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DYNAMIC OPTIMIZATION OF STYLIZED EQUITY PORTFOLIOS USING COPULAS

PhD Dissertation Summary
for the purpose of obtaining
Philosophy Doctor in Economics HSE

Academic supervisor:
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JEL: C14, C15, C58, G11,
G23

Moscow – 2018

1. General description of work

Relevance of the research topic

Investigation of the basic investment styles on the stock market, as well as tools for optimization of an investment portfolio under constraints (which can be determined by regulation in Russian Federation) can increase the degree of elaboration of portfolio theory and solve a number of actual problems in asset management. Investing by style assumes the division assets into groups by some criteria. Criteria can be financial indicators of companies (for example, the ratio of market capitalization of the company to its profits), statistical indicators of stock returns (for example, volatility), etc. Thus, stylized portfolios include stocks that have a specific characteristic that allows being attributed to a particular investment style. In addition to the style characteristic, in this study the portfolio is also optimized in terms of joint risk of assets. Taking into account both the risk and characteristics of the investment style allows one to operate with the concept of stylized optimization of the investment portfolio.

The paper is devoted to the problem of stylized portfolio optimization in relation to mutual funds in Russia. This on one hand, allows to bring a solution closer to the reality, as the mutual funds are subjected to certain regulatory restrictions, while on the other hand offers a solution to the problem in the context of the most common and transparent forms of mutual investment in Russia. According to the statistics provided by the "National League of Managers" as of June 2017 in Russia, there were more than 100 equity mutual funds with total assets under management of about 33 billion rubles.

Distinction between different investment styles is one of the logical steps in the development of the stock market and portfolio management in Russia. In developed markets such as US or UK, style investing is already quite widespread. There is a wide range of investment funds, including ETFs, with particular investment style (growth, value, momentum, dividends, etc.). Thus, the solution to the problem of portfolio optimization within specific investment style could be of practical importance, since for asset management companies it will allow diversifying the range of available investment products, and, consequently, increasing the attractiveness of mutual funds as a more flexible instrument. On the other hand, indicating particular investment style of a fund will add transparency to expected actions of its manager, which will also increase its attractiveness in comparison to mutual funds, without a particular declared strategy.

At present, mutual funds may acquire additional popularity not only due to the appearance of Individual Investment Accounts (IIS) offering tax incentives to Russian residents, but also because of reducing rates on bank deposits, which is the main and most preferred alternative to mutual funds in Russia. Reduction of rates can cause an outflow of money from bank deposits to the stock and bond markets, which is already happening (see Figure 1 below). In light of this, it is important to note the need for the development of competencies in asset management. The results of this study can become one of the key additions to the toolkit of traders, asset managers and risk managers, both private and professional. The use of the procedures for optimizing stylized investment portfolios within the framework of investment funds management will allow making more flexible investment products in terms of investor preferences, thus providing an additional competitive advantage for mutual funds over other options for financial investments.

Considering the relevance of the study from an academic point of view, it is appropriate to emphasize a number of things. The first one is the problem of optimizing an investment portfolio, which was solved generally within the framework of the Markowitz portfolio theory (1952). In the paper, it was proposed to compile investment portfolios as a function of expected return at a given level of risk (and vice versa). The study did not specify particular methods for estimation of risk and expected return. One may find the most famous and simple solution to this issue in Sharpe (1978). The solution is to use statistical indicators of returns of assets that make up a portfolio, in particular covariance and mean return. This method for portfolio optimization is often referred in the literature as the Markowitz portfolio. Individual works have shown the advantage of this approach to the naive approach, which builds a portfolio of assets with equal weights (Bernartzi & Thaler (2001), DeMiguel et al. (2009) and Windcliff & Boyle (2004)). A number of researchers (Haugen & Baker (1996), Pástor & Stambaugh (2000), Chou et al. (2006), among others) questioned the applicability of other characteristics of stocks (and/or issuers of stocks) not based on changes in asset prices (for example, market capitalization, multiples, different financial ratios etc.) to portfolio optimization. They also provided comparison of such methods with the more common optimization approach based on statistical indicators. Unfortunately, in most of these papers there is no specific methodology that would link the weights in the portfolio to the specific characteristics of the equities. The solution to this problem appeared somewhat later in the studies by Brandt et al (2009), Hjalmarsson & Manchev (2012), Flieberg et al. (2016), Fletcher (2017). While these works are close enough to the concept of this research, it is important to emphasize advantages of this study in comparison with its predecessors. First, while the researchers suggested their approaches to building a portfolio with asset weights dependent on the characteristics of companies / stocks, the risk was taken into account only in Flieberg et al. (2016) and Fletcher (2017). At the same time, the latter two only examine the risk of each asset individually, without taking into account possible interrelationships between returns of different assets. This can lead to erroneous estimation of the risk of the whole portfolio. In contrast, this study assesses the joint risk of assets, instead of only individual risks, thus allowing us to more accurately assess the portfolio risk and optimize it accordingly. Copulas are used in this research to assess the joint risk. Copulas basically can be defined as functions of the joint distribution of random variables, which in the case of this study are daily stock returns. The use of copulas is justified by a number of advantages. First, copulas do not impose any restrictions on the distribution of random variables, which is important in analyzing equity returns. Secondly, copulas allow tracking a nonlinear relationship, which makes them more preferable than linear models. This allows for a better assessment of the joint risk of stocks and, as a result, more thorough optimization. Thirdly, using the so-called Vine copulas (or pair copula constructions – PCC), significantly simplifies joint distribution modeling for a large number of random variables. It is important to emphasize that solution to the problems raised in the study is provided based on only one particular copula model – rotated Gumbel copula. Other models are mentioned in the text, but does not discussed in detail, as the main purpose of the study in stylized portfolio optimization. The problem of comparison between different copula models is left for other studies.

Previously mentioned studies do not consider the optimization of the portfolio within the framework of any investment style, but only explore a set of characteristics to use in portfolio optimization. In light of this, it seems reasonable to adopt a somewhat more systematic approach that would consider a specific characteristic (or set of characteristics) of equities corresponding to a certain investment style and build an investment portfolio with assets weights dependent on that particular characteristic. That results in a stylized investment portfolio, a portfolio that corresponds to specific investment style while taking into account assets risk as well, allowing for optimal balance between style and risk. The concept of optimality in this case will be defined later. The following styles are considered: value, growth, profitability, momentum and dividends.

