática: "Caos nos concursos = alunos sem aulas + 40.000 professores sem trabalho."

Our task

- Create a mathematical model of the teachers assignment problem
- Study its structural and algorithmic properties
- Create a user-friendly computer program for every-day use
- My friends and colleagues involved in the research:
  - Tamás Fleiner, Budapest
  - David Manlove, Ian McBride, Glasgow
  - Pavlos Eirinakis, Yiannis Mourtos, Dimitris Magos, Athens
  - Eva Oceľáková-Potpinková, Silvia Bodnárová, Michal Barančík - Košice






We also want to thank to COST action IC 1205

Outline of the talk

- **Maximizing the number of assigned trainee teachers:**
  - placement A and placement B
  - combinatorial representation and complexity results
  - results of ILP implementation
  - approximation algorithms
- **Two sided preferences - stability**
  - suitable stability notion
  - complexity results

The talk is based on publications

- K. Cechlárová, T. Fleiner, D. Manlove, I. McBride, E. Potpinková: **Modelling practical placement of trainee teachers to schools**, Central European Journal of Operations Research 23(3), 547-562, 2015.
- K. Cechlárová, P. Eirinakis, T. Fleiner, D. Magos, I. Mourtos, E. Oceľáková: **Approximation Algorithms for the Teachers A ssignment Problem**, Proc. 13th Int. Symposium on Operational Research in Slovenia, 479-484, 2015.
- K. Cechlárová, T. Fleiner, D. Manlove, I. McBride: **Stable matchings of teachers to schools**, arXiv:1501.05547

Formal model: TAP

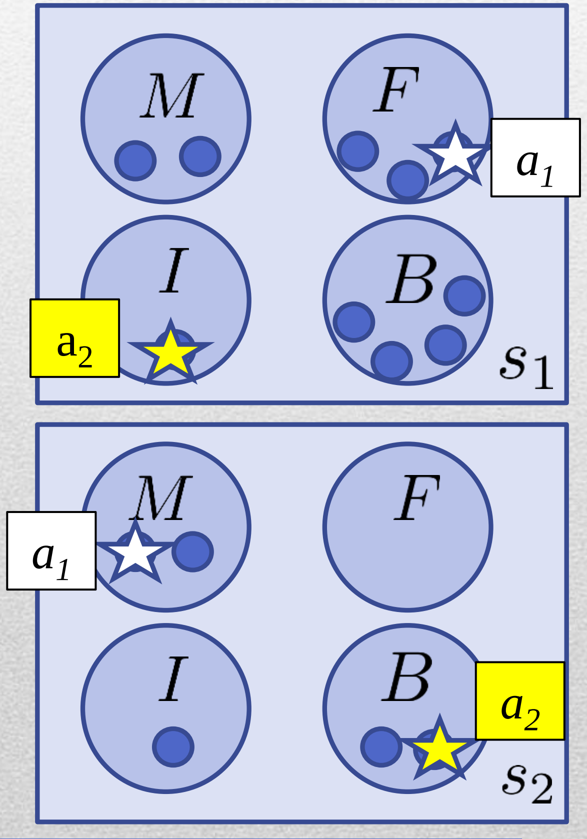
- An instance of TAP is a triple $I = (P, A, S)$
- set $P = \{M, F, B, I, \ldots\}$ of subjects
- set A of applicants = student teachers
- each $a \in A$ has pair of subjects $\mathbf{p}(a)$ and set of acceptable schools $S(a)$

	$\mathbf{p}(a_i)$	$\mathbf{s}(a_i)$
a_1	MF	$\{s_1, s_2\}$
a_2	IB	$\{s_1, s_2, s_5, \ldots\}$
$\vdots$	$\vdots$	$\vdots$

