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STRATEGIC TRADING AROUND ANTICIPATED SUPPLY/DEMAND SHOCKS

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I study the price/quantity effects of anticipated supply or demand shocks in a model of strategic trading, where imperfectly competitive traders share risk with price-takers. When there are at least two traders, anticipated shocks lead to the V-shaped pattern observed empirically: prices drift away from fundamentals before the shock, and slowly revert afterwards. How traders behave before the shock depends on whether they compete `a la Cournot (i.e. submit market orders) or in demand schedules (using limit orders). Consistent with empirical evidence, Cournot traders act as contrarians, while demand schedule traders first trade against, then with the shock.

Keywords: strategic trading, liquidity, price impact, thin markets, V-shaped price patterns, momentum and reversal, anticipated shocks

JEL codes: G12, G20, L12

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1 Introduction

Anticipated supply/demand shocks are predictable changes in supply or demand. They may be caused by passive investors' mechanical trading rules[1], scheduled equity or bond issuances (SEOs, bond reopenings), pre-announced trades, etc. These anticipated shocks are common, relatively frequent, and often uninformative about fundamentals. Yet, a large body of empirical literature documents temporary price pressure in the form of V-shaped price patterns around anticipated shocks. Prices drift away from fundamentals before the shock, and revert afterwards,[2] leading to costs for issuers and investors. For instance, to reduce tracking errors around index reconstitutions, passive investors are likely to sell deleted stocks at a deflated price, and buy added stocks at an inflated price.[3]

This evidence is hard to explain in frictionless markets. During these events, returns are partly predictable and are characterized by time-series momentum and reversal. Moreover, the data shows a diversity of behaviors by financial institutions, in particular before shocks take place. Some institutions are contrarians (e.g. buy ahead of a positive supply shock), others tend to trade with the wind or follow non-monotonic strategies. For instance, institutional investors tend to buy stocks ahead of SEOs (Chemmanur, He, and

[1] E.g. passive investors must rebalance portfolios around index inclusions/deletions (see Lynch and Mendenhall (1997), among many others), ETF roll over futures contracts at expiry (Bessembinder et al., 2016).

[2] These V-shaped price patterns have been documented around index reconstitutions (e.g. Lynch and Mendenhall, 1997, Chen, Naronha, and Singal, 2004), flow-induced price pressure by mutual funds (Coval and Stafford, 2007), SEOs (Kulak, 2008), Treasury issuances or reopenings (see Lou, Yan and Zhang (2013) for US evidence, and Sigaux (2016) for European evidence), corporate bond issuances (Newman and Rierson, 2003) and corporate bond index exclusions following downgrades (Dick-Nielsen and Rossi, 2019). See Duffie (2010) for a review of the empirical evidence of the price effects of anticipated shocks.

[3] In the case of bond index exclusions, Dick-Nielsen and Rossi (2019) state that the "reluctance to trade away from the exclusion date results in a hidden cost of indexing for final investors of approximately 34 bps annually". (p.4) In the case of Treasury issuances, Lou, Yan, and Zhang (2013) estimate a cost of "over half a billion dollars for note issuance alone in 2007".

Hu, 2009). However, market-makers or liquidity suppliers in futures market reduce inventories before anticipated liquidations (Cai, 2009) or ETF futures rolls (Bessembinder et al., 2016). Corporate bond dealers first increase and then reduce inventories before bond index exclusions (Dick-Nielsen and Rossi, 2019).

In this paper, I study a model of strategic trading, which qualitatively accounts for both the V-shaped price pattern and the diversity of trading behaviors around anticipated shocks. The key feature of the model is the heterogeneity in market power, and thus in price impact, between an oligopoly of strategic traders and a competitive fringe of price-takers. I consider two versions of the model: in the first one, traders compete à la Cournot; in the second one, they compete in demand schedule. I view Cournot traders as a proxy for traders using primarily market orders (e.g. opportunistic or directional traders, see Hagstromer and Norden, 2013) and demand schedule traders as institutions using mostly limit orders (e.g. market-makers).[4]

In both versions of the model, anticipated shocks may lead to a V-shaped price pattern provided there are at least two strategic traders, a competition effect. However, traders' strategies before a shock differ depending on the type of competition. Consistent with empirical evidence, Cournot traders act as contrarians, while demand schedule traders first trade against, then with the shock. Despite this potentially destabilizing behaviour, anticipated shocks have a smaller price impact under demand schedule competition.

Model and competitive benchmark. I consider a dynamic economy with one risk-free and one risky assets, and two types of investors, price-takers and strategic traders (traders, for short). All investors have exponential utility and the liquidating dividend of the risky asset is normally distributed.

[4]Hagstromer and Norden (2013) show how the use of different types of orders reveals the diversity of algorithmic traders. They find that traders following directional or opportunistic strategies use a large share of market orders, while those engaging in market-making activities use primarily limit orders. Chan and Lakonishok (1995) discuss how the investment style and other fund characteristics influence the type of execution strategies and in particular the type of orders used by the trading desks of institutional investors.

Traders and price-takers may differ in risk aversion, but all traders have identical risk aversion. Investors trade to share risk over T periods before consuming. They learn in the course of trading that the supply of the risky asset will increase at a later time, i.e. the *announcement* and the *realization* of the shock are distinct.[5] Information is complete. In particular, the supply shock is publicly announced and is uninformative about the value of the risky asset.[6]

Suppose first that all investors, including strategic traders, are competitive. In this case, there is no V-shaped price pattern around anticipated shocks. Indeed, in the competitive benchmark, gains from trade are realized in a single trading round, and the price is the sum of the expected dividend and a risk premium proportional to the risk-bearing capacity of the market. Thus, when a shock is announced, all investors understand that portfolios will be adjusted immediately at the realization. To preclude arbitrage opportunities, the risk premium immediately adjusts at the announcement.

Imperfect competition. Suppose next that traders are imperfectly competitive, and thus take into account the price impact of their trades.[7] Then, whether traders compete à la Cournot or in demand schedules, the price includes a liquidity premium in addition to the risk premium. The liquid-

[5]The positive supply shock is to fix ideas only. The model can also be written with demand shocks. In the Online Appendix, I show the equivalence between the model with anticipated supply shocks studied in the text and a model with anticipated shocks to price-takers' demand.

[6]Some shocks such as SEOs or even index reconstitutions (Denis, McConnell, Ovtchinnikov, and Yu, 2003) might convey information about fundamentals. However, the V-shaped price pattern occurs also in indices following mechanical rebalancing rules (e.g. FTSE, Russell) instead of discretionary rules (S&P500), ruling out a pure information effect (see Madhavan, 2003). My theory implies that even if the event conveys information, the mere difference of price impact between price-takers and strategic traders may lead to a V-shaped price pattern, controlling for the informational content of the shock.

[7]Price impact arises only due to market power in the model. As an illustration, Gabaix et al. (2006) calculate that in 2000, the liquidation by the 30$^{\text{th}}$ largest mutual fund of its position in its average stock would represent half of the daily turnover in that stock, implying that price impact is a concern for large traders even in the absence of superior information. In practice, strategic traders (e.g. large asset managers, dealers, and trading desks) rely on order execution techniques to minimize price impact (see, e.g. Chan and Lakonishok, 1995, Keim and Madhavan, 1995, 1997, and van Kervel and Menkveld, 2019)

ity premium stems from differences in price impact between traders and price-takers. It compensates price-takers for imperfect diversification, which arises as traders shade their bids to limit their price impact, causing delays in reaching Pareto-optimal allocations. Because of the liquidity premium, the trades of strategic traders have both a risk-sharing component, whereby traders progressively liquidate their initial positions and replace them with Pareto-optimal ones, and a speculative component, as traders take advantage of the liquidity premium.

Empirical studies typically control for the market factor, which corresponds to the risk premium in the model, hence I focus on the effects of anticipated shocks on the liquidity premium. To isolate the effects, I assume that initial endowments are Pareto-optimal, so that there is no reason to trade and no liquidity premium until the shock is announced.

When the shock is announced, a new trading motive emerges. Indeed, price-takers anticipate that the shock will not be immediately diversified at the realization, as traders break up orders. This gives them incentives to sell ahead of the realization to hedge their future over-exposure to the risky asset. Between the announcement and the realization, strategic traders trade off the effects of the anticipated shock on their current marginal trading profits and on their future marginal utilty. On the one hand, the shock lowers future prices, which reduces price-takers' demand today, thereby creating an opportunity for traders to buy at a lower price today. On the other hand, the anticipated shock increases future risk and liquidity premia. Thus, it increases profits from exploiting the liquidity premium in the future and affects the terms of trade at which traders can share risk in the future. For instance, a higher liquidity and risk premium make it more costly to liquidate positions as future prices are lower, and has an ambiguous effect on the cost of acquiring Pareto-optimal positions: traders must acquire larger positions, albeit at a lower price due to the higher premia. While the effect on the current trading profit gives traders incentives to buy ahead of the shock, the effect on their future marginal utility may induce them to buy less or even short. Which effect dominates depends on the type of competition.

Cournot competition. When traders compete à la Cournot, I show that a monopoly does not trade until the shock takes place, while oligopolistic traders start trading as soon as the shock is announced. The monopoly trades in such a way that both effects (on the current profit and on the future marginal utility) exactly offset each other. Her strategy is thus optimally myopic, in the sense that it does not depend on the shock, although the monopoly is aware of it. Instead, competition among oligopolistic traders induces them to trade ahead of each other to exploit the increase in the today's marginal profit. Thus, in equilibrium, all traders start buying from the announcement, a contrarian behaviour.

These different strategies lead to different liquidity premium dynamics. With a monopoly, price-takers understand that they cannot hedge their future overexposure in advance, so the liquidity premium immediately jumps at the announcement to the level it will have at realization, and remains constant until then.[8] It decreases after the realization, as the monopoly starts trading towards the new Pareto-optimal allocation. With an oligopoly, price-takers can start hedging by selling the asset to strategic traders. However, traders break up their orders to limit price impact. Thus, price-takers sell progressively as well, and each sale must be associated with a price decline (more precisely, an increase in the liquidity premium) for price-takers' demand to remain optimal and the market to clear. Indeed, price-takers are the marginal asset holders, and thus the marginal pricers in the model. Thus, before the realization, the liquidity premium increases gradually. After the realization, traders trade more aggressively towards Pareto-optimal positions, and the missalocation of the asset susbides, progressively reducing the liqudity premium. Hence, imperfect competition among strategic traders generates the gradual pattern observed in the data around antici-

[8]Strictly speaking, with Pareto-optimal endowments, there is no trading until the realization with a Cournot monopoly, thus the price and liquidity premium should be understood as price-takers' marginal valuation of the risky asset.

pated shocks.

Demand schedule competition. When traders compete in demand schedules, a V-shaped pattern also occurs only when traders are oligopolistic, but inventory dynamics are different. With oligopolistic traders, the effect of the anticipated shock on the current marginal profit dominates early on, while the effect on the future marginal utilty prevails just before the realization. Hence, traders start to buy from the announcement on, but revert their holdings and short one period before the realization. The reason why the effect of the shock on the current marginal profit becomes relatively smaller is that the price becomes less sensitive to the anticipated shock as the realization date approaches. Indeed, at the realization, traders buy more aggressively than in the Cournot case. As all traders submit downward-sloping schedules in equilibrium (instead of horizontal ones under Cournot), the residual demand curve steepens, and each trader faces a deeper market. Competition for liquidity provision is thus fiercer when traders submit schedules, and this improved liquidity at realization reduces the sensitivity of the price to the shock before the realization.

In spite of the different trading dynamics, the liquidity premium remains V-shaped. However, as traders short ahead of the shock, the liquidity premium starts shrinking one period ahead of the realization: this is because price-takers need to be compensated for holding the extra risk when they buy from traders. With a monopoly submitting a demand schedule, the price pattern and trading strategy depend on whether the shock occurs in the final trading round or not, but as under Cournot competition, there is no V-shaped price pattern.

The prediction that demand schedule traders short ahead of the realization and provide liquidity afterwards is in line with the empirical evidence about futures markets cited above, as well as evidence about dealers' behaviour in Treasury and corporate bond markets (Lou, Yan, and Zhang, 2013, Dick-Nielsen and Rossi, 2019). In all these cases, there is evidence that liquidity suppliers provide liquidity during shocks but offload inventories just before. Further, the non-monotonic strategy shown in panel *b* of Figure 2 is qualitatively similar to the inventory pattern documented by Dick-Nielsen and Rossi (2019) ahead of bond index exclusions (Figures 4b, 8b, and 9b in their paper). In their data, bond dealers first raise inventories some time before an anticipated shock, reduce them just before the shock, and increase them again during the shock.

Empirical evidence shows that large institutions buy ahead of SEOs, and that more aggressive buying reduces the SEO discount (Chemanur et al., 2009).[9] A potential test of the theory would thus consist to check whether these institutional investors make heavy use of market orders.

Related literature. This paper contributes to the literature in three ways. First, the paper provides a parsimonious mechanism, based only on heterogeneity in price impact, to explain the V-shaped price reaction to anticipated shocks. Second, the model delivers new predictions about inventory dynamics before the realization of anticipated shocks. Third, the paper gives an explicit comparison of price and trading dynamics in the Cournot and demand schedule cases.

From a theoretical point of view, it is difficult to explain V-shaped patterns, because these patterns imply short-term price predictability, which in a frictionless economy would be arbitraged away. To the best of my knowledge, only differences in price impact due to market power, as in this paper, and search frictions, can generate such price patterns in the absence of asymmetric information.[10]

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[9] Chemanur et al. (2009) argue in favor of an informational advantage of institutional investors. However, many of their results are also consistent with institutions having market power, so that a combination of informational advantage and market power cannot be excluded.

[10] With search frictions, Duffie (2010) is the closest to this paper in terms of theme and objectives, albeit with at least two differences. First, in Duffie's paper, diversification is not immediate due to exogenous delays in finding counterparties. Instead, in my paper these delays arise endogenously as the outcome of traders' optimal execution strategies. Second, in Duffie's model, the price rises before the V-shaped pattern, a phenomenon which is observed only for SEOs in the data. There is no such initial price increase in my setting. Indeed, in Duffie (2010), the price rise compensates traders who will be "stuck"

In the literature on trading with market power, the closest papers are Pritsker (2009) and Rostek and Weretka (2015).[11] Pritsker (2009) considers a Cournot setting with n traders and a competitive fringe, and studies numerically the effects of anticipated firesales by a distressed trader (one of the strategic traders), who is forced to hold onto his position until the firesale. In addition to Pritsker's results, I show that (i) the V-shaped price pattern arises only because of the difference in price impact;[12] (ii) competition among traders determines the occurrence of the V-shaped pattern, in particular, there is no such pattern with a single trader (this is an analytical result); and (iii) I relax the assumption that traders use market orders only, leading to new predictions about inventories. Rostek and Weretka (2015) study the price effects of anticipated shocks in a demand schedule game with n traders and no price-takers. The main difference with Rostek and Weretka (2015) is that I introduce price-takers in the demand schedule game, leading to heterogeneity in price impact.[13] Without such heterogeneity, the price effects of anticipated shocks are different and harder to reconcile with the empirical evidence. In particular, there is *no price drift* between the announcement and the realization of the shock, and the price returns to the competitive

with the asset for some time. In my setting, it is always possible to trade, although imperfect liquidity entails costs.

[11]The literature on demand schedule competition builds on the static model of Kyle (1989), extended to the dynamic case by Vayanos (1999). Recently published papers based on similar frameworks include Rostek and Weretka (2015), Du and Zhu (2017), and Kyle, Obizhaeva, and Wang (2017). The literature on Cournot competition among large traders, based on the inventory models of Grossman and Miller (1988), includes Kilhstrom (2000), Pritsker (2009), DeMarzo and Urosevic (2006), Edelstein, Sureda-Gomill, Urosevic and Wonder (2010), and Marinovic and Varas (2018). Capponi, Menkveld and Zhang (2019) consider a model where Cournot traders have only transitory price impact.

[12]Under Pritsker (2009)'s assumptions, the market anticipates three things: a change in competition, a change in the total risk-bearing capacity of the market, as the distressed trader exits the market after the firesale, and a change in supply/demand. Under these assumptions, *even the competitive price reaction can be V-shaped.* In my setting, there is a change in supply/ demand without change in the number of strategic traders, nor in the total risk-bearing capacity of the market. As a result, the V-shaped price reaction arises only due to the effect of the anticipated shock on the liquidity premium.

[13]In Rostek and Weretka's setting, all traders have identical risk aversion, leading to the same degree of market power and thus the same price impact.

level in one trading round after the shock.

Differences in price impact are thus key to explain the V-shaped price pattern. These differences have been documented at least since Chan and Lakonishok (1995). Heterogeneity in price impact is an important characteristics of financial markets, where large traders coexist with smaller institutions, passive investors, and/or retail investors.[14]

The second contribution of the paper is to provide new predictions about inventory dynamics around V-shaped price patterns. Several studies predict that traders sell as the price goes down, and buy when it rebounds. Vayanos (2001) obtains this prediction in a Cournot setting with a single privately-informed strategic trader. Papers on predatory trading (e.g. Brunnermeier and Pedersen, 2005, Carlin et al., 2007), which are based on a Cournot setting with *exogenous* demand curves and no asymmetric information, predict a similar pattern. Instead, in my setting price-takers generate an endogenous demand, a single Cournot trader abstains from trading before the shock, and multiple traders buy as the price goes down.

I am not aware of any paper predicting the non-monotonic inventory pattern of the demand schedule case, although this pattern is observed in the data. For instance, Rostek and Weretka (2015) predict that strategic traders *do not* trade on anticipated shocks until the realization.

Finally, the paper highlights quantitative and qualitative differences be-

[14] A few recent working papers share this emphasis on differences in price impact, albeit with different objectives. Glebkin (2016) studies a static model with asymmetric information, where price-takers and strategic traders compete in demand schedules. Sannikov and Skrzypacz (2016) consider a dynamic oligopoly of strategic traders with heterogeneous risk aversion and private trading needs. They extend the standard definition of the demand schedule equilibrium to allow for heterogeneous risk aversion among traders, allowing them to condition schedules on the outcome of other traders. I retain the standard equilibrium definition assuming that all strategic traders have identical risk aversion and focus on the effects of anticipated shocks. Rostek and Yoon (2019) consider a non-stationary model with n strategic traders competing in demand schedules, where traders have private information and heterogeneous risk aversion, but do not consider anticipated shocks in their setting. Note that since all traders have identical risk aversion in my setting, the equilibrium remains recursive in spite of the difference in price impact between traders and price-takers.

tween Cournot and demand schedule competitions, which have under-researched in the context of multiperiod financial markets.[15] These two settings can be easily tied to the strategies used by different institutions. I show that even though the price, value functions, and trading strategies are analytically similar for both types of competition, the coefficients of their different components are competition-specific, leading to different price and inventory dynamics.

After describing the setting, I study Cournot competition, and then demand schedule competition. The last section summarizes the main conclusions of the analysis. Proofs of the results on demand schedule competition are in the Online Appendix. All other results are proved in the Appendix.

2 A model of strategic trading and anticipated shocks

2.1 Set up

Time is discrete ($t = 0, \ldots, T$). Investors trade a risk-free asset and a risky asset between 0 and $T-1$, and consume at T. The risk-free asset is in perfectly elastic supply with return r_f normalized to zero. The risky asset, which trades at price p_t, pays off a liquidating dividend at T. The liquidating dividend is the sum of a fixed component D and a series of *iid* normally distributed dividend news ε_t: $D_T = D + \sum_{\tau=1}^{T} \varepsilon_\tau$, with $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$, for $1 \leq t \leq T$, with $\varepsilon_u \perp \varepsilon_t,\ u \neq t$. All investors observe the dividend news ε_t before trading at time t. Let $D_t \equiv \mathbb{E}_t(D_T) = D + \sum_{s=1}^{t} \varepsilon_s$ denote the conditional expected value of the dividend at time t.

[15] Vives (2011) provides a quite general albeit *static* framework to study supply functions equilibria with private information and their connection with the Cournot outcome. In their survey, Rostek and Yoon (2020) compare Cournot and demand schedule outcomes in both static and dynamic models with private information, but without price-takers when traders compete in demand schedule. They focus on the differences in terms of dynamic inference instead of the effects of anticipated shocks.

The risky asset is in net supply s_t at time t. It is convenient to write s_t as the sum of a fixed part and a series of shocks, $s_t = s + \sum_{\tau=1}^{t} \Delta s_\tau$. The supply shocks are anticipated: at time 0, all investors know the sequence of future shocks Δs_τ. In the main text, I focus on the effect of an *information release*, with a single pre-announced shock. Specifically, I assume that investors anticipate that the supply is constant ($s_t = s$) when they start trading at time 0. Then at time $t_1 \geq 0$, there is a public announcement before the market opens: investors learn that at time $t_2 \geq t_1$, the supply will jump from s to $s + \Delta s_{t_2}$. I refer to t_1 as the announcement date, and t_2 as the effective or realization date of the shock. In the special case $t_1 = t_2$, the announcement takes the market by suprise. If, instead, $t_1 < t_2$, the market anticipates the shock. It is easy to map this partition between the announcement and effective dates to events such as seasoned issuances and index changes studied by empiricists. The general case with an arbitrary sequence of shocks is treated in the Appendix.[16]

Two types of investors share risk in the market. First, there is a continuum mass one of risk-averse price-takers, indexed by m. Price-takers have exponential utility, $u(C_T^m) = -\exp(-aC_T^m)$, and start with endowments Y_{-1}^m in the risky asset. Second, there is an oligopoly of n strategic traders (traders, for short), indexed by i. Traders have market power and understand that their trades move prices. They also have exponential utility with constant absolute risk aversion b, $U(C_T^i) = -\exp(-bC_T^i)$. Traders start with endowments X_{-1}^i in the risky asset. Given the exponential utilities, we can normalize all investors' endowments in the risk-free asset to zero without loss of generality.

Price-takers enter time t with a total position $Y_{t-1} = \int_0^1 Y_{t-1}^m dm$ in the

[16]Some of my motivating examples, e.g. index reconstitutions, are demand shocks: when a stock or a bond is deleted from an index, passive index trackers such as index funds and ETFs liquidate their holdings on or close to the effective deletion date to minimize tracking errors (Blume and Edelen, 2004, Dick-Nielsen and Rossi, 2019). I show in the Appendix that the model can be rewritten with demand shocks by introducing endowment shocks to price-takers. I focus on supply shocks in the body of the paper.

risky asset and trade $y_t = \int_0^1 y_t^m dm$ at t. Their position after trading at t is $Y_t = Y_{t-1} + y_t$. Similarly, trader i enters time t with a position X_{t-1}^i in the risky asset, trades x_t^i, and ends up holding X_t^i, so that $X_t^i = X_{t-1}^i + x_t^i$. At each t, market-clearing for the risky asset implies that:

$$Y_t + \sum_{j=1}^{n} X_t^j = s_t \tag{1}$$

The trading process in each round depends on the type of competition between traders. In this section, I present the competitive benchmark. Then in Section 3, I consider Cournot competition, implying that traders use market or marketable orders. In Section 4, I consider demand schedule competition, implying that traders use a series of limit orders.

2.2 Momentum and reversal

I use the terms momentum if there is a gradual price adjustment before the effective date, and reversal when there is a gradual correction after the shock. In imperfectly competitive markets, the price will be the addition of the competitive price and a liquidity premium. Thus, in the body of the paper, I focus on the notion of momentum and reversal in the liquidity premium only. Additional justification for this weaker notion is that empirical studies usually evaluate the effects of anticipated shocks against a market factor, which corresponds to the competitive price in my setting. To fix ideas, all the results are given for a positive supply shock, but they would all go through with negative shocks.

Definition 1 (Momentum and Reversal) *For a shock $\Delta s_{t_2} > 0$, there is momentum and reversal in the liquidity premium around the effective date iff*

(Momentum): $\exists t_m \in \{t_1, \ldots, t_2 - 2\}$ *s.t.* $p_{t_m} - p^*_{t_m} > \cdots > p_{t_2} - p^*_{t_2}$, *and*

(Reversal): $p_{t_2} - p^*_{t_2} < \cdots < p_{T-1} - p^*_{T-1}$

There is momentum and reversal in the price if we replace $p_t - p_t^*$ by $\mathbb{E}(p_t)$.

In standard inventory models (e.g. Grosmann and Miller, 1988), a one-period price decline followed by a one-period rebound may easily occur. Here I am looking for a *gradual* price movement followed by a gradual correction of this movement, which implies a short-term drift in the price or liquidity premium followed by a reversal of this drift. This notion of momentum and reversal is in the time-series, different from the classic cross-sectional momentum of Jegadeesh and Titman (1993).

2.3 Competitive benchmark

The competitive equilibrium is a useful benchmark to understand the effects of traders' market power. From now on, a " * " superscript denotes the competitive outcome. Price-takers solve the following problem:

$$\mathcal{P}^*: \quad \max_{Y_t} -\mathbb{E}_0 \left(\exp(-a w_T)\right)$$
$$w_t = w_{t-1} + Y_{t-1}(p_t - p_{t-1})$$

where w_t denotes price-takers' wealth. Price-takers' optimal demand at time t is

$$Y_t = \frac{\mathbb{E}_t(p_{t+1}) - p_t}{a\sigma^2} \tag{2}$$

When they are competitive, traders solve a similar problem, but have absolute risk aversion b, and choose position X_t^i. The solutions to $\mathcal{P}^*$ for each type of investor and market-clearing (1) define a competitive equilibrium. Note that price-takers' risk-bearing capacity (or risk tolerance) is $\frac{1}{a}$, whereas trader's total risk-bearing capacity is $\frac{n}{b}$.

Proposition 1 (Competitive equilibrium with constant supply) *In the competitive equilibrium, investors hold the risky asset in proportion of their risk tolerances: $X_t = \frac{1/b}{1/a+n/b}s = \frac{a}{na+b}s \equiv X^*$. The competitive price is* *the sum of the conditional expected value of the dividend and a risk premium:*

$$p_t^{*,cs} = D_t - b\sigma^2(T-t)X^* \tag{3}$$

Equation (3) shows that the risk premium has a drift: this is because as time passes, uncertainty about the fundamental is gradually realized.

Corollary 1 (Fundamental effect of information release) *Let $\Delta X_{t_2}^* \equiv \frac{a}{na+b}\Delta s_{t_2}$ denote the change in Pareto-optimal holdings induced by the shock.*
In a competitive market, there is no momentum and reversal: investors adjust positions once and for all at the realization of the shock, and prices adjust as soon as the shock is announced.

$$\begin{aligned}
&\text{for } t < t_1,\ p_t^{*,as} = p_t^{*,cs} && \text{and } X_t^{*,as} = X^*,\\
&\text{for } t_1 \le t < t_2,\ p_t^{*,as} = p_t^{*,cs} - b\sigma^2(T-t_2)\Delta X_{t_2}^* && \text{and } X_t^{*,as} = X^*,\\
&\text{for } t > t_2,\ p_t^{*,as} = p_t^{*,cs} - b\sigma^2(T-t)\Delta X_{t_2}^* && \text{and } X_t^* = X^* + \Delta X_{t_2}^*,
\end{aligned}$$

In a competitive economy, the price immediately reflects the increase in the risk premium that will take place at the effective date (Figure 3). The risk premium increases, because the quantity of risk increases, while the risk-bearing capacity remains unchanged. When the shock takes place, investors increase their holdings to absorb the extra supply and ask for an extra risk premium $b\sigma^2(T-t_2)\Delta X_{t_2}^*$ to do so. At the announcement, the price drops by exactly this amount by the logic of absence of arbitrage. In line with Rostek and Weretka (2015), I refer to the competitive price reaction as the fundamental effect of the information release. The logic of absence of arbitrage implies that this effect is fixed (i.e. independent of t) and independent of how far in advance the shock is announced (i.e. independent of t_1, or $t_2 - t_1$). Thus, the competitive price reaction to an information release is hard to reconcile with the pattern observed in the data.

3 Cournot competition

I now relax the assumption of price-taking behaviour for traders, assuming Cournot competition instead.

3.1 Definitions

Equilibrium. In each trading round, price-takers submit a demand curve, while traders submit market orders to a Walrasian auctioneer, who determines the market-clearing price. Traders choose orders given the price schedule implied by the price-takers' demand and market-clearing. A price schedule $p_t\left(x_t^i; \sum_{j\neq i} x_t^j\right) : \mathbb{R} \to \mathbb{R}$ maps the effect of the order of trader i on the equilibrum price, given other traders' orders. Using (2) and (1), we obtain

$$p_t\left(x_t^i; \sum_{j\neq i} x_t^j\right) = \mathbb{E}_t(p_{t+1}) - a\sigma^2\left(s_t - \sum_{j=1}^{n} X_t^j\right) = \mathbb{E}_t(p_{t+1}) - a\sigma^2\left(s_t - \sum_{j=1}^{n} X_{t-1}^j - \sum_{j\neq i}^{n} x_t^j - x_t^i\right) \quad (4)$$

Traders' price impact is permanent: the price increases in traders' *positions*, $X_t^i = X_{t-1}^i + x_t^i$. Hence, the price depends on past and current trades.

Definition 2 (Cournot equilibrium) *A dynamic Cournot equilibrium is a collection of subgame-perfect Cournot Nash equilibria, which consists in prices and trades such that (i) given the anticipated price path, price-takers' demand maximizes the expected utility of final consumption. (ii) given other traders' orders, x_t^{-i}, and the price schedule, trader i chooses a quantity x_t^i to maximize expected utility.*[17]

[17]The superscript $-i$ denotes the actions taken by all traders, except trader i. I rule out deviations by a non-zero mass of price-takers. This restriction is standard in the durable goods literature (see, e.g. Gul, Sonnenschein, and Wilson, 1986). In the strategic trading literature, Kihlstrom (2000), Pritsker (2009), DeMarzo and Urosevic (2007), and Vayanos and Wang (2012) consider a similar notion of equilibrium.

Price-takers' problem is still given by $\mathcal{P}^*$, as in the competitive case. A trader's problem is now:

$$\begin{aligned}\mathcal{P}^C: \quad & \max_{x_t^i} \ \mathbb{E}_0\left(-\exp(-bW_T^i)\right) \\ & \text{s.t.} \quad W_T^i = X_T^i D_T + B_T^i \\ & \text{where} \quad B_t^i = B_{t-1}^i - x_t^i p_t \left(x_t^i; \sum_{j\neq i} x_t^j\right) \quad \text{and} \quad X_t^i = X_{t-1}^i + x_t^i\end{aligned}$$

We can restate $\mathcal{P}^C$ as a dynamic programming problem, introducing the value function (post-trade certainty equivalent) Ω_t^i:

$$\Omega_t^i = \max_{x_t^i} \ x_t^i \left(D_t - p_t\left(x_t^i; \sum_{j\neq i} x_t^j\right)\right) - \frac{b\sigma^2}{2}\left(X_t^i\right)^2 + \Omega_{t+1}^i \quad \text{s.t.} \quad (4) \tag{5}$$

The state variable for problem $\mathcal{P}^C$ is traders' aggregate positions $\sum_{j=1}^n X_t^j$. However, the model has a more intuitive form if we express the price and value function with an affine transformation of this state variable. To avoid moving the price against themselves, traders will trade less aggressively than in the competitive market, leading to imperfect and delayed risk-sharing. The key indicator of the imperfect risk-sharing in the model is the distance $\mathbf{\Lambda}_t$ between traders' aggregate positions when they enter time t and Pareto-optimal positions. With constant supply, $\mathbf{\Lambda}_t$ is simply a scalar and boils down to $\Lambda_t \equiv nX^* - \sum_{j=1}^n X_{t-1}^j$. With anticipated shocks, the Pareto-optimal position X_τ^* changes over time. Thus, the liquidity factor depends on the term structure of deviations from Pareto-optimal holdings:

$$\mathbf{\Lambda}_t \equiv \left(\Lambda_{t,t}, \quad \ldots \quad, \Lambda_{t,T-1}\right)^\top, \text{ with } \Lambda_{t,\tau} = nX_\tau^* - \sum_{j=1}^n X_{t-1}^j \tag{6}$$

The elements $\Lambda_{t,\tau}$ of $\mathbf{\Lambda}_t$ measure the distance between traders' current aggregate position and time-τ aggregate Pareto-optimal holdings. Note that

I use bold font for vectors and matrices. The guesses for the price p_t and traders' value function Ω_t^i, expressed as affine and quadratic functions of $\boldsymbol{\Lambda}_t$, are:

$$p_t = p_t^* - a\sigma^2 \boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t, \tag{7}$$

$$\frac{\Omega_t^i}{\sigma^2} = -\frac{b}{2} q_{1,t} (X_{t-1}^i)^2 - X_{t-1}^i \left(\boldsymbol{q}_{2,t}^\top \boldsymbol{\Lambda}_t + \boldsymbol{q}_{3,t}^\top \boldsymbol{X}^* \right) + \\ \frac{1}{2} \boldsymbol{\Lambda}_t^\top \boldsymbol{q}_{4,t} \boldsymbol{\Lambda}_t + \boldsymbol{\Lambda}_t^\top \boldsymbol{q}_{5,t} \mathbf{X}^* + \boldsymbol{X}^{*\top} \boldsymbol{q}_{6,t} \boldsymbol{X}^* \tag{8}$$

where $\boldsymbol{q}_{4,t}$ is a $(T-t) \times (T-t)$ upper diagonal matrix, and $\boldsymbol{q}_{5,t}$ and $\boldsymbol{q}_{6,t}$ are $(T-t) \times (T-t)$ matrices. Since the Cournot price departs from the competitive price due to market thinness, I refer to a$\sigma^2 \boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t$ as the liquidity premium and to $\boldsymbol{\Lambda}$ as the liquidity factor. Given this price and value function, the trade can be decomposed into a risk-sharing and speculative component:

$$x_t^i = \sum_{\tau=t}^{T-1} c_{t,\tau} (X_\tau^* - X_{t-1}^i) + \sum_{\tau=t}^{T-1} \eta_{t,\tau} \Lambda_{t,\tau} \tag{9}$$

The coefficients $c_{t,\tau}$ measure how aggressively traders trade towards future Pareto-optimal positions. Similarly, the coefficients $\eta_{t,\tau}$ measure how aggressively traders take advantage of future liquidity premia. The first component is standard in the strategic trading literature (see, e.g. Vayanos, 1999, Rostek and Weretka, 2015, Kyle et al., 2017), except for the anticipated suppy changes: traders target the Pareto-optimal portfolio, but shade bids to smooth their price impact, so that the coefficients $c_{t,\tau}$ will be smaller than one; the second component arises because of the asymmetry in market power between traders and price-takers, which leads to a temporary misallocation of the asset between the two groups. This misallocation generates a liquidity premium, offering traders the opportunity to make trading profits.