Fig. 1 Change in net asset value of mutual funds in Russia since 2012 (in billions rubbles)

The second aspect of the relevance of this research from an academic point of view is the main instrument of the research - copulas. The idea of applying copulas to optimize the investment portfolio can be justified on the one hand by a large number of new papers suggesting various modifications of copulas, and on the other hand, by continuing research on its applications in finance. The presence of a large number of modern studies in the field of finance with the use of copulas can be justified for a number of reasons. First, as shown by several studies (Erb et al. (1994), Longin & Solnik (2001), Ang & Bekaert (2002), Ang & Chen (2002), Bae et al. (2003)), time series which represent change in asset prices are generally not elliptically distributed, which limits the applicability of some alternative models. On the other hand, in the case of copulas, there are no requirements for the initial distribution of data, which makes them much more flexible and applicable. Secondly, copulas allow tracking the non-linear relationship between time series, which significantly increases their explanatory power in comparison with linear models (Patton (2006), Patton (2012), Creal et al. (2013)). In addition, copulas transfer the issue of the relationship of asset returns to the field of probability theory, allowing operating with such concepts as Value-at-Risk, which is widely used in assessing the risk of the investment portfolio (Kakouris & Rustem (2014)). Another important aspect of this study is the construction of a multidimensional copula using a Vine - copula (or construction of pair copulas - PCC). This approach has been used in a number of works, including Joe (1996), Bedford & Cooke (2001, 2002) and Kurowicka & Cooke (2006), Penicas (2014), Travkin (2013) and others. Vine - copulas significantly simplifies the calculations when constructing a multidimensional copula.

Object and subject of research

Equities and depositary receipts of issuers, domiciled in the Russian Federation, as well as stylized investment portfolios, made up of them under the restrictions imposed on the equity mutual funds in Russia, are the main objects of the study

The subject of the study is stylized portfolio optimization under given restraints and with implementation of copula to measure a portfolio risk.

Purpose and objectives of the study

The main purpose of the study is the solution to the problem of stylized portfolio optimization with development of the necessary procedure and utility functions, as well as assessing the feasibility of using copulas for this. At the same time, optimization is based on the characteristics of equities, which reflect each the aforementioned styles. In this case, optimization is understood as maximizing the ratio of the style characteristic (which, if one would draw analogy with the portfolio theory of Markowitz, can be interpreted as a proxy for the expected return) to measure of portfolio's risk.

The main objectives of the study:

• Build ratings for equities based on investment style characteristics

• Develop utility functions that take into account the risk and correspondence of equities to a particular investment style

• Justify the applicability of copulas to optimizing investment portfolios

• Justify the advantages of the developed optimization procedure in comparison with the market, using various coefficients of risk and expected return of the portfolios.

• Justify the advantage of using copulas in portfolio optimization versus the more traditional optimization approach – Markowitz portfolio.

• Present results and comparative analysis of stylized portfolios of domestic issuers, as well as justify the practicality of applying suggested methods.

Main Hypotheses

In the framework of this research, a number of hypotheses are tested:

H1: The hypothesis of the advantage of stylized portfolio optimization over the naive approach of building a portfolio with equal weights

H2: The hypothesis of the advantage of stylized portfolio optimization over the market, represented by stock index.

H3: The hypothesis of the advantage of using copula to assess the joint asset risk with stylized portfolio optimization over the more traditional optimization approach of using covariance and mean return (Markowitz portfolio).

Degree of elaboration of the problem.

The problem of this research stands at the junction of three areas - the portfolio theory, investment styles and the application of copulas in finance. Separately, each of these fields of study is quite developed, as indicated by the quantity and quality of the relevant literature.

Copulas were proposed by Sklar (1959) back in the last century, but the main papers on the use of copulas in finance began to appear much later - after 2000. It is important to note that the work by Sklar (1959) did not take into account the possibility of changing the structure of the joint distribution over time. Meanwhile, a number of research papers in the field of finance showed that the parameters of joint distribution can change with time, as some significant events may occur. So, Embrechts & Dias (2004) showed the inconsistency of the joint distribution (copula) parameter for the returns of currency pairs; Hu (2008) and Ning (2009) compared static and dynamic copulas using data on several national stock indices. Although static copulas are simpler in computation and may even have sometimes more explanatory power than dynamic ones, researchers come to the conclusion that dynamic copulas offer greater flexibility to changing conditions, which is quite important in the field of finance. Most modern works using copulas to assess the relationship of assets or portfolio risk involve the use of a dynamic copula (Ning (2009), Hu (2008), Patton (2013), Deng et al (2011), Bai M. & Sun (2007)). The first most significant paper presenting conversion from a static copula to a dynamic copula is Patton (2006). It also shows the structural shifts in the level of the relationship between currency pairs at the time of introduction of the euro, which was tracked by conditional SJC-copula. The conditional copula by Patton (2006) has been widely used by researchers for assessing the risks and the level of interdependence between asset returns (Chen & Fan (2006), Genest & Remilard (2008), Remilard (2010), Palaro & Hotta (2004), Huang et al 2009), Brechmann et al (2013) and Hu (2010) among others).

As noted in Patton (2012), the early studies on copulas were mostly related to the construction of a two-dimensional joint distributions. Although most modern works still consider two-dimensional or relatively small multidimensional joint distributions (for less than 10 random variables), there are also works that consider relatively larger dimensions for joint distributions. The most common approach for construction of multidimensional distributions is the Vine copulas (also known PCC copulas (Travkin (2013)) or hierarchical copulas (Penicas (2014)), in which a multidimensional copula is divided into a combination of conditional pair copulas and marginal distributions. The first findings on Vine-copulas can be found in Joe (1996), Bedford & Cooke (2001, 2002), and Kurowicka & Cooke (2006), while the later works study larger dimensions – Aas et al (2009), Heinen & Valdesogo (2009), Min & Czado (2010).

Considering portfolio optimization, the key paper is Markowitz (1952), which offers procedures for optimizing the investment portfolio of assets based on their statistical indicators. Various modifications based on Markowitz (1952) have been proposed, allowing the optimization of an investment portfolio with short positions, which was omitted in the original work (Jacobs et al (1999), Jacobs et al (2005), Konno et al. (2005), Jacobs et al (2006), Davidsson (2012)). A number of researchers also considered optimizing an investment portfolio under constraints - Cvitanić & KaratzasI (1992), Korn & Trautmann (1994), Chekhlov et al. (2000), Krokhmal et al (2001), Wu (2012), Cesarone et al (2013).

The convenience of using copulas in assessing risk of a portfolio and an interdependence of asset returns has become one of the reasons for the emergence of studies relating to the application of copulas in the optimization of an investment portfolio. The significant amount of research has been written on the subject (Patton (2004), Bartram et al. (2006), Hong et al (2007), Bay & Sun (2007), Ortobelly et al (2010), Deng et al. (2011), Michiels & De Schepper (2012), Ortobelly et al. (2012), Christoffersen & Langlois (2013), Kakouris & Rustem (2014), Han et al (2016) among others). Nevertheless, it is possible to identify several problems that have not yet been fully resolved, and are addressed in this study. First, there is no research on optimization of an investment portfolio of Russian equities. Second, there is no full-fledged study on portfolio optimization under constraints (for example, a restriction on the weight of a single issuer in a portfolio, such as in mutual investment funds) for optimization with copulas. It is worth noting the lack of full-fledged work, exploring various aspects of optimizing the investment portfolio with the use of copulas.