- set S of schools with partial capacities $c_p(s)$

	c_M	c_F	c_B	c_I	$\ldots$
s_1	2	3	4	1	$\ldots$
s_2	1	0	2	1	$\ldots$
$\vdots$					

Placement A:
separated subjects



Separated subjects – network flows

For instance I construct network $N(I) = (V, E, w)$

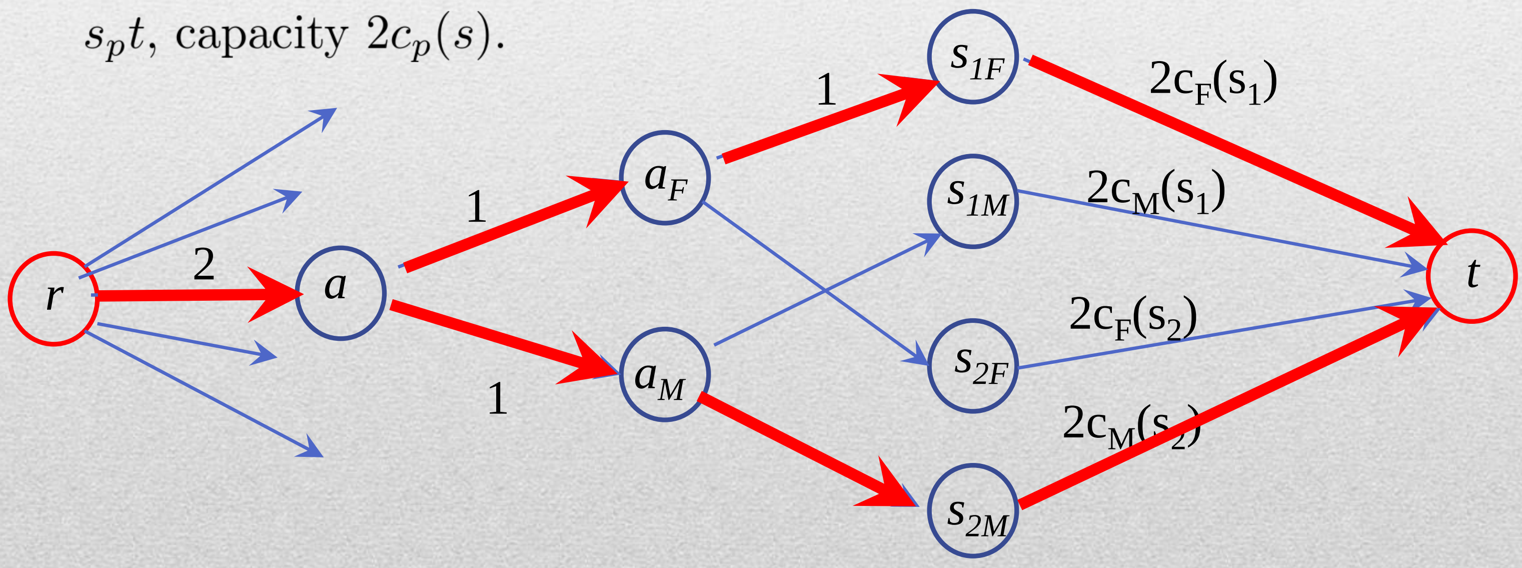
$V = \{a, a_p, a_q, \mathbf{p}(a) = \{p, q\}, a \in A\} \cup \{s_p, s \in S, p \in P, c_p(s) \neq \emptyset\} \cup \{r, t\}.$

Arcs: ra for each $a \in A$; capacity 2

aa_p, aa_q for $p, q \in \mathbf{p}(a)$, capacity 1

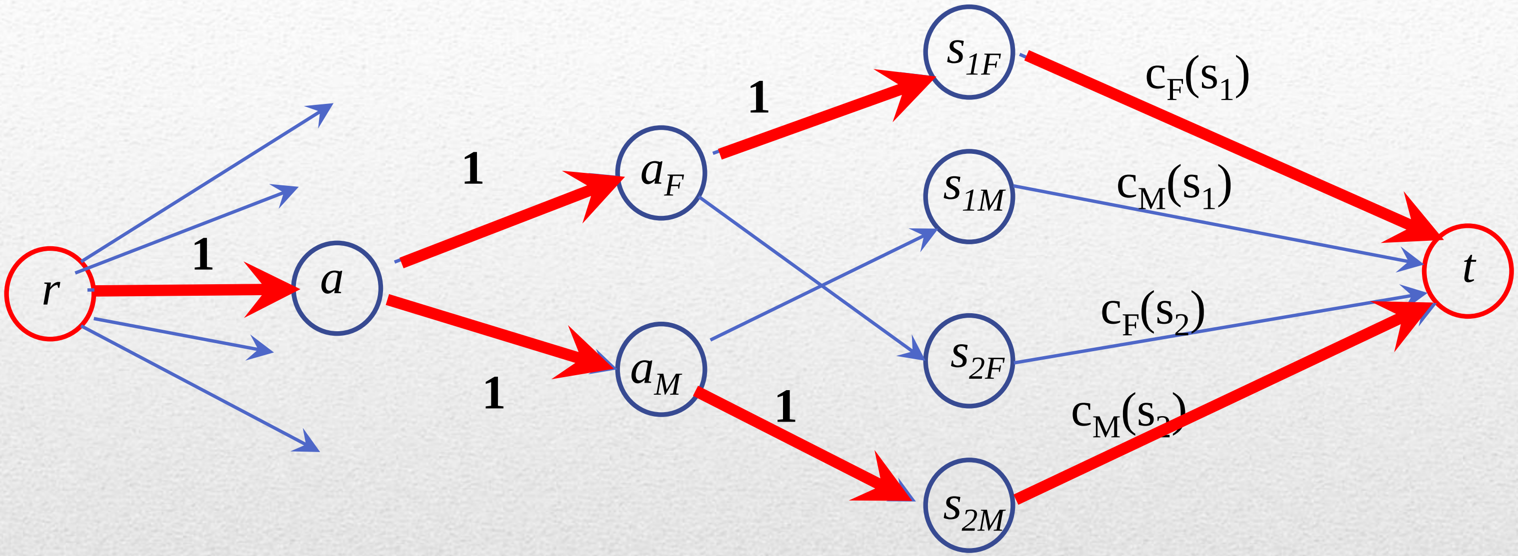
$a_p s_p$ for $s \in S(a)$, capacity 1

$s_p t$, capacity $2c_p(s)$.



