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Semenov Dmitry Pavlovich

ROBUST PROCEDURES FOR IDENTIFICATION OF NETWORK STRUCTURES AND THEIR CHARACTERISTICS

PhD Dissertation Summary

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Academic Supervisor: Koldanov Petr Alexandrovich, Doctor of Science, Professor of HSE University in Nizhny Novgorod.

Review on research topic and relevance Currently, network analysis is a rapidly developing area of analysis of complex systems. A complex system is as a system consisting of a large number of interacting elements. In the study of complex systems, analysis methods based on the corresponding network models are used. A network model is understood as a simple complete weighted graph, the vertices of which correspond to the elements of the object, and the weights of the edges are given by the measure of interaction between the elements [38]. Such models are used in the analysis of stock markets [34], [6], in genetics and biology [21], in climate analysis [49], etc. In order to highlight the main links in the object under study, various network structures are extracted from the network model, which are subgraphs of the network model. One of the most popular network structures is the concentration graph - a weighted graph obtained from a network model by removing edges with zero weights. Such a network structure is studied in problems of genetics [33]. Similarly, an edge is included in a threshold graph if and only if its weight is greater than a given threshold. A clique in a graph is a set of vertices such that every two nodes in the set are connected by an edge. An independent set is a set of vertices in a graph without an edge between them. The maximum spanning tree in a weighted graph is the spanning tree of the maximum total weight. A planar maximum filtered graph is a planar subgraph of a complete weighted graph of maximum total weight. The family of threshold graphs provides information about the change in the topology of pairwise connections for various thresholds. Cliques in a threshold graph are sets of closely related elements of a complex system. Independent sets in a threshold graph are sets of unrelated elements of a complex system. The maximum spanning tree and the planar maximum filtered graph make it possible to detect the hierarchical structure of clusters in a complex system.

Among complex objects, one can single out a class of objects of a random nature, the behavior of whose elements is characterized by random variables. An example of such an object is the stock market. When constructing a network model of the stock market, the vertices of the graph correspond to the stocks of the market under consideration, and the weights of the edges between the vertices are determined by the value of the chosen measure of dependence between stock returns. In this case, the prices and returns of stocks are described by random variables [3], [27], [1].

The network approach is a popular approach of analyzing the stock market and the subject of analysis are various network structures. The maximum spanning tree (MST) was used in [34] to describe the hierarchical structure of the US stock market. This approach was further developed in [40] for portfolio analysis and in [41] for stock market clustering. Planar Maximum Filtered Graph (PMFG) was introduced in [50] and used in [46] to develop a new technique for stock market stock clustering. MST for various stock markets have been studied in [9], [10], [15], [29], [52], [36], [54]. A popular network structure, the threshold or market graph, is also used to study stock markets. This approach was proposed in [6] and developed in [7], [8]. In these papers, it was shown that the cliques and independent sets of the threshold graph contain useful information about the stock market. Various stock markets were investigated using the threshold graph [12], [17], [22], [24], [19], [37], [51] . Other network characteristics of the stock market have also received much attention in the literature. The most influential stocks associated with the stock market index are explored in [13], [14], [20], [44], [48]. Connections with random matrix theory were studied in [39], [42], [53]. Clustering and dynamics of the market network has been studied in a large number of publications, see, for example, [18], [23], [30], [31], [43], [?]. Various communication measures for stock market networks were considered in [28], [45], [52], [55]. An overview on the topic and a large bibliography is provided in [35]. Most of the publications are related to numerical algorithms and economic interpretations of the results obtained. At the same time, there is a big drawback in these studies - the analysis of the robustness of the results.