Style investing classification is quite firmly entrenched in the world of finance, and researchers often try to compare different approaches to investing in order to choose the best one by some criteria (Graham & Dodd (1934), Basu (1977), Barberis & Shleifer (2003), Black & McMillian (2004), Wahal & Yavuz (2013)). Normally such studies consider the profitability of the so-called market neutral portfolio, in which equities of one style are taken into the portfolio with positive weights (long position), while equities of a different style are taken into the portfolio with negative weights (short position). The profitability of such market neutral portfolio should reflect the existence of the statistically proven advantage of one investment style over another. It is worth mentioning that distribution of weights between individual equities of the same style is not considered in such works. Instead so-called naive approach is implemented, when each of the equities has the same weight as others, without additional rules. Such studies do not take into account liquidity of shares, which may make the research results inapplicable in actual asset management. Even if one ignores liquidity, as its assessment is not always trivial, at least the weight in the portfolio can be distributed taking into account the market capitalization of the portfolio, which may partly be a liquidity proxy. In addition, most of the stock indices are calculated that way (for example, the whole group of MICEX indices). Thirdly, the balanced portfolio essentially gives all the assets in it equal weights, while individual stocks can be more vivid representatives of one or another investment style and, accordingly, should receive more weight in the final portfolio. In this study, the last of these problems is solved.

One of the most common breakdowns is stocks of growth versus stocks of value. As noted in a number of researches (Bourguignon & De Jong (2003) and Bird & Casavvechia (2007), Cahine (2008), Athanassakos (2009)) these are the two most well-known investment styles. Investors and portfolio managers, when making up an investment portfolio in the first place, tend to be guided by the perspectives of these two groups of stocks, preferring one over another (Bourguignon & De Jong (2003), Chan & Lakonishok (2004)). Other styles have also been studied quite well. One may find research on momentum, quality or dividends in some of the following papers – Jagadeesh & Titman (1993), Chan et al. (1996), Chan et al. (2000), Novy-Marx (2013), Asness et al. (2013), Bouchaud et al (2016). At the same time, as noted earlier, studies that link the characteristics of the stock's investment style with its weight in the portfolio are quite a few. Of these, one can check Brandt et al (2009), Hjalmarsson & Manchev (2012), Flieberg et al. (2016), Fletcher (2017). It is important to emphasize though, that in these works the issue of asset risk is either completely omitted or taken into account only partly (Flieberg et al. (2016), Fletcher (2017)), without considering possible interrelationship of asset returns – one of the problems that has been solved in this paper by applying CVaR, calculated on the basis of simulations on Vine copulas.

Methodology of the research and data

The research actively uses methods of financial analysis, econometric analysis and portfolio theory. The key tool for the study is the modification of copulas - Vine copulas, which allows building multidimensional joint distribution to identify the relationship of asset returns and to assess their combined risk. For comparison of stylized portfolios, graphical analysis and various coefficients of portfolio management efficiency are also used. Data collection was implemented in the Bloomberg API and Microsoft Excel. Data preprocessing is implemented in the Python programming language in the Spyder environment, while the main calculations are performed in the Matlab environment.

Scientific innovation of the study

The novelty of this study is justified by the combination of a number of research fields – investment styles, portfolio theory, implementation of characteristics to portfolio weights estimation, risk modeling with copulas. This study:

• Develops procedures for optimization of the investment portfolio under regulatory constraints and in application to the specifics of the Russian stock market.

• Uses a relatively new method of Vine copula; its application to portfolio optimization with a large number of assets presents some value, especially when applied to the Russian market.

• Puts and solves the problem of investing in style in new light - through the optimization of the stylized investment portfolio by risk, the level of interrelation and the level of compliance with particular investment style of each asset

• Compares the behavior of various stylized portfolios on the Russian stock market and determines the most efficient in terms of various coefficients.

Theoretical significance

Theoretical significance is built from the following sequence of results. First, a procedure was developed and its advantage is proved for optimizing the investment portfolio using copulas with restrictions on the weight of the asset.

Secondly, the problem of stylized optimization of the stock portfolio is posed. Stylized optimization allows obtaining risk-balanced portfolios that best match the specific investment style. Special utility functions were developed for that purpose, connecting weights in the investment portfolio with characteristics of each stock. This makes the investment process more flexible with respect to investor preferences and adds transparency to the strategy.

Thirdly, procedures have been developed for optimizing stylized portfolios of stocks for different investment styles-cost, growth, profitability, dividends, and momentum.

Finally, the behavior of stylized portfolios has been compared under different market conditions.

Practical significance.

The results of this research can be applied in practice by both professionals (portfolio managers, traders, risk managers) and private investors. The proposed procedures for optimizing stylized portfolios will allow market participants to design optimal investment portfolios in accordance with the preferred investment style, and use the results to better understand the advantages of a particular style in different market conditions. The last point still requires additional research though.

Approbation of the results

The main results of the research were presented at the conferences: "The Third Russian Economic Congress (REC-2016)", XVII April International Conference (Session Ea-13). The results of the research were also discussed at research seminars on economics at the Higher School of Economics.

Publications

The main results of this study are presented in three articles in the Russian journals "Applied Econometrics", "Finance and Credit" and "Financial Management". These journals at the time of publication are peer-reviewed scientific journals recommended by the Higher Attestation Commission for publication of the main scientific results of the Ph.D. thesis.

The structure of the paper

The paper consists of an introduction, three chapters, a list of used literature and appendix. The text of the dissertation 147 pages long, contains 17 figures, 15 tables. The bibliography includes 135 sources.

2. The main points of the dissertation

Classification of different investment styles occurs not only in the scientific papers, but also when certain professionals in the asset management industry define their strategy.

One may find a significant number of investment funds implementing particular investment style targeted at a specific range of investors. These funds can be both passive (for example, ETF-funds of companies such as Black Rock Inc., The Vanguarg Group), and active (for example, funds of Cornerstone Advisors, MFS Investment, AQR Capital, Wisdom Tree and others). The most widespread investing styles are value, growth, profitability, dividends, and momentum. At the same time, the overwhelming part of the literature on investment styles concentrates mainly on the criteria of classification and relative profitability and ignores the issue of constructing an optimal stylized portfolio, the weights in which are linked to the characteristics that determine the correspondence of each of the stocks to particular investment style.

In this paper, equity portfolios of 5 investment styles are considered:

• Growth stocks are generally defined as stocks of companies, which market value may seem to be overvalued relative to current company's fundamentals. At the same time, this apparent discrepancy can be explained by the fact that the expectations of company's profit growth in the future are significantly higher than the average growth rates for the market/sector /country. In this study, the price-to-book multiple is used to identify growth stocks. Among the papers using this multiple for defining growth stocks, some works use this multiple to define value stocks as well – Graham & Dodd (1934), Fama & French (1998), Bauman et al (1998), Fama & French (2007), Bragg (2007), Bodie et al (2009), Pinto et al, (2010)).