Theorem. All students can be placed iff $N(I)$ admits a flow of size $2 \cdot |A|$.

Separated subjects – network flows



To allocate subjects to periods: technique of capacity and flow halving

each capacity w replaced by $\lceil \frac{w}{2} \rceil$, each flow $f(e)$ replaced by $\frac{f(e)}{2}$

Integrality lemma. If f is a flow of integer value K then there exists an integer flow f' such that $\lfloor f(e) \rfloor \leq f'(e) \leq \lceil f(e) \rceil$ for each arc e.

Arcs with flow equal 1 correspond to the subject performed in period 1.

R. W. Irving, *Matching medical students to pairs of hospitals: a new variation on an old theme*, LNCS 1461, 381-392 (1998).

Separated subjects – create groups

Lemma. Let $n \geq 8$ be any integer. Then there exist integers x_1, x_2, x_3 such that $n = 4x_1 + 5x_2 + 6x_3$.

Proof. Let $y = \lfloor \frac{n}{6} \rfloor$.

x *mod* 6	x_1	x_2	x_3
0	y	0	0
1	$y-2$	1	2
2	$y-1$	0	2
3	$y-1$	1	1
4	y	0	1
5	y	1	0

Number of groups is $x_1 + x_2 + x_3$; for $n \geq 8$ we get minimum possible.

If the numbers of students whose specialization involves one subject is smaller than 8, then the minimizing the number of placement groups difficult.

Separated subjects – create groups

specialization	Students
MF	Anna, Boris
MI	Clyde, Daniel
IF	Eva, Frank

Group	Students
M	Anna, Boris, Clyde, Daniel
I	Clyde, Daniel, Eva, Frank
F	Anna, Boris, Eva, Frank

- these groups cannot be scheduled
- if M is in period 1, then both F and I must be in period 2
- Eva and Frank cannot complete both subjects

Separated subjects – create groups

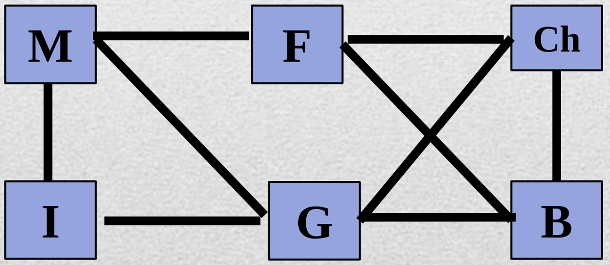
Theorem. If the numbers of students studying one subject are small then minimizing the number of placement groups is NP-hard.

Proof. Polynomial transformation from the problem BIPARTITE SUBGRAPH

Input: Graph $G = (V, H)$, integer k.

Question: Is it possible to delete at most k vertices to make G bipartite?

Transformation: Vertices are subject, edges are students



An instance (G, k) of BIPARTITE SUBGRAPH is a YES-instance $\Longleftrightarrow$ if $|V| + k$ groups suffice.

$\Longrightarrow$ Let us delete k vertices

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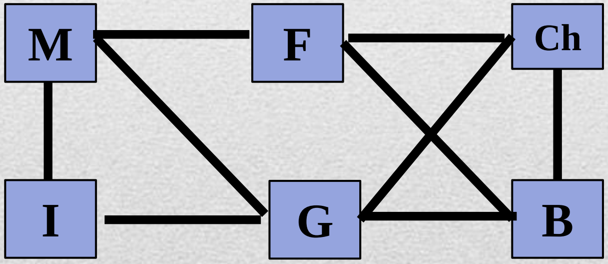

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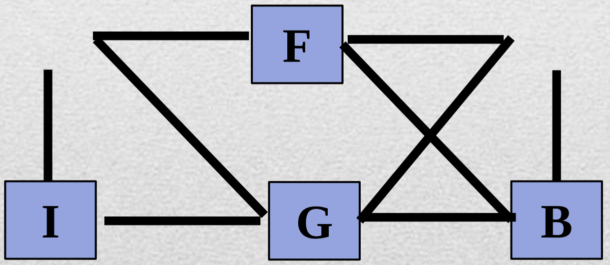

An instance (G, k) of BIPARTITE SUBGRAPH is a YES-instance $\Longleftrightarrow$ if $|V| + k$ groups suffice.