3.2 Equilibrium

I now provide conditions under which the guesses are correct in equilibrium. First note that for a vector $\mathbf{x}_t = \begin{pmatrix} x_{t,t}, & \dots & ,x_{t,T-1} \end{pmatrix}^\top$, $\bar{x}_t$ denotes the sum of its elements, i.e. $\bar{x}_t = \sum_{\tau=t}^{T-1} x_{t,\tau}$. The same applies to matrices (see Appendix A for a summary of the vector and matrix notations).

Proposition 2 (Dynamic Cournot Equilibrium) *For all $n \geq 1$, there exists a unique equilibrium in which the price, trade, and post-trade certainty equivalent (value function) are given by equations (7), (8), and (9) if the price and value function coefficients are defined recursively by the system $\mathcal{S}(q_k, \alpha)$ given in Lemma 2, and if for $t \in \{1, \dots, T-1\}$, the second-order condition holds*

$$2a(1 + \bar{\alpha}_{t+1}) + Q_{t+1} > 0, \tag{10}$$

where $Q_{t+1} \equiv \bar{Q}^{1,2}_{t+1} - n\bar{Q}^{2,4}_{t+1}$ measures the curvature of the value function, with $\bar{Q}^{1,2}_{t+1} \equiv b\sigma^2(1 + q_{1,t+1}) - \sigma^2 \bar{q}_{2,t+1}$ and $Q^{2,4}_{t+1,\tau} \equiv \sigma^2 \left(\bar{q}_{2,t+1} + \bar{q}_{4,t+1}\right)$. Boundary conditions for α and q_i given by the static version of the model in Proposition 6.

Proposition 2 provides a recursive characterization of the equilibrium. The equilibrium as a function of *primitives* takes a simple form in the special case of constant supply:

$$p^{cs}_t = p^{*,cs}_t - a\sigma^2 \bar{\alpha}_t l_{t-1} \Lambda_0 \tag{11}$$

$$X^* - X^i_t = c^\pi_t (X^* - X^i_{-1}) - \pi^{\eta,c,l}_t \Lambda_0, \tag{12}$$

With constant supply, the liquidity premium is determined by the distance between Pareto-optimal holdings and traders' endowments, $\Lambda_0 \equiv nX^* - \sum_{i=1}^n X^i_{-1}$. Further, the coefficients $\bar{\alpha}_t l_{t-1}$ depend on the number of strategic traders, n, and the number of remaining trading rounds, $T-t$. When $\Lambda_0 = 0$, the price matches the competitive price. Hence, only the initial distribution of asset ownership between price-takers and traders matters for pricing. If we set $\Lambda_0 = 0$ but $X^i_{-1} \neq X^*$ in equation (12), we see that some trade happens, but these trades do not move the price away from the competitive price, although the market is illiquid. Hence, risk-sharing within group does not affect the equilibrium price, but risk-sharing between groups does.

If $\Lambda_0 \neq 0$, traders start with inefficient positions, and the price converges gradually towards the competitive price. For instance, if $\Lambda_0 > 0$, traders initially hold smaller than efficient positions, so price-takers require a liquidity premium to hold the extra supply, and the price is below the competitive price. As time passes, traders gradually increase their positions and the price converges towards the competitive price. Equation (12) shows that traders target the Pareto-optimal holdings at rate c_t^{π} when the asset is correctly allocated between the two groups. When it is not the case, the convergence can be slower or faster, as the liqudity premium induces traders to exploit the price distortion and earn trading profits.

3.3 Information release

With a single anticipated shock, we have $X_\tau^* = X^*$ for $\tau < t_2$ and $X_\tau^* = X^* + \Delta X_{t_2}^*$ for $\tau \geq t_2$. Until the announcement, the equilibrium is the one with constant supply. After the announcement, trades and prices can be expressed as deviations from the constant supply case.

Trade decomposition. Trades can be written as if traders were trading on different accounts based on the constant supply, the anticipated shock, and the realized shock. Between the announcement and the realization of the shock, traders trade against the anticipated shock. After the realization, they treat the realized shock as a new layer of constant supply, simply starting from different endowments, i.e. the inventory accumulated on the anticipated shock account is transferred to the realized shock account and serves as its starting position.

When there is a single shock, it is either anticipated or realized, so that there is anticipated shock trading only between the announcement and the realization, and realized shock trading only after the realization of the shock:

$$\text{For } t < t_1,\; x_t^i = x_t^{i,cs},$$
$$\text{For } t_1 \le t < t_2,\; x_t^i = x_t^{i,cs} + \underbrace{x_t^{i,as}(t_2)}_{\textit{anticipated shock trading}}$$
$$\text{For } t \ge t_2,\; x_t^i = x_t^{i,cs} + \underbrace{x_t^{i,cs}(t_2)}_{\textit{realized shock trading}}, \quad \text{with } X_{t_2-1}^{cs}(t_2) = X_{t_2-1}^{as}(t_2) \tag{13}$$

The notation $x_t^{i,cs}(t_2)$ emphasizes that the realized shock becomes a new *c*onstant *s*upply after t_2, and will generate similar dynamics, nothwithstanding the different endowments. This partition of the trades implies the same partition for individual and aggregate holdings, with $X_{-1}^{i,cs} = X_{-1}^i$, and $X_{-1}^{as}(t_2) = X_{-1}^{cs}(t_2) = 0$. The partition of aggregate holdings leads to three types of liquidity factors Λ_t^{cs}, $\Lambda_t^{as}(t_2)$, and $\Lambda_t^{cs}(t_2)$. The first factor is the same as in the constant supply case given in Proposition 8. By analogy, $\Lambda_t^{cs}(t_2)$ denotes the liquidity factor associated with the realized shock afer t_2, $\Lambda_t^{cs}(t_2) \equiv n\Delta X_{t_2}^* - \mathcal{H}_t^{cs}(t_2)$, where $\mathcal{H}_t^{cs}$ is a shorthand to denote traders' aggregate position in the realized shock account, i.e. $\mathcal{H}_t^{cs} = \sum_i X_t^{i,cs}(t_2)$. Finally, $\mathbf{\Lambda}_t^{as}(t_2)$ is the vector of liquidity factors associated with the anticipated shock of time t_2, with $\Lambda_{t,j}(t_2) \equiv -\mathcal{H}_t^{as}(t_2)$ for $t \le j < t_2$, and $\Lambda_{t,j}^{as}(t_2) \equiv n\Delta X_{t_2}^* - \mathcal{H}_t^{as}$ for $t_2 \le j \le T-1$.

Fundamental and liquidity effects. Theorem 1 in the Appendix provides conditions under which trades, holdings and liquidity factors can be split as in equation (13) for an arbitrary sequence of shocks. This partition of trades yields a simple decomposition of the price effects of anticipated shocks in fundamental and liquidity effects.

Proposition 3 (Effects of Information Release under Cournot) *Before the announcement, the price and holdings are the same as in the constant* *supply case:* $\forall t < t_1$, $p_t = p_t^{cs}$, *and* $X_t^i = X_t^{i,cs}$.

1. *After the announcement, an information release leads to both a fundamental and a liquidity effect:*

$$\begin{aligned} \text{for } t_1 \le t \le t_2, \quad p_t &= p_t^{cs} - b\sigma^2(T-t)\Delta X_{t_2}^* - a\sigma^2 \boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t^{as}(t_2) \\ \text{for } t > t_2, \quad p_t &= p_t^{cs} - b\sigma^2(T-t)\Delta X_{t_2}^* - a\sigma^2 \bar{\alpha}_t \Lambda_t^{cs}(t_2) \end{aligned} \tag{14}$$

2. *The liquidity effect gradually decreases after the realization.*

3. *The price reaction between the announcement and the realization depends on competition:*

   (a) *It is optimal for a monopoly to trade myopically, ignoring the anticipated shock, i.e.* $x_t^{as}(t_2) = 0$, $t < t_2$*; the liquidity effect is constant over time between the announcement and the realization:* $\boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t^{as}(t_2) = \bar{\alpha}_{t_2} \Delta X_{t_2}^*$, *and there is no momentum. There is always reversal in at least the liquidity premium after the effective date if* Λ_0 *and* $\Delta X_{t_2}^*$ *have the same sign.*

   (b) *With an oligopoly, there is momentum and reversal iff*

$$C_{mr} : \begin{cases} \forall t \in \{t_m, \ldots, t_2\}, & l_t \Lambda_0 < \mathcal{S}_{t_1,t_2}^{t_2,T}(\delta, l) n \Delta X_{t_2}^* \\ \text{for } t > t_2, & l_t \Lambda_0 + l_{t_2,t} \left[1 - \mathcal{S}_{t_1,t_2}^{t_2,T}(\delta, l)\right] n \Delta X_{t_2}^* > 0, \end{cases}$$

   where $\mathcal{S}_{t_1,t}^{t_2,T}(\delta, l)$ *is the fraction of the supply shock that has been acquired by traders before the shock and* $\boldsymbol{\delta} = \boldsymbol{c} + n\boldsymbol{\eta}$.

Just as the price can be split between the competitive price and the liquidity premium, the effect of the anticipated or realized shock can be decomposed into the fundamental (i.e. competitive) and the liquidity (i.e. imperfectly competitive) effects. While this partition is standard (Rostek and Weretka, 2015), the difference in market power between traders and price-takers generates new dynamics for the liqudity effect. Equation (14) shows that after the shock, the liquidity premium will simply contract over time as in the constant supply case. The realized shock liquidity factor is $\Lambda_t^{cs}(t_2) = l_{t_2,t-1}\left[1 - \mathcal{S}_{t_1,t_2}^{t_2,T}(\delta, l)\right] n\Delta X_{t_2}^*$, so it contracts at rate $l_{t_2,t}/l_{t_2,t-1}$. The exact price pattern *before* the shock, in particuler the occurence of momentum, depends on the degree of competition among traders. There is no momentum ahead of the shock with a monopoly, while C_{mr} shows that momentum and reversal occur under mild conditions with an oligopoly. The first condition corresponds to momentum, the second to reversal. Each condition has two terms (from left to right) corresponding to the change over two consecutive periods in (i) the constant supply liquidity premium, (ii) either the anticipated shock, or the constant shock liquidity premium.The conditions show that there is a trade-off between the trend in the constant supply and the anticipated shock liquidity premium. Further, if we add that the numerical result that $0 < S_{t_1,t_2}^{t_2,T}(\delta, l) < 1$, it is easy to observe that momentum and reversal occurs for a wide range of parameters.

Claim 1 (Analytical / Numerical) *Consider an anticipated increase in supply ($\Delta s_{t_2} > 0$):*

1. *With Pareto-optimal endowments ($\Lambda_0 = 0$), there is momentum and reversal in the liquidity premium for any anticipated shock.*

2. *When traders start from inefficient positions ($\Lambda_0 \neq 0$), then there is momentum and reversal in the liquidity premium if the shock is sufficiently large relative to the liquidity premium that prevailed at the announcement. Momentum occurs mechanically when $\Lambda_0 < 0$, but not reversal, and vice versa when $\Lambda_0 > 0$.*

The conditions in this result are mild: supply shocks simply need to be large enough relative the existing liquidity factor to trigger a V-shaped pattern in the liquidity premium when $\Lambda_0 \neq 0$, and the conditions are automatically satisfied for $\Lambda_0 = 0$. Suppose, for instance, that the shock and Λ_0 have different signs. If the shock is sufficiently small, its effect will be dwarfed by the constant supply premium and the V shape in the liquidity premium disappears. Conditions for price momentum (instead of liquidity premium momentum) given in the Appendix are similar but stricter, since there is an additional trend in the risk premium to take into account.

The occurence of momentum before the realization of the shock is linked to the anticipated shock trading by strategic traders. The intuition is simple. Price-takers understand that due to market power, the shock will not be optimally diversified at the realization, so that they will have to hold more than they desire for some time. For this reason, they are willing to hedge in advance of the shock. A monopoly does not trade based on the anticipated shock. Thus, the price – more precisely, price-takers' valuation for the risky asset – drops immediately to the level it will reach at the realization: hence the liquidity effect is $\boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t^{as}(t_2) = \bar{\alpha}_{t_2} \Delta X_{t_2}^*$ at any time betwen the announcement and the realization; this is its level at the realization. Instead, oligopolistic traders do trade before the realization and $x_t^{i,as}(t_2) > 0$ in all the numerical solutions I examined. As a result, price-takers can hedge to some extent in advance the fact that they will have to hold an extra supply at t_2. Although traders are buying before the realization, numerical simulations show that the anticipated shock liquidity premium keeps *increasing*. The reason is that traders break up their orders. As price-takers are the marginal holders of the asset and therefore the marginal price-setters, the liquidity premium must increase each time they sell the asset to traders to ensure that their demand is optimal. After the shock, traders trade towards the new Pareto-optimal portfolio more aggressively, so that the liquidity premium progressively shrinks. The more aggressive trading cannot take place earlier, or it would eliminate the liquidity premium at the realization, and traders would have an incentive to deviate and trade more slowly.[18] Note that the average price pattern is predictable, but given that the market is

[18] This is why each buy by the traders pushes the liquidity premium up before the realization and down after the realization. Before the shock takes place, each trade brings price-takers closer to the suboptimal diversification that occurs at the realization, increasing the liquidity premium. After the realization, each trade brings postions closer to Pareto-optimal ones, reducing the liqudity premium.

thin, this pattern cannot be arbitraged away. Any deviating trader seeking to exploit the gradual price movement would adversely move the price, eliminating the benefit of deviating in the first place.[19]

To understand better the trading dynamics before the shock, it is useful to consider the expression of the trading on anticipated shocks:

$$x_t^{i,as}(t_2) = \sum_{k=t_2}^{T-1}(c_{t,k} + n\eta_{t,k})\Delta X_{t_2}^* - \bar{\eta}_t \mathcal{H}_{t-1}^{as}(t_2) - \bar{c}_t X_{t-1}^{i,as}(t_2), \quad t_1 \leq t \leq t_2 \tag{15}$$

The first term is a sum, because each coefficient $c_{t,k}$ and $\eta_{t,k}$ is associated with the supply of time k. Since the shock is permanent, affecting the supply from t_2 to the end, we must sum all the coefficients to compute the total effect. The second and third terms are related to smoothing price impact and sharing inventory risk. Quantitatively, however, the first term seems to drive the trading dynamics. It is easy to show that the terms related to the anticipated shock in equation (15) can be expressed as folllows, for $\tau \geq t+1$:

$$c_{t,\tau} + n\eta_{t,\tau} = \kappa_t \frac{\partial}{\partial X_\tau^*}\left[D_t - p_t\left(x_t^i; \sum_{j\neq i} x_t^j\right) + \frac{\partial \Omega_{t+1}^i}{\partial x_t^i}\right],$$
$$\text{with } \kappa_t = \frac{1}{(n+1)\lambda_t^{\mathcal{C}} + Q_{t+1}^{\mathcal{C}}} \tag{16}$$

where κ is a liquidity adjustment and $\lambda_t^{\mathcal{C}} = a\sigma^2(1+\bar{\alpha}_{t+1})$ is a trader's price

[19] Note that if an additional trader were to enter unexpectedly at the announcement, the total risk-bearing capacity of the market would increase. This would reduce the liquidity premium but not eliminate momentum and reversal around the shock. If a positive mass of price-takers were to enter unexpectedly, this would also increase the risk-bearing capacity, leading to a more muted V-shaped price pattern, but risk aversion would prevent price-takers from arbitraging it away. Below I consider comparative statics with respect to a and n, holding the total risk-bearing capacity constant.

impact. The marginal continuation value is

$$\frac{\partial \Omega^i_{t+1}}{\partial x^i_t} = -\left(b\sigma^2(q_{1,t+1} - \bar{q}_{2,t+1})X^i_t + \sum_{\tau=t+1}^{T-1} Q^{2,4}_{t+1,\tau}\Lambda_{t+1,\tau} + \sum_{\tau=t+1}^{T-1} Q^{3,5}_{t+1,\tau} X^* \right) \tag{17}$$

Thus, we can write:

$$c_{t,\tau} + n\eta_{t,\tau} = \kappa_t \left[a\sigma^2\theta_{t+1,\tau} - (nQ^{2,4}_{t+1,\tau} + Q^{3,5}_{t+1,\tau}) \right] \tag{18}$$

This equation shows that traders trade off the effect of the shock on the current marginal trading profit (via the price schedule) vs the effect on their future marginal utility. The first term in (18) is the total effect of the shock on next period's price, as $\theta_{t+1,\tau} = \frac{b+na\alpha_{t+1,\tau}}{a}$ includes both the effect on the risk premium and on the liquidity premium. An anticipated shock lowers future prices, which pushes price-takers' demand down today, and therefore pushes the price schedule down today (see (4)). As the first term shows, traders take advantage of this downward shift in price-takers' demand by taking the other side. Note that while traders submit price-insensitive orders under Cournot, they can condition them on future shocks. The effect on the price schedule is indirect, as it occurs through price-takers' demand and market-clearing. The anticipated shock, however, also has a direct effect on traders' future marginal utility, as shown by the second term of (18). Traders internalize the effect of their trades on their future investment opportunity sets, taking into account the impacts on both the risk-sharing and the speculative components. Since the shock increases future risk and liquidity premia, traders have an incentive to reduce their trades now to take advantage of future premia. However, the increase in future liquidity and risk premia also increases the cost of liquidating positions in the future, as prices will be lower than if there were no shocks. Similarly the shock increases the target Pareto-optimal positions X^*_τ, which, however, can be bought at lower prices than in the absence of shocks.

In the proof of Proposition 3, I show that with a Cournot monopoly, the two effects in (18) exactly cancel each other; namely $b\sigma^2 = Q^{3,5}_{t+1,\tau}$ and $a\sigma^2\alpha_{t+1\tau} = Q^{2,4}_{t+1,\tau}$. Hence the monopoly trades in such a way that the increase in the time-τ risk premium caused by the shock affects today's marginal profit and tomorow's marginal utility in exactly the same way. Similarly the increase the time-τ liquidity premium/factor has exactly the same impact on the current marginal profit and tomorrow's marginal utility. When there are multiple Cournot traders, however, the effect of the anticipated shock on today's marginal profit is larger than its effect on tomorrow's marginal utility (see Figure 5). As a result, traders rush to buy ahead of the shock. To sum up, imperfect competition among Cournot traders induces them to trade against the shock and leads under mild conditions to a V-shaped pattern in the liquidity premium. The gradual increase and decrease of the liquidity premium around the realization of the shock is due to traders breaking up their orders to smooth price impact. I next turn to comparative statics.

Comparative statics. The model delivers comparative statics with respect to competition, risk aversion, announcement date, and traders' endowments (see Figures 9-11). For the first two, it is necessary to normalize the risk-bearing capacity of the market, $R = \frac{1}{a} + \frac{n}{b}$. For instance, an increase in n increases both competition and the risk-bearing capacity, so it is necessary to adjust b as n increases to keep R constant. I show in the Appendix that anticipated shock trading is highest for $n = 2$ when traders are risk-neutral. Numerical simulations show that the same holds when traders are risk-averse. Numerical analysis also shows that the V-shaped price pattern is more pronounced when price-takers are more risk averse (holding the total risk-bearing capacity constant) and when there are fewer trading rounds between the announcement and the realization ($t_2 - t_1$ shorter, holding T fixed). Furter, in line with Claim 1, the V-shaped pattern is more pronounced if the traders hold smaller endowments in the risky asset (for a positive shock, and vice versa for a negative shock), because the initial misallocation reinforces price-takers' reluctance to hold the risky asset in anticipation of the shock. When traders hold more than the Pareto-optimal endowments, the price impact of the shock decreases, the V-shaped price pattern becomes more muted, and may entirely disappear as endowments become very large. Intuitively, price-takers would like to hold more of the asset and the misallocation of the shock is likely to bring them closer to this position. Consistent with this prediction of the model, Chemmanur, He, and Hu (2009) study the price effects of SEO and find a smaller SEO discount if institutional traders buy more (and thus hold larger inventories) before the offering.[20]

4 Demand schedule competition

Under Cournot competition traders are restricted to use market orders. I now relax this assumption by introducing competition in demand schedules, which we can interpret as series of limit orders. Analytically, the two models are very close. However, there are qualitative and quantitative differences between the two types of competition.

4.1 Definitions

I introduce price-takers in the standard framework of demand schedule competition (Vayanos, 1999, Rostek and Weretka, 2015). I consider the following schedules as candidate equilibrium strategies:

$$y_t^m(p_t) = \beta_t^y(D_t - p_t) - c_t^y Y_{t-1}^m + d_t^y \sum_{j=1}^{n} X_{t-1}^j + \sum_{\tau=t}^{T-1} f_t^{y,\tau} X_\tau^*, \; m \in [0,1] \tag{19}$$

[20] These comparative statics are specific to a model with price-takers. Without price-takers, and assuming that traders compete in demand schedules, Rostek and Weretka (2015) find a different price effect of anticipated shocks, without drift between the announcement and the effective dates, and an immediate reversal afterwards. Further, in their model the price effect is independent of traders' risk-aversion and of the number of trading rounds between the announcement and the effective date.

$$x_t^i(p_t) = \beta_t(D_t - p_t) - c_t X_{t-1}^i + d_t \sum_{j=1}^{n} X_{t-1}^j + \sum_{\tau=t}^{T-1} f_t^\tau X_\tau^*,\ i = 1,\ldots,n,$$
$$\text{with} \quad \beta_t > 0 \quad (20)$$

Unlike traders, price-takers do not internalize the effects of their strategies on the equilibrium price. A Walrasian auctioneer collects all demand schedules and determines the market-clearing price. I use the standard assumptions in case of ties, etc. (see, e.g. Kyle, 1989). It is well-known from the double auctions literature that, when information is complete, there is a continuum of equilibria in the standard demand schedule game with n traders, as slopes are indeterminate. The usual solution in the literature is to use a "trembling hand" refinement to select an equilibrium (Klemperer and Meyer, 1989, Vayanos, 1999). Similarly, I focus on the robust Nash equilibrium.

Definition 3 (Demand Schedule Equilibrium) *A dynamic equilibrium in downward-sloping demand schedules is a collection of subgame-perfect robust Nash equilibria in linear, downward-sloping demand schedules of the form* (19)-(20) *such that*

- $y_t^m(p_t)$ *maximizes the expected utility of price-taker* m*, given* p_t*, the anticipated price path, other price-takers' schedules* $y_t^{-m}(p_t)$*, and traders' schedules,*
- *trader is' schedule,* $x_t^i(p_t)$*, maximizes his expected utility, given price-takers' schedules, other strategic traders' schedules,* $x_t^{-i}(p_t)$*, and his and other traders' impact on the price.*

4.2 Equilibrium

Proposition 4 *For any* $n \geq 1$*, if for all* $t \in \{1,\ldots,T-1\}$*,* $Q_t \geq 0$*, there exists an equilibrium in demand schedules, where the price and value function are the same as in the Cournot case, the system defining value function* *coefficients $q_{k,t}$ remains the same, but the equilibrium vectors $\boldsymbol{c}_t$, $\boldsymbol{\eta}_t$, and $\boldsymbol{\alpha}_t$ and their boundary conditions are competition-specific.*

The decomposition of price effects in fundamental and liquidity effects and the decomposition of trades in constant supply and anticipated/realized shock trading remain the same.

Equilibrium demand schedule coefficients are

$$\begin{aligned} \beta_t &= \frac{1}{\lambda_t + Q_{t+1}}, & c_t &= \frac{\bar{Q}^{1,2}_{t+1}}{\lambda_t + \bar{Q}^{1,2}_{t+1}}, & d^y_t &= \bar{\alpha}_{t+1}\lambda_t\beta_t, \\ f_{t,\tau} &= -\beta_t(nQ^{2,4}_{t+1,\tau} + Q^{3,5}_{t+1,\tau}), & \beta^y_t &= \frac{1}{a\sigma^2} + n\bar{\alpha}_{t+1}\beta_t, & c^y_t &= 1, \\ d_t &= (1-c_t)\beta_t\bar{Q}^{2,4}_{t+1}, & f^y_{t,\tau} &= n\bar{\alpha}_{t+1}f_{t,\tau} - \theta_{t+1,\tau}, & f_{t,t} &= f^y_{t,t} = 0 \end{aligned} \tag{21}$$

where λ_t is the equilibrium price impact, $\lambda_t \equiv \frac{\partial p_t}{\partial x^i_t}$, defined by equation (131) in the Online Appendix.

In a standard framework without price-takers, existence requires at least three traders. This amount of competition ensures that traders do not have "too much" market power and do not bid too aggressively. When there are price-takers, the equilibrium may exist even for $n = 1$, provided that Q remains positive, i.e. that the value function remains sufficiently concave. In practice, this is the case if traders are sufficiently risk averse relative to price-takers.[21]

The equilibrium keeps the same form as in the Cournot case, because the residual demand curve remains linear under both types of competition, with traders' positions being a state variable. The differences between the Cournot and demand schedule competitions lie in the slope of the residual demand curve, i.e. the market depth, and in the initial conditions of the recursive system. The derivation of the equilibrium as a function of primitives

[21] Value function coefficients are defined by the same system as under Cournot, but are functions of competition-specific parameters, and thus their values differ from the Cournot case too.

does not rely on initial conditions, nor on the definitions of the equilibrium trade and price parameters. Therefore, the main results, in particular the first two points of Proposition 3 and Theorem 1, hold under demand schedule competition. The results that do *not* hold are Lemma 3 about the signs of value function and price coefficients under Cournot competition and Proposition 9 about the myopic trading of a single Cournot trader (as discussed in the nextion section, a weaker result holds).

Numerical solutions show that the liquidity premium is systematically smaller under demand schedule competition. Convergence to the competitive price seems to be always faster. Perhaps not surprisingly, market depth is larger. We can think of market orders as horizontal demand curves in the $(p, x(p))$ space. If strategic traders post downward-sloping schedules, the slope of the residual demand curve becomes steeper, reducing price impact. Risk-sharing is faster, in the sense that $c^{\mathcal{D}}_{t,\tau} \geq c^{\mathcal{C}}_{t,\tau}$ for any t, τ. Instead, the speculative motive may or may not be larger, depending on the point in time.

Further, market depth dynamics are reversed. Under Cournot, market depth improves over time, while it is the opposite under demand schedule competition. Two effects determine the dynamics of market depth: on the one hand, the asset becomes conditionally less risky, which improves liquidity as demands becomes more elastic; on the other hand, the number of trading opportunities decreases, which worsens liquidity, as demands become less elastic. Under Cournot, the first effect dominates, while it is opposite under demand schedule competition.

While the equilibrium may exist even with a single trader, the monopoly case does not typically correspond to the Cournot case, except in the one-shot version of the model (see Corollary 4 in the Online Appendix). The reason is that the slope chosen today by the monopolist affects the current trade and thus tomorrow's equilibrium allocation, and therefore the future price via the liquidity factor. For instance, a slight increase in the price (e.g. some price-takers trembled and acquired more shares than expected) leads to a decrease in the demand from trader i, but also from all other traders; this widens tomorrow's liquidity factor, which pushes tomorrow's price down and distorts the expected return on the asset; this distortion induces price-takers to adjust their demand as well. Instead, under Cournot, if some price-takers tremble, traders do not adjust their demand, because they submit price-insensitive orders. This mechanism involves future allocations and is thus inherently dynamic, which is why demand schedule and Cournot competitions yield the same outcome only in the static case.

4.3 Information release

As in the Cournot case, there is a sharp difference between the monopolistic and oligopolistic cases. A V-shaped pattern occurs only when there are at least two traders; however, the price rebounds one period before the realization, not from the realization. This is because traders buy from the announcement but sell right before the shock takes place. After the realization, traders buy as in the Cournot case, albeit more aggressively, due to the stronger emphasis on risk-sharing (Figure 8). As traders sell right before the realization, the liquidity premium must shrink to compensate price-takers for increasing their holdings. The V-shaped pattern nevertheless remains a consequence of competition.

The total trading on anticipated shock is given by (18) as in the Cournot case, but the coefficients of the first term are:

$$c_{t,\tau} + n\eta_{t,\tau} = \frac{\beta_t^y f_{t,\tau} - \beta_t f_{t,\tau}^y}{n\beta_t + \beta_t^y}$$

Under Cournot competition, traders condition their order on the anticipated shock. Under demand schedule competition, traders condition the intercept of their schedule on the anticipated shock. However, since all investors submit price-dependent schedules, the effect of the shock on $x_t^{as}(t_2)$ is not given directly by the sensitivities of the schedule to the shocks, f_t and f_t^y. These sensitivities are weighted by the price-elasticity of each type of investor as a fraction of the total price elasticity $\frac{\beta^y}{\beta^y+n\beta}$ and $\frac{\beta}{\beta^y+n\beta}$. Yet, as in the Cournot case, we can write

$$c_{t,\tau}^{\mathcal{D}} + n\eta_{t,\tau}^{\mathcal{D}} = \kappa_t^{\mathcal{D}} \frac{\partial}{\partial X_\tau^*} \left[D_t - p_t \left(x_t^i; \sum_{j \neq i} x_t^j \right) + \frac{\partial \Omega_{t+1}^i}{\partial x_t^i} \right],$$

$$\text{with } \kappa_t^{\mathcal{D}} = \frac{\beta}{a\sigma^2(\beta_t^y + n\beta_t)} = \frac{1}{\lambda_t^{\mathcal{D}} + Q_{t+1}^{\mathcal{D}} + na\sigma^2(1 + \bar{\alpha}_{t+1}^{\mathcal{D}})}$$

where $\kappa_t^{\mathcal{D}}$ is the demand schedule competition-specific liquidity adjustment. Thus, as before, we get

$$c_{t,\tau}^{\mathcal{D}} + n\eta_{t,\tau}^{\mathcal{D}} = \kappa_t^{\mathcal{D}} \left[a\sigma^2 \theta_{t+1,\tau}^{\mathcal{D}} - (nQ_{t+1,\tau}^{2,4,\mathcal{D}} + Q_{t+1,\tau}^{3,5,\mathcal{D}}) \right],$$

When traders are oligopolistic, the anticipated shock trade has initially the same pattern as under Cournot, i.e. traders buy from the announcement and their inventories increase. However, traders short just before the realization. Figure 5 (panel b) shows that the effect of the shock on today's profit first dominates and then declines, while the effect on the marginal utility terms remains increasing (see also Figure 6 for a term-by-term decomposition). The reason why the effect of the shock on the current profit declines is that the price becomes less sensitive to the anticipated shock. This is because traders compete more fiercely at realization to supply liquidity than under Cournot. As a result, the price becomes less sensitive to the liquidity factor. Quantitatively, it is the coefficients of the next period that dominate, so that at time $t_2 - 1$, the sign of $\sum_{\tau=t_2}^{T-1}(c_{t,\tau}^{\mathcal{D}} + n\eta_{t,\tau}^{\mathcal{D}})$ is the same as that of $c_{t,t_2}^{\mathcal{D}} + n\eta_{t,t_2}^{\mathcal{D}}$.

This effect relies on the market being imperfectly liquid at the realization, and is thus stronger when traders have more market power, i.e. when n is small, or when price-takers have a lower risk-bearing capacity, holding the total risk-bearing capacity constant.

This effect is also present with a single trader, except if the shock takes place in the final trading round (see Corollary 5 in the Online Appendix).

Indeed, in this case, the Cournot and demand schedule equilibria coincide. Thus, the effect of the shock on the current profit does not decrease and the monopolist behaves as under Cournot:s he does not trade on the anticipated shock before the realization.

The prediction of the model about inventory dynamics in the oligopolistic model is consistent with anecdotal and empirical evidence about market-makers and liquidity suppliers in various markets. In the Treasury futures market, Cai (2009) finds that market-makers trade in the same direction as impending liquidation trades from LTCM. In the oil futures market, Bessembinder et al. (2016) show that liquidity suppliers reduce inventories in anticipation of large ETF futures' rolls, while providing liquidity on the day of the roll. Interestingly, there is evidence of a similar behaviour by dealers ahead of seasoned issuances or index exclusions. Lou, Yan and Zhang (2013) discuss how dealers in the Treasury market reduce inventories in anticipation of scheduled bond issuances. Dick-Nielsen and Rossi (2019) study corporate bond index exclusions due to downgrades. They find that aggregate dealers' inventories first rise and then decline just before the scheduled exclusion, in particular for investment-grade bonds. This pattern is qualitatively similar to the prediction of the model.

5 Conclusion

In this paper, I study how markets absorb anticipated supply or demand schocks when investors differ in price impact. I consider a purposedly stylized setting, in which all investors optimize, to emphasize the effects of the market structure. The first main insight is about competition. The average price reactions to shocks in my model are qualitatively consistent with the V-shaped patterns observed in the data only if there is some competition among strategic traders. The second main insight of the model is about how traders with price impact trade in anticipation of shocks. Traders submitting market orders only – i.e. resembling investors following opportunistic or directional strategies in actual markets – trade against anticipated shocks. Instead, traders submitting demand schedules – i.e. investors comparable to market-makers or liquidty providers submitting series of limit orders – first trade against, then with the anticipated shock, just before it occurs. The empirical evidence is consistent with these predictions about inventory dynamics. Empiricist and practitioners alike should thus take into account the effects of the market structure: empiricists, by including proxies for traders' market power in studies on shock absorption and market resiliency; practitioners, by studying the market structure before launching ETFs, index funds, or issuing securities.