To study the robustness of the results of the analysis of stock markets, the market network is considered as a network of random variables and the uncertainty of statistical procedures for identifying network structures is investigated. A network of random variables is a pair of (X, γ) where $X = (X_1, \ldots, X_N)$ is a random vector (stock return vector) and γ is a measure of association (dependency) between components vector X (a measure of the dependence between stock returns) [25], [26]. The most popular measure of dependence between stock returns in the literature is the Pearson correlation. This measure is the most appropriate measure of dependency between random variables assuming Gaussian distributions as the joint distribution of stock returns. It should be noted that when developing an approach to constructing procedures for identifying network structures associated with the stock market, one should take into account that, as numerous studies show, empirical distributions of stock returns have heavier tails compared to the normal distribution. In this regard, a more appropriate probabilistic model for the distribution of stock market returns is a distribution model with a density that is constant on multidimensional ellipsoids (so called elliptical distributions) [2], [5]. At the same time, the adequacy of using this model is being studied in the literature and there are results that do not reject this model [32], and reject this model [11]. Thus, the investigation task of a more general mathematical model and the task of constructing procedures for identifying network structures and their characteristics that are robust in a wide class of models with a finite volume of observations become topical.

Robustness in this sense was studied in [4], [25]. In [25] two market network models (sign similarity network and Pearson correlation network) with elliptical distribution of the vector $X = (X_1, \ldots, X_N)$ were theoretically investigated. According to the article [25], a sign similarity network is a network of random variables, where the measure $\gamma_{i,j} = \gamma(X_i, X_j)$ is the probability that the signs of the random variables X_i, X_j coincide, and the network Pearson correlations is a network of random variables where the measure $\gamma_{i,j} = \gamma(X_i, X_j)$ is the Pearson correlation between random variables X_i, X_j. It has been proven that network models and network structures (market graph and MST) generated by signed similarity networks and Pearson's correlation network are equivalent. If the vector X has an elliptical distribution, it has been proven that the market graph identification and MST statistical procedures are robust in the sign similarity network. These results were obtained with the known mathematical expectation of the vector X. In [4], it was shown through simulation that the market graph and maximum spanning tree (MST) identification procedures are not robust in the Pearson correlation network, in contrast to the market graph and MST identification procedures over the sign matching network. A mixture of multivariate Gaussian and Student's distributions was used for modeling. The robustness of identification procedures for other network characteristics has not been previously studied.

Thus, the following questions remain open:

- How unstable are identification procedures based on other measures?
- Will the sign measure of similarity preserve robust for other network structures and network characteristics?
- Is robustness preserved if the expectation of the vector X is unknown?
- To what extent is the elliptic model acceptable for the class of problems under consideration?

Analytical solutions to the questions posed are unknown and difficult to obtain. The dissertation work partially answers these questions by developing programs and conducting numerical experiments using mathematical modeling.

Goals and objectives of the thesis. The purpose of the dissertation work is to study the robustness of procedures for identifying typical network structures and network characteristics in random variables networks with different similarity measures and to develop programs that allow one to study such procedures and use them to analyze random variables networks. To achieve these goals, the following specific tasks are being solved:

1. Design, implementation and testing of programs that implements robustness estimates of statistical procedures for identifying network structures for building network structures and network characteristics in random variables networks.

2. Estimates of the degree of instability of procedures based on the proximity measures of Pearson and Kendall, with deviation from the Gaussian distribution.

3. Development and application of numerical algorithms for checking the basic properties of an elliptical model using real data.

4. Development of new methods for identifying a threshold graph with a given level of confidence.

5. Analysis of the dynamics of network characteristics, in particular, analysis of the dynamics of the distribution of vertex degrees of a threshold graph.

Scientific novelty. All results are new and are as follows:

1. Robustness of identification procedures for the following characteristics of a market network is studied: the distribution of edge weights, the distribution of vertex degrees in the market graph,

cliques and independent sets of the market graph, and the distribution of vertex degrees of the maximum spanning tree. The true characteristics of the network, the losses from the error of their identification by observations, and the uncertainty of identification procedures as the expected value of losses are determined. Distributions from the class of elliptical distributions are used as a model for the multivariate distribution of stock returns. It is shown that the statistical identification procedures based on the similarity of signs are statistically robust, in contrast to the procedures based on the classical Pearson correlation.