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Now we can color the vertices by 2 colors

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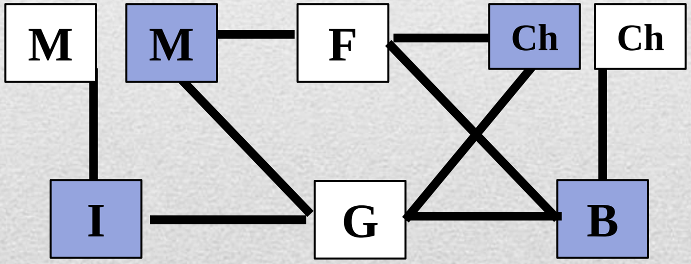

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$\Longrightarrow$ Let us delete k vertices

Now we can color the vertices by 2 colors

Duplicate the deleted vertices
and color them the by two different colors

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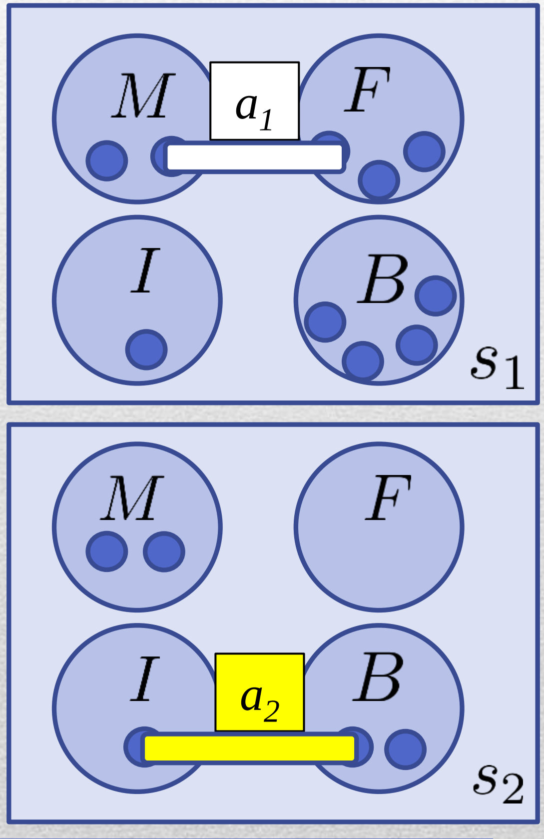
$\vdots$					

Placements B: inseparable subjects



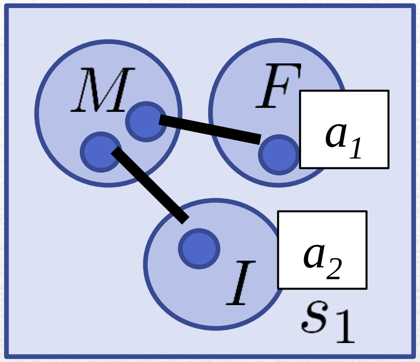
Inseparable subjects

Applicants:

a_1	MF	s_1
a_2	MI	s_1
a_3	FI	s_1, s_2

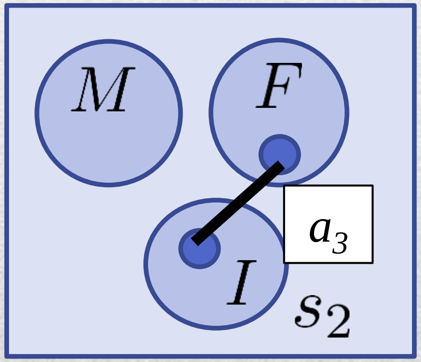
Schools:

	c_M	c_F	c_I
s_1	2	1	1
s_2	0	1	1



An assignment of students to schools is *feasible* if:

- each student is assigned to an acceptable school
- for each $s \in S$ and each $p \in P$: the number of students assigned to s whose specialization includes p does not exceed the capacity $c_p(s)$ of school s in subject p.



In this instance, all students can be placed.
However, if the first step is unfortunate ...

Inseparable subjects

Applicants:

a_1	MF	s_1
a_2	MI	s_1
a_3	FI	s_1, s_2

Schools:

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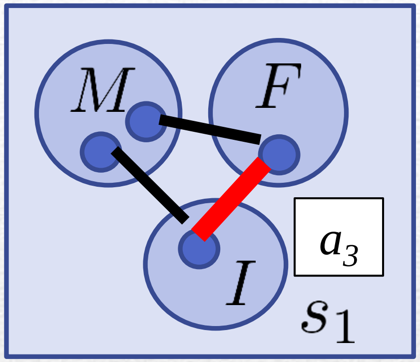

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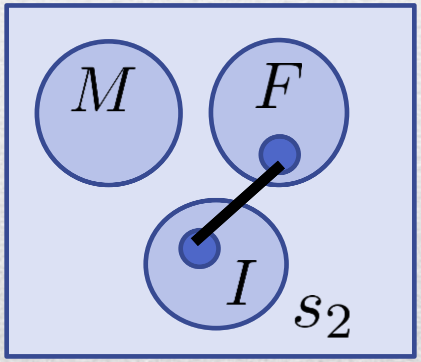


In this instance, all students can be placed.
However, if the first step is unfortunate …

… maximal matching assigning only 1/3 of maximum is obtained

Theorem. If μ and μ^* are maximal and maximum then $|\mu| \geq |\mu^*|/3$.