Proofs

A Notations

Notation 1 (Scalar / Vector notations)

1. *While* x_t *denotes a scalar,* $\mathbf{x}_t$ *denotes a vector of length* $T-t$*, with elements* $x_{t,\tau}$*,* $\tau = t, \ldots, T-1$*.*

2. *Let* $\bar{x}_t$ *denote the sum of the elements of* $\mathbf{x}_t$*, i.e.* $\bar{x}_t = \sum_{\tau=t}^{T-1} x_{t,\tau}$*.*

3. *Let* $y_t = \Big(0, \mathbf{x}_{t+1}\Big)$ *denote the vector in which the first element is zero, and the other elements those of* $\mathbf{x}_{t+1}$*.*

Guesses for the price and value function

$$p_t = p_t^* - a\sigma^2 \boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t, \tag{22}$$

$$\sigma^{-2}\Omega_t^i = -\frac{b}{2} q_{1,t}(X_{t-1}^i)^2 - X_{t-1}^i \left(\mathbf{q}_{2,t}^\top \boldsymbol{\Lambda}_t + \mathbf{q}_{3,t}^\top \mathbf{X}^*\right) + \frac{1}{2}\sum_{\tau=t}^{T-1}\sum_{j=\tau}^{T-1} q_{4,t}^{\tau,j}\Lambda_{t,\tau}\Lambda_{t,j}$$
$$+ \sum_{\tau=t}^{T-1}\sum_{j=t}^{T-1} q_{5,t}^{\tau,j}\Lambda_{t,\tau}X_j^* - \sum_{\tau=t}^{T-1}\sum_{j=\tau}^{T-1} q_{6,t}^{\tau,j} X_\tau^* X_j^* \tag{23}$$

Under these guesses, it is possible to write trades as

$$x_t^i = \sum_{\tau=t}^{T-1} c_{t,\tau}(X_\tau^* - X_t^i) + \boldsymbol{\eta}_t^\top \boldsymbol{\Lambda}_t \tag{24}$$

Notation 2 *Price, trade and value function coefficients:*

$$\bar{Q}^{1,2}_{t+1} \equiv b\sigma^2(1+q_{1,t+1}) - \sigma^2\bar{q}_{2,t+1},$$

$$Q^{2,4}_{t+1,\tau} \equiv \sigma^2\left(q^{\tau}_{2,t+1} + \frac{1}{2}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau})\right),$$

$$\iota_{t+1,\tau} \equiv \sum_{j=\tau}^{T-1} q^{\tau,j}_{4,t+1},$$

$$\hat{\iota}_{t+1,\tau} = \sum_{j=t+1}^{\tau} q^{j,\tau}_{4,t+1},$$

$$Q^{3,5}_{t+1,\tau} \equiv \sigma^2\left(q^{\tau}_{3,t+1} + \sum_{j=t+1}^{T-1} q^{j,\tau}_{5,t+1}\right),$$

$$\theta_{t+1,\tau} \equiv \frac{b + na\alpha_{t+1,\tau}}{a},$$

$$Q_{t+1} \equiv \bar{Q}^{1,2}_{t+1} - n\bar{Q}^{2,4}_{t+1},$$

$$A_{t+1} \equiv a\sigma^2(n(1+\bar{\alpha}_{t+1}) - 2)$$

$$\mathbf{h}_t = b\mathbf{1} - \left(0, \mathbf{q}_{3,t+1}\right) - 0.5b(1+q_{1,t+1})\mathbf{c}_t, \tag{25}$$

$$\mathbf{g}_t = a\boldsymbol{\alpha}_t - 0.5b(1+q_{1,t+1})\boldsymbol{\eta}_t - \left(0, \mathbf{q}_{2,t+1}\right) + \bar{q}_{2,t+1}\boldsymbol{\delta}_t \tag{26}$$

$$\delta_{t,\tau} = c_{t,\tau} + n\eta_{t,\tau}, \qquad \gamma_{t,\tau} = 1 - \delta_{t,\tau} \tag{27}$$

B Competitive equilibrium

Lemma 1 *Price-takers' demand at time t is given by equation (2).*

Proof. Let's show by induction that the price-takers' post-trade certainty equivalent is given by

$$CE_t = w_t + \sum_{s=t}^{T-1} \frac{(\mathbb{E}_s(\hat{p}_{s+1}) - \hat{p}_s)^2}{2aVar_s(\hat{p}_{s+1})} \tag{28}$$

where w_t is the price-takers wealth at t, and $\hat{p}_t$ denotes the equilibrium price (in this proof only). At $T-1$, the price-takers' objective is

$$\max_{Y_{T-1}} -\mathbb{E}_{T-1}\left[\exp -a(w_{T-1} + Y_{T-1}(D_T - p_{T-1}))\right]$$
$$\Leftrightarrow \max_{Y_{T-1}} -\exp\Big[-a\Big(w_{T-1} + Y_{T-1}(\mathbb{E}_{T-1}(D_T) - p_{T-1}) - \frac{a}{2}Var_{T-1}(D_T)Y_{T-1}^2\Big)\Big]$$

Therefore, the price-takers' demand is $Y_{T-1} = \frac{\mathbb{E}_{T-1}(D_T) - p_{T+1}}{aVar_{T-1}(D_T)}$. Substituting back the demand, we obtain the certainty equivalent after trading at $T-1$ as a function of the equilibrium price $\hat{p}_{T-1}$: $CE_{T-1} = w_{T-1} + \frac{\mathbb{E}_{T-1}(D_T - \hat{p}_{T-1})}{2aVar_{T-1}(D_T)}$. Thus the property holds at $T-1$.

Let's now assume that at t, the post-trade certainty equivalent of the price-taker is given by (28) and show that this property holds at $t-1$. Let's first substitute the dynamic budget constraint, $w_t = w_{t-1} + Y_{t-1}(\hat{p}_t - p_{t-1})$, into (28) to obtain the price-takers' objective at $t-1$:

$$\max_{Y_{t-1}} -\mathbb{E}_{t-1}\exp\left[-a(w_{t-1} + Y_{t-1}(\hat{p}_t - p_{t-1}) + \sum_{s=t}^{T-1}\frac{\mathbb{E}_s(\hat{p}_{s+1}) - p_s)^2}{2aVar_s(\hat{p}_{s+1})}\right]$$

As we will verify below, for both types of competition among strategic traders, for any $s \geq t$, $\mathbb{E}_s(\hat{p}_{s+1}) - \hat{p}_s$ is non-stochastic and $Var_s(\hat{p}_{s+1}) = \sigma^2$. Thus, the objective function boils down to

$$\max_{Y_{t-1}} -\exp\left[-a(w_{t-1} + Y_{t-1}(\mathbb{E}_{t-1}(\hat{p}_t) - p_{t-1}) - \frac{a}{2}Var_{t-1}(\hat{p}_t)Y_{t-1}^2 + \sum_{s=t}^{T-1}\frac{(\mathbb{E}_s(\hat{p}_{s+1}) - \hat{p}_s^*)^2}{2aVar_s(\hat{p}_{s+1})}\right]$$

From the first order condition, we get $Y_{t-1} = \frac{\mathbb{E}_{t-1}(\hat{p}_t) - p_{t-1}}{aVar_{t-1}(\hat{p}_t)}$. Substituting back into the objective function, we get the certainty equivalent of the price-taker at $t-1$, $CE_{t-1} = w_{t-1} + \sum_{s=t-1}^{T-1} \frac{(\mathbb{E}_s(\hat{p}_{s+1}) - \hat{p}_s)^2}{2aVar_s(\hat{p}_{s+1})}$. So the property holds at $t-1$, and thus it holds for any $t \in \{0, \dots, T-1\}$.

B.1 Proposition 5

The proposition in the text is a special case of the following result:

Proposition 5 (Competitive Equilibrium) *In the competitive equilibrium:*

- *Traders hold a Pareto-optimal position in the risky asset, in proportion of their risk-bearing capacity, at any time:* $X_t^i = \frac{\frac{1}{b}}{\frac{1}{a}+\frac{n}{b}} = \frac{a}{na+b} s_t \equiv X_t^*$.
- *When the supply changes, traders immediately adjust their portfolios by trading*

$$x_t^{i,*} = X_t^{i,*} - X_{t-1}^{i,*} = \frac{a}{na+b}(s_t - s_{t-1}) = \frac{a}{na+b}\Delta s_t \equiv \Delta X_t^* \quad (29)$$

- *The competitive price is the expected value of the dividend minus a risk premium, which is proportional to supply shocks:*

$$p_t^* = D_t - b\sigma^2(T-t)\left(X^* + \sum_{\tau=1}^{t} \Delta X_\tau^*\right) - b\sigma^2 \sum_{\tau=t+1}^{T-1} (T-\tau)\Delta X_\tau^*,$$
$$\textit{where } X^* \equiv \frac{a}{na+b}s. \quad (30)$$

Proof. The strategic trader's optimal demand is analogous to price takers', except that traders' risk aversion is b. Hence Lemma 1 applies, and a strategic trader's demand at time t is $X_t^i = \frac{\mathbb{E}_t(p_{t+1}) - p_t}{b\sigma^2}$. Thus using market clearing (1) and solving for p_t, we obtain the equilibrium price stated in the proposition. Substituting the equilibrium price in the strategic trader's demand yields X_t^*.

When there are shocks, equation (30) shows that the risk premium has two components. The first one is the risk premium related to the current supply, including shocks that have already occured. The second one is the risk premium due to future shocks. Because future shocks will be absorbed only later, when uncertainty will be smaller than today, they command a smaller premium today (since $T-\tau < T-t$ for $\tau \geq t+1$).

C Cournot competition

C.1 Static model

Proposition 6 *In the static Cournot model, there exists a unique equilibrium for all $n \geq 1$, where the price, trade and holding of trader i are*

$$p_{T-1} = p^*_{T-1} - a\sigma^2 \alpha_{T-1} \Lambda_{T-1} \tag{31}$$

$$x^i_{T-1} = \eta_{T-1}\Lambda_{T-1} + c_{T-1}\left(X^* - X^i_{T-2}\right) \tag{32}$$

$$X^i_{T-1} = \eta_{T-1}\Lambda_{T-1} + c_{T-1}X^* + (1-c_{T-1})X^i_{T-2} \tag{33}$$

with parameters are $\alpha_{T-1} = \frac{a}{(n+1)a+b}$, $\eta_{T-1} = \frac{a}{a+b}\alpha_{T-1}$, $c_{T-1} = \frac{b}{a+b}$. The post-trade equilibrium certainty equivalent (value function) is given by (8), with $t=\tau=T-1$. The coefficients of the value function are $q_{1,T-1} = (1-c_{T-1})^2$, $q_{2,T-1} = ac_{T-1}\alpha_{T-1} + b(1-c_{T-1})\eta_{T-1}$, $q_{3,T-1} = bc_{T-1}(2-c_{T-1})$, $\frac{1}{2}q_{4,T-1} = \eta_{T-1}\left(a\alpha_{T-1} - \frac{b}{2}\eta_{T-1}\right)$, $q_{5,T-1} = q_{2,T-1}$, and $q_{6,T-1} = bc(1-\frac{c}{2})$.

Proof. Since price-takers' demand is given by Lemma 1, to solve for the equilibrium, we simply need to solve strategic traders' optimization problem.

Price schedule. The first step is to derive the price schedule faced by traders. By inverting price-takers' demand, and imposing market clearing (setting $t = T-1$ in equation (1)), we obtain:

$$p_{T-1} = D_{T-1} - a\sigma^2 \left(s_{T-1} - \sum_{j=1}^{n} X_{T-1}^{j} \right) \tag{34}$$

Traders' optimization problems. Traders' wealth at $T-1$ is given by

$$W_T^i = B_T^i + X_T^i D_T = B_{T-2}^i - x_{T-1}^i p_{T-1} + X_{T-1}^i D_{T-1}$$

Therefore, trader i solves the following problem, taking as given other traders' orders, $\sum_{-i} x_{T-1}^{-i}$:

$$\max_{x_{T-1}^i} -\mathbb{E}\left(-\exp(-bW_T^i)\right) \quad \text{s.t. } (34)$$

After substituting the price schedule into the maximand, and using the project theorem for normal variables, the problem boils down to:

$$\max_{x_{T-1}^i} -\exp\left[-b\left(B_{T-2}^i + X_{T-2}^i D_{T-1} + a\sigma^2 x_{T-1}^i \left(s_{T-1} - \sum_{j=1}^{n} X_{T-1}^{j} \right) - \frac{b\sigma^2}{2}(X_{T-1}^i)^2 \right) \right] \tag{35}$$

where $\sum_{j=1}^n X_{T-1}^j = \sum_{j=1}^n X_{T-2}^j + \sum_{j=1}^n x_{T-1}^j = \sum_{j=1}^n X_{T-2}^j + \sum_{-i} x_{T-1}^{-i} + x_{T-1}^i$. From the FOC, we obtain:

$$a\left(s_{T-1} - \sum_j X_{T-2}^j - \sum_i x_{T-1}^{-i} - 2x_{T-1}^i \right) = bX_{T-1}^i \tag{36}$$

Equilibrium trade and price. Summing over all i, and using $X_{T-1}^i =$

$X^i_{T-2} + x^i_{T-1}$, we get:

$$\sum_{j=1}^{n} x^j_{T-1} = \frac{ans_{T-1} - (na+b)\sum_{j=1}^{n} X^j_{T-2}}{(n+1)a+b} \tag{37}$$

We then rewrite (36) as $(a+b)x^i_{T-1} = -bX^i_{T-2} + a\big(s_{T-1} - \sum_{j=1}^{n} X^i_{T-2} - \sum_{j=1}^{n} x^i_{T-1}\big)$. Substituting (37) into this equation, we obtain the equilibrium of the subgame:

$$x^i_{T-1} = \frac{a}{(n+1)a+b}\left(s_{T-1} - \frac{a}{a+b}\sum_{j=1}^{n} X^j_{T-2}\right) - \frac{b}{a+b}X^i_{T-2} \tag{38}$$

We can rewrite the equilibrium trade as in the proposition by adding $(\frac{b}{na+b}s_{T-1} - \frac{b}{na+b}s_{T-1})$, recognizing X^* (defined in Proposition 5) and rearranging the terms.
Then from (37), we get the total time-T-1 position of the traders:

$$\sum_{j=1}^{n} X^j_{T-1} = \frac{nas_{T-1}}{(n+1)a+b} + \frac{a}{(n+1)a+b}\sum_{i=1}^{n} X^i_{T-2} \tag{39}$$

Substituting into the price schedule (34) yields the equilibrium price of the subgame:

$$p_{T-1} = \mathbb{E}_{T-1}(D_T) - a\sigma^2 \frac{(a+b)s_{T-1} - a\sum_{i=1}^{n} X^i_{T-2}}{(n+1)a+b} \tag{40}$$

To write the price as in the proposition, note that from (30), we can write $D_{T-1} = p^*_{T-1} + b\sigma^2 X^*_{T-1}$. Substituting this expression into (40) and rearranging the terms gives

$$p_{T-1} = p^*_{T-1} - a\sigma^2 \frac{a}{na+b}\left[nX^* - \sum_{j=1}^{n} X^j_{t-1}\right]$$

which we can write as in the proposition using the definition of the liqudity factor $\Lambda_{T-1} = nX^* - \sum_{j=1}^{n} X_{t-1}^{j}$.

Equilibrium certainty equivalent. The traders' expected utility at time 1 is given by

$$\mathbb{E}u(C_T^i) = -\exp\left\{-b\left[B_{T-2}^i + X_{T-2}^i D_{T-1} + a\sigma^2 x_1^i \left(s_{T-1} - \sum_{j=1}^{n} X_{T-1}^j\right) - \frac{b\sigma^2}{2} X_{T-1}^{i2}\right]\right\} \quad (41)$$

Substituting for the equilibrium trade (32) and the price (31), we can write $\Omega_{T-1}^i \equiv a\sigma^2 x_1^i \left(s_{T-1} - \sum_{j=1}^{n} X_{T-1}^j\right) - \frac{b\sigma^2}{2} X_{T-1}^{i2}$ as (8) by defining the coefficients as in the proposition.

C.2 Anticipated supply shocks

C.2.1 Recursive characterization

I providea more detailed result than in the text:

Proposition 7 (Dynamic Cournot Equilibrium)

1. *For all $n \geq 1$, there exists a unique equilibrium in which the price, trade, and post-trade certainty equivalent (value function) are given by equations (7), (8), and (9) with, for all $t \in \{0, \ldots, T-1\}$*

$$\mathcal{P}_{as}: \; \bar{q}_{2,t} = \bar{q}_{5,t}, \; \textit{and} \; bq_{1,t} + \bar{q}_{3,t} = (T-t)b,$$

if the price and value function coefficients are defined recursively by the system $\mathcal{S}(q_k, \alpha)$ given in Lemma 2, and if for $t \in \{1, \ldots, T-1\}$, the second-order condition holds

$$2a(1 + \bar{\alpha}_{t+1}) + Q_{t+1} > 0, \quad (42)$$

where $Q_{t+1} \equiv \bar{Q}^{1,2}_{t+1} - n\bar{Q}^{2,4}_{t+1}$ *measures the curvature of the value function, with* $\bar{Q}^{1,2}_{t+1} \equiv b\sigma^2(1+q_{1,t+1}) - \sigma^2\bar{q}_{2,t+1}$ *and* $Q^{2,4}_{t+1,\tau} \equiv \sigma^2\left(\bar{q}_{2,t+1} + \bar{q}_{4,t+1}\right)$*. Boundary conditions for* α *and* q_i *given by the static version of the model in Proposition 6.*

2. *The liquidity factor evolves as follows:*

$$\text{for } \tau = t, \ldots, T-1, \;\; \Lambda_{t+1,\tau} = \Lambda_{t,\tau} - \sum_{j=t}^{T-1} \delta_{t,j}\Lambda_{t,j} \tag{43}$$

3. *The parameters are defined as follows:*

$$\eta_{t,t} = \frac{a - (a+\bar{\mu}_{t+1})\delta_{t,t}}{\vartheta_t}, \eta_{t,\tau} \equiv \frac{\mu_{t+1,\tau} - (a+\bar{\mu}_{t+1})\delta_{t,\tau}}{\vartheta_t}, \;\; \tau \geq t+1 \tag{44}$$

$$c_{t,t} \equiv \frac{b}{\vartheta_t}, \qquad c_{t,j} \equiv \frac{b - \sum_{\tau=t+1}^{T-1} q^{\tau,j}_{5,t+1} - q^{j}_{3,t+1}}{\vartheta_t} \text{ for } j \geq t+1 \tag{45}$$

$$\delta_{t,t} \equiv \frac{\frac{na}{\vartheta_t} + c_{t,t}}{\tilde{\vartheta}_t}, \qquad \delta_{t,\tau} \equiv \frac{\frac{n\mu_{t+1,\tau}}{\vartheta_t} + c_{t,\tau}}{\tilde{\vartheta}_t}, \text{ for } \tau \geq t+1 \tag{46}$$

$$\vartheta_t = a(1+\bar{\alpha}_{t+1}) + b(1+q_{1,t+1}) - \bar{q}_{2,t+1}, \tag{47}$$

$$\tilde{\vartheta}_t \equiv 1 + \frac{n(a+\bar{\mu}_{t+1})}{\vartheta_t} \tag{48}$$

$$\mu_{t+1,\tau} = a\alpha_{t+1,\tau} - q^{\tau}_{2,t+1} - \frac{1}{2}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}) \tag{49}$$

Note: For the sake of comparison with demand schedule competition, parameters can be rewritten using Notation 2 as

$$\lambda_t^{\mathcal{C}} \equiv a\sigma^2(1+\bar{\alpha}_{t+1}), \quad \bar{\eta}_t = -\frac{\lambda_t^{\mathcal{C}}}{\lambda_t^{\mathcal{C}} + \bar{Q}_{t+1}^{1,2}} \frac{a\sigma^2(1+\bar{\alpha}_{t+1}) - \bar{Q}_{t+1}^{2,4}}{(n+1)\lambda_t^{\mathcal{C}} + Q_{t+1}}$$

$$c_{t,t} = \frac{b\sigma^2}{\lambda_t^{\mathcal{C}} + \bar{Q}_{t+1}^{1,2}}, \qquad c_{t,\tau} = \frac{b\sigma^2 - Q_{t+1,\tau}^{3,5}}{\lambda_t^{\mathcal{C}} + \bar{Q}_{t+1}^{1,2}}$$

Lemma 2 (Recursive system $\mathcal{S}(q_k, \alpha)$) *The price and value function coefficients are defined recursively by the following system for $t \in \{0, \ldots, T-2\}$:*

$$\alpha_{t,t} = 1 - (1+\bar{\alpha}_{t+1})\delta_{t,t} \tag{50}$$

$$\alpha_{t,\tau} = \alpha_{t+1,\tau} - (1+\bar{\alpha}_{t+1})\delta_{t,\tau} \tag{51}$$

$$q_{1,t} = (1+q_{1,t+1})(1-c_t)^2$$

$$\mathbf{q}_{2,t} = a\bar{c}_t\boldsymbol{\alpha}_t + (1-\bar{c}_t)\left[b(1+q_{1,t+1})\boldsymbol{\eta}_t + \left(0, \mathbf{q}_{2,t+1}\right)^\top - \bar{q}_{2,t+1}\boldsymbol{\delta}_t\right]$$

$$\mathbf{q}_{3,t} = b\bar{c}_t\mathbf{1} + (1-\bar{c}_t)\left[b(1+q_{1,t+1})\mathbf{c}_t + \left(0, \mathbf{q}_{3,t+1}\right)^\top\right]$$

$$\frac{1}{2}q_{4,t}^{t,t} = \eta_{t,t}g_{t,t} + \frac{1}{2}\bar{q}_{4,t+1}\delta_{t,t}^2$$

$$\frac{1}{2}q_{4,t}^{t,\tau} = \eta_{t,t}g_{t,\tau} + g_{t,t}\eta_{t,\tau} + \delta_{t,t}\left(\bar{q}_{4,t+1}\delta_{t,\tau} - \frac{1}{2}\left(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}\right)\right), \quad \text{for } t+1 \le \tau \le T-1$$

$$\frac{1}{2}q_{4,t}^{\tau,\tau} = \eta_{t,\tau}g_{t,\tau} + \frac{1}{2}\left[q_{4,t+1}^{\tau,\tau} + \bar{q}_{4,t+1}\delta_{t,\tau}^2 - \delta_{t,\tau}\left(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}\right)\right], \quad \text{for } t+1 \le \tau \le T-1$$

$$\frac{1}{2}q_{4,t}^{\tau,j} = \eta_{t,\tau}g_{t,j} + g_{t,\tau}\eta_{t,j} + \frac{1}{2}\big[q_{4,t+1}^{\tau,j} + 2\bar{q}_{4,t+1}\delta_{t,\tau}\delta_{t,j} - \delta_{t,\tau}\left(\iota_{t+1,j} + \hat{\iota}_{t+1,j}\right) - \delta_{t,j}\left(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}\right)\big], \text{for } t+1 \le \tau \le T-2, \text{ and } \tau+1 \le j \le T-1$$

$$q_{5,t}^{t,t} = \eta_{t,t} h_{t,t} + c_{t,t} g_{t,t}$$

$$q_{5,t}^{t,j} = \eta_{t,t} h_{t,j} + c_{t,j} g_{t,t} - \delta_{t,t} \sum_{u=t+1}^{T-1} q_{5,t+1}^{u,j}, \ for\ j \geq t+1$$

$$q_{5,t}^{\tau,j} = \eta_{t,\tau} h_{t,j} + g_{t,\tau} c_{t,j} + q_{5,t+1}^{\tau,j} - \delta_{t,\tau} \sum_{u=t+1}^{T-1} q_{5,t+1}^{u,j},$$

$$for\ \tau \geq t+1,\ and\ j \geq t+1$$

$$q_{5,t}^{\tau,t} = \eta_{t,\tau} h_{t,t} + g_{t,\tau} c_{t,t}, \ for\ \tau \geq t+1$$

$$q_{6,t}^{t,t} = \frac{b}{2}(1 + q_{1,t+1})(c_{t,t})^2$$

$$q_{6,t}^{t,\tau} = c_{t,t} q_{3,t+1}^{\tau}, \quad for\ \tau \geq t+1$$

$$q_{6,t}^{\tau,\tau} = q_{3,t+1}^{\tau} c_{t,\tau} + \frac{b}{2}(1 + q_{1,t+1})(c_{t,\tau})^2 + q_{6,t+1}^{\tau,\tau}, \quad for\ \tau \geq t+1$$

$$q_{6,t}^{\tau,j} = b(1 + q_{1,t+1}) c_{t,\tau} c_{t,j} + c_{t,\tau} q_{3,t+1}^{j} + q_{6,t+1}^{\tau,j},$$

$$for\ t+1 \leq \tau \leq T-2,\ \tau+1 \leq j \leq T-1$$

Proof. The proof is by induction. Let's assume that the expression of the price and value function, (7) and (8), and the properties $\mathcal{P}_{as}$ hold at $t+1$ and all periods up to $T-1$. We can now show that this implies that they also hold at t. The main steps of the derivation are: (i) obtaining the price schedule; (ii) solving the traders' optimization problem at t; (iii) calculating the equilibrium price and traders' value function at t, and (iv) showing that $\mathcal{P}_{as}$ holds at t.

Step 1: Price schedule. First, let's invert the price-takers' demand (lemma 1) and impose market-clearing (1) to get $p_t = \mathbb{E}_t(p^*_{t+1}) - a\sigma^2\alpha^\top_{t+1}\mathbf{\Lambda}_{t+1} - a\sigma^2\left(s_t - \sum_{j=1}^n X^j_t\right)$. Proposition 5 implies that

$$p^*_t = \mathbb{E}(p^*_{t+1}) - b\sigma^2 X^*_t, \tag{52}$$

Substituting (52) and grouping terms, we obtain the price schedule:

$$p_t(\cdot) = p^*_t - a\sigma^2\left(\Lambda_{t+1,t} + \alpha^\top_{t+1}\Lambda_{t+1}\right), \text{ with } \Lambda_{t+1,t} = nX^*_t - \textstyle\sum_j X^j_t. \tag{53}$$

Step 2: Traders' optimization. Using (53), and (52), we can write trader i's time t maximization problem as follows:

$$\max_{x^i_t} \sigma^2 x^i_t \left(b\sum_{j=t}^{T-1} X^*_j + a(\Lambda_{t+1,t} + \alpha^\top_{t+1}\mathbf{\Lambda}_{t+1})\right) - 0.5b\sigma^2(X^i_t)^2 + \Omega^i_{t+1}$$

where trader i takes the orders of other traders, $\sum_{-i} x^{-i}_t$, as given. From the FOC, we obtain:

$$\begin{aligned} &- \left[b(1+q_{1,t+1}) - \bar{q}_{2,t+1}\right] X^i_{t-1} - \vartheta_t x^i_t + D_t - p_t(\cdot) \\ &\quad - \mathbf{q}^\top_{2,t+1}\mathbf{\Lambda}_{t+1} - \frac{1}{2}\sum_{\tau=t+1}^{T-1}\sum_{j=\tau}^{T-1} q^{\tau,j}_{4,t+1}\left(\Lambda_{t+1,\tau} + \Lambda_{t+1,j}\right) - \\ &\quad\quad \sum_{j=t+1}^{T-1}\left(\sum_{\tau=t+1}^{T-1} q^{\tau,j}_{5,t+1} + q^j_{3,t+1}\right) X^*_j = 0 \end{aligned} \tag{54}$$

where $D_t - p_t(\cdot) = \sum_{j=t}^{T-1} bX^*_j + a\left(\Lambda_{t+1,t} + \alpha^\top_{t+1}\mathbf{\Lambda}_{t+1}\right)$ and $\vartheta_t = a(1+\bar{\alpha}_{t+1}) + b(1+q_{1,t+1}) - \bar{q}_{2,t+1}$. From the SOC, x^i_t is a maximum if inequality (42) holds.

Step 3: Equilibrium trade, liquidity factor, price, and value function. We first rearrange the terms in $q_{4,t+1}$ in the FOC. After some algebra, we get:

$$\sum_{\tau=t+1}^{T-1}\sum_{j=\tau}^{T-1} q_{4,t+1}^{\tau,j}\left(\Lambda_{t+1,\tau}+\Lambda_{t+1,j}\right)=\sum_{\tau=t+1}^{T-1}\left(\iota_{t+1,\tau}+\hat{\iota}_{t+1,\tau}\right)\Lambda_{t+1,\tau}$$

where $\iota_{t+1,\tau}=\sum_{j=\tau}^{T-1} q_{4,t+1}^{\tau,j}$ and $\hat{\iota}_{t+1,\tau}=\sum_{j=t+1}^{\tau} q_{4,t+1}^{j,\tau}$. Therefore, using this expression and the induction hypothesis, we can write

$$x_t^i=\frac{1}{\vartheta_t}\left[a\Lambda_{t+1,t}+\sum_{\tau=t+1}^{T-1}\mu_{t+1,\tau}\Lambda_{t+1,\tau}\right]+\sum_{j=t}^{T-1} c_{t,j}\left(\bar{X}_j^*-X_{t-1}^i\right) \tag{55}$$

where $\mathbf{c}_t$ and $\mu_{t+1,\tau}$ are given by equations (45) and (49). From (55), we can obtain the trader's aggregate equilibrium trade by summing over i. We have

$$\sum_{i=1}^{n} x_t^i=\frac{n}{\vartheta_t}\left[a\Lambda_{t+1,t}+\sum_{\tau=t+1}^{T-1}\mu_{t+1,\tau}\Lambda_{t+1,\tau}\right]+\sum_{j=t}^{T-1}\left(nX_j^*-\sum_{i=1}^{n}X_{t-1}^i\right)$$

Since $\Lambda_{t+1,\tau}=\Lambda_{t,\tau}-\sum_i X_t^i$ and $nX_j^*-\sum_{i=1}^{n}X_{t-1}^i=\Lambda_{t,j}$, the equilibrium aggregate trade is:

$$\sum_{j=1}^{n} x_t^j=\sum_{\tau=t}^{T-1}\delta_{t,\tau}\Lambda_{t,\tau} \tag{56}$$

where δ_t is defined by (46). Thus we can express $\Lambda_{t+1,\tau}$ as a function of $\Lambda_{t,\tau}$

$$\Lambda_{t+1,\tau}=\Lambda_{t,\tau}-\sum_{j=t}^{T-1}\delta_{t,j}\Lambda_{t,j} \tag{57}$$

From (57) and (53), we can derive the equilibrium price. To do so, we first need to compute $\Lambda_{t+1,t}+\alpha_{t+1}^{\top}\mathbf{\Lambda}_{t+1}$. From (57), we have:

$$\begin{aligned}\alpha_{t+1}^{\top}\mathbf{\Lambda}_{t+1}&=\sum_{\tau=t+1}^{T-1}\alpha_{t+1,\tau}\Lambda_{t+1,\tau}=\sum_{\tau=t+1}^{T-1}\alpha_{t+1,\tau}\Lambda_{t,\tau}-\sum_{\tau=t+1}^{T-1}\alpha_{t+1,\tau}\sum_{j=t}^{T-1}\delta_{t,j}\Lambda_{t,j}\\&=\begin{pmatrix}0, & \alpha_{t+1}^{\top}\end{pmatrix}\mathbf{\Lambda}_t-\bar{\alpha}_{t+1}\delta_t^{\top}\mathbf{\Lambda}_t\end{aligned}$$

Therefore, $\Lambda_{t+1,t} + \alpha_{t+1}^\top \mathbf{\Lambda}_{t+1} = \Lambda_{t,t} + \left\{ \begin{pmatrix} 0, & \alpha_{t+1}^\top \end{pmatrix} - (1 + \bar{\alpha}_{t+1})\delta_t^\top \right\} \mathbf{\Lambda}_t$. So we can write the equilibrium price as (7) if we define $\boldsymbol{\alpha}_t$ by (51). Then combining (55) and (57), we get the equilibrium trade. First, note that

$$a\Lambda_{t+1,t} + \sum_{\tau=t+1}^{T-1} \mu_{t+1,\tau}\Lambda_{t+1,\tau} = a\Lambda_{t,t} + \left\{ \begin{pmatrix} 0, & \mu_{t+1} \end{pmatrix}^\top - (a + \bar{\mu}_{t+1})\delta_t^\top \right\} \mathbf{\Lambda}_t$$

Then we can write the equilibrium trade as
$x_t^i = \eta_t^\top \mathbf{\Lambda}_t + \sum_{\tau=t}^{T-1} c_{t,\tau} \left(X_\tau^* - X_{t-1}^i \right)$ by defining η_t as in the proposition.