• Value stocks are defined as shares of companies which market value does not fully reflect the company's financial performance in comparison with the average of the market / sector / country. That is, the company is undervalued compared to others and therefore has a greater growth potential. Value stocks are often opposed to growth stocks. Bourguignon & De Jong (2003) note that the most common way of classifying shares by investment style is using multiples. Price-to-book multiple is used in this study to define values stocks, the lower the multiple the better a stock suits value investment style (Graham & Dodd (1934), Fama & French (1998) , Bauman et al (1998), Fama & French (2007), Bragg (2007), Bodie et al (2009), Pinto et al, (2010))).

• Momentum stocks in this paper are defined as stocks, with the largest change in the price over a certain period of time. These are market leaders which, through their historically high returns, attract the attention of new investors who expect that their capitalization will continue to grow.

• Profitability stocks in this study are defined as stocks of companies with highest financial indicators such as profitability or return on capital. In this study, the ROE coefficient is used to select such stocks (preference is given to ROE, since there are more data available on it).

• Dividend stocks in this study are defined as stocks with the highest current dividend yield. The questions of the predictive power of the dividend yield in relation to the total return on stocks were investigated among others by Blume (1980), Visscher & Filbeck (2003), Connover et al. (2016). Although investors in dividend stocks in addition to dividend yield usually pay attention to other characteristics, due to the limited data on the Russian market, a simplified approach has been chosen allowing covering a longer period of time for analysis.

As mentioned earlier, copulas are used in this work to optimize weights of the investment portfolio. They allow tracking the interrelation of asset returns and assess their joint risk, which can be used to build a portfolio with the optimal structure and an appropriate ratio of expected return to risk. This study uses one of the latest copula modifications to track the change in the relationship between assets over time, which is quite important in the case of financial time series.

In this study modeling of copulas is divided into 2 stages (1) Construction of uniform marginal distributions of daily logarithmic returns of financial instruments and (2) Construction of joint distribution of returns of financial assets based on the uniform marginal distributions obtained on the first stage.

For the first stage, one needs logarithmic returns of assets, which requires a number of additional transformations of the data. To model the marginal distributions of returns of each stock, the following model is used:

$$Y_{it} = \mu_i(Z_{t-1}) + \sigma(Z_{t-1}) \times \varepsilon_{it};\ i = [1, N];\ Z_{t-1} \in F_{t-1}$$

$$\varepsilon_{it} | F_{t-1} \sim F_i(0,1) \forall t$$

Where Y_{it} is the daily logarithmic return of asset i at time t;

$\mu_i(Z_{t-1})$ is the mathematical expectation of the return on asset i based on the data available before the time t;

$\sigma_i(Z_{t-1})$ is the corresponding standard deviation of the daily logarithmic return on asset i, based on the data available before time t;

ε_{it} is the corresponding error;

F_{t-1} is the set of information available before time t.

Mean is modeled in ARMA, while GJR-GARCH is used to model the standard deviation. These models are further used to calculate standardized error:

$$\hat{\varepsilon}_{it} = [Y_{it} - \mu_i(Z_{t-1}; \hat{\alpha})]/\sigma(Z_{t-1}; \hat{\alpha})$$

Where α is the vector of optimal parameters for the models of mean and standard deviation.

As the distribution of the residuals is unknown, one needs to either make an assumption about their distribution or use a semiparametric model. In the case of this study the most of the data is stock prices. Stock prices commonly tend to change their statistical characteristics over time. In addition different price shocks tend to happen from time to time which hampers making correct assumptions about their distribution. Taking that into account it is logical to utilize semiparametric copula model, which uses empirical distribution function for modeling of the residuals. One may also assume that from the study by Kim & Silvapulle (2007), who prove that fully parametric models are non-robust against misspecification of marginal distributions and semiparametric models are better overall. Therefore, empirical distribution function (EDF) is used in this study to model residuals distribution (Patton(2012)):

$$\hat{F}_i(x) = \frac{1}{T+1}\sum_{t=1}^{T} 1\{\hat{\varepsilon}_{it} \leq x\}$$

EDF are further used for building joint distribution with Vine copulas which are based on Gumbel copula.

The resulting standardized residuals are then used to construct a joint distribution of a copula, based on the Gumbel inverse copula, which places greater emphasis on the relationship between negative returns. The choice of this copula is related to several of its advantages. First, dynamic copulas allow you to track changes in the characteristics of the joint distribution of quantities. Secondly, since the risks of the financial instruments under consideration are of the greatest interest, and the risk can be defined as the decrease in the asset value, the choice of the reverse copula of Gumbel, which focuses on the relationship of negative returns, also appears as a reasonable choice. In that sense one may also consider other more common alternative – Clayton copula, which also emphasizes interdependence of negative values. The decision for choosing rotated Gumbel copula over Clayton copula is based on study by Patton(2012) that proves better performance of the first. Patton(2012) also shows that t-copula is best in terms of performance, followed by rotated Gumbel copula. Still in terms of computation simplicity rotated Gumbel copula is better and that is the reason for picking it over t-copula in this study.

Vine - copulas allow constructing a multidimensional copula based on pair copulas of a large number of random variables, thus solving the problem of constructing a joint multidimensional distribution. The use of vine copulas for building an optimal investment portfolio has been addressed in a number of studies (Deng et al (2011), Low et al (2013)). The most frequent variations are D-Vine and C-Vine and imply a different hierarchy structure.

The joint distribution density of n the random variables $u_1, \dots, u_n$ can be decomposed without loss of generality using the following formula:

$$f(u_1, \dots, u_n) = f(u_1) \times f(u_2|u_1) \times f(u_3|u_1, u_2) \times \dots \times f(u_n|u_1, \dots, u_{n-1})$$

Each conditional density in the product can be further decomposed using a copula. For example, the second multiplier can be represented as:

$$f(u_2|u_1) = \mathrm{c}_{12}(F_1(u_1), F_2(u_2)) \times f_2(u_2)$$

Thus, expanding each factor, and using the C-Vine structure, the joint density formula can be represented in the following form (Deng et al (2011)):

$$f(u_1, \dots, u_n) = \prod_{i=1}^{n} f_i(u_i) \prod_{j=1}^{n-1} \prod_{t=1}^{n-j} c_{j,j+t|1,\dots j-1}(F\big(u_j|u_1, \dots, u_{j-1}\big), F(u_{j+t}|u_1, \dots, u_{j-1}))$$

The formula for D-Vine can be written as

$$f(u_1, \dots, u_n) = \prod_{i=1}^{n} f_i(u_i) \prod_{j=1}^{n-1} \prod_{t=1}^{n-j} c_{t,t+j|1,\dots j-1}(F\big(u_t|u_{t+1}, \dots, u_{t+j-1}\big), F(u_{t+j}|u_{t+1}, \dots, u_{t+j-1}))$$

Where the marginal conditional distributions $F(u|v)$ for each j can be represented as:

$$F(u|v) = \frac{\partial \mathrm{C}_{u,v_j|v_{j-1}}(F\big(u|v_{-j}\big), F(v_j|v_{-j}))}{\partial F\big(v_j|v_{-j}\big)}$$

Where v is a vector, v_j is a separate element of the vector, and v_{-j} is a vector v without the element v_j.