Computational complexity of TAP

Solvable in polynomial time if there are only 2 subjects.

Theorem 1. The problem of maximizing the number of assigned students is NP-hard even in the cases when each student lists at most 3 schools and

- there are 3 subjects and no partial capacity exceeds 2;
- there are 4 subjects and no partial capacity exceeds 1.

Theorem 2. If each applicant is allowed to list at most 2 acceptable schools and all partial capacities are at most 1 then it can be decided in polynomial time whether all applicants can be placed. However, the problem of maximizing the number of assigned students is NP-hard.

Proof: 38 subjects

Open problem: What is the minimum number of subjects for NP-completeness?

Computational complexity of TAP

Theorem 3. The problem of maximizing the number of assigned students is NP-hard even in the case when each school is acceptable for each student and no partial capacity exceeds 2.

Proof: number of subjects is not constant

Open problem: Complexity with the constant number of subjects?

Open problem: Eficiently solvable special cases?

Open problem: Other parameters for efficient solvability?

Integer linear program for TAP

Students $A = \{a_1, \ldots, a_n\}$, schools $S = \{s_1, \ldots, s_m\}$, subjects $P = \{p_1, \ldots, p_k\}$
Student a_i has a k-vector $\mathbf{y}$: $y_{ir} = 1$ iff a_i studies subject p_r
Student a_i has an ordered list of acceptable schools of length $\ell(a_i)$
Let $s(a_i, \rho)$ be the school in the ρ-th place of a_i's list
Binary variables $x_{i,\rho}$ for each a_i and each $\rho = 1, 2, \ldots, \ell(a_i) + 1$

Interpretation: $x_{i,\rho} = \begin{cases} 1 & \text{if } a_i \text{ is assigned to the school in position } \rho \\ 0 & \text{otherwise} \end{cases}$

Cost function: $\sum_{i=1}^{n} \sum_{\rho=1}^{\ell(a_i)} x_{i\rho} \to max$

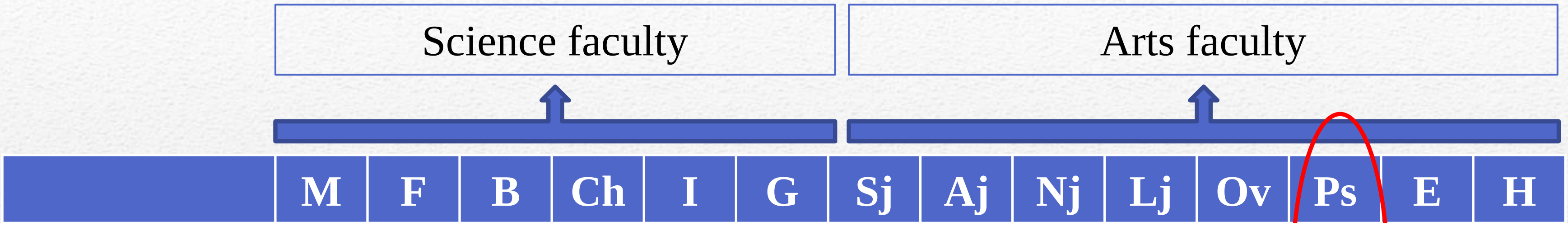
Constraints:

$$\sum_{\rho=1}^{\ell(a_i)+1} x_{i\rho} = 1$$

$$\sum_{i=1}^{n} \sum_{\rho=1}^{\ell(a_i)} \{x_{i\rho} : s(a_i, \rho) = s_j \ \& \ y_{i\rho} = 1\} \leq c_\rho(s_j)$$

$$x_{i\rho} \in \{0, 1\}$$

P.J.Šafárik university in numbers



Usual practice: students of Ps are allocated to E or Ov.

year	# of students	# of schools	# of assigned	time
2015	82	59	82	8 sec
2014	138	197	137	21 sec
2014	138	59	120	6 minutes
2014+2015	220	197	208	13 minutes
2014+2015	220	59	no result	$>$ 7 hours