Value function. Next, we use the equilibrium trade, holding, and liquidity factor to calculate the value function. Ω_t^i is the sum of equilibrium J_t^i and equilibrium (post-trade) Ω_{t+1}^i:

$$\Omega_t^i = \max_{x_t^i} J_t^i + \Omega_{t+1}^i$$

with $J_t^i \equiv \sigma^2 x_t^i \left(b \sum_{j=t}^{T-1} X_j^* + a \left(\Lambda_{t+1,t} + \alpha_{t+1}^\top \mathbf{\Lambda}_{t+1} \right) \right) - \frac{1}{2} b\sigma^2 (X_t^i)^2$. Substituting the equilibrium trade and holding, and the liquidity factor (57) into this expression, and rearranging terms, we get:

$$\begin{aligned} J_t^i = &\left(\eta_t^\top \mathbf{\Lambda}_t + \mathbf{c}_t^\top \mathbf{X}^* \right) \left[b \left(\mathbf{1}^\top - 0.5\mathbf{c}^\top \right) \mathbf{X}^* + \left(a\alpha_t^\top - 0.5b\eta_t^\top \right) \mathbf{\Lambda}_t \right] \\ &- X_{t-1}^i \left[\left(b\bar{c}_t \mathbf{1}^\top + b(1-\bar{c}_t)\mathbf{c}_t^\top \right) \mathbf{X}^* + \left(a\bar{c}_t\alpha_t^\top + b(1-\bar{c}_t)\eta_t^\top \right) \mathbf{\Lambda}_t \right] \\ &- \frac{b}{2}(1-\bar{c}_t)^2 \left(X_{t-1}^i \right)^2 \end{aligned} \tag{58}$$

Next, we compute the equilibrium value of Ω_{t+1}^i as a function of X_{t-1}^i and

$\mathbf{\Lambda}_t$. Starting with the terms in X_t^i, we get:

$$
\begin{aligned}
&- \left[\eta_t^\top \mathbf{\Lambda}_t + \mathbf{c}_t^\top \mathbf{X}^*\right] \Big[\Big(0.5bq_{1,t+1}\eta_t^\top + \Big(0\mathbf{q}_{2,t+1}^\top\Big) - \bar{q}_{2,t+1}\delta_t^\top\Big) \mathbf{\Lambda}_t + \\
&\qquad \Big\{0.5bq_{1,t+1}\mathbf{c}_t^\top + \Big(0\mathbf{q}_{3,t+1}^\top\Big)\Big\} \mathbf{X}^*\Big] \\
&-X_{t-1}^i(1-\bar{c}_t)\Big[\Big\{bq_{1,t+1}\eta_t^\top - \bar{q}_{2,t+1}\delta_t^\top + \Big(0\mathbf{q}_{2,t+1}^\top\Big)\Big\} \mathbf{\Lambda}_t + \\
&\Big\{bq_{1,t+1}\mathbf{c}_t^\top + \Big(0\mathbf{q}_{3,t+1}^\top\Big) \mathbf{X}^*\Big\}\Big] - \frac{b}{2}(1-\bar{c}_t)^2 q_{1,t+1}\left(X_{t-1}^i\right)^2
\end{aligned}
\tag{59}
$$

Then we can compute the terms in $q_{4,t+1}$ and $q_{5,t+1}$. Using (57), developing and rearranging terms, we get (skipping a few lines of algebra):

$$
\begin{aligned}
Q_4 &\equiv \frac{1}{2}\sum_{\tau=t+1}^{T-1}\sum_{j=\tau}^{T-1} q_{4,t+1}^{\tau,j}\Lambda_{t+1,\tau}\Lambda_{t+1,j} \\
&= \frac{1}{2}\bar{q}_{4,t+1}\delta_{t,t}^2\Lambda_{t,t}^2 + \delta_{t,t}\Lambda_{t,t}\sum_{\tau=t+1}^{T-1}\left\{\bar{q}_{4,t+1}\delta_{t,\tau} - \frac{1}{2}\left(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}\right)\right\}\Lambda_{t,\tau} \\
&+ \frac{1}{2}\sum_{\tau=t+1}^{T-1}\left\{q_{4,t+1}^{\tau,\tau} + \bar{q}_{4,t+1}\delta_{t,\tau}^2 - \delta_{t,\tau}\left(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}\right)\right\}\Lambda_{t,\tau}^2 \\
&+ \frac{1}{2}\sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\Big\{q_{4,t+1}^{\tau,j} - \left(\delta_{t,\tau}\left(\iota_{t+1,j} + \hat{\iota}_{t+1,j}\right) + \delta_{t,j}\left(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}\right)\right) + \\
&2\bar{q}_{4,t+1}\delta_{t,\tau}\delta_{t,j}\Big\}\Lambda_{t,\tau}\Lambda_{t,j}
\end{aligned}
$$

where $\iota_{t+1,j} \equiv \sum_{u=j}^{T-1} q_{4,t+1}^{j,u}$ and $\hat{\iota}_{t+1,j} \equiv \sum_{u=t+1}^{j} q_{4,t+1}^{u,j}$. Similarly, we compute the terms in $q_{5,t}$.

$$Q_5 = \sum_{\tau=t+1}^{T-1} \sum_{j=t+1}^{T-1} q_{5,t+1}^{\tau,j} \Lambda_{t+1,\tau} X_j^*$$

$$= -\delta_{t,t}\Lambda_{t,t} \sum_{j=t+1}^{T-1} \left(\sum_{u=t+1}^{T-1} q_{5,t+1}^{u,j} \right) X_j^* +$$

$$\sum_{\tau=t+1}^{T-1} \sum_{j=t+1}^{T-1} \left\{ q_{5,t+1}^{\tau,j} - \delta_{t,\tau} \left(\sum_{u=t+1}^{T-1} q_{5,t+1}^{u,j} \right) \right\} \Lambda_{t,\tau} X_j^*$$

Thus, adding terms in X_{t-1}^i and Q_4 and Q_5, we obtain

$$\begin{aligned} \Omega_{t+1}^i = \\ -\left[\eta^\top \mathbf{\Lambda}_t + \mathbf{c}_t^\top \mathbf{X}^*\right] \Big[\Big\{ 0.5bq_{1,t+1}\eta_t^\top + \Big(0, \mathbf{q}_{2,t+1}^\top\Big) - \bar{q}_{2,t+1}\delta_t^\top \Big\} \mathbf{\Lambda}_t + \\ \Big\{ 0.5bq_{1,t+1}\mathbf{c}_t^\top + \Big(0, \mathbf{q}_{3,t+1}^\top\Big) \Big\} \mathbf{X}^* \Big] \\ -X_{t-1}^i(1-\bar{c}_t) \Big[\Big\{ bq_{1,t+1}\eta_t^\top + \Big(0, \mathbf{q}_{2,t+1}^\top\Big) - \bar{q}_{2,t+1}\delta_t^\top \Big\} \mathbf{\Lambda}_t + \\ \Big\{ bq_{1,t+1}\mathbf{c}_t^\top + \Big(0, \mathbf{q}_{3,t+1}^\top\Big) \Big\} \mathbf{X}^* \Big] \\ -\frac{b}{2}(1-\bar{c}_t)^2 \left(X_{t-1}^i\right)^2 + Q_4 + Q_5 \end{aligned} \quad (60)$$

Thus, adding (58) and (60), we obtain the value function

$$\begin{aligned}
\Omega_t^i = \left[\eta_t^\top \boldsymbol{\Lambda}_t + \mathbf{c}_t^\top \mathbf{X}^*\right] \times \\
\left[\left\{a\alpha_t^\top - 0.5b(1+q_{1,t+1})\eta_t^\top - \left(0, \mathbf{q}_{2,t+1}^\top\right) + \bar{q}_{2,t+1}\delta_t^\top\right\}\boldsymbol{\Lambda}_t \right. \\
\left. + \left\{b\left(\mathbf{1}^\top - 0.5(1+q_{1,t+1})\mathbf{c}_t^\top\right) - \left(0\mathbf{q}_{3,t+1}^\top\right)\right\}\mathbf{X}^*\right] \\
-X_{t-1}^i \left[\left\{a\bar{c}_t\alpha_t^\top + b(1-\bar{c}_t)\left[(1+q_{1,t+1})\eta_t^\top + \left(0, \mathbf{q}_{2,t+1}^\top\right) - \bar{q}_{2,t+1}\delta_t^\top\right]\right\}\boldsymbol{\Lambda}_t \right. \\
\left. + \left\{b\bar{c}_t\mathbf{1}^\top + b(1-\bar{c}_t)\left[(1+q_{1,t+1})\mathbf{c}_t^\top + \left(0, \mathbf{q}_{3,t+1}^\top\right)\right]\right\}\mathbf{X}^*\right] \\
-\frac{b}{2}(1+q_{1,t+1})(1-\bar{c}_t)^2\left(X_{t-1}^i\right)^2 + Q_4 + Q_5
\end{aligned} \tag{61}$$

Let's define $\mathbf{g}$ and $\mathbf{h}$ as in (25)-(26).Using this notation, we can rewrite (61) as (8) by defining the coefficients $q_{i,t}$, $i = 1, \ldots, 5$ as in the proposition.

Step 4: Property $\mathcal{P}_{as}$. To complete the proof, we need to show that property $\mathcal{P}_{as}$ holds at time t. Using the recursive definition of $q_{5,t}$ given in the proposition, we have:

$$\begin{aligned}
\sum_{\tau=t}^{T-1}\sum_{j=t}^{T-1} q_{5,t}^{\tau,j} = \sum_{\tau=t}^{T-1}\sum_{j=t}^{T-1}\left(\eta_{t,\tau}h_{t,j} + c_{t,\tau}g_{t,j}\right) + \\
\sum_{\tau=t+1}^{T-1}\sum_{j=t+1}^{T-1}\left(q_{5,t+1}^{\tau,j} - \delta_{t,\tau}\sum_{u=t+1}^{T-1} q_{5,t+1}^{u,j}\right) - \delta_{t,t}\sum_{j=t+1}^{T-1}\left(\sum_{u=t+1}^{T-1} q_{5,t+1}^{u,j}\right)
\end{aligned} \tag{62}$$

We first calculate the first term, using the definitions of $\mathbf{h}_t$ and $\mathbf{g}_t$,

$$\sum_{j=t}^{T-1} h_{t,j} = (T-t)b - 0.5b\bar{c}_t(1+q_{1,t+1}) - \bar{q}_{3,t+1}$$

$$\sum_{j=t}^{T-1} g_{t,j} = a\bar{\alpha}_t - 0.5b(1+q_{1,t+1})\bar{\eta}_t - \bar{q}_{2,t+1} + \bar{q}_{2,t+1}\bar{\delta}_t$$

Thus, we have $\sum_{\tau=t}^{T-1}\sum_{j=t}^{T-1}\left(\eta_{t,\tau}h_{t,j}+c_{t,\tau}g_{t,j}\right)=(T-t)b\bar{\eta}_t-b(1+q_{1,t+1})\bar{c}_t\bar{\eta}_t-\bar{q}_{3,t+1}\bar{\eta}_t+a\bar{\alpha}_t\bar{c}_t-\bar{q}_{2,t+1}\bar{c}_t(1-\bar{\delta}_t)$. Further, the second and third terms are equal to

$$\begin{aligned}
&\sum_{\tau=t+1}^{T-1}\sum_{j=t+1}^{T-1}q_{5,t+1}^{\tau,j}-\sum_{\tau=t+1}^{T-1}\delta_{t,\tau}\sum_{j=t+1}^{T-1}\sum_{u=t+1}^{T-1}q_{5,t+1}^{u,j}-\delta_{t,t}\sum_{j=t+1}^{T-1}\sum_{u=t+1}^{T-1}q_{5,t+1}^{u,j}\\
&=\bar{q}_{5,t+1}-\sum_{\tau=t+1}^{T-1}\delta_{t,\tau}\bar{q}_{5,t+1}-\delta_{t,t}\bar{q}_{5,t+1}\\
&=\bar{q}_{5,t+1}\left(1-\sum_{\tau=t}^{T-1}\delta_{t,\tau}\right)=\bar{q}_{5,t+1}(1-\bar{\delta}_t)
\end{aligned}$$

Therefore, adding the two terms, we get:

$$\begin{aligned}
\bar{q}_{5,t}\equiv\sum_{\tau=t}^{T-1}\sum_{j=t}^{T-1}q_{5,t}^{\tau,j}=b(T-t)\bar{\eta}_t-b(1+q_{1,t+1})\bar{c}_t\bar{\eta}_t-\bar{q}_{3,t+1}\bar{\eta}_t+a\bar{\alpha}_t\bar{c}_t\\
-\bar{q}_{2,t+1}\bar{c}_t(1+\bar{\delta}_t)+\bar{q}_{5,t+1}(1-\bar{\delta}_t)
\end{aligned}$$

Then, by summation, we have from the definition of $\mathbf{q}_{2,t}$: $\bar{q}_{2,t}=a\bar{c}_t\bar{\alpha}_t+(1-\bar{c}_t)\left[b(1+q_{1,t+1})\bar{\eta}_t+\bar{q}_{2,t+1}(1-\bar{\delta}_t)\right]$. Hence, using the induction hypothesis $\bar{q}_{2,t+1}=\bar{q}_{5,t+1}$ we get that $\bar{q}_{2,t}=\bar{q}_{5,t}$ is equivalent to
$\bar{\eta}_t\left[(T-t)b-b(1+q_{1,t+1})-\bar{q}_{3,t+1}\right]$, which is equal to 0 by $\mathcal{P}_{as}$. Thus $\bar{q}_{2,t}=\bar{q}_{5,t}$.
Then, we show the second property of $\mathcal{P}_{as}$. From the recursive definitions of $q_{1,t}$ and $q_{3,t}$:

$$\begin{aligned}
b(1+q_{1,t})+\bar{q}_{3,t}&=b\left[1+(1-\bar{c}_t)^2(1+q_{1,t+1})\right]\\
&\quad+b\bar{c}_t(T-t)+(1-\bar{c}_t)b(1+q_{1,t+1})\bar{c}_t+(1-\bar{c}_t)\bar{q}_{3,t+1}\\
&=b+(1-\bar{c}_t)\left[b(1+q_{1,t+1})+\bar{q}_{3,t+1}\right]+b\bar{c}_t(T-t)\\
&=b+(T-t)b-\bar{c}_t\left(b(T-t)-b(T-t)\right)
\end{aligned}$$

Thus, $bq_{1,t} + \bar{q}_{3,t} = (T-t)b$. This completes the proof.

C.3 Constant supply

C.3.1 Recursive characterization

Using Proposition 2 in the special case of constant supply, the system $\mathcal{S}(\alpha, q)$ boils down to

$$\bar{\alpha}_t = \bar{\gamma}_t(1+\bar{\alpha}_{t+1}) \tag{63}$$

$$q_{1,t} = (1-\bar{c}_t)^2(1+q_{1,t+1}) \tag{64}$$

$$\bar{q}_{2,t} = a\bar{c}_t\bar{\gamma}_t(1+\bar{\alpha}_{t+1}) + b\bar{\eta}_t(1-\bar{c}_t)(1+q_{1,t+1}) + (1-\bar{c}_t)\bar{\gamma}_t\bar{q}_{2,t+1} \tag{65}$$

$$\bar{q}_{3,t} = b\bar{c}_t(T-t) + (1-\bar{c}_t)\left(\bar{q}_{3,t+1} + b\bar{c}_t(1+q_{1,t+1})\right) \tag{66}$$

$$\frac{1}{2}\bar{q}_{4,t} = \frac{1}{2}\bar{\gamma}_t^2\bar{q}_{4,t+1} + a\bar{\eta}_t\bar{\alpha}_t - \bar{\gamma}_t\bar{\eta}_t\bar{q}_{2,t+1} - \frac{b}{2}\bar{\eta}_t^2(1+q_{1,t+1}) \tag{67}$$

The coefficients of the trade and liquidity factors are given by:

$$\bar{\eta}_t \equiv \frac{a(1+\bar{\alpha}_{t+1}) - \bar{q}_{2,t+1} - \bar{q}_{4,t+1}}{\vartheta_t}\bar{\gamma}_t, \qquad \bar{c}_t \equiv \frac{b(1+q_{1,t+1}) - \bar{q}_{2,t+1}}{\vartheta_t} \tag{68}$$

$$\vartheta_t \equiv a(1+\bar{\alpha}_{t+1}) + b(1+q_{1,t+1}) - \bar{q}_{2,t+1}, \qquad \bar{\gamma}_t \equiv \frac{\vartheta_t(1-\bar{c}_t)}{\bar{\vartheta}_t}, \tag{69}$$

$$\hat{\vartheta}_t \equiv \vartheta_t + n\left(a(1+\bar{\alpha}_{t+1}) - \bar{q}_{2,t+1} - \bar{q}_{4,t+1}\right) \tag{70}$$

Proof. The expressions of the recursive system and equilibrium parameters derive from summing the corresponding expressions in Proposition 7.

Equilibrium parameters. Let's first define $\hat{\vartheta}_t \equiv \vartheta_t + n(a+\bar{\mu}_{t+1})$, with $\bar{\mu}_{t+1} = a\bar{\alpha}_{t+1} - \bar{q}_{2,t+1} - \bar{q}_{4,t+1}$, so that we can rewrite $\tilde{\vartheta}_t = 1 + \frac{n(a+\bar{\mu}_{t+1})}{\vartheta_t} = \frac{\hat{\vartheta}_t}{\vartheta_t}$.

Then, using the definitions of Proposition 2, we compute:

$$\bar{\delta}_t = \delta_{t,t} + \sum_{\tau=t+1}^{T-1} \delta_{t,\tau} = \frac{\bar{c}_t + \frac{n(a+\bar{\mu}_{t+1})}{\vartheta_t}}{\tilde{\vartheta}_t}$$

$$\bar{\gamma}_t = 1 - \bar{\delta}_t = \frac{\tilde{\vartheta}_t - c_t - \frac{n(a+\bar{\mu}_{t+1})}{\vartheta_t}}{\tilde{\vartheta}_t} = \frac{\vartheta_t(1-\bar{c}_t)}{\hat{\vartheta}_t}, \quad \text{from the definition of } \hat{\vartheta}_t$$

$$\bar{\eta}_t = \frac{a(1-\delta_{t,t}) - \bar{\mu}_{t+1}\delta_{t,t} + \bar{\mu}_{t+1} - (a+\bar{\mu}_{t+1})\sum_{\tau=t+1}^{T-1}\delta_{t,\tau}}{\vartheta_t} = \frac{(a+\bar{\mu}_{t+1})}{\vartheta_t}\gamma_t$$

$$\bar{c}_t = \frac{b + b(T-t-1) - \bar{q}_{3,t+1} - \bar{q}_{5,t+1}}{\vartheta_t} = \frac{b(1+q_{1,t+1}) - \bar{q}_{2,t+1}}{\vartheta_t}, \quad \text{from } \mathcal{P}_{as}$$

Further, $\Lambda_{t+1,\tau} = \Lambda_{t,\tau} - \sum_{j=t}^{T-1}\delta_{t,j}\Lambda_{t,j} = nX^*_\tau - \sum_i X^i_t - \sum_{j=t}^{T-1}\delta_{t,j}(nX^*_j - \sum_i X^i_t)$. So when $s_\tau = s$, $X^*_\tau = X^*$, $\Lambda_{t+1,\tau} = \Lambda_{t+1} = (1-\bar{\delta}_t)(nX^* - \sum_i X^i_t) = \bar{\lambda}_t(nX^* - \sum_i X^i_t) = \bar{\gamma}_t\Lambda_t$.

Recursive system. For q_1, $\bar{q}_2$, $\bar{q}_3$, the computation is straightforward. For $\bar{q}_4$, let's proceed by adding groups of terms. First, the terms in $g\eta$ give:

$$\begin{aligned}
&\eta_{t,t}\left(g_{t,t} + \sum_{\tau=t+1}^{T-1} g_{t,\tau}\right) + \sum_{\tau=t+1}^{T-1} g_{t,\tau}\eta_{t,\tau} + \sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\eta_{t,\tau}g_{t,j} \\
&\qquad + \sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\eta_{t,j}g_{t,\tau} + g_{t,t}\sum_{\tau=t+1}^{T-1}\eta_{t,\tau} \\
&= \eta_{t,t}\sum_{\tau=t}^{T-1} g_{t,\tau} + \sum_{\tau=t+1}^{T-1} g_{t,\tau}\eta_{t,\tau} + \sum_{\tau=t+1}^{T-1}\eta_{t,\tau}\left(\sum_{j=t+1, j\neq\tau}^{T-1} g_{t,j}\right) + g_{t,t}\sum_{\tau=t+1}^{T-1}\eta_{t,\tau} \\
&= \eta_{t,t}\sum_{\tau=t}^{T-1} g_{t,\tau} + \sum_{\tau=t+1}^{T-1}\eta_{t,\tau}\left(\sum_{j=t+1}^{T-1} g_{t,j}\right) + g_{t,t}\sum_{\tau=t+1}^{T-1}\eta_{t,\tau} \\
&= \eta_{t,t}\sum_{\tau=t}^{T-1} g_{t,\tau} + \sum_{\tau=t+1}^{T-1}\eta_{t,\tau}\left(\sum_{j=t}^{T-1} g_{t,j}\right) = \bar{g}_t\bar{\eta}_t \qquad (71)
\end{aligned}$$

The second line follows from adding the fourth and fifth terms together, the third line from adding the second and third terms. The terms in δ^2 give

$$\frac{1}{2}\bar{q}_{4,t+1}\left(\delta_{t,t}^2 + 2\sum_{\tau=t+1}^{T-1}\delta_{t,t}\delta_{t,\tau} + \sum_{\tau=t+1}^{T-1}\delta_{t,\tau}^2 + 2\sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\delta_{t,\tau}\delta_{t,j}\right)$$
$$= \frac{1}{2}\bar{q}_{4,t+1}\left(\sum_{\tau=t}^{T-1}\delta_{t,\tau}^2 + 2\sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\delta_{t,\tau}\delta_{t,j}\right) = \frac{1}{2}\bar{q}_{4,t+1}\left(\sum_{\tau=t}^{T-1}\delta_{t,\tau}\right)^2 \quad (72)$$

Next, denote $I_{t+1,\tau} \equiv \frac{1}{2}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau})$, and note that $\sum_{\tau=t+1}^{T-1} I_{t+1,\tau} = \bar{q}_{4,t+1}$. Then summing the terms in $I_{t+1,\tau}$ gives:

$$\sum_{\tau=t+1}^{T-1}\delta_{t,t}I_{t+1,\tau} + \sum_{\tau=t+1}^{T-1}\delta_{t,\tau}I_{t+1,\tau} + \sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\delta_{t,\tau}I_{t+1,j}$$
$$+ \sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\delta_{t,j}I_{t+1,\tau} = \sum_{\tau=t+1}^{T-1}\delta_{t,t}I_{t+1,\tau} + \sum_{\tau=t+1}^{T-1}\delta_{t,\tau}I_{t+1,\tau}$$
$$+ \sum_{\tau=t+1}^{T-1}\delta_{t,\tau}\sum_{j=t+1,j\neq\tau}^{T-1}I_{t+1,j} = \sum_{\tau=t+1}^{T-1}\delta_{t,t}I_{t+1,\tau} + \sum_{\tau=t+1}^{T-1}\delta_{t,\tau}\sum_{j=t+1}^{T-1}I_{t+1,j}$$
$$= \sum_{\tau=t+1}^{T-1}\delta_{t,\tau}\bar{q}_{4,t+1} \quad (73)$$

The remaining terms are $\sum_{\tau=t+1}^{T-1}\frac{1}{2}q_{4,t+1}^{\tau,\tau} + \sum_{\tau=t+1}^{T-2}\sum_{j=\tau+1}^{T-1}\frac{1}{2}q_{4,t+1}^{\tau,\tau} = \frac{1}{2}\bar{q}_{4,t+1}$. Adding (71), (72), and (73), we get

$$\frac{1}{2}\bar{q}_{4,t} = \bar{\eta}_t\bar{g}_t + \frac{1}{2}\bar{q}_{4,t+1}\left(1 + \left(\sum_{\tau=t}^{T-1}\delta_{t,\tau}\right)^2 - 2\sum_{\tau=t}^{T-1}\delta_{t,\tau}\right)$$
$$= \bar{\eta}_t\bar{g}_t + \frac{1}{2}\bar{q}_{4,t+1}\left(1 - \sum_{\tau=t}^{T-1}\delta_{t,\tau}\right)^2 = \bar{\eta}_t\bar{g}_t + \frac{1}{2}\bar{q}_{4,t+1}\bar{\lambda}_t^2 \quad (74)$$

Substituting for $\bar{g}_t$ yields the expression given in the proposition. Finally, $\bar{\alpha}_t = \alpha_{t,t} + \sum_{\tau=t+1}^{T-1}\alpha_{t,\tau} = (1+\bar{\alpha}_{t+1})(1-\bar{\delta}_t) = (1+\bar{\alpha}_{t+1})\bar{\gamma}_t$.

C.3.2 Signs of value function coefficients with constant supply

Lemma 3 (Coefficients Signs in the Cournot Equilibrium) *Under Cournot competition, if* $\forall t \in \{0, \ldots, T-1\}$*,* $a(1+\bar{\alpha}_t) > \bar{q}_{2,t} + \bar{q}_{4,t}$*, then for any* $n \geq 1$

$$\bar{c}_t \in]0,1[\,, \quad \bar{\eta}_t \in]0,1[\,, \quad \bar{\gamma}_t \in]0,1[$$
$$q_{1,t}, \quad \bar{q}_{2,t}, \ q_{3,t} > 0, \quad \bar{\alpha}_t > 0, \quad b(1+q_{1,t}) \geq \bar{q}_{2,t}$$

Proof. The proof is by induction. It is sufficient to show that under our assumption, the following conditions hold:

$$\mathcal{C}_1 : \begin{cases} b(1+q_{1,t}) \geq q_{2,t} \\ q_{1,t}, q_{2,t}, q_{3,t}, \bar{\alpha}_t > 0 \end{cases}$$

At $T-1$, from proposition 6, for any $a, b > 0$, and $n \geq 1$, we have $c_{T-1} = \dfrac{b}{a+b} \in]0,1[$ and $\alpha_{T-1} = \dfrac{a}{(n+1)a+b} > 0$. This implies that $\eta_{T-1} = (1-c_{T-1})\alpha_{T-1} > 0$, $q_{1,T-1} = (1-c_{T-1})^2 > 0$, $q_{2,T-1} = ac_{T-1}\alpha_{T-1} + b(1-c_{T-1})\eta_{T-1} > 0$, and $q_{3,T-1} = bc_{T-1}(2-c_{T-1}) > 0$. Further $b(1+q_{1,T-1}) \geq q_{2,T-1}$ is equivalent to $b(1-c_{T-1})^2\,(1-\alpha_{T-1}) \geq ac_{T-1}\alpha_{T-1}$. Substituting for c_{T-1} and α_{T-1} and simplifying, this condition boils down to $(n-1)a \geq 0$, which holds for any $n \geq 1$. Thus $\mathcal{C}_1$ holds at $T-1$.

Next, let's assume that $\mathcal{C}_1$ holds at $t+1$, for a given t, and show that this implies it also holds at t. First, note that if $\mathcal{C}_1$ holds at $t+1$, then from the definition of ϑ_t and $\bar{\vartheta}_t$ in Proposition **??**

$$\hat{\vartheta}_t > \vartheta_t > 0, \ c_t = \frac{b(1+q_{1,t+1}) - q_{2,t+1}}{\vartheta_t} \in]0,1[\,, \ \gamma_t = \frac{\vartheta_t(1-c_t)}{\bar{\vartheta}_t} \in]0,1[$$

Further, $c_t \in]0,1[$ and $a(1+\alpha_{t+1}) - q_{2,t+1} - q_{4,t+1} > 0$ imply that $\eta_t \in]0,1[$. Since $\gamma_t > 0$ and $\alpha_{t+1} > 0$, $\alpha_t = \gamma_t(1+\alpha_{t+1}) > 0$. And $c_t \in]0,1[$, $q_{1,t+1} > 0$ imply that $q_{1,t} > 0$. Thus we also have $q_{2,t} > 0$ (as a sum of positive terms) and $q_{3,t} > 0$ (for the same reason). It remains to show that $b(1+q_{1,t}) \geq q_{2,t}$.

Using the recursive definition of $q_{1,t}$ and $q_{2,t}$, we compute:

$$\begin{aligned} b(1+q_{1,t}) - q_{2,t} &= b - ac_t\gamma_t(1+\alpha_{t+1}) \\ &\quad + b(1-c_t)(1-c_t-\eta_t)(1+q_{1,t+1}) - \gamma_t(1-c_t)q_{2,t+1} \end{aligned} \tag{75}$$

It is sufficient to show that $-ac_t\gamma_t(1+\alpha_{t+1}) + b(1-c_t)(1-c_t-\eta_t)(1+q_{1,t+1}) - \gamma_t(1-c_t)q_{2,t+1} > 0$. Since $c_t\gamma_t = \dfrac{(1-c_t)(b(1+q_{1,t+1}) - q_{2,t+1})}{\bar{\vartheta}_t}$, we get

$$\begin{aligned} &-ac_t\gamma_t(1+\alpha_{t+1}) + b(1-c_t)(1-c_t-\eta_t)(1+q_{1,t+1}) - \gamma_t(1-c_t)q_{2,t+1} \\ &\quad = (1-c_t)\Big[b(1+q_{1,t+1})\left(1-c_t-\eta_t - \frac{a(1+\alpha_{t+1})}{\bar{\vartheta}_t}\right) \\ &\qquad\qquad - q_{2,t+1}\left(\gamma_t - \frac{a(1+\alpha_{t+1}}{\bar{\vartheta}_t}\right)\Big] \end{aligned}$$

By definition, $\gamma_t = 1 - c_t - n\eta_t$. Hence, $\eta_t > 0$ implies that $1 - c_t - \eta_t \geq \gamma_t$ for any $n \geq 1$. Thus, given that $q_{1,t+1} > 0$ and $q_{2,t+1} > 0$ (induction hypothesis),

$$\begin{aligned} b(1+q_{1,t+1})\left(1-c_t-\eta_t - \frac{a(1+\alpha_{t+1})}{\bar{\vartheta}_t}\right) - q_{2,t+1}\left(\gamma_t - \frac{a(1+\alpha_{t+1})}{\bar{\vartheta}_t}\right) \\ > \left(\gamma_t - \frac{a(1+\alpha_{t+1})}{\bar{\vartheta}_t}\right)(b(1+q_{1,t+1}) - q_{2,t+1}) \end{aligned}$$

But from the definition of c_t, γ_t and d_t, $\gamma_t - \dfrac{a(1+\alpha_{t+1})}{\bar{\vartheta}_t} = 0$. Thus $-a(1+\alpha_{t+1})c_t\gamma_t + b(1-c_t)(1-c_t-\eta_t)(1+q_{1,t+1}) - \gamma_t(1-c_t)q_{2,t+1} > 0$, and so $b(1+q_{1,t}) > q_{2,t}$.

C.3.3 Equilibrium price and holdings as a function of primitives with constant supply

To express the price and trading dynamics, it is convenient to define $c^{\pi}_{k,t}$ and $l_{k,t}$ as

$$c^{\pi}_{k,t} \equiv \prod_{\tau=k}^{t}(1-\bar{c}_{\tau}) \quad \text{and} \quad l_{k,t} \equiv \prod_{\tau=k}^{t}\bar{\gamma}_{\tau}, \tag{76}$$

with the convention that $c_{t+1,t} = l_{t+1,t} = 1$. For brevity, I write c^{π}_{t} and l_t when $k = 0$. Lemma 3 implies that $c_{k,t}$ and $l_{k,t}$ belong to the interval $(0, 1)$, and decrease as time passes.

Proposition 8 (Constant Supply Equ. as a Function of Primitives) *With constant supply (cs), the equilibrium price and quantities are*

$$\Lambda^{cs}_{t} = l_{t-1}\Lambda_0 \tag{77}$$

$$p^{cs}_{t} = p^{*,cs}_{t} - a\sigma^2\bar{\alpha}_t l_{t-1}\Lambda_0 \tag{78}$$

$$X^* - X^i_t = c^{\pi}_{t}(X^* - X^i_{-1}) - \pi^{\eta,c,l}_{t}\Lambda_0, \tag{79}$$

where $\pi^{\eta,c,l}_{t} \equiv \left[\sum_{k=0}^{t}\bar{\eta}_k c^{\pi}_{k+1,t} l_{k-1}\right]$. *Thus, the liquidity factor contracts at rate* $\bar{\gamma}_t$ *and the price converges to the competitive price at rate* $a\sigma^2 l_t\Lambda_0$.

Proof. Equation (77) follows from iterating equation (43), which recursively defines the liquidity factor in the constant supply case. Substituting the liquidity factor in equation (22) gives the equilibrium price (78). From (120), we have

$$X^* - X^i_t = -\bar{\eta}_t\Lambda_t + (1-\bar{c}_t)(X^* - X^i_{t-1})$$

Iterating backward this equation and substituting (77) for Λ_t gives equilibrium holdings (79). Given that $\bar{\alpha}_t = \bar{\lambda}_t(1+\bar{\alpha}_{t+1})$ (from equation (63)), the convergence to the competitive price is $\mathbb{E}_t(p_{t+1}-p^*_{t+1})-(p_t-p^*_t) = a\sigma^2 l_t\Lambda_0$.

C.4 T liquidity factors and T-account separation (Theorem 1)

To generalize the discussion in the text to an arbitrary sequence of shocks, it is useful to decompose time t-supply as a series of permanent shocks, $s_t = s + \sum_{l=1}^{t} \Delta s_l$. Then following the same logic as in the text, we can decompose the trade x_t^i as follows:

$$x_t^i = x_t^{i,cs} + \sum_{\tau=1}^{t} x_t^{i,cs}(\tau) + \sum_{\tau=t+1}^{T-1} x_t^{i,as}(\tau) \tag{80}$$

where $x_t^{i,cs}(\tau)$ denotes the part of the time-t trade based on the shock realized at $\tau \leq t$, and $x_t^{i,as}(\tau)$ the part based on the anticipated shock, which will occur at $\tau > t$. This partition of the trades implies the same partition for individual and aggregate holdings, with

$$X_t^i = X_t^{i,cs} + \sum_{\tau=1}^{t} X_t^{i,cs}(\tau) + \sum_{\tau=t+1}^{T-1} X_t^{i,as}(\tau) \tag{81}$$

$$\mathcal{H}_t = \mathcal{H}_t^{cs} + \sum_{\tau=1}^{t} \mathcal{H}_t^{cs}(\tau) + \sum_{\tau=t+1}^{T-1} \mathcal{H}_t^{as}(\tau) \tag{82}$$

where $X_t^{i,cs} = \sum_{l=0}^{t} x_l^{i,cs} + X_{-1}^{i,cs}$, $X_t^{i,cs}(\tau) = \sum_{l=0}^{t} x_l^{i,cs}(\tau) + X_{-1}^{i,cs}(\tau)$, and $X_t^{i,as}(\tau) = \sum_{l=0}^{t} x_l^{i,as}(\tau) + X_{-1}^{i,as}(\tau)$, and denoting aggregating holdings $\mathcal{H}_t \equiv \sum_{j=1}^{n} X_t^j$ for brevity. The initial holdings of the different accounts are a trader's endowment for the constant supply account, and zero for the others:

$$X_{-1}^{i,cs} \equiv X_{-1}^i \quad \text{and} \quad X_{-1}^{i,cs}(\tau) = X_{-1}^{i,as}(\tau) \equiv 0, \quad \tau \geq 1 \tag{83}$$

This partition of aggregate holdings leads to three types of liquidity factors Λ_t^{cs}, $\Lambda_t^{cs}(\tau)$, and $\Lambda_t^{as}(\tau)$. The first factor, Λ_t^{cs} – that I sometimes denote $\Lambda_t^{cs}(0)$ – is the same as in the constant supply case given in Proposition 8. By analogy, $\Lambda_t^{cs}(\tau)$ denotes the liquidity factor associated with the constant shock which occurred at time $\tau \leq t$, $\Lambda_t^{cs}(\tau) \equiv n\Delta X_\tau^* - \mathcal{H}_t^{cs}(\tau)$. Finally, $\mathbf{\Lambda}_t^{as}(\tau)$ is the vector of liquidity factors associated with the anticipated shock that will occur at time τ, with $\Lambda_{t,j}(\tau) \equiv -\mathcal{H}_t^{as}(\tau)$ for $t \leq j < \tau$, and $\Lambda_{t,j}^{as}(\tau) \equiv n\Delta X_\tau^* - \mathcal{H}_t^{as}$ for $\tau \leq j \leq T-1$.

To split the trades, holdings, and liquidity factors into T parts as in equations (80), (81), and (82) it is sufficient to set initial endowments as in (83), and define recursively individual holdings as follows:

$$X^* - X_t^{i,cs} = (1-\bar{c}_t)(X^* - X_{t-1}^{i,cs}) - \bar{\eta}_t \Lambda_t^{cs} \tag{84}$$

$$\begin{aligned} \Delta X_\tau^* - X_t^{i,cs}(\tau) &= (1-\bar{c}_t)(\Delta X_\tau^* - X_{t-1}^{i,cs}(\tau)) - \bar{\eta}_t \Lambda_t^{cs}(\tau), \quad \tau \leq t, \\ \text{with } \; X_{\tau-1}^{i,cs}(\tau) &= X_{\tau-1}^{i,as}(\tau) \; \text{ and } \; X_k^{i,cs}(\tau) = 0, \text{ for } k \leq \tau - 2 \end{aligned} \tag{85}$$

$$X_t^{i,as}(\tau) = \left(\sum_{k=\tau}^{T-1} c_{t,k} \right) \Delta X_\tau^* + \boldsymbol{\eta}_t^\top \mathbf{\Lambda}_t^{as}(\tau) + (1-\bar{c}_t) X_{t-1}^{i,as}(\tau), \; \tau \geq t+1 \tag{86}$$

Equation (84) is the same as in the constant supply case. Equation (85) says that the account for the shock occurred at time $\tau \leq t$ evolves as in the constant supply case as soon as the shock takes place, but with different initial conditions due to anticipated trading before the shock. Equation (86) describes the dynamics of the account due to anticipated trading on the shock that will occur at time τ, starting from a zero position. These account dynamics lead to the following separation result.

Theorem 1 (T Liquidity Factors and T-Account Separation) *If we set initial inventories as in (83), then it suffices to define individual holdings recursively as in (84)-(86) to decompose the trade of time t into $T+1$*

accounts, where, at time t,

- *Account 0 (constant supply account) is similar to the constant supply holdings of Prop. 8.*
- *Accounts 1 to t (constant shock accounts) contain a trader's positions with respect to constant shocks realized up to time t. These account remains empty until one period before the shock takes place, where the inventory due to anticipated trading is transferred to the corresponding account. From the time the shock takes place until $T-1$, the account dynamics are given by equation (85).*
- *Accounts $t+1$ to $T-1$ (anticipated shocks accounts) contain a trader's positions with respect to future shocks, due to past and current trading against anticipated shocks. These accounts start with a zero position and their dynamics are given by equation (86) up to one period before the shock takes place, where inventories are transferred to a constant shock account.*

As a result, the equilibrium price is the sum of the competitive price and T liquidity premia associated with past and future shocks:

$$p_t = p_t^* - a\sigma^2 \bar{\alpha}_t \sum_{\tau=0}^{t} \Lambda_t^{cs}(\tau) - a\sigma^2 \sum_{\tau=t+1}^{T-1} \boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t^{as}(\tau), \tag{87}$$

where p_t^* is given by (30) and the equilibrium liquidity factors are equation (77) for Λ_t^{cs}, and equations (96) and (97) for $\Lambda_t^{cs}(\tau)$ and $\boldsymbol{\Lambda}_t^{as}(\tau)$.