The optimal parameters for the joint distribution are selected using the maximum likelihood function. For C-Vine:

$$\sum_{j=1}^{n-1} \sum_{t=1}^{n-j} \sum_{k=1}^{K} log\, c_{j,j+t|1,\dots,j-1}(F\big(u_{j,k}|u_{1,k}, \dots, u_{j-1,t}\big), F(u_{j+t,k}|u_{1,k}, \dots, u_{j-1,k})$$

For D-Vine:

$$\sum_{j=1}^{n-1} \sum_{t=1}^{n-j} \sum_{k=1}^{K} log\, c_{t,t+j|t+1,\dots,t+j-1}(F_j\big(u_{t,k}|u_{t+1,k}, \dots, u_{t+j-1,k}\big), F_{j+t}(u_{t+j,k}|u_{t+1,k}, \dots, u_{t+j-1,k})$$

As noted in the work of Deng et al. (2011), when calculating the optimal parameters in the formulas above, it is first necessary to estimate the optimal parameters for each step in the hierarchy, and use them as initial data.

As it is emphasized in Deng et al. (2011), D-Vine and C-Vine are suitable for different datasets. The structure of C-Vine assumes the presence of one key element that determines the behavior of the rest of the elements. If there is no such element, then D-Vine is more suitable. This paper studies the relationship between returns of various stocks, issuers of which are represented in various sectors of the economy. Therefore it is not possible to determine some key stock that seems to define the behavior of the others. Consequently, for this study, it is more appropriate to use D-Vine structure of a copula.

It is important to mention that Vine-copulas allow using different copula models in the same construction. Still, in order to keep the focus of the study on stylized portfolio optimization, this study is conducted basing on only one model – rotated Gumbel copula. The problem of comparison between other copula models and pair constructions is left for other studies. In addition to Vine-copula in order to build multivariate distribution one may use Archimedian hierarchical copulas (as mentioned in Penicas(2014)), still the problem of applicability of these structure to stylized optimization is also left for the other studies.

After fitting parameters of the copula, one can switch to building the optimal portfolio. Since the study aims to develop a procedure for stylized optimization of investment portfolios, a separate procedure was performed for each of the five styles. Nevertheless, the general optimization procedure can be presented as follows:

1. Let the investment portfolio is being optimized at time T. The data on returns in period [T-365, T] is used to estimate the marginal distributions and parameters of the Vine-copula, which determines the joint distribution of financial instruments.
2. Based on the rating according to a specific criterion and according to the newest data available at time T, the Top-30%, Top-40% or Top-50% of stocks are selected. Further steps are taken for these assets.
3. ARMA is estimated for each of the stock in the groups defined on step 2
4. Joint distribution for ARMA residuals is built by optimizing Vine copula parameters
5. Residuals are generated via simulation with copula and further used to generate asset return scenarios using formulas (1) and (2).

$$R = \begin{pmatrix} r_{11} & \cdots & r_{1N} \\ \cdots & r_{ij} & \cdots \\ r_{P1} & \cdots & r_{PN} \end{pmatrix}$$

Where r_{ij} is the return on asset i on the simulation j. There are 1000 simulations for each stock.

6. Based on the simulated scenarios, which imply the specific returns for each of the stocks, one can estimate the return on the entire portfolio at each of the simulations, assuming particular weights:

$$PR = \begin{pmatrix} r_{11} & \cdot\cdot & r_{1N} \\ \cdot\cdot & r_{ij} & \cdot\cdot \\ r_{P1} & \cdot\cdot & r_{PN} \end{pmatrix} \times \begin{pmatrix} w_1 \\ \cdot\cdot \\ w_P \end{pmatrix}$$

Where PR is the portfolio return vector for given weights W = $(w_1, \ldots, w_P)$ for N independent simulations ($\sum_1^P w_i = 1$)

7. Assuming confidence level of 99%, CVaR is defined as the average value of elements of PR vector, which are less than 99% of the remaining values.
8. The portfolio is optimized by changing weight vector W to achieve minimum CVaR of the portfolio:

$$min_W(\mathrm{C}VaR_{PR}^W)$$

Where $CVaR_{PR}^W$ is CVaR of the portfolio, calculated on the basis of the PR vector, as indicated in step 4.

9. The resulting weight vector W is used as the portfolio structure until the next moment T at which the portfolio composition is revised using steps 1-8

The following restriction is imposed on the weights of individual assets in the portfolio to meet the norms of the regulation of mutual funds in Russia:

$$15\% \geq w_i \geq 0$$

In determining the rebalancing frequency of the portfolio structure, the main prerequisite was the regulation of mutual funds in Russia. In particular there is the provision that if the weight of one issuer in the portfolio exceeds 15% due to a change in the market value, the violation must be liquidated within a month. Accordingly, portfolios are reviewed on a monthly basis, so that there are no serious regulatory violations.

Steps 2 and 8 in the above-mentioned procedure should be altered in order to address specific stylize of the investment portfolio. The required modifications for each style are described below. It is important to note that although the stylization criteria defined in this study can be considered rather naive, since there are fresh studies with a more rigorous set of criteria, the main purpose of this study is not to select criteria for each investment style, but to develop a procedure for stylized portfolio optimization and introduce stylized utility functions. Expanding the set of criteria for defining each investment style would not only significantly dilute the focus of the research, but also complicate the calculations and reduce the number of observations due to limited data on Russian stock market.

The growth stocks in this study are defined as the stocks of companies with market value being high relative to current fundamental indicators. The Price-to-Book multiple is used to classify the growth stocks in this study. The higher the multiple, the more the stock fits the growth stock criteria. This multiple was also used in papers by Graham & Dodd (1934), Fama & French (1998), Bauman et al (1998), Fama & French (2007), Bragg (2007), Bodie et al (2009), Pinto et al (2010))

Stylized portfolio in this case will be optimized using the following utility function:

$$max_W\left(M_p^W/VaR_p^W\right)$$

Where M_p^W – is the average multiple of the portfolio in accordance with the weights W of the assets.

Value stocks are defined in this study as stocks of companies with market value that does not fully reflect the company's financial performance in comparison with the average for the market /

sector / country, that is, the company is undervalued. As in the previous case, the Price-to-Book multiple is used to find value stocks. Although in case of value, stocks with the lowest multiple are selected. The utility function for stylized value portfolio is as follows:

$$max_W\left(1/\left(M_p^W \times VaR_p^W\right)\right)$$

Profitability stocks are characterized by high profitability indicators. When selecting shares of profitability, stocks of companies with relatively high ROE (Return on Equity) or ROA (Return on Assets) ratios are selected. Similarly, these coefficients were used in the works of Greenblatt & Tobias (2010), Asness et al. (2013), Bouchaud et al (2016) In the case of this study, the ROE coefficient is used, since there was more data on it. The utility function for profitability portfolio is as follows:

$$max_W\left(Q_p^W/VaR_p^W\right)$$

Where Q_p^W is weighted ROE value for a portfolio with weights W.