unassigned combinations EH, ENj, GPs, GOv, OvH

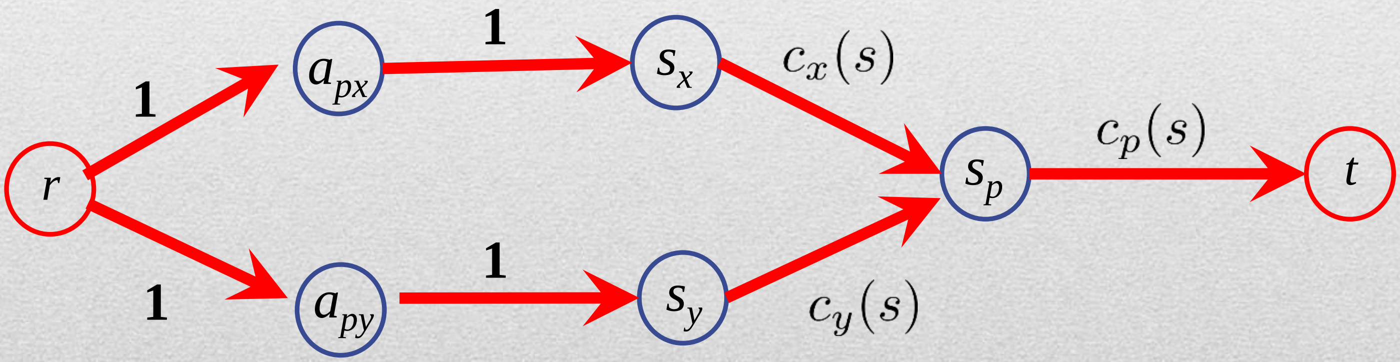
Approximation algorithms

Approximation algorithm: does not output the maximum number of asignees

Approximation algorithm $\mathcal{A}$ for a maximization problem $\mathcal{P}$ has an **appoximation guarantee** α if $Opt(I) \leq \alpha \cdot \mathcal{A}(I)$ for each instance I of $\mathcal{P}$.

Notation: A_p is the set of applicants whose specialization involves subject p

Algorithm to find μ^p: maximum cardinality matching for A_p: flow network

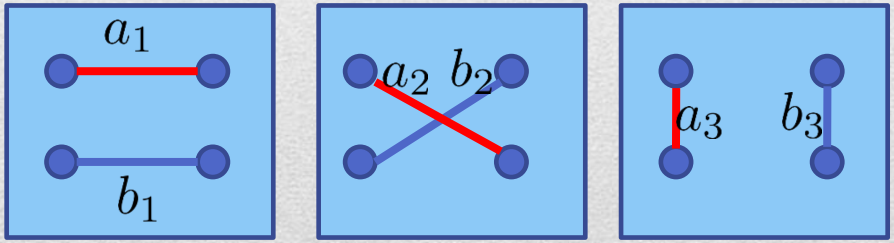


Approximation algorithms: Greedy1

begin fix the order of subjects $1, 2, \ldots, k$;
for $p := 1$ to k **do**
    begin find a maximum cardinality matching μ^p for A_p;
        reduce the set of applicants and partial capacities of schools accordingly
    end
end

Theorem. The approximation guarantee of algorithm Greedy1 is 2.

applicant	type	acceptable schools
b_1	$\{3,4\}$	s_1
b_2	$\{2,4\}$	s_2
b_3	$\{2,3\}$	s_3
a_1	$\{1,2\}$	s_1, s_2, s_3
a_2	$\{1,3\}$	s_1, s_2, s_3
a_3	$\{1,4\}$	s_1, s_2, s_3



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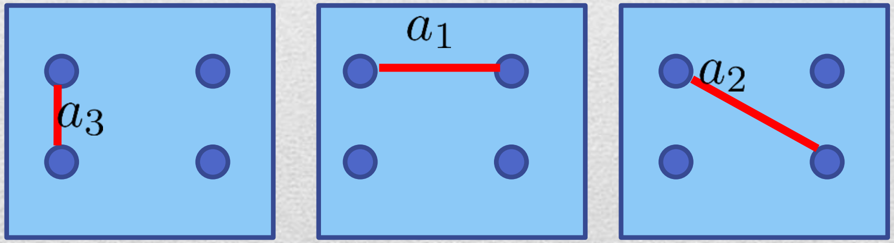


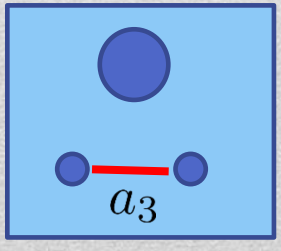
If the first stage outputs μ^1 as follows ...
matching of size $3 = \frac{1}{2}$ of optimum is output.
Approximation bound is tight.

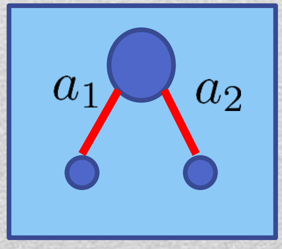
Approximation algorithms: Greedy2

begin for $p := 1$ to k find a maximum cardinality matching μ^p of applicants A_p;
keep μ^j whose size is maximum;
add applicants from $A \setminus A_j$ arbitrarily to get a maximal matching
end

Theorem. The approximation guarantee of algorithm Greedy2 is $\frac{k}{2}$.
Better only for $k = 3$.

school	capacities for 1	2	3	applicant	type	acceptable schools
s_1	2	1	1	a_1	$\{1,2\}$	s_1, s_2
s_2	2	1	1	a_2	$\{1,3\}$	s_1, s_2
				a_3	$\{2,3\}$	s_1





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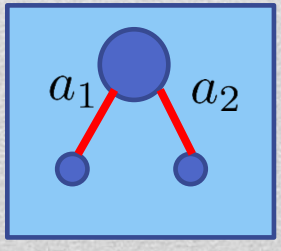

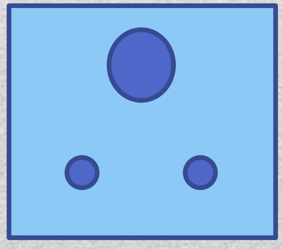

If the first stage outputs μ^1 as follows . . .
matching of size $2 = \frac{2}{3}$ of optimum is output.
Approximation bound is tight.