Proof. The proof is by induction. The induction hypothesis is that the partition (80) of the trade holds for all dates from 0 to $t-1$. Further, assume that inventories are given by (83). This implies that equations (81)-(82) for individual and aggregate inventories also hold between 0 and $t-1$. We will show that equations (80), (81), and (82) then also hold until t, and derive equilibrium holdings and liquidity premia from the recursive definitions obtained during the induction.

Aggregate holdings. The starting point is the recursive definition of aggregate trades (56) in Proposition 2, which says that $\sum_{j=1}^{n} x_t^j = \sum_{\tau=t}^{T-1} \delta_{t,\tau}\Lambda_{t,\tau}$. Using the definition of $\Lambda_{t,\tau}$ (equation (6)), adding $\sum_{j=1}^{n} X_{t-1}^j$, and using the notation $\mathcal{H}_t$ for brevity, we obtain the following recursive relationship for traders' aggregate holdings:

$$\mathcal{H}_t = n \sum_{\tau=t}^{T-1} \delta_{t,\tau} X_\tau^* + (1-\bar{\delta}_t)\mathcal{H}_{t-1} \tag{88}$$

Since $X_\tau^* = X^* + \sum_{l=1}^{\tau} \Delta X_l^*$, we can rewrite the first term of (88) as

$$n \sum_{\tau=t}^{T-1} \delta_{t,\tau} X_\tau^* = n\bar{\delta}_t \left(X^* + \sum_{\tau=1}^{t} \Delta X_\tau^* \right) + n \sum_{\tau=t+1}^{T-1} \left(\Delta X_\tau^* \left(\sum_{k=\tau}^{T-1} \delta_{t,k} \right) \right)$$

Then using the induction hypothesis, we can rewrite aggregate holdings as follows:

$$\begin{aligned} \mathcal{H}_t = n\bar{\delta}_t X^* + (1-\bar{\delta}_t)\mathcal{H}_{t-1}^{cs} + \sum_{\tau=1}^{t} \left(\Delta X_\tau^* + (1-\bar{\delta}_t)\mathcal{H}_{t-1}^{cs}(\tau) \right) \\ + \sum_{\tau=t+1}^{T-1} \left(\Delta X_\tau^* \left(\sum_{k=\tau}^{T-1} \delta_{t,k} \right) + (1-\bar{\delta}_t)\mathcal{H}_{t-1}^{as}(\tau) \right) \end{aligned} \tag{89}$$

Then it is enough to identify terms by terms and define aggregate holdings recursively as:

$$\mathcal{H}_t^{cs} = n\bar{\delta}_t X^* + (1-\bar{\delta}_t)\mathcal{H}_{t-1}^{cs}, \quad \mathcal{H}_{-1}^{cs} = \mathcal{H}_{-1} \tag{90}$$

$$\mathcal{H}_t^{cs}(\tau) = n\bar{\delta}_t \Delta X_\tau^* + (1-\bar{\delta}_t)\mathcal{H}_{t-1}^{cs}(\tau), \quad \text{with} \quad \mathcal{H}_{\tau-1}^{cs}(\tau) = \mathcal{H}_{\tau-1}^{as}(\tau) \tag{91}$$

$$\mathcal{H}_t^{as}(\tau) = n \left(\sum_{k=\tau}^{T-1} \delta_{t,k} \right) \Delta X_\tau^* + (1-\bar{\delta}_t)\mathcal{H}_{t-1}^{as}(\tau), \quad \text{with} \quad \mathcal{H}_{-1}^{as}(\tau) = 0 \tag{92}$$

Liquidity premia. We can then use this partition of aggregate holdings and the recursive definitions to split liquidity premia into T parts and obtain their expression as a function of primitives. The defintion of the liquidity premium is $\sum_{\tau=t}^{T-1} \alpha_{t,\tau}\Lambda_{t,\tau} = \sum_{\tau=t}^{T-1} \alpha_{t,\tau}(nX_\tau^* - \mathcal{H}_{t-1})$. Developing the terms in X_τ^* as before, and splitting $\mathcal{H}_{t-1}$ in T parts using the induction hypothesis, we obtain:

$$\sum_{\tau=t}^{T-1} \alpha_{t,\tau}\Lambda_{t,\tau} = \bar{\alpha}_t n \left(X^* + \sum_{\tau=1}^{t} \Delta X_\tau^* \right) + n \sum_{\tau=t+1}^{T-1} \left(\Delta X_\tau^* \left(\sum_{k=\tau}^{T-1} \alpha_{t,k} \right) \right) - \bar{\alpha}_t \left[\sum_{\tau=0}^{t-1} \mathcal{H}_{t-1}^{cs}(\tau) + \sum_{\tau=t}^{T-1} \mathcal{H}_{t-1}^{as}(\tau) \right] \quad (93)$$

where we denote $\mathcal{H}_{t-1}^{cs}(0) = \mathcal{H}_{t-1}^{cs}$. Then by grouping terms and using the definitions of the liquidity factors Λ_t^{cs}, $\Lambda_t^{cs}(\tau)$, and $\Lambda_t^{as}(\tau)$, we get

$$\sum_{\tau=t}^{T-1} \alpha_{t,\tau}\Lambda_{t,\tau} = \bar{\alpha}_t \sum_{\tau=0}^{t} \Lambda_t^{cs}(\tau) + \sum_{\tau=t+1}^{T-1} \alpha_t^\top \Lambda_t^{as}(\tau)$$

To obtain the liquidity premia as a function of primitives, we need to iterate the recursive definitions of the equilibrium aggregate holdings. Starting from (91), we get:

$$n\Delta X_\tau^* - \mathcal{H}_t^{cs}(\tau) \equiv \Lambda_{t+1}^{cs}(\tau) = (1 - \bar{\delta}_t)\Lambda_t^{cs}(\tau) = l_{\tau,t}\Lambda_\tau^{cs}(\tau) = l_{\tau,t}(n\Delta X_\tau^* - \mathcal{H}_{\tau-1}^{cs}(\tau)) \quad (94)$$

Since $\mathcal{H}_{\tau-1}^{cs}(\tau) = \mathcal{H}_{\tau-1}^{as}(\tau)$, we need to derive $\mathcal{H}_{\tau-1}^{as}(\tau)$. Starting from equation (92), iterating backward, and using the fact that anticipated shocks account start from zero inventoriy, we get:

$$\mathcal{H}_{t-1}^{as}(\tau) = n \sum_{k=\tau}^{T-1} \left(\sum_{q=0}^{t} \delta_{q,k} l_{q+1,t} \right) \Delta X_\tau \quad (95)$$

Thus, $\mathcal{H}_{\tau-1}^{as}(\tau) = n \sum_{k=\tau}^{T-1} \left(\sum_{q=0}^{\tau-1} \delta_{q,k} l_{q+1,\tau-1} \right) \Delta X_\tau$. Substituting this expression in (94), we obtain the equilibrium constant shock liquidity factor:

$$\Lambda_t^{cs}(\tau) = l_{\tau,t-1}\Lambda_\tau^{cs}(\tau) = l_{\tau,t-1}\left[1 - \sum_{k=\tau}^{T-1}\left(\sum_{q=0}^{\tau-1}\delta_{q,k} l_{q+1,\tau-1}\right)\right] n\Delta X_\tau^* \tag{96}$$

We have $\alpha_t^\top \Lambda_t^{as} = n\Delta X_\tau^* \left(\sum_{k=\tau}^{T-1}\alpha_{t,k}\right) - \bar{\alpha}_t \mathcal{H}_{t-1}^{as}$. Substituting (95) for $\mathcal{H}_{t-1}^{as}$ gives the equilibrium anticipated shock liquidity factor

$$\boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t^{as}(\tau) = \sum_{k=\tau}^{T-1}\left(\alpha_{t,k} - \bar{\alpha}_t \sum_{q=0}^{t-1}\delta_{q,k} l_{q+1,t-1}\right) n\Delta X_\tau^*, \quad \tau > t \tag{97}$$

The constant supply liquidity factor is given in Proposition 8. The competitive price is given by (30).

Individual trade/holding. To complete the induction, it remains to show that these consequences for aggregate holdings and liquidity premia lead to the same partition for individual holdings or trades as in the induction hypothesis. We start from equation (**??**), develop terms in X_τ^* as before, use the induction hypothesis to subsitute for the different components of aggregate holdings, and the different liquidity factors. This gives

$$\begin{aligned} x_t^i = \bar{c}_t(X^* - X_{t-1}^{i,cs}) + \bar{\eta}_t \Lambda_t^{cs} + \sum_{\tau=1}^{t}\left(\bar{c}_t(\Delta X - \tau^* - X_{t-1}^{i,cs}(\tau)) + \bar{\eta}_t \Lambda_t^{cs}(\tau)\right) \\ + \sum_{\tau=t+1}^{T-1}\left(\sum_{k=\tau}^{T-1} c_{t,k}\Delta X_\tau^* - \bar{c}_t X_{t-1}^{i,as}(\tau) + \eta_t^\top \boldsymbol{\Lambda}_t^{as}(\tau)\right) \end{aligned} \tag{98}$$

which we can indenfy term by term as $x_t^i = x_t^{i,cs} + \sum_{\tau=1}^{t} x_t^{i,cs}(\tau)$
$+ \sum_{\tau=t+1}^{T-1} x_t^{i,as}(\tau)$. Hence, the induction hypothesis holds at time t. Further,

adding X^i_{t-1} and grouping terms, we obtain:

$$X^* - X^{i,cs}_{t-1} + \sum_{\tau=1}^{t}(\Delta X^*_\tau - X^{i,cs}_{t-1}(\tau)) - \sum_{\tau=t+1}^{T-1} X^{i,as}_{t-1}(\tau) =$$

$$(1-\bar{c}_t)(X^* - X^{i,cs}_{t-1}) - \bar{\eta}_t \Lambda^{cs}_t$$

$$+ \sum_{\tau=1}^{t}\Big((1-\bar{c}_t)(\Delta X - \tau^* - X^{i,cs}_{t-1}(\tau)) - \bar{\eta}_t \Lambda^{cs}_t(\tau)\Big)$$

$$- \sum_{\tau=t+1}^{T-1}\left(\sum_{k=\tau}^{T-1} c_{t,k}\right)\Delta X^*_\tau + (1-\bar{c}_t)X^{i,as}_{t-1}(\tau) + \eta_t^\top \mathbf{\Lambda}^{as}_t(\tau)$$

Then, it suffices to define recursively individual holdings as in (84)-(86). Iterating these equations gives the equilibrium holdings given in the Theorem. It is simple to verify that by aggregating (84)-(86), we get back the recursive relationships for aggregate holdings (90)-(92).

C.5 Myopic trading by a Cournot monopoly

To establish the main result, I first prove an auxiliary lemma about price and value function coefficients. The result is based on three properties.

Lemma 4 *If $n = 1$:*

$$bq_{1,t} = \bar{q}_{2,t} \tag{99}$$

$$\forall j \geq t,\ b - \sum_{\tau=t}^{T-1} q^{\tau,j}_{5,t} - q^j_{3,t} = 0 \tag{100}$$

$$\forall \tau \geq t,\ \mu_{t,\tau} = a\alpha_{t,\tau} - q^\tau_{2,t} - \frac{1}{2}(\iota_{t,\tau} + \hat{\iota}_{t,\tau}) = 0 \tag{101}$$

Proof. All three properties are proved by induction.

Initial values. Property 99 holds at $T-1$, as can be verified from Proposition 6. Further, when $n = 1$, $q_{3,T-1} = \frac{b^2(2a+b)}{(a+b)^2}$. Since $q_{5,T-1} = q_{2,T-1} = \dfrac{ba^2}{(a+b)^2}$ (Proposition 6), we have $q_{3,T-1} + q_{5,T-1} = b$. So Property 100 holds at $T-1$. Further, $\mu_{T-1,T-1} = a\alpha_{T-1} - q_{2,T-1} - \frac{1}{2}(\iota_{T-1} + \hat{\iota}_{T-1})$. Since $\frac{1}{2}(\iota_{T-1} + \hat{\iota}_{T-1}) = q_{4,T-1}$ and $q_{4,T-1} = \frac{a^4}{(a+b)^2(2a+b)}$, we have $a\alpha_{T-1} - q_{2,T-1} - q_{4,T-1} = \frac{a^2}{2a+b} - \frac{a^2 b}{(a+b)^2} - \frac{a^4}{(a+b)^2(2a+b)} = 0$, so Property 101 holds at $T-1$.

Preliminary remarks. Assume now that the three properties hold at some time $t+1$. I will show that this implies that they hold at t. First, notice that if Properties 99, 100, 101 hold at t, then they imply that $\forall \tau \geq t+1$, $c_{t,\tau} = \eta_{t,\tau} = \delta_{t,\tau} = 0$ (this can be seen by using the definitions of c, η and δ given in Proposition 2). The properties further imply that

$$\bar{c}_t = c_{t,t} = \frac{b}{\vartheta_t},\ \vartheta_t = b + a(1+\bar{\alpha}_{t+1}),\ \tilde{\vartheta}_t = 1 + \frac{a}{\vartheta_t},$$
$$\bar{\delta}_t = \delta_{t,t} = \frac{a+b}{a+\vartheta_t},\ \bar{\eta}_t = \eta_{t,t} = \frac{a(1-\bar{\delta}_t)}{\vartheta_t}$$

Thus for any $\tau \geq t+1$, equation (120) becomes

$$x_t = \sum_{\tau=t}^{T-1}(c_{t,\tau} + \eta_{t,\tau})\Lambda_{t,\tau} = (\eta_{t,t} + c_{t,t})(X_t^* - X_{t-1}),$$
$$\text{with } c_{t,t} + \eta_{t,t} = \frac{b + a\bar{\gamma}_t}{b + a(1+\bar{\alpha}_{t+1})}. \qquad (102)$$

Besides, we have from Proposition 2, $\bar{\gamma}_t = 1 - \bar{\delta}_t$. Since when $n = 1$, $\bar{\gamma}_t = 1 - \bar{c}_t - \bar{\eta}_t$, we obtain $\bar{\delta}_t = \bar{c}_t + \bar{\eta}_t$. (This can also be verified by direct calculation). Further, $\delta_{t,\tau} = 0$ implies that

$$\alpha_{t,t} = 1 - \bar{\delta}_t(1+\bar{\alpha}_{t+1}),\ \alpha_{t,\tau} = \alpha_{t+1,\tau},\ \forall \tau \geq t+1$$

The latter implies that $\alpha_{t,\tau} = \alpha_{\tau,\tau}$, which in turn means that

$$\sum_{\tau=t}^{T-1}\alpha_{t,\tau}\Lambda_{t,\tau} = \alpha_{t,t}\Lambda_{t,t} + \sum_{\tau=t+1}^{T-1}\alpha_{\tau,\tau}\Lambda_{t,\tau} = \sum_{\tau=t}^{T-1}\alpha_{\tau,\tau}\Lambda_{t,\tau}.$$

Induction from $t+1$ to t. Given these preliminary remarks, we can now show that Properties 99, 100 and 101 hold at t.

- Property 99: First, from the definitions of $q_{1,t}$ and $\mathbf{q}_{2,t}$ given in proposition 2, we get

$$\bar{q}_{2,t} = bq_{1,t} \Leftrightarrow a\bar{c}_t\bar{\alpha}_t + (1-\bar{c}_t)\big[b(1+q_{1,t+1})(\bar{c}_t+\bar{\eta}_t-1) + (1-\bar{\delta}_t)\bar{q}_{2,t+1}\big] = 0$$

  Using $\bar{\gamma}_t = 1-\bar{\delta}_t = 1-\bar{c}_t-\bar{\eta}_t$, $\bar{\alpha}_t = \bar{\gamma}_t(1+\bar{\alpha}_{t+1})$, and the induction hypothesis $bq_{1,t+1} = \bar{q}_{2,t+1}$, we can simplify this expression as follows: $\bar{\gamma}_t(a\bar{c}_t(1+\bar{\alpha}_{t+1}) - b(1-\bar{c}_t)) = 0$. This equality holds true since $\bar{c}_t = \dfrac{b}{\vartheta_t}$ and $\vartheta_t = b + a(1+\bar{\alpha}_{t+1})$. Thus $\forall t \in \{0,\dots,T-1\}$, $bq_{1,t} = \bar{q}_{2,t}$.

- Property 100: We must show that $\forall j \geq t$, $q_{3,t}^{j} + \sum_{\tau=t}^{T-1} q_{5,t}^{\tau,j} = b$.

  - First case: $j = t$. Let's first compute $\sum_{\tau=t}^{T-1} q_{5,t}^{\tau,t} = q_{5,t}^{t,t} + \sum_{\tau=t+1}^{T-1} q_{5,t}^{\tau,t}$. Using the recursive definition of $q_{5,t}^{\tau,j}$ (Proposition 2) and the fact that $\forall \tau \geq t+1$ $\eta_{t,\tau} = 0$, we get

$$\begin{aligned}\sum_{\tau=t}^{T-1} q_{5,t}^{\tau,t} &= \eta_{t,t}h_{t,t} + c_{t,t}g_{t,t} + c_{t,t}\sum_{\tau=t+1}^{T-1} g_{t,\tau} \\ &= \bar{\eta}_t\left(b - \frac{b}{2}\bar{c}_t(1+q_{1,t+1})\right) + \bar{c}_t\bar{g}_t \qquad (103) \\ &= b\bar{\eta}_t + a\bar{c}_t\bar{\alpha}_t - \bar{q}_{2,t+1}\bar{c}_t(1-\bar{\delta}_t) - b\bar{c}_t\bar{\eta}_t(1+q_{1,t+1})\end{aligned}$$

    The second step follows from using the definition of $h_{t,t}$ and the last one from using the definition of $\bar{g}_t$ (both given in Proposition 2). Then we use the recursive definition of $\mathbf{q}_{3,t}$ (Proposition 2), and the facts that $\bar{\gamma}_t = 1-\bar{c}_t-\bar{\eta}_t = 1-\bar{\delta}_t$, and group terms. We get:

$$q^t_{3,t} + \sum_{\tau=t} q^{\tau,t}_{5,t} = b(\bar{c}_t + \bar{\eta}_t) + b\bar{c}_t\bar{\gamma}_t(1 + q_{1,t+1})$$
$$+ a\bar{c}_t\bar{\gamma}_t(1 + \bar{\alpha}_{t+1}) - \bar{\gamma}_t\bar{c}_t\bar{q}_{2,t+1}$$

Then using the induction hypothesis $bq_{1,t+1} = \bar{q}_{2,t+1}$, and simplifying, we obtain

$$\sum_{\tau=t}^{T-1} q^{\tau,t}_{5,t} + q^t_{3,t} = b + \bar{\gamma}_t \left[b(\bar{c}_t - 1) + a\bar{c}_t(1 + \bar{\alpha}_{t+1})\right],$$

where we used the fact that $\bar{\gamma}_t = 1 - \bar{c}_t - \bar{\eta}_t$. Given the definition of $\bar{c}_t$ and ϑ_t, the term in bracket equals 0. Thus, property 100 holds at t for $j = t$.

- Second case: $j \geq t+1$. Since $\forall \tau \geq t+1$, $c_{t,\tau} = \eta_{t,\tau} = \delta_{t,\tau} = 0$, using the definition of $q^{\tau,j}_{5,t}$ we get $\sum_{\tau=t}^{T-1} q^{\tau,j}_{5,t} = \bar{\eta}_t h_{t,j} + (1 - \bar{\delta}_t)\sum_{u=t+1}^{T-1} q^{u,j}_{5,t+1}$. So substituting for $h_{t,j}$ and adding $q^j_{3,t}$, we obtain

$$q^j_{3,t} + \sum_{\tau=t}^{T-1} q^{\tau,j}_{5,t} = b(\bar{c}_t + \bar{\eta}_t) + \bar{\gamma}_t \left(q^j_{3,t+1} + \sum_{u=t+1}^{T-1} q^{u,j}_{5,t+1}\right)$$

Thus, using $\bar{c}_t + \bar{\eta}_t = 1 - \bar{\gamma}_t$ and the induction hypothesis, we obtain $\forall j \geq t+1$, $\sum_{\tau=t}^{T-1} q^{\tau,j}_{5,t} + q^j_{3,t} = b$. Hence Property 100 holds $\forall t \in \{0, \ldots, T-1\}$ and $\forall j \geq t$.

- Property 101: We must show that $\forall \tau \geq t$, $\mu_{t,\tau} = a\alpha_{t,\tau} - q^{\tau}_{2,t} - \frac{1}{2}\left(\iota_{t,\tau} + \hat{\iota}_{t,\tau}\right) = 0$

– First case: $\tau = t$. We first compute

$$
\begin{aligned}
\frac{1}{2}\iota_{t,t} &= \frac{1}{2}\sum_{u=t}^{T-1} q_{4,t}^{u,t} = \frac{1}{2}q_{4,t}^{t,t} + \frac{1}{2}\sum_{\tau=t+1}^{T-1} q_{4,t}^{t,\tau} \\
&= \eta_{t,t}g_{t,t} + \frac{1}{2}\bar{\delta}_t^2\bar{q}_{4,t+1} + \sum_{\tau=t+1}^{T-1}\left(\eta_{t,t}g_{t,\tau} - \frac{1}{2}\bar{\delta}_t(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau})\right) \\
&= \bar{\eta}_t\bar{g}_t + \frac{1}{2}\bar{\delta}_t^2\bar{q}_{4,t+1} - \bar{\delta}_t\frac{1}{2}\sum_{\tau=t+1}^{T-1}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}), \quad (104)
\end{aligned}
$$

where we used the fact that $\delta_{t,\tau} = \eta_{t,\tau} = 0$ and $\eta_{t,t} = \bar{\eta}_t$ and $\delta_{t,t} = \bar{\delta}_t$. Note that

$$
\begin{aligned}
\sum_{\tau=t+1}^{T-1}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}) &= \sum_{\tau=t+1}^{T-1}\sum_{u=\tau}^{T-1} q_{4,t+1}^{\tau,u} + \sum_{\tau=t+1}^{T-1}\sum_{u=t+1}^{\tau} q_{4,t+1}^{u,\tau} \\
&= \bar{q}_{4,t+1} + q_{4,t+1}^{t+1,t+1} \\
+ \left(q_{4,t+1}^{t+1,t+2} + q_{4,t+1}^{t+2,t+2}\right) &+ \cdots + \left(q_{4,t+1}^{t+1,T-1} + \cdots + q_{4,t+1}^{T-1,T-1}\right) \\
&= \bar{q}_{4,t+1} + \sum_{\tau=t+1}^{T-1}\sum_{j=\tau}^{T-1} q_{4,t+1}^{\tau,j} = 2\bar{q}_{4,t+1}
\end{aligned}
$$

Thus, substituting (105) into (104), we get $\frac{1}{2}\iota_{t,t} = \bar{\eta}_t\bar{g}_t + \bar{\delta}_t\left(\frac{1}{2}\bar{\delta}_t - 1\right)\bar{q}_{4,t+1}$. Since $\frac{1}{2}\hat{\iota}_{t,t} = \frac{1}{2}q_{4,t}^{t,t} = \eta_{t,t}g_{t,t} + \frac{1}{2}\bar{\delta}_t^2\bar{q}_{4,t+1}$, we obtain $\frac{1}{2}(\iota_{t,t} + \hat{\iota}_{t,t}) = \bar{\eta}_t(g_{t,t} + \bar{g}_t) + \bar{\delta}_t\left(\bar{\delta}_t - 1\right)\bar{q}_{4,t+1}$. Then using the definition of $\mathbf{g}_t$ (Proposition 2), we get $g_{t,t} + \bar{g}_t = a\bar{\alpha}_t + a\alpha_{t,t} - b\bar{\eta}_t(1 + q_{1,t+1}) + (2\bar{\delta}_t - 1)\bar{q}_{2,t+1}$. Since $q_{2,t}^t = a\bar{c}_t\alpha_{t,t} +$

$(1-\bar{c}_t)\left[b\bar{\eta}_t(1+q_{1,t+1})-\bar{\delta}_t\bar{q}_{2,t+1}\right]$, we compute

$$\begin{aligned}\mu_{t,t} &= a\alpha_{t,t} - q^t_{2,t} - \frac{1}{2}\left(\iota_{t,t}+\hat{\iota}_{t,t}\right)\\ &= a(1-\bar{c}_t)\alpha_{t,t} - (1-\bar{c}_t)b\bar{\eta}_t(1+q_{1,t+1}) + (1-\bar{c}_t)\bar{\delta}_t\bar{q}_{2,t+1}\\ &\qquad - \bar{\eta}_t(g_{t,t}+\bar{g}_t) + \bar{\delta}_t\bar{\gamma}_t\bar{q}_{4,t+1}\\ &= a(1-\bar{c}_t-\bar{\eta}_t)\alpha_{t,t} - a\bar{\eta}_t\bar{\alpha}_t - b\bar{\eta}_t(1-\bar{c}_t-\bar{\eta}_t)(1+q_{1,t+1})\\ &\qquad + \bar{\gamma}_t(\bar{\delta}_t+\bar{\eta}_t)\bar{q}_{2,t+1} + \bar{\delta}_t\bar{\gamma}_t\bar{q}_{4,t+1}\\ &\qquad = a\bar{\gamma}_t\alpha_{t,t} - a\bar{\eta}_t\bar{\alpha}_t - b\bar{\eta}_t\bar{\gamma}_t + \bar{\delta}_t\bar{\gamma}_t\left(\bar{q}_{2,t+1}+\bar{q}_{4,t+1}\right)\end{aligned}$$

The first step follows from substituting $q^t_{2,t}$ and $\iota_{t,t}+\hat{\iota}_{t,t}$, the second step from substituting $g_{t,t}+\bar{g}_t$, the third step from using $bq_{1,t+1}=\bar{q}_{2,t+1}$.
Then using $\alpha_{t,t}=1-\bar{\delta}_t(1+\bar{\alpha}_{t+1})$ and $\bar{\alpha}_t=\bar{\gamma}_t(1+\bar{\alpha}_{t+1})$ and grouping terms, we obtain

$$\begin{aligned}\mu_{t,t} &= a\bar{\gamma}_t(1-\bar{\delta}_t) - a\bar{\delta}_t\bar{\gamma}_t\bar{\alpha}_{t+1} - a\bar{\eta}_t\bar{\gamma}_t(1+\bar{\alpha}_{t+1}) - b\bar{\eta}_t\bar{\gamma}_t\\ &\quad + \bar{\delta}_t\bar{\gamma}_t(\bar{q}_{2,t+1}+\bar{q}_{4,t+1}) = a\bar{\gamma}_t^2 - a\bar{\eta}_t\bar{\gamma}_t(1+\bar{\alpha}_{t+1})\\ &\qquad - b\bar{\eta}_t\bar{\gamma}_t + \bar{\gamma}_t\bar{\delta}_t\left(\bar{q}_{2,t+1}+\bar{q}_{4,t+1}-a\bar{\alpha}_{t+1}\right)\end{aligned} \tag{105}$$

Note that by summing from $\tau=t+1$ to $T-1$, and using (105), the induction hypothesis implies that $\sum_{\tau=t+1}^{T-1}\mu_{t+1,\tau}=a\bar{\alpha}_{t+1}-\bar{q}_{2,t+1}-\bar{q}_{4,t+1}=0$, thus the previous expression boils down to $\mu_{t,t}=\bar{\gamma}_t\left[a\bar{\gamma}_t-a\bar{\eta}_t(1+\bar{\alpha}_{t+1})-b\bar{\eta}_t\right]$. Since $\bar{\eta}_t=\dfrac{a(1-\bar{\delta}_t)}{\vartheta_t}=\dfrac{a\bar{\gamma}_t}{\vartheta_t}$ and $\vartheta_t=b+a(1+\bar{\alpha}_{t+1})$, we get: $a\bar{\gamma}_t-a\bar{\eta}_t(1+\bar{\alpha}_{t+1})-b\bar{\eta}_t=a\bar{\gamma}_t-\bar{\eta}_t\vartheta_t=a\bar{\gamma}_t\left(1-\dfrac{\vartheta_t}{\vartheta_t}\right)=0$. Thus $\mu_{t,t}=0$.

– Second case: $\tau\geq t+1$. We compute $\mu_{t,\tau}=a\alpha_{t,\tau}-q^\tau_{2,t}-$

$\frac{1}{2}\left(\iota_{t,\tau}+\hat{\iota}_{t,\tau}\right)$. First, let's compute the terms in ι

$$\frac{1}{2}\iota_{t,\tau}=\frac{1}{2}\sum_{j=\tau}^{T-1}q_{4,t}^{\tau,j}=\frac{1}{2}q_{4,t}^{\tau,\tau}+\frac{1}{2}\sum_{j=\tau+1}^{T-1}q_{4,t}^{\tau,j}=\frac{1}{2}\sum_{j=\tau}^{T-1}q_{4,t+1}^{\tau,j}$$

The last equality follows from substituting $q_{4,t}^{\tau,\tau}$ and $q_{4,t}^{\tau,j}$ and using $\delta_{t,\tau}=\eta_{t,\tau}=0$ for $\tau\geq t+1$. We now compute

$$\begin{aligned}\frac{1}{2}\hat{\iota}_{t,\tau}&=\frac{1}{2}q_{4,t}^{t,\tau}+\sum_{j=t+1}^{\tau-1}\frac{1}{2}q_{4,t}^{j,\tau}+\frac{1}{2}q_{4,t}^{\tau,\tau}\\&=\bar{\eta}_t g_{t,\tau}-\frac{1}{2}\bar{\delta}_t\left(\iota_{t+1,\tau}+\hat{\iota}_{t+1,\tau}\right)+\sum_{j=t+1}^{\tau-1}\frac{1}{2}q_{4,t+1}^{j,\tau}+\frac{1}{2}q_{4,t+1}^{\tau,\tau}\\&=\bar{\eta}_t g_{t,\tau}-\frac{1}{2}\bar{\delta}_t\left(\iota_{t+1,\tau}+\hat{\iota}_{t+1,\tau}\right)+\sum_{j=t+1}^{\tau}\frac{1}{2}q_{4,t+1}^{j,\tau}\end{aligned}$$

Thus, we obtain, for $\tau\geq t+1$,

$$\begin{aligned}&\frac{1}{2}\left(\iota_{t,\tau}+\hat{\iota}_{t,\tau}\right)=\bar{\eta}_t g_{t,\tau}-\frac{1}{2}\bar{\delta}_t\left(\iota_{t+1,\tau}+\hat{\iota}_{t+1,\tau}\right)\\&+\frac{1}{2}\left(\sum_{j=\tau}^{T-1}q_{4,t+1}^{\tau,j}+\sum_{j=t+1}^{\tau}q_{4,t+1}^{j,\tau}\right)=(1-\bar{\delta}_t)\frac{1}{2}\left(\iota_{t+1,\tau}+\hat{\iota}_{t+1,\tau}\right)+\bar{\eta}_t g_{t,\tau}\end{aligned}\tag{106}$$

Substituting for $g_{t,\tau}$, we get

$$\frac{1}{2}\left(\iota_{t,\tau}+\hat{\iota}_{t,\tau}\right)=\frac{1}{2}(1-\bar{\delta}_t)\left(\iota_{t+1,\tau}+\hat{\iota}_{t+1,\tau}\right)+\bar{\eta}_t\left(a\alpha_{t,\tau}-q_{2,t+1}^{\tau}\right)$$

Then using the recursive definition $q_{2,t}^{\tau}=a\bar{c}_t\alpha_{t,\tau}-(1-\bar{c}_t)q_{2,t+1}^{\tau}$ and substituting the expressions of $\frac{1}{2}\left(\iota_{t,\tau}+\hat{\iota}_{t,\tau}\right)$, we obtain, after simplifying:

$$\mu_{t,\tau} = (\bar{c}_t + \bar{\eta}_t - 1)\, q^{\tau}_{2,t+1} - (1 - \bar{\delta}_t)\frac{1}{2}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau})$$
$$+ a\alpha_{t,\tau}(1 - \bar{c}_t - \bar{\eta}_t) = \bar{\gamma}_t \left[a\alpha_{t,\tau} - q^{\tau}_{2,t+1} - \frac{1}{2}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau})\right]$$

Since for $n = 1$, $\alpha_{t,\tau} = \alpha_{t+1,\tau}$, the induction hypothesis implies that $\forall \tau \geq t+1$, $\mu_{t,\tau} = 0$. This completes the proof.

Proposition 9 ("Myopic" Trading by a Cournot Monopoly) *For a Cournot single trader:*

1. *It is optimal to trade "myopically", ignoring anticipated shocks. In equilibrium, the anticipated shock accounts remain zero, and the constant shock accounts start from a zero inventory, i.e.* $X^{cs}_{\tau-1}(\tau) = 0$ *for the shock occuring at* τ*.*

2. *As a result, the effects of anticipated shocks on the liquidity premium are constant until their realization*

$$\sum_{\tau=t}^{T-1} \alpha_{t,\tau}\Lambda_{t,\tau} = \bar{\alpha}_t \left[\Lambda^{cs}_t + \sum_{\tau=1}^{t} \Lambda^{cs}_t(\tau)\right] + \sum_{\tau=t+1}^{T-1} \bar{\alpha}_\tau \Delta X^*_\tau \qquad (107)$$

with $\Lambda^{cs}_t(\tau) = l_{\tau,t-1}\Delta X^*_\tau$*.*

Proof. We can now use Lemma 4 to prove the main result. Proceeding by induction as in the proof of Theorem 1, it is easy to show that $x^{as}_t(\tau) = 0$ for any $\tau \geq t+1$.