Momentum stocks are characterized by a relatively high price change over a certain period of time. This study examines momentum for 3, 6 and 12 months' timeframes. For each periodicity a portfolio is built. The utility function for momentum portfolio is as follows:

$$max_W\left(TR_p^W/VaR_p^W\right)$$

Where TR_p^W is the weighted average return on the portfolio with weights W during previous period of time.

Dividend stocks in this study are defined as stocks with the highest dividend yield. Stylized optimization in this case results in a portfolio that is balanced by risk and dividend yield. Utility function in this case can be written as:

$$max_W\left(DY_p^W/VaR_p^W\right)$$

Where DY_p^W is the weighted average dividend yield of the portfolio with weights W.

Since the correspondence of a stock to a particular investment style is determined relatively to all other stocks in the sample, it seems reasonable to consider several rankings - Top-30%, Top-40% and Top-50% of stocks that best suit a particular investment style. In this case, for each style, except for "momentum" there are 3 portfolios, optimized based on the each of the ranks. In the case of momentum stocks, 3 periodicities (3, 6 and 12 months) are also considered, making 9 different stylized momentum portfolios. Based on the performance of each portfolio the top portfolio for each style is selected for further study. The criteria for ranking of each style are as follows:

1) Growth stocks are ranked in descending order by multiple Price-To-Book

2) Value stocks are ranked in ascending order by multiple Price-To-Book.

3) Momentum stocks are ranked in descending order by return for the previous period (which is chosen variably 3, 6, 12 months)

4) Profitability stocks are ranked in descending order by ROE coefficient

5) Dividend stocks are ranked in descending order by annualized dividend yield

To test the H3 hypothesis, one also needs to determine the stylized Markowitz portfolio. Stocks are selected similarly to stylized groups as described above, still the utility function is the same for all stylized Markowitz portfolios:

$$max_W \left(\frac{\mu_W^p}{\sigma_W^p} \right)$$

Where μ_W^p is the mean portfolio return:

$$\mu_W^p = \sum_{i=1,N} w_i \mu_i$$

σ_W^p is the standard deviation of the portfolio return:

$$\sigma_W^p = \sqrt{\sum_{i=1,N} \sum_{j=1,N} \rho_{ij} \times w_i \sigma_i \times w_j \sigma_j}$$

Where $\mu_i$is the mean return on stock i, σ_i is the standard deviation of returns of stock i, ρ_{ij} is the correlation between returns of stocks i and j, w_i is the weight of stocks i in the portfolio.

For naïve portfolio construction, which is required to test H1 hypothesis, stocks are also selected via ratings, mentioned above, but weights in the portfolio are assigned equally.

In order to compare resulting portfolios a number of ratios is used. Some of them are described below in detail.

The simplest and most common indicator is total return of a portfolio over the whole period (TR):

$$TR = \left(\frac{PS_N}{PS_0} - 1 \right) \%$$

Where PS_0 and PS_N are the portfolio values at the beginning and at the end of the period.

From total return one may also calculate average annual return of the portfolio (AR):

$$AR = \left((TR + 1)^{\frac{365}{T}} - 1 \right) \%$$

Where T is the total number of days when portfolio was managed, including weekends and holidays.

A number of coefficients are used to assess the risk of a portfolio as well. Basing on Sharpe (1978), one may find it enough to use only the standard deviation of return, but there are more informative indicators that might be useful. For example, the maximum drawdown shows the size of the maximum loss that an investor could suffer if investing in a particular investment portfolio at certain periods in the past. The maximum drawdown is determined on the basis of historical data. The formula can be written as follows:

$$MDD = \left| Min_i \left(\frac{PS_t}{Max_{j<t}(PS_j)} - 1 \right) \% \right|$$

Where PS_t is the portfolio price at time $t \in [1, T]$, $Max_{j<t}(PS_j)$ is the maximum portfolio price for the entire time before moment t.

In fact, comparing portfolios by coefficients that take into account only risk or only expected return is not advisable, since there can be different combinations of risk and return that are suitable for one investor and do not for the others. Therefore, it is appropriate to consider the coefficients combining risk and expected return in one. In addition to the standard performance ratios of Sharpe and Sortino, the coefficient Gain-to-Pain, which was proposed in Fitshen (2013), is used:

$$GP = \frac{AR}{MDD}$$

Based on the results of optimization of various types of stylized portfolios, most effective ones were chosen for each of the styles. Next, selected 5 portfolios corresponding to each of the five styles (growth, cost, profitability, dividends and "momentum") were compared with MICEX Total Return Index, naive stylized portfolios and stylized Markowitz portfolios. Also 5 portfolios were compared and among themselves.

The dynamics of the stylized portfolios total return are shown in Figure 2.