Stability definition

Definition. Let μ be a matching. We say that a pair $(a, s) \in A \times S$ with $\mathbf{p}(a) = \{p_1, p_2\}$ is *blocking* if a is not assigned in μ or a prefers s to $\mu(a)$ and one of the following conditions hold:

(i) s is undersubscribed in both p_1 and p_2,

(ii) s is undersubscribed in p_i and it prefers a to one applicant in $\mu_{p_{3-i}}(s)$ for some $i \in \{1, 2\}$,

(iii) s prefers a to one applicant in $\mu_{p_1,p_2}(s)$,

(iv) s prefers a to two different applicants a_1, a_2 such that $a_1 \in \mu_{p_1}(s)$ and $a_2 \in \mu_{p_2}(s)$.

A matching is *stable* if it admits no blocking pair.

school	capacities for M	I	F	preferences	applicant	type	preferences
s_1	1	1	2	a_3, a_4, a_1, a_2	a_1	MF	s_1, s_3
s_2	1	1	1	a_4, a_3	a_2	MF	s_1, s_3
s_3	1	1	2	a_4, a_1, a_2	a_3	MI	s_1, s_2
					a_4	IF	s_3, s_2, s_1

blocking pairs:

(a_4, s_3)

(a_4, s_2)

(a_1, s_1)

(a_3, s_1)

Stability definition

Definition. Let μ be a matching. We say that a pair $(a, s) \in A \times S$ with $\mathbf{p}(a) = \{p_1, p_2\}$ is *blocking* if a is not assigned in μ or a prefers s to $\mu(a)$ and one of the following conditions hold:

(*i*) s is undersubscribed in both p_1 and p_2,

(*ii*) s is undersubscribed in p_i and it prefers a to one applicant in $\mu_{p_{3-i}}(s)$ for some $i \in \{1, 2\}$,

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A matching is *stable* if it admits no blocking pair.

school	capacities for M	I	F	preferences
s_1	1	1	2	a_3, a_4, a_1, a_2
s_2	1	1	1	a_4, a_3
s_3	1	1	2	a_4, a_1, a_2

applicant	type	preferences
a_1	MF	s_1, s_3
a_2	MF	s_1, s_3
a_3	MI	s_1, s_2
a_4	IF	s_3, s_2, s_1

blocking pairs:
(a_4, s_3)
(a_4, s_2)
(a_1, s_1)
(a_3, s_1)

Stability definition

Definition. Let μ be a matching. We say that a pair $(a, s) \in A \times S$ with $\mathbf{p}(a) = \{p_1, p_2\}$ is *blocking* if a is not assigned in μ or a prefers s to $\mu(a)$ and one of the following conditions hold:

- (i) s is undersubscribed in both p_1 and p_2,
- (ii) s is undersubscribed in p_i and it prefers a to one applicant in $\mu_{p_{3-i}}(s)$ for some $i \in \{1, 2\}$,
- (iii) s prefers a to one applicant in $\mu_{p_1,p_2}(s)$,
- (iv) s prefers a to two different applicants a_1, a_2 such that $a_1 \in \mu_{p_1}(s)$ and $a_2 \in \mu_{p_2}(s)$.

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school	capacities for M	I	F	preferences	applicant	type	preferences
s_1	1	1	2	a_3, a_4, a_1, a_2	a_1	MF	s_1, s_3
s_2	1	1	1	a_4, a_3	a_2	MF	s_1, s_3
s_3	1	1	2	a_4, a_1, a_2	a_3	MI	s_1, s_2
					a_4	IF	s_3, s_2, s_1

blocking pairs:

(a_4, s_3)

(a_4, s_2)

(a_1, s_1)

(a_3, s_1)

Stability definition

Definition. Let μ be a matching. We say that a pair $(a, s) \in A \times S$ with $\mathbf{p}(a) = \{p_1, p_2\}$ is *blocking* if a is not assigned in μ or a prefers s to $\mu(a)$ and one of the following conditions hold:

(i) s is undersubscribed in both p_1 and p_2,

(ii) s is undersubscribed in p_i and it prefers a to one applicant in $\mu_{p_{3-i}}(s)$ for some $i \in \{1, 2\}$,

(iii) s prefers a to one applicant in $\mu_{p_1,p_2}(s)$,

(iv) s prefers a to two different applicants a_1, a_2 such that $a_1 \in \mu_{p_1}(s)$ and $a_2 \in \mu_{p_2}(s)$.