Constant liquidity premium. Next, we derive the liquidity factor and the liquidity premium. Starting from (102), we have

$$x_t = (c_{t,t} + \eta_{t,t}) \left[X^* + \sum_{\tau=1}^{t} \Delta X^*_\tau - X^i_{t-1}\right]$$

From this equation, we obtain $\Lambda^{cs}_{t+1} + \sum_{\tau=1}^{t} \Lambda^{cs}_{t+1}(\tau) = \lambda_{t,t}\left(\Lambda^{cs}_t + \sum_{\tau=1}^{t} \Lambda^{cs}_t(\tau)\right)$. So, proceeding as in the proof of Theorem 1, it is sufficient to define recursively

$$\Lambda^{cs}_{t+1} = \gamma_{t,t}\Lambda^{cs}_t, \quad \Lambda^{cs}_{t+1}(\tau) = \gamma_{t,t}\Lambda^{cs}_t(\tau), \quad 1 \le \tau \le t$$

In equilibrium, $\Lambda^{cs}_t(\tau) = l_{\tau,t-1}\Lambda^{cs}_\tau(\tau)$. But since there is no trading on anticipated shocks $\mathcal{H}^{cs}_{\tau-1}(\tau) = \mathcal{H}^{as}_{\tau-1}(\tau) = 0$. Thus $\Lambda^{cs}_t(\tau) = l_{\tau,t-1}\Delta X^*_\tau$. The liquidity premium is, by definition, $\sum_{\tau=t}^{T-1} \alpha_{t,\tau}\Lambda_{t,\tau} = \bar{\alpha}_t\left(X^* + \sum_{\tau=1}^{t} \Delta X^*_\tau\right) + \sum_{\tau=t+1}^{T-1}\left(\sum_{k=\tau}^{T-1} \alpha_{t,k}\right)\Delta X^*_\tau - \bar{\alpha}_t\mathcal{H}_{t-1}$. From Lemma 4, for any $\tau \ge t+1$, $\alpha_{t,\tau} = \alpha_{t+1,\tau}$, so $\alpha_{t,\tau} = \alpha_{\tau,\tau}$. Thus, $\sum_{k=\tau}^{T-1} \alpha_{t,k} = \sum_{k=\tau}^{T-1} \alpha_{k,k} = \bar{\alpha}_\tau$. Therefore, splitting $\mathcal{H}_t$ in two pieces, substituting and rearranging the terms, we obtain (107).

First-order condition when $n = 1$. Starting from the first-order condition(54), and using $n = 1$, we get

$$a\sigma^2(1 + \bar{\alpha}_{t+1})x_t + b\sigma^2\left[1 + q_{1,t+1} - \bar{q}_{2,t+1}\right]X_t + \sum_{\tau=t+1}^{T-1}(Q^{t+1,\tau}_{2,4} + Q^{t+1,\tau}_{3,5})X^*_\tau - \bar{Q}^{t+1}_{2,4}X_t = D_t - p_t \tag{108}$$

Applying Properties 99 and 101, we simplify the expression further to obtain

$$a\sigma^2 x_t + b\sigma^2 X_t + \sum_{\tau=t+1}^{T-1}(Q^{t+1,\tau}_{2,4} + Q^{t+1,\tau}_{3,5})X^*_\tau - \bar{Q}^{t+1}_{2,4}X_{t-1} = D_t - p_t$$

Subsituting for $Q^{t+1,\tau}_{2,4} + Q^{t+1,\tau}_{3,5}$ using Properties 100 and 101, and rearranging the terms, we get

$$a\sigma^2 x_t + b\sigma^2 X_t = D_t - p_t - b\sigma^2\sum_{\tau=t+1}^{T-1} X^*_\tau - a\sigma^2\sum_{\tau=t+1}^{T-1}\alpha_{t+1,\tau}\Lambda_{t+1,\tau}$$

Then, we simply have to recognize that the right-hand side is equal to $\mathbb{E}_t(p_{t+1}) - p_t$.

C.6 Information release under Cournot

In this section, I prove the rest of the results given in Proposition 3 and Claim 1.

C.6.1 Momentum and reversal with a single trader

Proof. The absence of price and liquidity premium momentum is obvious given definition 1 and Proposition 3. After the shock, $\Lambda_t^{cs}(0)$ and $\Lambda_t^{cs}(t_2)$ follow the same dynamics if Λ_0 and Δs_{t_2} have the same sign.

C.6.2 Conditions for momentum and reversal with multiple traders

The full result is:

Corollary 2 (Momentum and Reversal under Cournot Oligopoly) *Let* $\mathcal{S}_{t_1,t_2}^{t,T}(\delta, l) \equiv \sum_{k=t_2}^{T-1} \sum_{q=t_1}^{t} \delta_{q,k} l_{q+1,t}$. *There is momentum and reversal (for* $\Delta X_{t_2}^* \geq 0$*) iff*

$$C_{mr} : \begin{cases} \forall t \in \{t_m, \dots, t_2\}, \quad bX^* + al_t\Lambda_0 < a\mathcal{S}_{t_1,t_2}^{t_2,T}(\delta, l)n\Delta X_{t_2}^* \\ \text{for } t > t_2, \quad b(X^* + \Delta X_{t_2}^*) + al_t\Lambda_0 \\ +al_{t_2,t}\left[1 - S_{t_1,t_2}^{t_2,T}(\delta, l)\right] n\Delta X_{t_2}^* > 0 \end{cases}$$

If $\Delta X_{t_2}^* < 0$*, the conditions have the opposite sign. For momentum and reversal in the liquidity premium only, the conditions are the same, but withouth the first term.*

Proof. Momentum conditions. Let's first determine the change in the competitive price and liquidity premium before t_2. For $t_1 \leq t \leq t_2$,

$$\mathbb{E}_t(p^*_{t+1} - p^*_t) = -\frac{ab\sigma^2(T-t-1)}{na+b}s + \frac{ab\sigma^2(T-t)}{na+b}s = b\sigma^2 X^* \tag{109}$$

$$\bar{\alpha}_t \Lambda^{cs}_t(0) - \bar{\alpha}_{t+1}\Lambda^{cs}_{t+1}(0) = \bar{\alpha}_t l_{t-1}\Lambda_0 - \bar{\alpha}_{t+1} l_t \Lambda_0 = \bar{\lambda}_t l_{t-1}\Lambda_0 = l_t \Lambda_0 \tag{110}$$

In the second line, the first equality follows from the definition of the liquidity premium, the second from the recursive definition of $\bar{\alpha}_t$, and the third from the definition of l_t in equation (76).

Let's now determine the change in the anticipated shock liquidity premium. For $t \leq t_2$,

$$\alpha_t^\top \Lambda^{as}_t(t_2) - \alpha_{t+1}^\top \Lambda^{as}_{t+1}(t_2) = \sum_{k=t_2}^{T-1}\Bigg(\alpha_{t,k} - \alpha_{t+1,k} + \bar{\alpha}_{t+1}\sum_{q=t_1}^{t}\delta_{q,k} l_{q+1,t} \\ - \bar{\alpha}_t \sum_{q=t_1}^{t-1}\delta_{q,k} l_{q+1,t-1}\sum_{k=t_2}^{T-1}\Bigg) n\Delta X^*_{t_2}$$

Using (51) and rearranging terms, the terms in parenthesis become

$$-(1+\bar{\alpha}_{t+1})\delta_{t,k} + \sum_{q=t_1}^{t-1}(\bar{\alpha}_{t+1}\delta_{q,k} l_{q+1,t} - \bar{\alpha}_t \delta_{q,k} l_{q+1,t-1}) + \bar{\alpha}_{t+1}\delta_{t,k} l_{t+1,t} \\ = -\delta_{t,k} - \bar{\gamma}_t \sum_{q=t_1}^{t-1}\delta_{q,k} l_{q+1,t-1}$$

The equality follows from the convention that $l_{t+1,t} = 1$, and the recursive definition of $\bar{\alpha}_t$. Then, since by definition $\bar{\lambda}_t l_{q+1,t-1} = l_{q+1,t}$, and using again the convention that $l_{t+1,t} = 1$, the change in anticipated shock liquidity premium boils down to

$$\alpha_t^\top \Lambda^{as}_t(t_2) - \alpha_{t+1}^\top \Lambda^{as}_{t+1}(t_2) = -\mathcal{S}^{t,T}_{t_1,t_2}(\delta, l) n\Delta X^*_{t_2} \tag{111}$$

For $t_1 \leq t \leq t_2$, the price change is $\mathbb{E}_t(p_{t+1} - p_t) = \mathbb{E}_t(p^*_{t+1} - p^*_t) + a\sigma^2 \left(\bar{\alpha}_t \Lambda^{cs}_t(0) - \bar{\alpha}_{t+1}\Lambda^{cs}_{t+1}(0)\right) + a\sigma^2 \left(\alpha_t^\top \Lambda^{as}_t(t_2) - \alpha_{t+1}^\top \Lambda^{as}_{t+1}(t_2)\right)$. Substituting (109), (110), and (111) gives the first condition in C_{mr}.

Reversal conditions. The price change over two consecutive periods after the realization is again the sum of three terms: the change in p^*, the change in $\Lambda^{cs}(0)$, and the change in $\Lambda^{cs}(t_2)$. For $t > t_2$, the change in the competitive price is $\mathbb{E}_t(p^*_{t+1} - p^*_t) = b\sigma^2(X^* + \Delta X^*_{t_2})$. The change in $\Lambda^{cs}(0)$ is still given by (110). The change in $\Lambda^{cs}(t_2)$ can be computed using (**??**)

$$\begin{aligned}\bar{\alpha}_t \Lambda^{cs}_t(t_2) - \bar{\alpha}_{t+1}\Lambda^{cs}_{t+1}(t_2) &= (\bar{\alpha}_t l_{t_2,t-1} - \bar{\alpha}_{t+1} l_{t_2,t}) \left[1 - \mathcal{S}^{t_2-1,T}_{t_1,t_2}(\delta, l)\right] n\Delta X^*_{t_2} \\ &= l_{t_2,t}\left[1 - \mathcal{S}^{t_2-1,T}_{t_1,t_2}(\delta, l)\right] n\Delta X^*_{t_2}\end{aligned}$$

Adding these three changes gives the second condition in C_{mr}.

C.6.3 Competition effect under risk neutrality

Corollary 3 *Suppose that there are only two trading rounds ($t = 0, 1$, so $t_1 = 0$, $t_2 = 1$) and that traders are risk-neutral ($b = 0$). Then $x_0^{i,cs} = \bar{\eta}_0 \Lambda^{cs}_0$ and $x_0^{i,as}(1) = n\eta_{0,1}\Delta X^*_1$. Holding $\mathcal{H}_{-1}$ constant, $x_0^{i,cs}$ always decreases with competition. However, the anticipated shock trade first increases and then decreases with competition, with a maximum for $n = 2$. Further, $x_0^{as}(1) = 0$ for $n = 1$ and $\lim_{n\to\infty} x_0^{i,as}(1) = 0$.*

Proof. With $b = 0$, the vectors $\mathbf{c}$, $\mathbf{h}$, $\mathbf{q}_2$, $\mathbf{q}_3$ and $\mathbf{q}_5$ are equal to the vector of zeros. Further, the vectors $\boldsymbol{\delta}$, $\boldsymbol{\eta}$, $\mathbf{q}_4$ and auxiliary parameters ϑ, $\tilde{\vartheta}$, $\boldsymbol{\mu}$ and $\mathbf{g}$ simplify as follows:

$$\vartheta = a(1 + \bar{\alpha}_{t+1}), \quad \tilde{\vartheta}_t = \frac{1 + n(a + \bar{\mu}_{t+1})}{\vartheta_t},$$

$$\mu_{t+1,\tau} = (n+1)a(1 + \bar{\alpha}_{t+1}) - \frac{1}{2}(\iota_{t+1,\tau} + \hat{\iota}_{t+1,\tau}) \quad (112)$$

This implies that $\vartheta_t\tilde{\vartheta}_t = (n+1)a(1+\bar{\alpha}_{t+1}) - n\bar{q}_{4,t+1}$. Then we have:

$$\delta_{t,t} = \frac{na}{\vartheta_t\tilde{\vartheta}_t}, \quad \delta_{t,\tau} = \frac{n\mu_{t+1,\tau}}{\vartheta_t\tilde{\vartheta}_t}, \quad \eta_{t,t} = \frac{a-(a+\bar{\mu}_{t+1})\delta_{t,t}}{a(1+\bar{\alpha}_{t+1})},$$
$$\eta_{t,\tau} = \frac{\mu_{t+1,\tau}-(a+\bar{\mu}_{t+1})\delta_{t,\tau}}{a(1+\bar{\alpha}_{t+1})} \qquad (113)$$

The initial conditions are $\eta_1 = \alpha_1 = \frac{1}{n+1}$, $\frac{1}{2}q_{4,1} = a\eta_1^2 = \frac{a}{(n+1)^2}$. Then substituting initial conditions into the parameter defintions and using notation $\phi_n = n^3+4n^2+2n+1$, we get:

$$\delta_{0,0} = \frac{n(n+1)^2}{\phi_n}, \quad \delta_{0,1} = \frac{n(n-1)}{\phi_n}, \quad \alpha_{0,0} = \frac{n^2+n+2}{\phi_n}, \ \alpha_{0,1} = \frac{3n+2}{\phi_n},$$
$$\eta_{0,0} = \frac{(n+1)^2}{\phi_n}, \ \eta_{0,1} = \frac{n-1}{\phi_n} \qquad (114)$$

From Theorem 1, $x_0^i = x_0^{i,cs} + x_0^{i,as}$, with $x_0^{i,cs} = \bar{\eta}_0\Lambda_0^{cs}$ and $x_0^{i,as} = n\eta_{0,1}\Delta X_1^*$. For $n=1$, $\eta_{0,1}=0$. Further, $n\eta_{0,1}$ increases from $n=1$ to $n=2$ and decreases afterwards, and $\lim_{n\to\infty} n\eta_{0,1} = 0$. When $b=0$, $\Lambda_0^{cs} = s - \mathcal{H}_{-1}$. Thus, holding $\mathcal{H}_{-1}$ constant, $\frac{\partial |x_0^{i,cs}|}{\partial n} = \frac{\partial \bar{\eta}_0}{\partial n} \leq 0$, for all $n \geq 1$.

Online Appendix

D Demand schedule competition

Definition 4 (Linear schedules) *With demand schedule competition, we consider schedules of the form*

$$y_t^m(p_t) = \beta_t^y(D_t - p_t) - c_t^y Y_{t-1}^m + d_t^y \sum_j X_{t-1}^j + \sum_{\tau \geq t+1} f_{t,\tau}^y X_\tau^* \tag{115}$$

$$x_t^i(p_t) = \beta_t(D_t - p_t) - c_t X_{t-1}^i + d_t \sum_j X_{t-1}^j + \sum_{\tau \geq t+1} f_{t,\tau} X_\tau^*, \textit{ with } \beta_t > 0 \tag{116}$$

As usual, the equilibrium is not assumed to be linear; rather, as others submit linear schedules, it is optimal for an investor to submit a linear schedule as well. Note that unlike the Cournot case, $y_t^m(p_t)$ and $x_t^i(p_t)$ do not depend on X_t^*. This is without loss of generality: if schedules were allowed to depend on the current supply, the coefficient would be zero in equilibrium. This implies that Similarly, allowing for different coefficient for the impact of D_t and $-p_t$ would yield that these coefficients are equal in equilibrium.

Definition 5 (Demand Schedule Equilibrium) *A Nash equilibrium in demand schedule competition is such that (i) every price-taker's demand schedule $y_t^m(p_t)$ is optimal given prices; (ii) every strategic trader's schedule $x^i(p_t)$ is optimal given price-takers' and other strategic traders' schedules, (iii) markets clear.*

Following Rostek and Weretka (2015) and Rostek and Yoon (2020), we can equivalently derive the equilibrium as one in which strategic traders optimize given their assumed price impact, provided price impacts are consistent. Additionally here, price-takers's schedules must be optimal given prices. Because price-takers are competitive, they do not take into account their price impact, and the consistency condition thus concerns only strategic traders.

Lemma 5 (Rostek and Yoon, 2020) *A collection of demand schedules $y_t^m(\cdot), x_t^i(\cdot)$ is a Nash equilibrium if and only if (i) each price-taker's schedule satisfies pointwise the price-taker's first-order condition; (ii) each strategic trader's schedule satisfies pointwise his best-response function, given his assumed price impact λ^i; (iii) strategic trader's price impacts are consistent, i.e. each strategic trader's price impact is equal to the slope of his inverse supply function,*

$$\lambda_t^i = \lambda_t \equiv \frac{\partial p_t}{\partial x_t^i} = -\frac{1}{\frac{\partial \int_0^1 y_t^m dm}{\partial p_t} + \sum_{j \neq i} \frac{\partial x^j}{\partial p_t}}, \quad \forall i \tag{117}$$

All strategic traders have the same price impact due to identical preferences and information set.

Guesses for the price and value function

$$p_t = p_t^* - a\sigma^2 \boldsymbol{\alpha}_t^\top \boldsymbol{\Lambda}_t, \tag{118}$$

$$\sigma^{-2}\Omega_t^i = -\frac{b}{2} q_{1,t}(X_{t-1}^i)^2 - X_{t-1}^i \left(\mathbf{q}_{2,t}^\top \boldsymbol{\Lambda}_t + \mathbf{q}_{3,t}^\top \mathbf{X}^* \right) + \frac{1}{2} \sum_{\tau=t}^{T-1} \sum_{j=\tau}^{T-1} q_{4,t}^{\tau,j} \Lambda_{t,\tau} \Lambda_{t,j}$$
$$+ \sum_{\tau=t}^{T-1} \sum_{j=t}^{T-1} q_{5,t}^{\tau,j} \Lambda_{t,\tau} X_j^* - \sum_{\tau=t}^{T-1} \sum_{j=\tau}^{T-1} q_{6,t}^{\tau,j} X_\tau^* X_j^* \tag{119}$$

Under these guesses, it is possible to write trades as

$$x_t^i = \sum_{\tau=t}^{T-1} c_{t,\tau}(X_\tau^* - X_t^i) + \boldsymbol{\eta}_t^\top \boldsymbol{\Lambda}_t \tag{120}$$

D.1 Static model

Proposition 10 (Static Demand Schedule Equilibrium) *Suppose $T = 1$. Then,*

1. *In equilibrium, the schedules are*

$$y^m(p) = \frac{D-p}{a\sigma^2} - Y^m \tag{121}$$

$$x^i(p) = \frac{1}{b\sigma^2+\lambda}(D-p-b\sigma^2 X^i), \tag{122}$$

i.e. $\beta^y = \frac{1}{a\sigma^2}$, $c^y = 1$, $d^y = f^y = d = f = 0$, $\beta = \frac{1}{b\sigma^2+\lambda}$, *and* $c = b\sigma^2\beta$.

2. *A trader's price impact is*

$$\lambda = \begin{cases} a\sigma^2 & \text{if } n = 1 \\ \frac{\sqrt{\varphi}-(b+(n-2)a)}{2}\sigma^2 & \text{if } n \geq 2 \end{cases}$$

where $\varphi = b^2 + (n-2)^2 a^2 + 2nab$.

3. *Similar to the Cournot case, by defining* η *and* α *as follows, we can decompose equilibrium trades as* (120)*, and write prices and value functions as* (118)*-*(119)*. The definition of value function coefficients is the same as in the Cournot case.*

$$\eta_{T-1} = \frac{ac_{T-1}(1-c_{T-1})}{b+nac_{T-1}} \tag{123}$$

$$\alpha_{T-1} = \frac{b(1-c_{T-1})}{b+nac_{T-1}} \tag{124}$$

Proof. With linear schedules, the price impact function will depend linearly on $x^i + \sum_{j\neq i} x^j$. Given the residual supply $\mathcal{S} = s_1 - \sum_{j\neq i} x^j(p)$, the maximisation problems are

$$\max_{x^i}(X^i + x^i)(D - p(x^i)) - \frac{b\sigma^2}{2}(X^i + x^i)^2 \tag{125}$$

$$\max_{y^m}(Y^m + y^m)(D - p) - \frac{a\sigma^2}{2}(Y^m + y^m)^2 \tag{126}$$

The first-order conditions give

$$D - p - a\sigma^2 y^m - a\sigma^2 - Y^m = 0 \tag{127}$$

$$D - p - b\sigma^2 X^i - (b\sigma^2 + \frac{\partial p}{\partial x^i})x^i = 0 \tag{128}$$

So we obtain the following relationships for every p

$$y^m(p) = \frac{1}{a\sigma^2}(D - p) - Y^m$$
$$x^i(p) = \frac{1}{b\sigma^2 + \lambda}(D - p - b\sigma^2 X^i)$$

By pointwise identification, we obtain the demand schedule coefficients.

Using these relationships, we get for every $j \neq i$, $\frac{\partial p}{\partial x^j} = -\frac{1}{b\sigma^2+\lambda}$, and for every m, $\frac{\partial y_t^m}{\partial p_t} = -\frac{1}{a\sigma^2}$. Substituting into the consistency condition (117), we get the equation defining price impact:

$$\lambda = \frac{1}{\frac{1}{a\sigma^2} + \frac{n-1}{b\sigma^2+\lambda}}$$

There are two cases. If $n = 1$, then $\lambda = a\sigma^2$. If $n > 1$, then λ is defined as

$$\lambda^2 + \sigma^2(b + (n-2)a)\lambda - ab\sigma^4 = 0$$

There are always two roots, one positive and one negative, but λ is required to be positive, so in equilibrium

$$\lambda = \frac{\sqrt{\varphi} - (b + (n-2)a)}{2}\sigma^2$$

where φ is given in the result.

We can write the equilibrium price and trade as in the Cournot case, with $\eta_{T-1}^{\mathcal{D}}$ and $\alpha_{T-1}^{\mathcal{D}}$ defined as in the proposition. The expression of the value function follows, as in the Cournot case.

D.2 Equivalence with Cournot static model

Corollary 4 *If there is a single strategic trader and a single trading round, the equilibria under Cournot and demand schedule competitions coincide. If there are multiple traders, then all else equal,*

- *at the aggregate level, traders trade a larger quantity in absolute value under demand schedule competition;*
- *at the individual level, the risk-sharing component of the trade increases, while the speculative component decreases relative to Cournot competition;*
- *the market is deeper under demand schedule competition.*

Proof. Single trader. Since the $q_{i,t}$'s are defined similarly under either type of competition (Proposition 10), it is sufficient to show that c, α, and η coincide when $n = 1$ in the static model. Recall that under Cournot competition, if $n = 1$,

$$c_{T-1} = \frac{b}{a+b}, \ \alpha_{T-1} = \frac{a}{2a+b}, \ \eta_{T-1} = \frac{a^2}{(a+b)(2a+b)}$$

Under demand schedule competition, using Proposition 10, when $n = 1$, we get

$$c^{\mathcal{D}}_{T-1} = \frac{b}{a+b} = c_{T-1}, \quad \alpha^{\mathcal{D}}_{T-1} = \frac{b\dfrac{a}{a+b}}{b + na\dfrac{b}{a+b}} = \frac{a}{2a+b} = \alpha_{T-1},$$
$$\eta^{\mathcal{D}}_{T-1} = \frac{ab}{a+b}\frac{1}{b}\alpha^{\mathcal{D}}_{T-1} = \frac{a^2}{(a+b)(2a+b)} = \eta_{T-1}$$

Mulitple traders. For demand schedule competition, if $n \geq 2$, we compute using Proposition 10

$$c^{\mathcal{D}}_{T-1} = \frac{(n-2)a - b + \varphi^{\frac{1}{2}}}{2(n-1)a}, \quad \alpha^{\mathcal{D}}_{T-1} = \frac{b(na+b-\varphi^{\frac{1}{2}})}{a\left[(n-2)(b+na) + n\varphi^{\frac{1}{2}}\right]},$$
$$\eta^{\mathcal{D}}_{T-1} = \frac{a}{b} c^{\mathcal{D}}_{T-1}\alpha^{\mathcal{D}}_{T-1}$$

Then we compute the following inequalities:

Risk-sharing motive: $c^{\mathcal{D}}_{T-1} > c_{T-1} \Leftrightarrow \frac{b((n+1)a+b)-(n-2)a^2}{a+b}$. If a/b is large enough the RHS is negative and $c^{\mathcal{D}}_{T-1} > c_{T-1}$. Otherwise, we can raise both sides to the square. After rearranging and simplifying terms, the condition becomes $a^2 b(nb + (n^2 - 2n + 1)a) > 0$, which holds true for any $a, b > 0$ and $n \geq 2$.

Liquidity premium: $\alpha^{\mathcal{D}}_{T-1} < \alpha_{T-1} \Leftrightarrow \frac{a}{(n+1)a+b} > \frac{b(na+b-\varphi^{\frac{1}{2}})}{a\left[(n-2)(b+na)+n\varphi^{\frac{1}{2}}\right]}$, which is equivalent to

$$(na+b)\left[b((n+1)a+b) - (n-2)a^2\right] < \left[b((n+1)a+b) + na^2\right]\varphi^{\frac{1}{2}}$$

If a/b is large enough, the LHS is negative and the condition holds true. Otherwise, raise both sides to the square, and rearrange terms to get

$$a^2(b+2na) + a((n+1)a+b)\left[n(n-2)a + (n-1)b\right] > 0,$$

which holds true for any $a, b > 0$ and $n \geq 2$.

Speculative motive: first, note that $\frac{a}{b} c^{\mathcal{D}}_{T-1} < \frac{a}{a+b}$ is equivalent to $c^{\mathcal{D}}_{T-1} < \frac{b}{a+b} = c_{T-1}$, which was just showed. Further, since $\alpha^{\mathcal{D}}_{T-1} < \alpha_{T-1}$, we can write for any $a, b > 0$ that $\frac{a}{b} c^{\mathcal{D}}_{T-1}\alpha^{\mathcal{D}}_{T-1} < \frac{a}{a+b}\alpha_{T-1}$, which, by definition of the variables, means $\eta^{\mathcal{D}}_{T-1} < \eta_{T-1}$.

Aggregate trade: $\delta_{T-1} = c_{T-1} + n\eta_{T-1} = \frac{1}{a+b}\frac{na^2+b((n+1)a+b)}{(n+1)a+b}$, and $\delta^{\mathcal{D}}_{T-1} =$

$\frac{(na+b)\left[(n-2)a-b+\varphi^{\frac{1}{2}}\right]}{a\left[(n-2)(b+na)+n\varphi^{\frac{1}{2}}\right]}$. The inequality $\delta^{\mathcal{D}}_{T-1} < \delta_{T-1}$ is equivalent to

$$\left[((n+1)a+b)\left[b^2+a(na+b)\right]-n^2a^3\right]\varphi^{\frac{1}{2}} > (na+b)(a+b) \times \left[b((n+1)a+b)-(n-2)a^2\right]$$

This condition can be further simplified to

$$\left[b((n+1)a+b)+na^2\right]\varphi^{\frac{1}{2}} > (na+b)\left[b((n+1)a+b)-(n-2)a^2\right]$$

If a/b is large enough, the condition is satisfied. Otherwise, raising both sides to the square, we obtain

$$a^2\left[(n-1)b+(n^2+n-2)a\right]+((n+1)a+b)(n-1)ab > 0$$

which is equivalent to $(n-1)(n+2)ab+(n^2+n-2)a^2+(n-1)b^2 > 0$. This condition holds true for any $a, b > 0$ and $n \geq 2$.

Market depth: under Cournot, price impact (the inverse of depth) in the static model is $a\sigma^2$. Under demand schedule competition, price impact is $\chi^{-1}_{T-1} = \frac{1}{(n-1)\beta_{T-1}+\beta^y_{T-1}}$ (off-equilibrium), and $\hat{\chi}^{-1}_{T-1} = \frac{1}{n\beta_{T-1}+\beta^y_{T-1}}$, on the equilibrium path. Since $\beta^y_{T-1} = \frac{1}{a\sigma^2}$, in either case, price impact is lower and thus market depth larger with demand schedule competition.

D.3 Dynamic model

Lemma 6 (Existence in the dynamic equilibrium) *A dynamic equilibrium in demand schedules exists if and only if, for every t, (i) there exists a positive solution λ_t to the following equation*

$$\lambda_t^2 + \lambda_t(Q_{t+1} + A_{t+1}) - Q_{t+1} = 0 \tag{129}$$

and (ii) the second-order condition

$$\lambda_t + Q_{t+1} > 0 \tag{130}$$

is satisfied.

Proposition 11 (Dynamic equilibrium)

1. *Existence and uniqueness:*

   (a) *For any $n \geq 1$, if for every t, $Q_{t+1} \geq 0$, there is a unique equilibrium in demand schedules, in which price impact is*

   $$\lambda_t = \frac{1}{2}(\sqrt{\varphi_{t+1}} - (Q_{t+1} + A_{t+1})) \tag{131}$$

   where $\varphi_{t+1} \equiv (Q_{t+1} + A_{t+1})^2 + 4a\sigma^2 Q_{t+1}$.

   (b) *If $Q_{t+1} < 0$, then if $\lambda_t + Q_{t+1} > 0$ and $\varphi_{t+1} > 0$, there is an additional equilibrium in which price impact is given by*

   $$\lambda_t = -\frac{1}{2}(\sqrt{\varphi_{t+1}} + (Q_{t+1} + A_{t+1})) \tag{132}$$

   Otherwise, there is no equilibrium.

2. *Equilibrium coefficients are*

$$\begin{aligned}
\beta_t^y &= \frac{1}{a\sigma^2} + n\bar{\alpha}_{t+1}\beta_t, & c_t &= \frac{\bar{Q}_{t+1}^{1,2}}{\lambda_t + \bar{Q}_{t+1}^{1,2}}, & d_t^y &= \bar{\alpha}_{t+1}\lambda_t\beta_t, \\
f_{t,\tau} &= -\beta_t(nQ_{t+1,\tau}^{2,4} + Q_{t+1,\tau}^{3,5}), & \beta_t &= \frac{1}{\lambda_t + Q_{t+1}}, & c_t^y &= 1, \\
f_{t,\tau}^y &= n\bar{\alpha}_{t+1}f_{t,\tau} - \theta_{t+1,\tau}, & d_t &= (1 - c_t)\beta_t\bar{Q}_{t+1}^{2,4} &&
\end{aligned} \tag{133}$$

3. *When an equilibrium exists, prices, value functions and trades can be written as in the Cournot case, provided that price coefficients are defined as (**??**)-(143), trade coefficients are defined as (148)-(150), and the value function coefficients are defined as in the Cournot case.*

Proof. The proof is by induction. The results hold at $T-1$ given proposition 10. We then show that if the induction properties hold at $t+1$, then they also hold at t. Specifically, assume that between $t+1$ and $T-1$, equations (118) and (119) for the price and value function, and property $\mathcal{P}_{as}$ hold. The proof proceeds in three steps.

Step 1: optimization given price impact

Strategic traders. Given his own price impact, a trader solves

$$\max_{(x_t^i} \quad x_t^i(D_t - p_t(x_t^i;\lambda_t)) - \frac{b\sigma^2}{2}(X_{t-1}^i + x_t^i)^2 + \Omega_{t+1}^i$$

The first-order condition gives

$$\begin{aligned}0 = D_t - p_t - \left(\frac{\partial p_t}{\partial x_t^i} + \bar{Q}_{t+1}^{1,2}\right) x_t^i - \bar{Q}_{t+1}^{1,2} X_{t-1}^i \\ - \sigma^2 \sum_{\tau=t+1}^{T-1} \left\{ q_{2,t+1}^{\tau} + \frac{1}{2}\left(\hat{\iota}_{t+1,\tau} + \iota_{t+1,\tau}\right)\right\} \Lambda_{t+1,\tau} \\ - \sigma^2 \sum_{\tau=t+1}^{T-1} \left(q_{3,t+1}^{\tau} + \sum_{j=t+1}^{T-1} q_{5,t+1}^{j,\tau} \right) X_\tau^*\end{aligned}$$

Using abbreviations from Notation 2, we can rewrite the FOC as

$$D_t - p_t - \left(\lambda_t + \bar{Q}_{t+1}^{1,2}\right) x_t^i - \bar{Q}_{t+1}^{1,2} X_{t-1}^i - \sum_{\tau=t+1}^{T-1} Q_{t+1,\tau}^{2,4} \Lambda_{t+1,\tau} - \sum_{\tau=t+1}^{T-1} Q_{t+1,\tau}^{3,5} X_\tau^* = 0 \quad (134)$$

Note that $\Lambda_{t+1,\tau} = nX_\tau^* - \sum_{j=1}^n X_{t-1}^j - \sum_{j\neq i} x_{t-1}^j - x_t^i$, so that this equation can be rewritten as

$$M^i(p_t) = \left(\lambda_t + \bar{Q}_{t+1}^{1,2}\right) x_t^i - \bar{Q}_{t+1}^{2,4} \sum_{j=1}^n x_t^j \quad (135)$$

where $M^i(p_t) = D_t - p_t - \bar{Q}^{1,2}_{t+1} X^i_t - \sum_{\tau=t+1}^{T-1} (nQ^{2,4}_{t+1,\tau} + Q^{3,5}_{t+1,\tau}) X^*_\tau +$ $\bar{Q}^{2,4}_{t+1} \sum_{j=1}^n X^j_{t-1}$. The problem is strictly concave iff condition (130) holds.

Price-takers. From Lemma 3 in the main text, the FOC of price-takers' problem is $\mathbb{E}_t(p_{t+1} - p_t) = a\sigma^2 Y^m_t$. We use the induction hypothesis for p_{t+1} and replace p^*_{t+1} by its value to obtain the following condition:

$$D_t - p_t - a\sigma^2 \sum_{\tau=t+1}^{T-1} \alpha_{t+1,\tau}\Lambda_{t+1,\tau} - a\sigma^2 Y^m_{t-1} - a\sigma^2 y^m_t - b\sigma^2 \sum_{\tau=t+1}^{T-1} X^*_\tau = 0$$

$$a\sigma^2(Y^m_{t-1} + y^m_t) = D_t - p_t - \sum_{\tau=t+1}^{T-1} (b\sigma^2 + na\sigma^2\alpha_{t+1,\tau}) X^*_\tau + a\sigma^2\bar{\alpha}_{t+1}\left(\sum_{j=1}^n X^j_{t-1} + \sum_{j=1}^n x^j_t\right) \quad (136)$$

Solving for optimal schedules. While equation (136) for price-takers depends on $\sum_j x^j_t$, equation (135) for strategic traders does not depend on y^m_t, and can thus be solved independently. Equation (135) holds for any i and any p. Then summing equation (135) over i, we get:

$$\sum_{j=1}^n M^j(p_t) = n(D_t - p_t) + (n\bar{Q}^{2,4}_{t+1} - \bar{Q}^{1,2}_{t+1}) \sum_{j=1}^n X^j_{t-1} - n \sum_{\tau=t+1}^{T-1} (nQ^{2,4}_{t+1,\tau} + Q^{3,5}_{t+1,\tau}) X^*_\tau = (\lambda_t + Q_{t+1}) \sum_{j=1}^n x^j_t$$

Substuting back into (135) and rearranging the terms gives the strategic trader's optimal schedule:

$$x_t^i(p_t) = \frac{1}{\lambda_t + Q_{t+1}} \left[D_t - p_t - \sum_{\tau=t+1}^{T-1} (nQ_{t+1,\tau}^{2,4} + Q_{t+1,\tau}^{3,5}) X_\tau^* - \frac{\lambda_t \bar{Q}_{t+1}^{2,4}}{\lambda_t + \bar{Q}_{t+1}^{1,2}} \sum_{j=1}^{n} X_{t-1}^j \right] - \frac{\bar{Q}_{t+1}^{1,2}}{\lambda_t + \bar{Q}_{t+1}^{1,2}} X_{t-1}^i, \quad \forall\, i, p_t \tag{137}$$

By pointwise identification, we obtain the equilibrium coefficients (133) for price-takers. Then summing (137) over strategic traders, we get the strategic traders' aggregate demand schedules:

$$\sum_{j=1}^{n} x_t^j(p_t) = \frac{n}{\lambda_t + Q_{t+1}} \left[D_t - p_t - \sum_{\tau=t+1}^{T-1} (nQ_{t+1,\tau}^{2,4} + Q_{t+1,\tau}^{3,5}) X_\tau^* \right] - \frac{Q_{t+1}}{\lambda_t + Q_{t+1}} \sum_{j=1}^{n} X_{t-1}^j \tag{138}$$

Substituting into (136), we obtain the price-takers' schedules

$$y_t^m(p_t) = \left(\frac{1}{a\sigma^2} + \frac{n\bar{\alpha}_{t+1}}{\lambda_t + Q_{t+1}} \right) (D_t - p_t) + \frac{\bar{\alpha}_{t+1}\lambda_t}{\lambda_t + Q_{t+1}} \sum_{j=1}^{n} X_{t-1}^j - \sum_{\tau=t+1}^{T-1} \{ \theta_{t+1,\tau} + \frac{n\bar{\alpha}_{t+1}}{\lambda_t + Q_{t+1}} (nQ_{t+1,\tau}^{2,4} + Q_{t+1,\tau}^{3,5}) \} X_\tau^*, \quad \forall\, m, p_t \tag{139}$$

By pointwise identification, we obtain the equilibrium coefficients (133) for strategic traders.