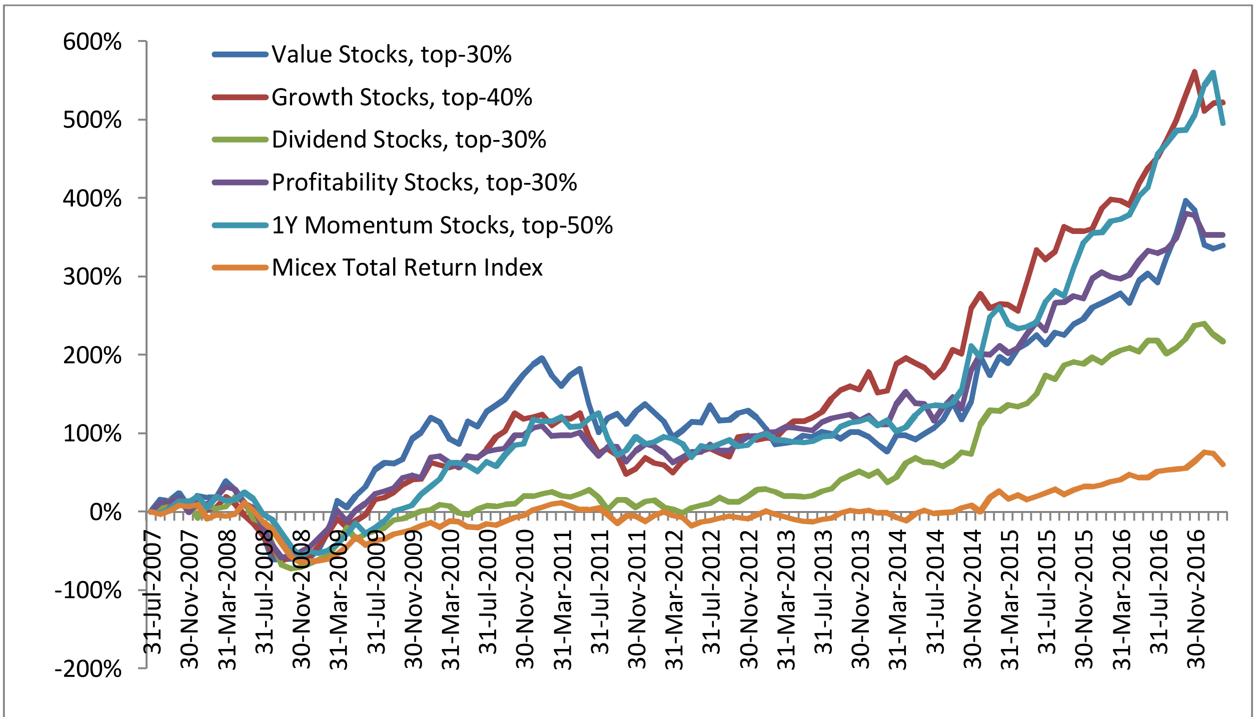

Fig. 2 Stylized CVaR portfolios performance versus Micex Total Return Index

As can be seen from the figure, the proposed procedure for optimizing the investment portfolio allows obtaining results that exceed the benchmark (MICEX Total Return Index) for each style.

The most profitable in the domestic stock market are stylized portfolios of Top-30% growth stocks and Top-50% 12 month momentum stocks. However, in the first half of the analyzed period the leading style was value stocks. From the figure above one may also draw two interesting conclusions regarding the financial crisis in 2007-2008:

• Investment style that first began to decline before the global financial crisis was growth stocks.

• Investment style, which started to recover after the global financial crisis of 2008 faster than the rest, was profitability stocks and value stocks.

Table 1 shows performance indicators for each of the stylized portfolios and the benchmark - MICEX Total Return Index. As can be seen from the table, portfolios of growth and momentum stocks are close to each other not only in terms of total return, but also in terms of risk. They also have quite similar Gain-To-Pain, Sharpe and Sortino ratios, yet momentum stocks are slightly better. MICEX Total Return Index is the least risky, but the higher risk of stylized portfolios is justified by higher return ratio. Thus, H2 hypothesis about the advantage of stylized portfolios against the market, which is represented by MICEX Total Return Index, is accepted.

Table 1 Performance indicators of optimized portfolios of different styles vs MICEX Total Return Index

	Value Top-30%	Growth Top-40%	Dividends Top-30%	Profitability Top-30%	Momentum 12m, Top-50%	MICEX Total Return
Total Return	339.8%	522.1%	217.8%	352.8%	495.7%	60.4%
Annualized Return	16.86%	21.21%	12.94%	17.22%	20.66%	5.10%
Annualized Standard Deviation	36.17%	29.45%	31.96%	27.97%	27.93%	26.34%
Sharpe Ratio	0.47	0.72	0.40	0.62	0.74	0.19
Maximum Drawdown	72.06%	68.53%	76.64%	68.35%	65.19%	67.91%
Annualized Standart Negative Deviation	22.0%	19.8%	20.9%	20.1%	21.3%	22.5%
Sortino Ratio	0.77	1.07	0.62	0.86	0.97	0.23
Gain-To-Pain Ration	0.23	0.31	0.17	0.25	0.32	0.08

In addition, it is appropriate to consider the possible costs associated with managing stylized portfolios. Table 2 shows the average annual turnover of the portfolios and the average transaction costs as a percentage of the portfolio value. Transaction costs are assumed to be the fee for the transaction (accepted at 0.15% per transaction, which is a rather high estimate for the Russian brokerage services) and slippage, that is, the difference between the current price and the price at which it was actually possible make a trade (accepted at the level of 0.2%, which is expert estimate). As can be seen from Table 2, the impact of transaction costs on the return of stylized portfolios is limited, although in the case of momentum portfolios, costs may be more than 1% per year. Therefore for the purpose of cost efficiency growth stocks portfolio will be more preferable as the performance is similar. On the other hand, such insignificant transaction costs can be associated with a relatively small number of stocks, from which the constituents of the portfolio are selected. Perhaps the application of the proposed methodologies in markets with a larger number of stocks will result in higher transaction costs.

Table 2 Analysis of transaction costs for stylized portfolios

Investment Style	Average annual Turnover	Average Annual Transaction Costs
Value Stocks Top-30%	25.9%	0.09%
Growth Stocks Top-40%	41.2%	0.14%
Dividend Stocks Top-30%	28.1%	0.12%
Profitability Stocks Top-30%	16.3%	0.06%
12 month Momentum Stocks Top-50%	287.4%	1.01%

This study also has to consider whether the use of copulas improves optimizing of stylized portfolios. For that the comparison of CVaR stylized portfolios versus Markowitz stylized portfolios is provided in the table below. As can be seen from the table, in most cases, the new method offers better performance (besides the standard deviation of return, which makes it more risky, although that is justified by higher return). This confirms the **H3** hypothesis: copulas do allow for better tracking of the relationship and joint risk of assets, and therefore they allow building a portfolio that is more effective in terms of risk and return than the traditional approach that uses covariance for risk assessment. The only style that showed a slightly higher maximum drawdown for the new portfolio format is profitability stocks, however, this high drawdown is offset by a higher expected return and new method still has advantage in terms of the return-to-risk ratio. Therefore, the advantage of the new method is confirmed for all considered styles and **H3** hypothesis is accepted fully.

Table 3 Comparison of the results of Stylized CVaR-Copula portfolios and Stylized Markowitz portfolios

Investment Styles	Portfolio method	Total Rturn	Annualized return	Annualized standard deviation	Sharpe Ration	Maximum Drawdown	Annualized standard negative deviation	Sortino Ration	Gain-To-Pain Ratio
Momentum	Markowitz	307.6%	13.51%	26.12%	0.52	67.48%	-22.4%	0.60	0.20
	Copula CVaR	587.9%	19.00%	26.80%	0.71	65.19%	-20.8%	0.91	0.29
Profitability	Markowitz	285.9%	13.0%	27.9%	0.46	67.7%	-21.0%	0.62	0.19
	Copula CVaR	492.6%	17.4%	28.2%	0.62	68.4%	-20.1%	0.87	0.26
Growth	Markowitz	296.3%	13.23%	29.17%	0.45	71.20%	-20.4%	0.65	0.19
	Copula CVaR	678.3%	20.34%	29.45%	0.69	68.53%	-19.8%	1.02	0.30
Value	Markowitz	364.5%	14.86%	35.45%	0.42	72.88%	-22.2%	0.67	0.20
	Copula CVaR	513.7%	17.78%	36.17%	0.49	72.06%	-22.0%	0.81	0.25
Dividends	Markowitz	174.8%	9.55%	31.05%	0.31	77.68%	-21.5%	0.44	0.12
	Copula CVaR	327.5%	14.00%	31.96%	0.44	76.64%	-20.9%	0.67	0.18

Table 4 below provides comparison of the proposed copula CVaR procedure of stylized optimization and the naive method of constructing a stylized portfolio. A naive method is the construction of a portfolio with all assets in it weighted equally. For better comparison naïve portfolios are also stylized in a way that they contain Top-30%, Top-40% or Top 50% or assets of a particular investment style. As can be seen from the Table 4, almost for all styles, a Copula CVaR approach with a stylized portfolio optimization gives better results in terms of return to risk ratio than naïve approach. However, there are 2 caveats:

• Naive construction of a growth stocks portfolio gives better results than the stylized optimization approach in terms of all coefficients except standard negative deviation and maximum drawdown. Although the advantage is not significant in order to simplify investment process, it makes sense to choose the naive approach over the Copula CVaR approach

• For momentum and dividend styles, stylized Copula CVaR optimization may appear more risky in terms of the standard deviation, although higher risks are justified by higher return.