A matching is *stable* if it admits no blocking pair.

school	capacities for M	I	F	preferences	applicant	type	preferences
s_1	1	1	2	a_3, a_4, a_1, a_2	a_1	MF	s_1, s_3
s_2	1	1	1	a_4, a_3	a_2	MF	s_1, s_3
s_3	1	1	2	a_4, a_1, a_2	a_3	MI	s_1, s_2
					a_4	IF	s_3, s_2, s_1

blocking pairs:
(a_4, s_3)
(a_4, s_2)
(a_1, s_1)
(a_3, s_1)

Intractability

If there are only two subjects: All applicants are equivalent; the problem is in fact the hospital/residents matching problem:

- a stable matching always exists
- Rural hospitals theorem

Theorem. Given an instance of TAP, the problem of deciding whether a stable matching exists, is NP-complete. This result holds even if

- there are at most three subjects,
- each partial capacity of a school is at most 2,
- the preference list of each teacher is of length at most 3.

Master lists

Master list of teachers: the preferences of all schools are the same: $a_1, a_2, \ldots, a_n$

begin $\mu := \emptyset$;
  for $i = 1, 2, \ldots, n$
  if a_i's list contains a school with enough free capacity
  $\{s :=$ first such school on a_i's list ;
  $\mu := \mu \cup \{(a_i, s)\}$;
  $\}$
end

Algorithm Serial dictatorship

Theorem. Let J be an instance of TAP with the master list of teachers $a_1, a_2, \ldots, a_n$. Then J admits a unique stable matching that may be found by an application of Serial Dictatorship.

Master lists

Master list of schools: preferences of all teachers are the same: $s_1, s_2, \ldots, s_m$

begin
  $\mu := \emptyset$;
  for $j = 1, 2, \ldots, m$
    /* let s_j's list be $a_{i_1}, \ldots, a_{i_\ell}$ */
    for $r = 1, 2, \ldots, \ell$
      if a_{i_r} is unassigned and s_j has enough capacity for a_{i_r} **then**
        $\mu := \mu \cup \{(a_{i_r}, s_j)\}$;
end

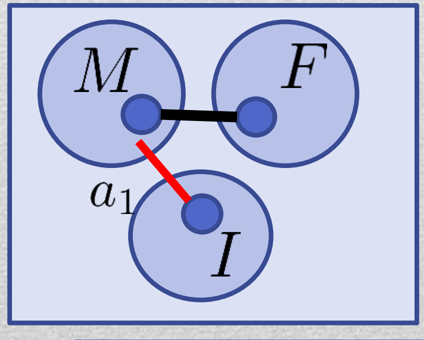
Algorithm Double Serial dictatorship

Theorem. Let J be an instance of TAP with the master list of schools $s_1, s_2, \ldots, s_m$. Then J admits a unique stable matching that may be found by an application of Double Serial Dictatorship.

Subject specific preferences

Definition. Let μ be a matching. We say that a pair (a, s) with $\mathbf{p}(a) = \{p_1, p_2\}$ and $s \in S(A)$ is blocking if a is not assigned in μ or a prefers s to $\mu(a)$, and one of the following conditions hold:

- (i) s is undersubscribed in both p_1 and p_2,
- (ii) s is undersubscribed in p_i and it prefers a in subject p_{3-i} to one applicant in $\mu_{p_{3-i}}(s)$ for some $i \in \{1, 2\}$,
- (iii) s prefers a in both subjects p_1, p_2 to one applicant in $\mu_{p_1,p_2}(s)$,
- (iv) s prefers a in subject p_1 to applicant $a_1 \in \mu_{p_1}(s)$ and in subject p_2 to another applicant $a_2 \in \mu_{p_2}(s)$.



Applicants:

a_1	MI
a_2	MF
a_3	FI

Preferences of school s:

M	a_1, a_2
F	a_2, a_3
I	a_3, a_1

At most one applicant can be assigned

Let a_2 be assigned

Then (a_1, s) blocks, as

- there is free place in I
- s prefers a_1 to a_2

Subject specific preferences

Theorem. If there are subject specific preference lists, the problem of deciding whether a stable matching exists is NP-complete. This result holds even if

- there are at most three subjects,
- each partial capacity of a school is at most 1,
- the preference lists of the schools are derived from subject-specific master lists of the applicants,
- and the preference lists of the applicants are derived from a single master list of schools.

Minimizing instability

Abraham, Biró, Manlove(2006): stable roommates

Biró, Manlove, Mittal (2010): stable marriage

P: any stable matching problem, deciding existence NP-complete.

MIN BP P: find a matching with the minimum number of blocking pairs for P.

$opt(I)$= 1 plus the minimum number of blocking pairs of any matching in I.

Theorem. MIN BP P is not approximable within $n^{1-\varepsilon}$, where n is the number of agents in a given instance, for any $\varepsilon > 0$ unless P=NP.

This result holds even if there are at most three subjects, each partial capacity of a school is at most 1, the preference lists of the schools are derived from subject-specific master lists of the applicants, and the preference lists of the applicants are derived from a single master list of schools.

Thank you for your attention!