Step 2: Solving for price impact

This part provides the proof to point (i) in Lemma 6. The second-order condition is given with strategic traders' optimization problems in Step 1.

Given the optimal schedules, we have:

$$-\sum_{j\neq i}\frac{\partial x_t^j}{\partial p_t}=(n-1)\beta_t=\frac{n-1}{\lambda_t+Q_{t+1}},\quad \text{and}\quad -\frac{\partial\int_0^1 y_t^m dm}{\partial p_t}=\beta_t^y$$
$$=\frac{1}{a\sigma^2}+\frac{n\bar{\alpha}_{t+1}}{\lambda_t+Q_{t+1}}=\frac{1}{a\sigma^2}+n\bar{\alpha}_{t+1}\beta_t \quad (140)$$

Substituting these values into the price impact consistency condition (117), and rearranging terms gives the second-order equation in λ_t, (129). The discriminant of the polynomial is φ_{t+1} given in Lemma 6. Thus, an equilibrium exists with positive price impact if and only if there exists at least one positive root and the second-order condition holds. Thus, there are two cases:

(a) If $Q_{t+1}\geq 0$, the disciminant is non negative, and there is a single positive root, given by (131). The second-order condtion (130) is trivially satisfied.

(b) If $Q_{t+1}<0$, then the discriminant may be negative, so the condition $\varphi_{t+1}\geq 0$ must be added. The second-order condition is also no longer trivial and must be added. The second positive root is given (132).

Step 3: Equilibrium representation and system of difference equations

The last step is to to show that equations (118) and (119) for the price and value function hold at t, that trades can be decomposed as in the Cournot case, and that property $\mathcal{P}_{as}$ holds. Doing so, we will obtain the recursive system defining the coefficients α, c, η. Value function coefficients are defined as a function of these and their prior values, so the system defining them is similar to the Cournot case.

Equilibrium price. Market clearing requires that $\sum_j x_t^j(p_t)+\int_0^1 y_t^m dm =$

$s_t - s_{t-1}$. So from equations (138) and (139), we obtain

$$D_t - p_t = \left[\frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}\right]^{-1} \times \\ \left[\frac{na+b}{a}X_t^* + \sum_{\tau=t+1}^{T-1}\{\theta_{t+1,\tau} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}(nQ_{t+1,\tau}^{2,4} + Q_{t+1,\tau}^{3,5})\}X_\tau^*\right] \\ - \left[\frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}\right]^{-1} \frac{(1+\bar{\alpha}_{t+1})\lambda_t}{\lambda_t + Q_{t+1}} \sum_{j=1}^{n} X_{t-1}^j \quad (141)$$

Equation (118) implies $D_t - p_t = \sum_{\tau\geq t} b\sigma^2 X_t^* + a\sigma^2 \sum_{\tau\geq t}(nX_\tau^* - \sum_{j=1}^n X_{t-1}^j)$. Thus, for (118) to hold at time t, we must define:

$$b\sigma^2 + na\sigma^2\alpha_{t,t} = \left[\frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}\right]^{-1} \frac{na+b}{a} \quad (142)$$

$$b\sigma^2 + na\sigma^2\alpha_{t,\tau} = \left[\frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}\right]^{-1} \times \\ \left[\frac{na\alpha_{t+1,\tau} + b}{a} + \frac{n(1+\bar{\alpha}_{t+1})(nQ_{t+1,\tau}^{2,4} + Q_{t+1,\tau}^{3,5})}{\lambda_t + Q_{t+1}}\right], \tau \geq t+1 \quad (143)$$

Further, it must be that

$$a\sigma^2\bar{\alpha}_t = \left[\frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}\right]^{-1} \frac{\lambda_t(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}$$

Thus, the price can be written in the form of equation (118), if

$$b\sigma^2 + na\sigma^2\alpha_{t,t} + \sum_{\tau=t+1}^{T-1}(b\sigma^2 + na\sigma^2\alpha_{t,\tau}) = b\sigma^2(T-t) + a\sigma^2\bar{\alpha}_t \\ = b\sigma^2(T-t) + \left[\frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}\right]^{-1} \frac{\lambda_t(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}$$

For brevity, denote $\chi_t = \frac{1}{a\sigma^2} + \frac{n(1+\bar{\alpha}_{t+1})}{\lambda_t + Q_{t+1}}$. Using (**??**)-(143) for the left-hand side and multiplying each side by $\chi_t \sigma^{-2}$, gives

$$\frac{na(1+\bar{\alpha}_{t+1}) + b(T-t)}{a} + \frac{n(1+\bar{\alpha}_{t+1})(nQ^{2,4}_{t+1,\tau} + Q^{3,5}_{t+1,\tau})}{\lambda_t + Q_{t+1}} = \frac{b(T-t)(\lambda_t + Q_{t+1} + na\sigma^2(1+\bar{\alpha}_{t+1})) + na(1+\bar{\alpha}_{t+1})\lambda_t}{a(\lambda_t + Q_{t+1})} \quad (144)$$

Putting the left-hand side on the same denominator, several times cancel out, and we get after a few lines of simple algebra $Q^{1,2}_{t+1} + Q^{3,5}_{t+1} = b\sigma^2(T-t)$, which is implied by the property $\mathcal{P}_{as}$ at $t+1$, which is part of the induction hypothesis.

Equilibrium trade. To decompose trades into the risk-sharing and speculative components, it is easier to first write the equilibrium price as a function of the β, c, d, and f coefficients and then to subsitute it back into the schedule (116). We get:

$$D_t - p_t = \frac{1}{n\beta_t + \beta^y_t}\Big(\frac{na+b}{a} X^*_t - (1 - c_t + nd_t + d^y_t)\sum_{j=1}^{n} X^j_{t-1} - \sum_{\tau \geq t+1}(nf_{t,\tau} + f^y_{t,\tau})X^*_\tau\Big) \quad (145)$$

This implies the following equilibrium trade:

$$x^i_t = \frac{\beta}{n\beta_t + \beta^y_t}\frac{na+b}{a}X^*_t + \sum_{\tau \geq t+1}\frac{\beta^y_t f_{t,\tau} - \beta_t f^y_{t,\tau}}{n\beta_t + \beta^y_t}X^*_\tau + \frac{\beta_t(c_t - 1 - d^y_t) + \beta^y_t d_t}{n\beta_t + \beta^y_t}\sum_{j=1}^{n} X^j_{t-1} - c_t X^i_{t-1} \quad (146)$$

To write $x_t^i = \sum_{\tau \geq t} c_{t,\tau}(X_\tau^* - X_{t-1}^i) + \sum_{\tau \geq t} \eta_{t,\tau}\Lambda_{t,\tau}$, we thus need to define

$$c_{t,t} + n\eta_{t,t} = \frac{\beta}{n\beta_t + \beta_t^y}\frac{na+b}{a} \tag{147}$$

$$c_{t,\tau} + n\eta_{t,\tau} = \frac{\beta_t^y f_{t,\tau} - \beta_t f_{t,\tau}^y}{n\beta_t + \beta_t^y}, \quad \tau \geq t+1 \tag{148}$$

$$\sum_{\tau \geq t} c_{t,\tau} = \bar{c}_t = \frac{\bar{Q}_{t+1}^{1,2}}{\lambda_t + \bar{Q}_{t+1}^{1,2}} \tag{149}$$

$$\sum_{\tau \geq t} \eta_{t,\tau} = \bar{\eta}_t = -\frac{\beta_t(c_t - 1 - d_t^y) + \beta_t^y d_t}{n\beta_t + \beta_t^y}, \tag{150}$$

and we need to check that

$$c_{t,t} + n\eta_{t,t} + \sum_{\tau \geq t+1}(c_{t,\tau} + n\eta_{t,\tau}) = \bar{c}_t + n\bar{\eta}_t$$

By substituting the equilibrium values of the schedule coefficients for f^y as a function of f, and then substituting for $\theta_{t+1,\tau}$ and summing over τ, we get for the left-hand side:

$$\begin{aligned} &c_{t,t} + n\eta_{t,t} + \sum_{\tau \geq t+1}(c_{t,\tau} + n\eta_{t,\tau}) \\ &\quad = \frac{\beta}{n\beta_t + \beta_t^y}(n(1 + \bar{\alpha}_{t+1}) + \frac{b}{a}(T-t) - \frac{1}{a\sigma^2}(n\bar{Q}_{t+1}^{2,4} + Q_{t+1}^{3,5})) \end{aligned} \tag{151}$$

To reduce the right-hand side, first note that from the definition of the schedule coefficients d, d^y and c, we can write $d_t^y = -\bar{\alpha}_{t+1}(c_t - nd_t - 1)$, implying that

$$\bar{c}_t + n\bar{\eta}_t = \frac{\bar{Q}_{t+1}^{1,2}}{\lambda_t + \bar{Q}_{t+1}^{1,2}} + \frac{n\beta_t(1-c_t)(1+\bar{\alpha}_{t+1}) - \frac{1}{a\sigma^2}nd_t}{n\beta_t + \beta_t^y}$$

Then substituting for the equilibrium value of d and $1-c$ gives

$$\bar{c}_t + n\bar{\eta}_t = \frac{n\beta_t(1+\bar{\alpha}_{t+1})(\lambda_t + \bar{Q}^{1,2}_{t+1}) + \frac{1}{a\sigma^2}(\bar{Q}^{1,2}_{t+1} - n\beta_t\lambda_t\bar{Q}^{2,4}_{t+1})}{(\lambda_t + \bar{Q}^{1,2}_{t+1})(n\beta_t + \beta^y_t)}$$

This equation is equal to the left-hand side (151) if and only if

$$\begin{aligned}\frac{\beta}{n\beta_t + \beta^y_t}&\left(n(1+\bar{\alpha}_{t+1}) + \frac{b}{a}(T-t) - \frac{1}{a\sigma^2}(n\bar{Q}^{2,4}_{t+1} + Q^{3,5}_{t+1})\right)\\ &= \frac{n\beta_t(1+\bar{\alpha}_{t+1})(\lambda_t + \bar{Q}^{1,2}_{t+1}) + \frac{1}{a\sigma^2}(\bar{Q}^{1,2}_{t+1} - n\beta_t\lambda_t\bar{Q}^{2,4}_{t+1})}{(\lambda_t + \bar{Q}^{1,2}_{t+1})(n\beta_t + \beta^y_t)}\end{aligned}$$

Canceling and regrouping terms then gives

$$\beta_t\frac{b}{a}(T-t) = \frac{1}{a\sigma^2}\beta_t\bar{Q}^{3,5}_{t+1} + \frac{1}{a\sigma^2}\frac{\bar{Q}^{1,2}_{t+1}}{\lambda_t + \bar{Q}^{1,2}_{t+1}}(1 + n\beta_t\bar{Q}^{2,4}_{t+1})$$

However, the definition of c and β imply that $\lambda_t + \bar{Q}^{1,2}_{t+1} = \frac{1+n\beta_t\bar{Q}^{2,4}_{t+1}}{\beta_t}$, so that the equation boils down to $Q^{1,2}_{t+1} + Q^{3,5}_{t+1} = b\sigma^2(T-t)$, which is implied by $\mathcal{P}_{as}$ at $t+1$, and confirms that we can decompose trades as in the Cournot case also at time t.

Uniqueness of equilibrium representation. The system given by (147)-(150) does not uniquely define the two vectors $\mathbf{c}_t$ and $\boldsymbol{\eta}_t$. However, if we define the vector $\mathbf{c}_t$, then the vector $\boldsymbol{\eta}_t$ is uniquely pinned down. Note that the equilibrium definition of c_t, $c_t = \frac{\bar{Q}^{t+1}_{1,2}}{\lambda_t + \bar{Q}^{t+1}_{1,2}}$, involves a sum, since $\bar{Q}^{t+1}_{1,2}$ is a sum:

$$\begin{aligned}\bar{Q}^{t+1}_{1,2} \equiv& b\sigma^2(1 + q_{1,t+1}) - \sigma^2\bar{q}_{2,t+1}\\ =& b\sigma^2 + b\sigma^2 q_{1,t+1} - \sigma^2\bar{q}_{5,t+1}\\ =& \sigma^2(b + b(T-t-1) - \bar{q}_{5,t+1} - \bar{q}_{3,t+1})\end{aligned}$$

where the second and third line follow from $\mathcal{P}_{as}$ at t+1. By definition, the parameter $\bar{q}_{i,t+1}$ is a sum. Thus, in analogy with the Cournot case, we can exhibit a vector $\mathbf{c}_t$ satisfying (149):

$$
\begin{aligned}
c_{t,t} =& \frac{b\sigma^2}{\lambda_t + \bar{Q}_{1,2}^{t+1}} \\
c_{t,\tau} =& \frac{\sigma^2(b - q_{3,t+1}^{\tau} - \sum_{j=t+1}^{T-1} q_{5,t+1}^{j,\tau})}{\lambda_t + \bar{Q}_{1,2}^{t+1}} = \frac{b\sigma^2 - Q_{3,5}^{t+1,\tau}}{\lambda_t + \bar{Q}_{1,2}^{t+1}}, \quad \tau \geq t+1
\end{aligned}
$$

Value function. To complete the induction, we need to show that property $\mathcal{P}_{as}$ holds at time t and that the value function can be expressed as a linear quadratic function. These two steps are exactly the same as in the Cournot case.

E Alternative representation

We can rewrite the price, trade, and value function as follows:

$$
x_t^i = \sum_{\tau=t}^{T-1} \nu_{t,\tau} s_\tau - \eta_t \sum_{j=1}^{n} X_{t-1}^j - c_t X_{t-1}^i \tag{152}
$$

$$
p_t = D_t - a\sigma^2 \left[\sum_{\tau=t}^{T-1} \tilde{\theta}_{t,\tau} s_\tau - \alpha_t \sum_{j=1}^{n} X_{t-1}^j \right] \tag{153}
$$

$$
\begin{aligned}
\Omega_t^i = & -\frac{b\sigma^2}{2} r_{1,t} \left(X_{t-1}^i\right)^2 - \sigma^2 X_{t-1}^i \left[r_{2,t} \sum_{j=1}^{n} X_{t-1}^j + \sum_{\tau=t}^{T-1} r_{3,t}^{\tau} s_\tau \right] \\
& + \frac{\sigma^2}{2} r_{4,t} \left(\sum_{j=1}^{n} X_{t-1}^j \right)^2 + \sigma^2 \sum_{\tau=t}^{T-1} r_{5,t}^{\tau} s_\tau \sum_{j=1}^{n} X_{t-1}^j + \sigma^2 \sum_{\tau=t}^{T-1} \sum_{u=\tau}^{T-1} r_{6,t}^{\tau,u} s_\tau s_u
\end{aligned} \tag{154}
$$

These alternative expressions are useful to prove that (i) the quadratic representation is unique, and (ii) that the anticipated shock liquidity premium is constant when the shock occurs in the last period, as in the Cournot case (see Corollary 5 below). In this part, I use the following notation:

Notation 3

$$\gamma_t = 1 - c_t - n\eta_t, \qquad R_{1,2}^{t+1} = b\sigma^2(1 + r_{1,t+1}) + \sigma^2 r_{2,t+1},$$
$$R_{3,5}^{t+1,\tau} = \sigma^2 \left(r_{3,t+1}^{\tau} - r_{5,t+1}^{\tau}\right), \quad R_{2,4}^{t+1} = \sigma^2 \left(r_{4,t+1} - r_{2,t+1}\right)$$

E.1 Equilibrium

Proposition 12 *The equilibrium in demand schedules can be expressed by* (152)-(154) *provided we define the price parameters by* (170)-(172)*, the trade pararmeters by* (173)-(175) *and the value function coefficients as follows:*

$$r_{1,t} = (1 - c_t)^2(1 + r_{1,t+1}) \tag{155}$$

$$r_{2,t} = (1 - c_t)\left[\gamma_t r_{2,t+1} - b\eta_t(1 + r_{1,t+1})\right] - ac_t\alpha_t \tag{156}$$

$$r_{3,t}^{t} = ac_t\theta_{t,t} + (1 - c_t)\left[b(1 + r_{1,t+1}) + nr_{2,t+1}\right]\nu_{t,t} \tag{157}$$

$$r_{3,t}^{\tau} = ac_t\theta_{t,\tau} + (1 - c_t)\left[b(1 + r_{1,t+1}) + nr_{2,t+1}\right]\nu_{t,\tau} + (1 - c_t)r_{3,t+1}^{\tau} \tag{158}$$

$$\frac{1}{2}r_{4,t} = \frac{1}{2}r_{4,t+1}\gamma_t^2 - \eta_t\left[\frac{b}{2}\eta_t(1 + r_{1,t+1}) - a\alpha_t - \gamma_t r_{2,t+1}\right] \tag{159}$$

$$r_{5,t}^{t} = -a\eta_t\theta_{t,t} + \nu_{t,t}\left[n\gamma_t r_{4,t+1} + b(1 + r_{1,t+1})\eta_t - a\alpha_t + r_{2,t+1}(n\eta_t - \gamma_t)\right] \tag{160}$$

$$r_{5,t}^{\tau} = \gamma_t r_{5,t+1}^{\tau} + \eta_t r_{3,t+1}^{\tau} - a\eta_t\theta_{t,\tau} + \nu_{t,\tau}\left[n\gamma_t r_{4,t+1} + b(1 + r_{1,t+1})\eta_t - a\alpha_t + r_{2,t+1}(n\eta_t - \gamma_t)\right] \tag{161}$$

Demand schedule equilibrium parameters are

$$f^y_{t,\tau} = -n\alpha_{t+1}\beta_t R^{t+1,\tau}_{3,5} - \tilde{\theta}_{t+1,\tau}, \quad d^y_t = \lambda_t \alpha_{t+1}\beta_t, \quad c^y_t = 1,$$
$$\beta^y_t = \frac{1}{a\sigma^2} + n\alpha_{t+1}\beta_t, \quad \beta_t = \left(\lambda_t + R_{t+1}\right)^{-1}, \quad c_t = \frac{R^{t+1}_{1,2}}{\lambda_t + R^{t+1}_{1,2}},$$
$$f_{t,\tau} = -R^{t+1,\tau}_{3,5}\beta_t, \quad d_t = (1-c_t)R^{t+1}_{2,4}\beta_t \tag{162}$$

The initial conditions are:

$$c_{T-1} = b\sigma^2\beta_{T-1}, \quad \tilde{\theta}_{T-1} = \frac{b}{b + nac_{T-1}}, \quad \alpha_{T-1} = \frac{b(1-c_{T-1})}{b+nac_{T-1}}, \tag{163}$$

$$\nu_{T-1} = \frac{ac_{T-1}}{b+nac_{T-1}}, \quad \eta_{T-1} = \frac{ac_{T-1}(1-c_{T-1})}{b+nac_{T-1}} \tag{164}$$

Existence conditions are the same as in the baseline model (with $\bar{Q}^{t+1}_{1,2} = R^{t+1}_{1,2}$ and $\bar{Q}^{t+1}_{2,4} = R^{t+1}_{2,4}$.)

Proof. The proof is by induction. I first show that (152)-(154) hold in the static model (at $T-1$), and then show that if these expessions hold at $t+1$, they also hold at t.

Final period $(T-1)$

We simply need to rewrite the equilibrium price, trade, and value function from Proposition 10. Using the definition of the equilibrium demand schedule parameters and the expression of the implied equilibrium price, we obtain

$$p_{T-1} = D_{T-1} - a\sigma^2\left[\frac{b}{b+nac_{T-1}}s_{T-1} - \frac{b(1-c_{T-1})}{b+nac_{T-1}}\sum_{j=1}^{n} X^j_{T-2}\right],$$

from which we obtain the definitions of θ_{T-1} and α_{T-1}. Similarly, substituting the equilibrium parameters into the implied equilibrium trade gives

$$x^i_{T-1} = \frac{ac_{T-1}}{b + nac_{T-1}} \left[s_{T-1} - (1 - c_{T-1}) \sum_{j=1}^{n} X^j_{T-2} \right] - c_{T-1} X^i_{T-2},$$

from which we get the definitions of ν_{T-1} and η_{T-1}. Substituting the equilibrium price and trade into a strategic trader's certainty equivalent yields (154) (evaluated at $T-1$), with coefficients

$$\begin{aligned}
r_{1,T-1} &= (1 - c_{T-1})^2 \equiv q_{1,T-1}, \\
r_{2,T-1} &= -b\eta_{T-1}(1 - c_{T-1}) - a\alpha_{T-1}c_{T-1}, \\
r_{3,T-1} &= ac_{T-1}\tilde{\theta}_{T-1} + b(1 - c_{T-1})\nu_{T-1}, \\
\frac{1}{2} r_{4,T-1} &= a\alpha_{T-1}\eta_{T-1} - \frac{b}{2}\eta^2_{T-1}, \\
r_{5,T-1} &= -a\tilde{\theta}_{T-1}\eta_{T-1} - a\alpha_{T-1}\nu_{T-1} + b\nu_{T-1}\eta_{T-1}
\end{aligned}$$

Induction from $t+1$ to t

First note that by using the definition of $\Lambda_{t,\tau}$, we can rewrite the price, trade and value function of the model as (152), (153) and (154). Rearranging the terms implies the following equivalence:

$$\begin{aligned}
\nu_{t,\tau} &\equiv \frac{a}{na+b}(c_{t,\tau} + n\eta_{t,\tau}), & \tilde{\theta}_{t,\tau} &\equiv \frac{b + na\alpha_{t,\tau}}{na+b}, \quad r_{1,t} \equiv q_{1,t}, \\
r^\tau_{3,t} &\equiv \frac{a}{na+b}(nq^\tau_{2,t} + q^\tau_{3,t}), & r_{2,t} &\equiv -\bar{q}_{2,t}, \qquad r_{4,t} \equiv \bar{q}_{4,t}, \\
r^\tau_{5,t} &\equiv -\frac{a}{na+b}\left(\sum_{j=t}^{T-1} q^{j,\tau}_{5,t} + \frac{n}{2}(\iota_{t,\tau} + \hat{\iota}_{t,\tau}) \right)
\end{aligned}$$

We thus simply need to rewrite the equilibrium coefficients as a function of the new value function and price parameters, and derive the recursive definition of the value function and price parameters. The steps are the same as for the quadratic representation of the equilibrium.

Optimization given price impact. The optimization problem of price-takers is unchanged, so using the price expression (153) at $t+1$, their first-order condition is

$$D_t - p_t - a\sigma^2 \sum_{\tau \geq t+1} \theta_{t+1,\tau} s_\tau + a\sigma^2 \alpha_{t+1} \left(\sum_j X_{t-1}^j + \sum_j x_t^j \right) - a\sigma^2 (Y_{t-1}^m + y_t^m) = 0 \quad (165)$$

Given the new expression for the value function, and using Notation 3 strategic traders' first-order conditions become, for all i:

$$D_t - p_t - \left(\lambda_t + R_{t+1}^{1,2} \right) x_t^i - R_{t+1}^{1,2} X_{t-1}^i - \sum_{\tau \geq t+1} R_{t+1,\tau}^{3,5} s_\tau + R_{t+1}^{2,4} \left(\sum_j X_{t-1}^j + \sum_j x_t^j \right) = 0 \quad (166)$$

We proceed as before and sum over i, which gives:

$$(\lambda_t + R_{t+1}) \sum_j x_t^j = n \left(D_t - p_t - \sum_{\tau \geq t+1} R_{t+1,\tau}^{3,5} s_\tau \right) - R_{t+1} \sum_j X_{t-1}^j \quad (167)$$

Substituting back into the first-order conditions gives the equilibrium schedules of strategic traders and price-takers:

$$x_t^i = \frac{1}{\lambda_t + R_{t+1}} \left(D_t - p_t - \sum_{\tau \geq t+1} R_{t+1,\tau}^{3,5} s_\tau \right) + \frac{\lambda_t R_{t+1}^{2,4}}{(\lambda_t + R_{t+1}^{1,2})(\lambda_t + R_{t+1})} \sum_j X_{t-1}^j - \frac{R_{t+1}^{1,2}}{\lambda_t + R_{t+1}^{1,2}} X_{t-1}^i y_t^m = \left(\frac{1}{a\sigma^2} + \frac{n\alpha_{t+1}}{\lambda_t + R_{t+1}} \right) (D_t - p_t) - \sum_{\tau \geq t+1} \left(\tilde{\theta}_{t+1,\tau} + \frac{n\alpha_{t+1} R_{t+1,\tau}^{3,5}}{\lambda_t + R_{t+1}} \right) s_\tau \quad + \frac{\lambda_t \alpha_{t+1}}{\lambda_t + R_{t+1}} \sum_j X_{t-1}^j - Y_{t-1}^m \tag{168}$$

Identifying pointwise with the schedules (115)-(116) gives equilibrium coefficients (162).

Price impact. This step is exactly the same as in the previous case, with $\bar{Q}_{1,2}^{t+1} = R_{1,2}^{t+1}$ and $\bar{Q}_{2,4}^{t+1} = R_{2,4}^{t+1}$.

Recursive system. The final step consists in deriving recursive relationships for the price and value function parameters and show that the equilibrium trade can be written as postulated.

The equilibrium price follows from market-clearing and using (167) and (168), and using $Y_{t-1} + \sum_j X_{t-1}^j = s_{t-1}$. We get:

$$D_t - p_t = \left(\frac{1}{a\sigma^2} + \frac{n\alpha_{t+1}}{\lambda_t + R_{t+1}} \right)^{-1} \times \left(s_t + \sum_{\tau \geq t+1} (\tilde{\theta}_{t+1,\tau} + n(1 + \alpha_{t+1}) R_{3,5}^{t+1,\tau} \beta_t) s_\tau - (1 + \alpha_{t+1}) \lambda_t \beta_t \sum_j X_{t-1}^j \right) \tag{169}$$

Thus, we simply need to define:

$$a\sigma^2\tilde{\theta}_{t,t} = \left(\frac{1}{a\sigma^2} + \frac{n\alpha_{t+1}}{\lambda_t + R_{t+1}}\right)^{-1} \tag{170}$$

$$a\sigma^2\tilde{\theta}_{t,\tau} = \left(\frac{1}{a\sigma^2} + \frac{n\alpha_{t+1}}{\lambda_t + R_{t+1}}\right)^{-1} (\tilde{\theta}_{t+1,\tau} + n(1+\alpha_{t+1})R_{3,5}^{t+1,\tau}\beta_t), \qquad \tau \geq t+1 \tag{171}$$

$$a\sigma^2\alpha_t = \left(\frac{1}{a\sigma^2} + \frac{n\alpha_{t+1}}{\lambda_t + R_{t+1}}\right)^{-1} (1+\alpha_{t+1})\lambda_t\beta_t \tag{172}$$

Then substituting the equilibrium price into the equilibrium demand schedule gives the equilibrium trade:

$$\begin{aligned} x_t^i = {} & \frac{a\sigma^2}{\lambda_t + R_{t+1} + na\sigma^2(1+\alpha_{t+1})} s_t + \sum_{\tau \geq t+1} \frac{a\sigma^2\tilde{\theta}_{t+1,\tau} - R_{3,5}^{t+1,\tau}\beta_t}{\lambda_t + R_{t+1} + na\sigma^2(1+\alpha_{t+1})} s_\tau \\ & - \frac{\lambda_t}{\lambda_t + R_{t+1}^{1,2}} \frac{\left(a\sigma^2(1+\alpha_{t+1}) - R_{2,4}^{t+1}\right)}{\lambda_t + R_{t+1} + na\sigma^2(1+\alpha_{t+1})} \sum_j X_{t-1}^j - \frac{R_{t+1}^{1,2}}{\lambda_t + R_{t+1}^{1,2}} X_{t-1}^i \end{aligned}$$

Thus, it suffices to define

$$\nu_{t,t} = \frac{a\sigma^2}{\lambda_t + R_{t+1} + na\sigma^2(1+\alpha_{t+1})} \tag{173}$$

$$\nu_{t,\tau} = \frac{a\sigma^2\tilde{\theta}_{t+1,\tau} - R_{3,5}^{t+1,\tau}\beta_t}{\lambda_t + R_{t+1} + na\sigma^2(1+\alpha_{t+1})} \tag{174}$$

$$\eta_t = \frac{\lambda_t}{\lambda_t + R_{t+1}^{1,2}} \frac{\left(a\sigma^2(1+\alpha_{t+1}) - R_{2,4}^{t+1}\right)}{\lambda_t + R_{t+1} + na\sigma^2(1+\alpha_{t+1})} \tag{175}$$

$$c_t = \frac{R_{t+1}^{1,2}}{\lambda_t + R_{t+1}^{1,2}} \tag{176}$$

Value function. Finally, we calculate the value function. We start by calculating $J_t^i = x_t^i(D_t - p_t) - \frac{b\sigma^2}{2}(X_t^i)^2$. We obtain:

$$\sigma^{-2} J_t^i = \left(\sum_{\tau=t}^{T-1} \nu_{t,\tau} s_\tau - \eta_t \sum_{j=1}^{n} X_{t-1}^j\right) \Big[\sum_{\tau=t}^{T-1} (a\tilde{\theta}_{t,\tau} - \frac{b}{2}\nu_{t,\tau}) s_\tau + \left(\frac{b}{2}\eta_t - a\alpha_t\right) \sum_{j=1}^{n} X_{t-1}^j \Big]$$
$$- X_{t-1}^i \left[\sum_{\tau=t}^{T-1} \left(ac_t\tilde{\theta}_{t,\tau} + b(1-c_t)\nu_{t,\tau}\right) s_\tau - (ac_t\alpha_t + b(1-c_t)\eta_t) \sum_{j=1}^{n} X_{t-1}^j \right] - \frac{b}{2}(1-c_t)^2 (X_{t-1}^i)^2$$

We now express Ω_{t+1}^i as a function of time $T-1$ variables. We break Ω_{t+1}^i in two parts. First, we have $\underline{\Omega}_{t+1}^i = -\frac{b\sigma^2}{2}\left(X_j^i\right)^2 - X_t^i\big(r_{2,t+1}\sum_{j=1}^n X_t^j + \sum_{\tau=t+1}^{T-1} r_{3,t+1}^\tau s_\tau\big)$. Using $\sum_{j=1}^n X_t^j = n\sum_{\tau=j}^{T-1}\nu_{t,\tau}s_\tau + (1-c_t-n\eta_t)\sum_{j=1}^n X_{t-1}^j$ by summing the equilibrium trade over i, we obtain after a few lines of algebra:

$$\sigma^{-2}\underline{\Omega}^i_{t+1} = -\frac{b}{2}(1-c_t)^2 r_{1,t+1}(X^i_{t-1})^2 - (1-c_t)X^i_{t-1} \times \Big[\sum_{\tau=t}^{T-1}(br_{1,t+1}+nr_{2,t+1})\nu_{t,\tau}s_\tau + \sum_{\tau=t+1}^{T-1} r^\tau_{3,t+1}s_\tau + \{(1-c_t-n\eta_t)r_{2,t+1} - br_{1,t+1}\eta_t\}\sum_{j=1}^n X^j_{t-1}\Big]$$

$$-\left[\sum_{\tau=t}^{T-1}\nu_{t,\tau}s_\tau - \eta_t\sum_{j=1}^n X^j_{t-1}\right]\left[\sum_{\tau=t}^{T-1}\left(\frac{b}{2}r_{1,t+1}+nr_{2,t+1}\right)\nu_{t,\tau}s_\tau + \sum_{\tau=t}^{T-1} r^\tau_{3,t+1}s_\tau + \left\{r_{2,t+1}(1-c_t-\eta_t n) - \frac{b}{2}r_{1,t+1}\eta_t\right\}\sum_{j=1}^n X^j_{t-1}\right] \quad (177)$$

Next, we calculate the terms in $r_{4,t+1}$ and $r_{5,t+1}$.

$$\frac{1}{2}r_{4,t+1}\left(\sum_{j=1}^n X^j_t\right)^2 = \frac{1}{2}r_{4,t+1}\left[n^2\left(\sum_{\tau=t}^{T-1}\nu_{t,\tau}s_\tau\right)^2 + (1-c_t-n\eta_t)^2 \times \left(\sum_{j=1}^n X^j_{t-1}\right)^2 + 2n(1-c_t-n\eta_t)\left(\sum_{\tau=t}^{T-1}\nu_{t,\tau}s_\tau\right)\left(\sum_{j=1}^n X^j_{t-1}\right)\right] \quad (178)$$

$$\sum_{\tau=t+1}^{T-1} r^\tau_{5,t+1}s_\tau\left(\sum_{j=1}^n X^j_t\right) = \left(\sum_{\tau=t+1}^{T-1} r^\tau_{5,t+1}s_\tau\right) \times \left(n\sum_{\tau=t}^{T-1}\nu_{t,\tau}s_\tau + (1-c_t-n\eta_t)\sum_{j=1}^n X^j_{t-1}\right] \quad (179)$$

Adding these two terms to (177), and simplifying, we obtain (154) by defining the coefficients $r_{i,t}$ as in the proposition. This completes the induction.

E.2 Special case: supply shock in the final period

Corollary 5 *Suppose that $n = 1$. If the anticipated shock occurs at $T-1$, then (i) the monopoly does not trade on the shock, i.e. for any t, $\nu_{t,T-1} = 0$, (ii) the anticipated shock liquidity premium is the same as under Cournot; and thus (iii) there is no momentum in the price or liquidity premium.*

Proof. The proof has two parts. I first rewrite the equilibrium in a more compact way, then prove the result (point (i)) by induction. The two other points are direct consequences of (i).