Table 4 Comparison of the results of portfolios constructed by the naive and the Copula CVaR method proposed in this paper

Investment Styles	Portfolio method	Total Rturn	Annualized return	Annualized standard deviation	Sharpe Ration	Maximum Drawdown	Annualized standard negative deviation	Sortino Ration	Gain-To-Pain Ratio
Momentum	Naive	285%	13%	27%	0.49	70%	-24%	0.55	0.18
	Copula CVaR	587.9%	19.00%	26.80%	0.71	65.19%	-20.8%	0.91	0.29
Profitability	Naive	391.9%	15.46%	28.49%	0.54	69.58%	-19.6%	0.79	0.22
	Copula CVaR	492.6%	17.4%	28.2%	0.62	68.4%	-20.1%	0.87	0.26
Growth	Naive	739.8%	21.16%	29.34%	0.72	70.84%	-20.2%	1.05	0.30
	Copula CVaR	678.3%	20.34%	29.45%	0.69	68.53%	-19.8%	1.02	0.30
Value	Naive	-53.5%	-6.68%	38.72%	-0.17	89.60%	-28.3%	-0.24	-0.07
	Copula CVaR	513.7%	17.78%	36.17%	0.49	72.06%	-22.0%	0.81	0.25
Dividends	Naive	173.4%	9.50%	31.17%	0.30	76.66%	-21.3%	0.44	0.12
	Copula CVaR	327.5%	14.00%	31.96%	0.44	76.64%	-20.9%	0.67	0.18

Thus, the hypothesis **H1** about the advantage of stylized optimization over the naive method of building a portfolio can be accepted, but with the above-mentioned exception for growth stocks

3. Main conclusions of the study

In this paper, methods for optimizing stylized investment portfolios were developed and analyzed. The introduced methods allow obtaining effective portfolios corresponding to one of the specific investment styles - value, growth, profitability, dividends, and momentum. At the core of the methods is the optimization with the utility function that uses the ratio of characteristic of a particular investment style to a measure of risk of the portfolio. That way an investor can obtain a portfolio that suits particular investment style, yet is also balanced in terms of risk. The introduced procedure was defined as a stylized optimization of the investment portfolio. Stylized portfolios were built from stocks of Russian companies during the period from 2006 to 2017, and then compared with the benchmarks - the MICEX Total Return Index, naive stylized portfolios and stylized Markowitz portfolios. Also, portfolios were constructed under particular restrictions, corresponding to the regulation of mutual funds in the Russian Federation.

Three hypotheses were tested within the framework of the study:

H1: The hypothesis of the advantage of stylized portfolio optimization over the naive approach of building a portfolio with equal weights for each of the styles in question - value, growth, profitability, momentum, dividends.

H2: The hypothesis of the advantage of stylized portfolio optimization before the market, as the proxy of which is the stock index.

H3: The hypothesis of the advantage of using copula to assess the joint asset risk with stylized portfolio optimization before the more traditional approach using covariance.

An advantage of one portfolio over another was defined by higher ratios of expected return to risk. A number of performance coefficients were used for comparison of the portfolios. Based on the results of the study, it was proved for all the considered styles the advantage of the introduced methodology both over market benchmark and over stylized Markowitz investment portfolio. Therefore the hypotheses H2 and H3 have been fully accepted. The H1 hypothesis is partially accepted, since for growth style a naive approach of construction a portfolio yielded slightly better results than stylized optimization.

In addition, the stylized portfolios were compared to each other. In terms of the ratio of expected return to risk, the most efficient in the analyzed period were portfolios of growth momentum stocks, although during the period from 2007 to 2011, value portfolio was the top performer. These results generally coincide with the conclusions of modern studies on the behavior of investment styles during periods of crisis, recovery, growth and slowdown of the economy. The long-term advantage of the growth stocks and momentum stocks can be explained not only by such factors as overstated expectations and reactionary decision-making by investors on the Russian market, but also by a strong imbalance of liquidity, that is, value stocks can be ignored due to the fact that they are not liquid enough. An analysis of possible costs was carried out separately. Even if one takes them into account, stylized copula CVaR portfolios show significant outperformance over the benchmark.

The results of this study may be of practical interest to both professional portfolio managers and risk managers, as well as to private investors with a mathematical mindset, since managing stylized portfolios according to the proposed methodology requires complex calculations. However, it is important to note that this optimization procedure should rather serve as an additional tool in the process of making investment decisions. Important issues that cannot be solved solely by the proposed methodology include, but are not limited to: exposure management for individual asset styles/classes, hedging of individual risks, loss limitation, market timing and other. The solution of these issues should allow, among other things, to limit the size of the maximum drawdown, which is rather large for the introduced stylized portfolios, although smaller that for benchmark.

There are several possible developments of the study. Firstly, it makes sense to consider more strict criteria for determining a particular investment style; this might improve the overall performance of a stylized portfolio. Secondly, the problems raised in the previous paragraph are also worthy of separate and detailed consideration. Thirdly, it is quite interesting to conduct a study that would allow developing a procedure for optimizing a portfolio not by asset class, but by style, so that at each particular point in time the portfolio would be concentrated in the most promising investment styles. Finally, it is logical to expand the example of the rotated Gumbel copula to other copula models in order to reveal the most effective one for the purpose of stylized portfolio optimization.

The main results of the dissertation were published in the following articles

Articles published in leading peer-reviewed scientific journals recommended by the Higher Attestation Commission of the Ministry of Education and Science of the Russian Federation:

Atskanov I.A., 2017. Stylized optimization of stock portfolios with the help of copula. *Financial Management*, 5, pp. 97 - 107

Atskanov I.A., 2015. Dynamic optimization of the investment portfolio with the use of paired copulas on the example of the main European stock markets. *Applied Econometrics*, 4(40), pp. 84-105.

Atskanov I.A., 2016. Application of GAS-copulas for optimization of the investment portfolio of shares of Russian companies. *Finances and credit*, 704(32), pp. 25-37.