Rewriting the equilibrium

To show this result, it is convenient to rederive the alternative representation of the equilibrium in the special case $n = 1$. With a monopoly, the candidate equilibrium strategies boil down to

$$y_t^m(p_t) = \beta_t^y(D_t - p_t) - c_t^y Y_{t-1}^m + d_t^y X_{t-1} + \sum_{\tau=t}^{T-1} f_{t,\tau}^y s_\tau, \ m \in [0,1] \quad (180)$$

$$x_t^i(p_t) = \beta_t(D_t - p_t) - \hat{c}_t X_{t-1} + \sum_{\tau=t}^{T-1} f_{t,\tau} s_\tau, \ i = 1, \dots, n, \text{with} \quad \beta_t > 0 \quad (181)$$

with $\hat{c} = c - nd = c - d$. Further, the value function and equilibrium trade become $\Omega_t = -\frac{1}{2}\hat{Q}_t (X_t)^2 - \left(\sum_{\tau=t}^{T-1} \hat{R}_{t,\tau} s_\tau\right) X_t$ and $x_t = \sum_{\tau=t}^{T-1} \nu_{t,\tau} s_\tau - \hat{\eta}_t X_t$, where $\hat{Q}_t \equiv R_{1,2}^t - R_{2,4}^t - b\sigma^2$, $\hat{R}_{t,\tau} = R_{3,5}^{t,\tau}$ to simplify the notation, and $\hat{\eta}_t = c_t + n\eta_t = c_t + \eta_t$. The equilibrium price is unchanged. With these notations, the equilibrium parameters become

$$d_t^y = \alpha_{t+1}(\hat{c}_t - 1), \quad \hat{c}_t = (b\sigma^2 + \hat{Q}_{t+1})\beta_t,$$
$$\beta_t\left(\lambda_t + b\sigma^2 + \hat{Q}_{t+1}\right) = 1, \quad \hat{\eta}_t = \frac{a\sigma^2(1+\alpha_{t+1})\beta_t + \hat{c}_t}{1 + a\sigma^2(1+\alpha_{t+1})\beta_t} \tag{182}$$

The recursive system becomes

$$\alpha_t = \frac{(1+\alpha_{t+1})(1-\hat{c}_t)}{1 + a\sigma^2(1+\alpha_{t+1})\beta_t}$$
$$\frac{1}{2}\hat{Q}_t = -a\sigma^2\alpha_t\hat{\eta}_t + \frac{1}{2}(b\sigma^2 + \hat{Q}_{t+1})(1-\hat{\eta}_t)^2$$
$$\hat{R}_{t,t} = a\sigma^2\hat{\eta}_t\tilde{\theta}_{t,t} + a\sigma^2\alpha_t\nu_{t,t} + (b\sigma^2 + Q_{t+1})(1-\hat{\eta}_t)\nu_{t,t}$$
$$\hat{R}_{t,\tau} = a\sigma^2\hat{\eta}_t\tilde{\theta}_{t,\tau} + a\sigma^2\alpha_t\nu_{t,\tau} + (b\sigma^2 + \hat{Q}_{t+1})(1-\hat{\eta}_t)\nu_{t,\tau} + (1-\hat{\eta}_t)\hat{R}_{t+1,\tau} \tag{183}$$

The initial conditions are $\hat{\eta}_{T-1} = \frac{a+b}{2a+b}$, $\hat{R}_{T-1} = a\sigma^2(\tilde{\theta}_{T-1}\hat{\eta}_{T-1} + \alpha_{T-1}\nu_{T-1}) + b\sigma^2\nu_{T-1}(1-\hat{\eta}_{T-1})$, and $\frac{1}{2}\hat{Q}_{T-1} = \frac{b\sigma^2}{2}(1-\hat{\eta}_{T-1})^2 - a\sigma^2\hat{\eta}_{T-1}\alpha_{T-1}$. The other equilibrium parameters and initial conditions keep the same form.

Showing that for all $t < T-1$, $\nu_{t,T-1} = 0$

The proof is by induction. It is easy to check using the initial conditions that $\nu_{T-2,T-1} = 0$.

Then suppose that $\nu_{t+1,T-1} = 0$ for some t. Since $\nu_{t,\tau} = \frac{\beta_t\left(a\sigma^2\tilde{\theta}_{t+1,\tau} - \hat{R}_{t+1,\tau}\right)}{1+a\sigma^2(1+\alpha_{t+1})\beta_t}$, $\nu_{t+1,T-1} = 0$ implies that $a\sigma^2\tilde{\theta}_{t+2,T-1} - \hat{R}_{t+2,T-1} = 0$. The induction hypothesis also implies that $\hat{R}_{t+1,T-1} = a\sigma^2\hat{\eta}_{t+1}\tilde{\theta}_{t+1,T-1} + (1-\hat{\eta}_{t+1})\hat{R}_{t+2,T-1}$. Thus the numerator of $\nu_{t,T-1}$ is $(1-\hat{\eta}_{t+1})(a\sigma^2\tilde{\theta}_{t+1,T-1} - \hat{R}_{t+2,T-1}) = (1-\hat{\eta}_{t+1})a\sigma^2(\tilde{\theta}_{t+1,T-1} - \tilde{\theta}_{t+2,T-1})$. But the recursive definition of $\tilde{\theta}$ implies that

$$\tilde{\theta}_{t+1,T-1} = \frac{\theta_{t+2,T-1} + (1+\alpha_{t+1})\beta_t\hat{R}_{t+2,T-1}}{1 + a\sigma^2(1+\alpha_{t+1})\beta_t}$$
$$= \frac{\tilde{\theta}_{t+2,T-1}(1 + a\sigma^2(1+\alpha_{t+1})\beta_t)}{1 + a\sigma^2(1+\alpha_{t+1})\beta_t} = \tilde{\theta}_{t+2,T-1} \tag{184}$$

Therefore $\nu_{t,T-1} = 0$, and the induction hypothesis for any t, and $\tilde{\theta}_{t,T-1} = \tilde{\theta}_{t+1,T-1} = \tilde{\theta}_{T-1}$.

Consequences for the quadratic representation of the model and the liquidity premium

By definition, $\tilde{\theta}_{t,\tau} = \frac{b+na\alpha_{t,\tau}}{b+na}$, where $\alpha_{t,\tau}$ is the coefficient of the quadratic model. First note that $\alpha_{T-2,T-1} = \alpha_{T-1}$. The previous induction implies that $\alpha_{t,T-1} = \alpha_{t+1,T-1} = \alpha_{T-1}$.

This further implies that the anticipated shock liquidity premium is constant over time, and since when $n = 1$, Cournot and demand schedule initial conditions coincide (Corollary 4 in the main appendix), the anticipated shock liquidity premium is the same for both types of competition when the shock occurs at $T - 1$, which implies that there is no momentum.

F Equivalence between endowment and supply shock models

Suppose that the risky asset is in fixed supply s and that *price-takers* receive endowment shocks $\Delta s_\tau \epsilon_{\tau+1}$ at time τ, where Δs_τ is known by all investors from time 0. Because these endowment shocks are correlated with the dividend news, they mechanically increase price-takers' exposure to the risky asset, reducing their demand. Tracking error constraints work in a similar way on index trackers. In this interpretation of the model, endowment shocks to price-takers proxy for demand for immediacy of index trackers.[22] I

[22] Given their passive strategies and mechanical trading rules, index trackers are often unable to strategically manage their price impact. ETFs rolling over futures contracts, may be forced to "fire-sell" or "fire-purchase" assets to satisfy institutional constraints. For instance, Bessembinder, Carrion, Tuttle, and Ventakaram (2016) note that the oil-future ETF USO rolls over its front-month contract in less than a day, with the total volume to be rebalanced often exceeding the average daily volume. Similarly, funds replicating bond indices "for most bonds, spread their selling activity within the exclusion date" (Nick-Nielsen and Rossi, 2019, p. 8), and thus require a high level of immediacy during these events.

show in the online appendix that, for any type of competition, the model with endowment shocks is equivalent to the model with supply shocks. The equivalence between the two settings relies on the fact that the endowment shocks affect price-takers. Without price-takers, supply and endowment shocks are not equivalent (Rostek and Weretka, 2015).

I consider two specifications for the model with endowment shocks.

- Model 1: The risky asset is in positive net supply s. At time t, price-tackers receive endowment shocks $\left(\sum_{l=1}^{t-1} \Delta s_l\right) \epsilon_t$ at time t.
- Model 2: The risky asset is in zero net supply. Price-takers receive endowment shocks $s_{t-1}\epsilon_t$ at time t.

Proposition 13

1. *With Cournot competition, the model with supply shocks is equivalent to both endowment shocks models.*
2. *With demand schedule competition, the model with supply shocks and the second specification of the endowment shocks model are equivalent.*

I first prove the following auxiliary result.

Lemma 7 (Price-takers' optimal demand with endowment shocks)

- *Under Model 1, price-takers' optimal demand at t is*

$$Y_t = \frac{\mathbb{E}_t(p_{t+1}) - p_t}{a\sigma^2} - \sum_{l=1}^{t} \Delta s_l \tag{185}$$

- *Under Model 2, it is*

$$Y_t = \frac{\mathbb{E}_t(p_{t+1}) - p_t}{a\sigma^2} - s_t \tag{186}$$

Proof. Price-takers' dynamic budget constraint is

$$w_{t+1} = w_t + Y_t(p_{t+1} - p_t) + \sum_{l=1}^{t} \Delta s_l \epsilon_{t+1} \quad \text{under Model 1}$$
$$w_{t+1} = w_t + Y_t(p_{t+1} - p_t) + s_t \epsilon_{t+1} \quad \text{under Model 2}$$

Then we can show by induction that price takers' post-trade certainty equivalent is

$$CE_t = w_t + \sum_{q=t}^{T-1} \left[\frac{(\mathbb{E}_q(\hat{p}_{q+1}) - \hat{p}_q)^2}{2avar_q(\hat{p}_{q+1})} - \sum_{l=1}^{q} \Delta s_l \left(\mathbb{E}_q(\hat{p}_{q+1}) - \hat{p}_q\right) \right] \quad \text{in Model 1}$$
$$CE_t = w_t + \sum_{q=t}^{T-1} \left[\frac{(\mathbb{E}_q(\hat{p}_{q+1}) - \hat{p}_q)^2}{2avar_q(\hat{p}_{q+1})} - s_q \left(\mathbb{E}_q(\hat{p}_{q+1}) - \hat{p}_q\right) \right] \quad \text{in Model 2}$$

Where $\hat{p}_q$ denotes the equilibrium price. From these certainty equivalents and the dynamic budget constraints, taking the first-order conditions yields the demands (185) and (186). The rest of the induction is the same as in Lemma 3 in the main appendix.

We can then prove Proposition 13.

Proof.

Cournot Competition. From (185) and (186), we obtain the same price schedule as in the supply shock model. In Model 1, market clearing implies that $\sum_{j=1}^{n} X_t^k + Y_t = s$ so

$$\mathbb{E}_t(p_{t+1} - p_t) + \sum_{j=1}^{n} X_t^j = a\sigma^2 \left(s + \sum_{l=1}^{t} \Delta s_l \right) = a\sigma^2 s_t$$

In Model 2, market-clearing implies that $\sum_{j=1}^{n} X_t^j + Y_t = 0$, so

$$\mathbb{E}_t(p_{t+1}) - p_t + \sum_{j=1}^{n} X_t^j = a\sigma^2 s_t$$

In either case, we recognize the recursive definition of the price schedule. The rest of the steps are the same as in the supply shock model.

Demand schedule competition. Under Model 2, market clearing requires $\sum_i x_t^i(p_t) + \int y_t^m(p_t)dm = 0$. Price-takers will now condition their schedule on the current endowment shock, so our candidate linear schedule must be adjusted as follows:

$$y_t^m(p_t) = \beta_t^y(D_t - p_t) - c_t^y Y_{t-1}^m + d_t^y \sum_j X_{t-1}^j + \sum_{\tau \geq t} f_{t,\tau}^y X_\tau^* \quad (187)$$

There is now a coefficient $f_{t,t}^y$. For strategic traders, schedules are unchanged. Their optimization problem also remains the same. Their equilibrium schedules do not change. Price-takers' first-order conditions must be adjusted for the endowment shock:

$$D_t - p_t - a\sigma^2 \sum_{\tau \geq t+1} \theta_{t+1,\tau} s_\tau + a\sigma^2 \alpha_{t+1} \left(\sum_j X_{t-1}^j + \sum_j x_t^j \right) - a\sigma^2(Y_{t-1}^m + y_t^m) - a\sigma^2 s_t = 0 \quad (188)$$

If we substitute (167), we get:

$$y_t^m = \left(\frac{1}{a\sigma^2} + \frac{n\alpha_{t+1}}{\lambda_t + R_{t+1}} \right)(D_t - p_t) - \frac{na+b}{a} X_t^* - \sum_{\tau \geq t+1} \left(\tilde{\theta}_{t+1,\tau} + \frac{n\alpha_{t+1} R_{t+1,\tau}^{3,5}}{\lambda_t + R_{t+1}} \right) s_\tau + \frac{\lambda_t \alpha_{t+1}}{\lambda_t + R_{t+1}} \sum_j X_{t-1}^j - Y_{t-1}^m \quad (189)$$

All the coefficients are the same as before, and $f_{t,t}^y = -\frac{na+b}{a}$. Then, using market clearing, and substituting equilibrium schedules we get the equilibrium price, which corresponds to (7). Since the strategic traders' schedules and the equilibrium price are both the same as in the supply shock model, it is possible to write the equilibrium trade as before, and the rest of the proof proceeds as in the supply shock model.

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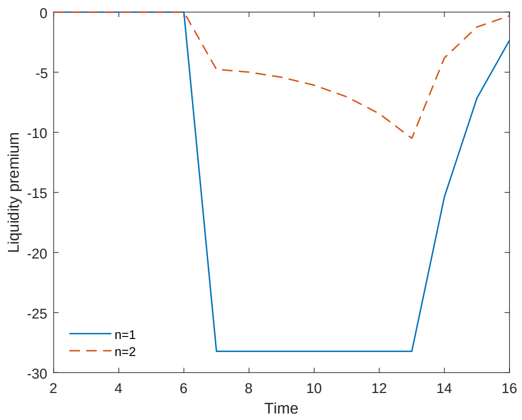
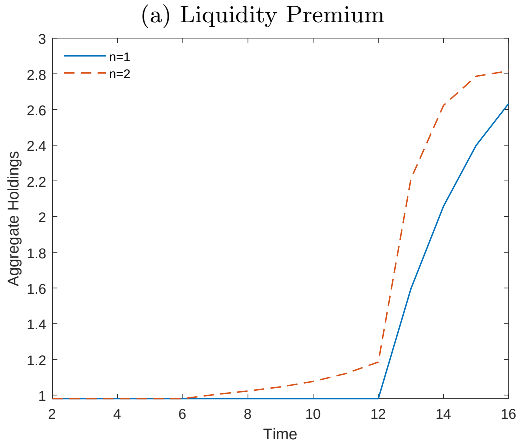
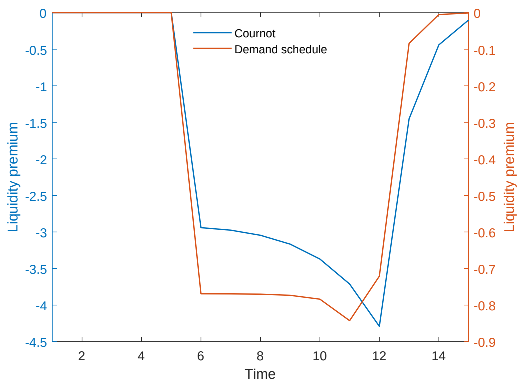


(a) Liquidity Premium

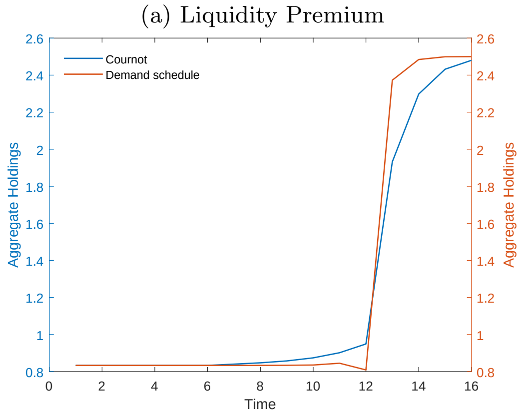


(b) Aggregate Holdings

Figure 1: Liquidity premium and aggregate holdings under Cournot competition: monopoly vs oligopoly ($n = 2$). The risk-bearing capacity is fixed at 5 in both cases, with $a = 10$. The liquidity premium is defined as the distance between the market price and the competitive price. The shock is announced at $t_1 = 5$ and occurs at $t_2 = 11$. Traders start with Pareto-optimal endowments.

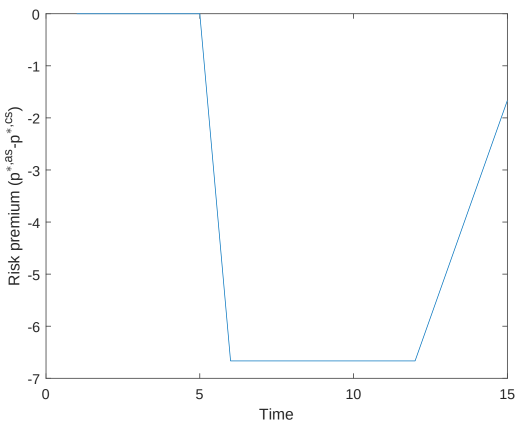


(a) Liquidity Premium

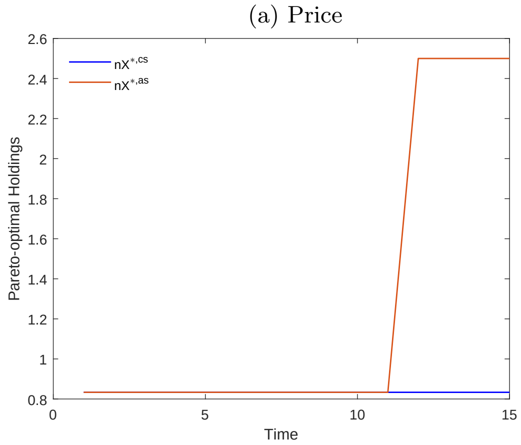


(b) Inventories

Figure 2: Liquidity premium and Inventories under Cournot and Demand Schedule competition, with multiple traders ($n \geq 2$). The shock is announced at $t_1 = 5$ and occurs at $t_2 = 11$ ($a = 5$, $b = 2$). All traders start with Pareto-optimal endowments.

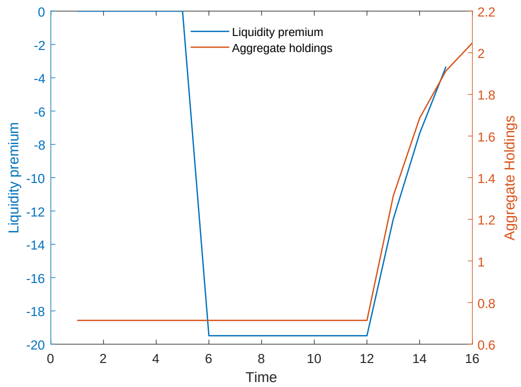


(a) Price

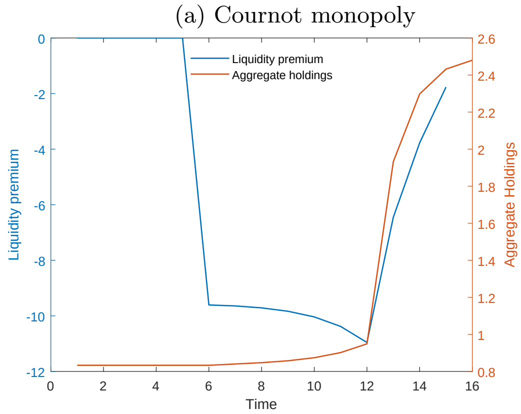


(b) Inventories

Figure 3: Price and inventory effects of an anticipated shock in a competitive market. Parameters are $D = 0$, $T = 15$, $a = 5$, $b = 2$, $n = 2$, $t_1 = 5$, $t_2 = 11$, $s = \sigma = 1$, $\Delta s_{t_2} = 2$.

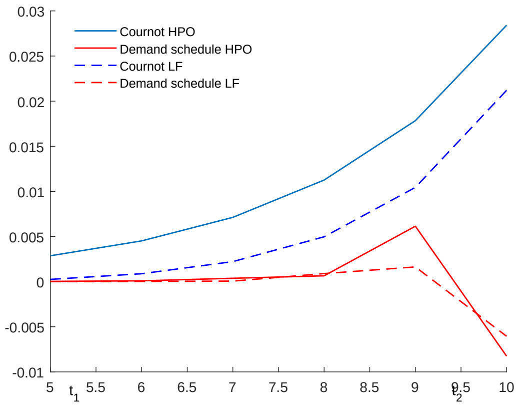


(a) Cournot monopoly

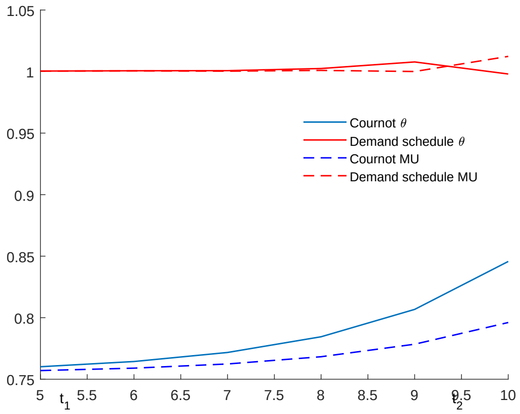


(b) Cournot oligopoly

Figure 4: Liquidity premium and inventory effects of an anticipated shock: Cournot monopoly vs oligopoly. Parameters are $D = 0$, $T = 15$, $a = 5$, $b = 2$, $t_1 = 5$, $t_2 = 11$, $s = \sigma = 1$, $\Delta s_{t_2} = 1$, $n = 1$ for panel (a) and $n = 2$ for panel (b).

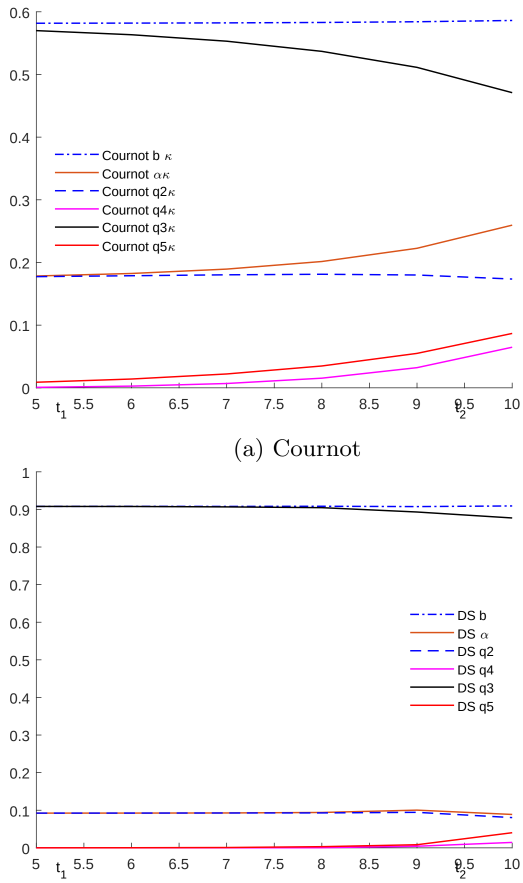


(a)



(b)

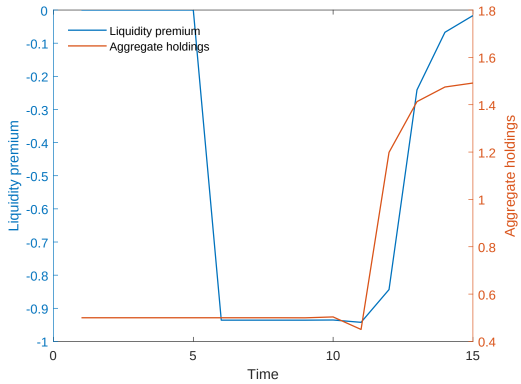
Figure 5: Two different partitions of equation (18). On the left panel, the graph shows $\sum_{\tau=t_2}^{T-1} \kappa_t^k (b\sigma^2 - Q_{t+1,\tau}^{3,5,k})$ (denoed HPO for Pareto-optimal related terms) and $\sum_{\tau=t_2}^{T-1} \kappa_t^k (na\sigma^2\alpha_{t+1,\tau}^k - nQ_{t+1,\tau}^{2,4,k})$ (denoted LF for liquidity factor-related terms), in the Cournot ($k = \mathcal{C}$) and Demand schedule ($k = \mathcal{D}$) cases. On the right panel, the graph shows $\sum_{\tau=t_2}^{T-1} \kappa_t^k a\sigma^2\alpha_{t+1,\tau}^k$ (abbreviated as θ) and $\sum_{\tau=t_2}^{T-1} \kappa_t^k (Q_{t+1,\tau}^{3,5,k}) + nQ_{t+1,\tau}^{2,4,k})$, denoted MU for marginal utility. In both panels, $t_1 = 5$, $t_2 = 11$, $s = \sigma = 1$, $\Delta s_{t_2} = 2$, $n = b = 2$, $a = 4$.



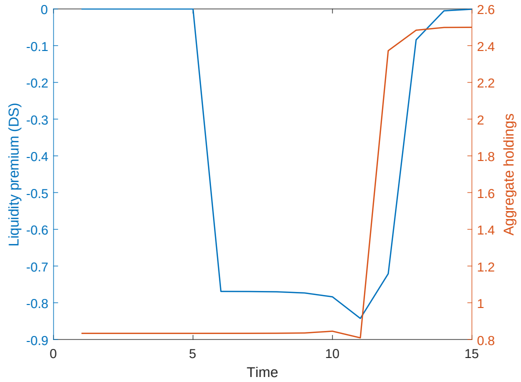
(a) Cournot

(b) Demand schedule

Figure 6: Term-by-term decomposition of equation (18). Parameters are the same as in Figure 5.

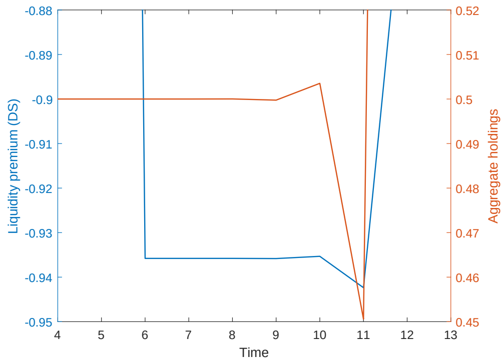


(a) Single trader

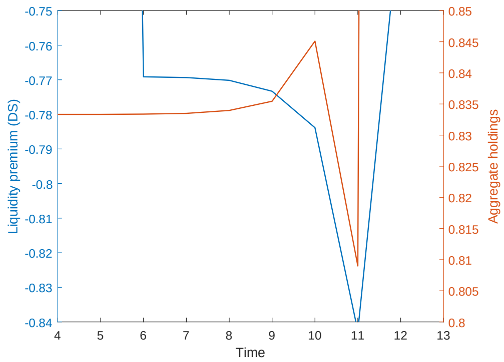


(b) Multiple traders

Figure 7: Liquidity premium and aggregate trades in the demand schedule case. In panel a, there is a single trader ($n = 1$, $a = 2 = b = 2$). In panel b, there are multiple traders ($n = 2$, $b = 2$, $a = 5$). In both cases, $t_1 = 5$, $t_2 = 10$, $s = \sigma = 1$, $\Delta s_{t_2} = 2$. All traders start with Pareto-optimal endowments.

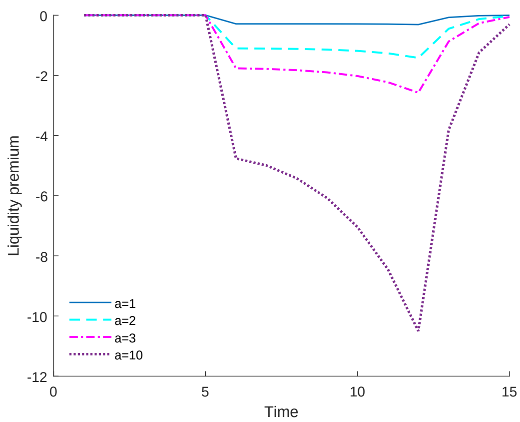


(a) Single trader

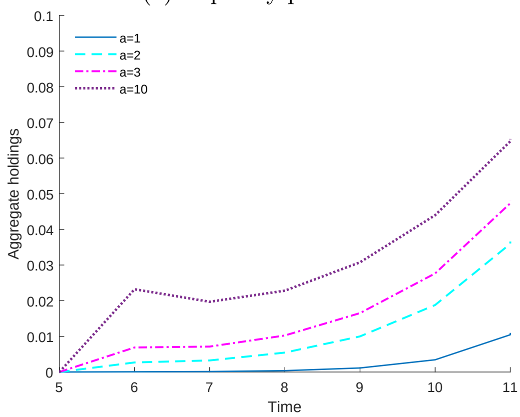


(b) Multiple traders

Figure 8: Both panels show the previous a zoomed version of the previous graphs.

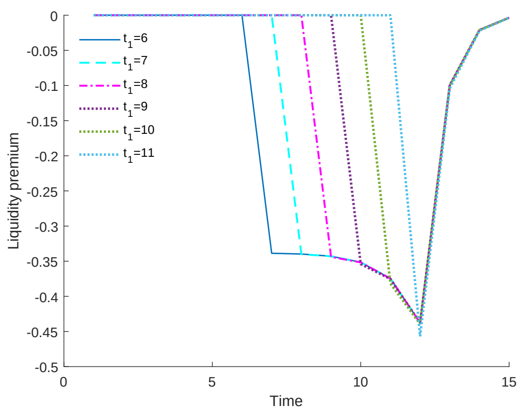


(a) Liquidity premium

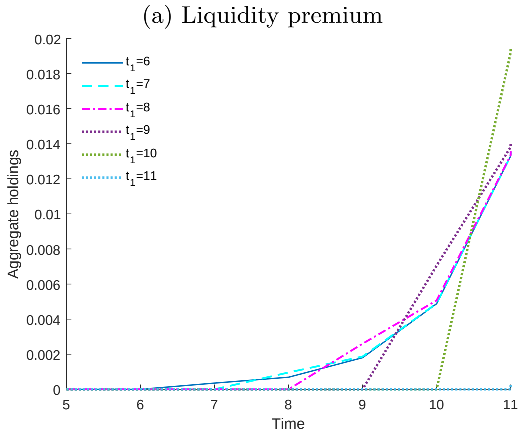


(b) Aggregate trades

Figure 9: Liquidity premium and trades as a function of price-takers' risk-aversion a. The total risk-bearing capacity is fixed at $R = 5$. Parameters are $D = 0$, $T = 15$, $n = 2$, $b = \frac{n}{R-1/a}$, $t_1 = 5$, $t_2 = 10$, $s = \sigma = 1$, $\Delta s_{t_2} = 2$. Holdings are shown only between $t = 4$ and $t = 10$.

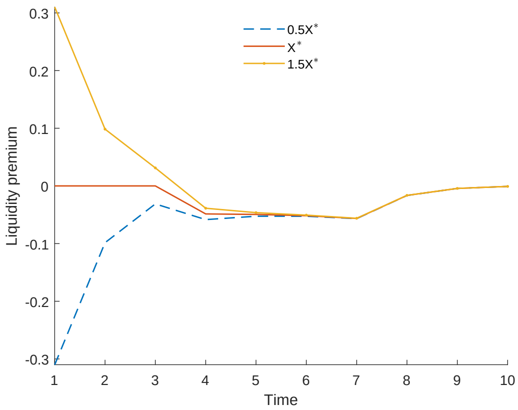


(a) Liquidity premium



(b) Trades

Figure 10: Liquidity premium and trades as a function of the announcement date t_1. The realization date is fixed at $t_2 = 11$. Parameters are $D = 0$, $T = 15$, $n = 3$, $a = b = 1$, $s = \sigma = 1$, $\Delta s_{t_2} = 2$. Aggregate holdings are shown only between $t = 5$ and $t = 10$.



(a) Liquidity premium

Figure 11: Liquidity premium as a function of traders' initial endowments X^i_{-1}. Parameters are $D = 0$, $T = 10$, $n = 2$, $a = b = 2$, $s = \sigma = 1$, $\Delta s_{t_2} = 0.1$. The realization of dividend news is 0 for all ε_t, so prices and average prices coincide.

Фардо, Винсент*.

Стратегическая торговля в условиях ожидаемых шоков спроса/предложения [Электронный ресурс] : препринт WP9/2022/02 / В. Фардо ; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (850 Кб). – М. : Изд. дом Высшей школы экономики, 2022. – (Серия WP9 «Исследования по экономике и финансам»). – 128 с. (На англ. яз.)

Исследуется влияние ожидаемых шоков спроса или предложения на цену/количество в модели стратегической торговли, в которой трейдеры с несовершенной конкуренцией делят риск с теми, кто принимает цены. Когда есть по крайней мере два трейдера, ожидаемые шоки приводят к V-образной модели, наблюдаемой эмпирически: цены отклоняются от фундаментальных показателей до шока и медленно возвращаются после него. То, как трейдеры ведут себя до шока, зависит от того, конкурируют ли они в стиле Курно (то есть подают рыночные запросы) или по кривым спроса (используя лимитные запросы). В соответствии с эмпирическими данными, трейдеры Курно действуют как противники, в то время как трейдеры по графику спроса сначала торгуют против, а затем в направлении шока.

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*Данная статья основывается на и заменяет более ранний документ, распространенный под названием «Динамический стратегический арбитраж». Статья подготовлена в рамках Программы фундаментальных исследований НИУ ВШЭ при финансовой поддержке Российского проекта академического превосходства «5–100». За полезные комментарии я хотел бы поблагодарить Мигеля Антона, Бруно Бье, Сильвана Карре, Джерома Дюгаста (участник дискуссии), Дениса Громба, Димитрия Ливдана, Семена Маламуда, Альберта Менквельда, Мартина Эмке (участник обсуждения), Юки Сато, Нормана Шурхоффа, Роберто Стери (участник дискуссии), Димитриоса Цомокоса, Костаса Захариадиса и участников семинара и конференции в Высшей школе экономики, Университете Намюра, Университете Цюриха, Франкфуртской школе финансов и менеджмента, АФА Сан-Диего, ЕФА Осло, Шестой Московской финансовой конференции, Летнего форума в Барселоне и Европейского летнего симпозиума по финансовым рынкам. Особая благодарность Алексею Булатову, Дмитрию Ваяносу и Мареку Веретке за подробные обсуждения.

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Препринт WP9/2022/02
Серия WP9
Исследования по экономике и финансам

Фардо Винсент

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(*на английском языке*)

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