2. A new procedure for checking the property of an elliptical model based on the symmetry condition for the tails of two-dimensional marginal distributions is proposed. A statistical procedure for multiple hypothesis testing is proposed for testing the elliptical model. The multiple hypothesis testing procedure is applied on real market data.

3. Methods for constructing a confidence set of edges for a truncated graph are developed. To construct such sets, multiple hypothesis testing procedures were used. One-step statistical procedures constructed using three types of individual tests: Pearson, Kendall, Fechner are investigated. The properties of such procedures are studied by the method of statistical modeling. The results showing the robustness of the procedures based on the combination of Kendall's and Fechner's individual tests are presented. At the same time, it is shown that the simultaneous application of hypotheses testing tests on the value of the classical Pearson correlation coefficient does not lead to robustness when the distribution deviates from normal.

4. Methods for testing the hypothesis about the homogeneity of the distributions of degrees of vertices of a threshold graph are devel-

oped.

Theoretical significance. The following results are of theoretical significance:

1. A new procedure for testing an elliptical model based on multiple hypothesis testing about the symmetry property of distribution tails;

2. Methodology for checking the robustness of procedures for identifying network structures and their characteristics;

3. An approach to constructing a confidence set for edges of a threshold graph.

Practical significance. The developed mathematical models, algorithms and programs are of practical value in the study of specific stock markets. The results of practical importance include: verification of the robustness of the procedures for identifying network structures and their characteristics; verification of compliance with the symmetry property of the elliptic model used to describe the joint distribution of stock market returns; robustness analysis of confidence set identification procedures; study of the dynamics of the distribution of degrees of vertices of the stock market graphs of various countries.

Methodology and research methods. The dissertation work uses methods of mathematical modeling of complex objects of a random nature, methods of network analysis and graph theory, methods of probability theory and mathematical statistics. When developing and implementing numerical methods, methods for constructing effective algorithms and object-oriented programming are used. When developing programs, the MatLab and Python packages were used.

Provisions submitted for defense.

1. Study of the robustness of identification procedures for various characteristics of the market network.

2. A new procedure for testing an elliptical model based on multiple testing of hypotheses about the symmetry property of distribution tails.

3. Methods for constructing a confidence set for edges of a threshold graph.

4. Methods for testing the hypothesis about the homogeneity of the distributions of degrees of vertices of a threshold graph.

Reliability of work results. The reliability of the results of the work is ensured by the strict application of the mathematical apparatus used, the correct choice of methods for conducting statistical modeling, as well as the consistency of the results of the dissertation with some well-known results published in the works [11], [25], [4], [16].

Approbation of the work. The main results of the dissertation were reported and discussed at the following international conferences, symposiums and seminars:

1. Report "Testing Hypotheses of Homogeneity of the Topology of Stock Market Network Structures"; Third Russian Economic Congress (Moscow, 2016).

2. Report "Rejection graph for multiple testing of elliptical model for market network"; XIX April international scientific conference "Modernization of the economy and society" (Moscow, 2018).

3. Report ”Characteristics of comparisons of stock market networks”; XX April International Scientific Conference ”Modernization of the Economy and Society” (Moscow, 2019).

4. Report ”How to measure dynamics of stock market network?”; XXI April International Scientific Conference ”Modernization of the Economy and Society” (Moscow, 2020).

5. Report ”Building a confidence set of connected stocks”; International Scientific Conference on Network Analysis NET 2020.

6. Report ”Investigation of influence dynamics in networks”; XXII April International Scientific Conference ”Modernization of the Economy and Society” (Moscow, 2021).

7. Report ”Detection of the dynamics of degrees of vertices of the market graph”; VI All-Russian Scientific Student Conference of the National Research University Higher School of Economics - Nizhny Novgorod, 2022.

8. Report ”Building a set of connected stocks with given confidence”; International Scientific Conference on Network Analysis NET 2022.

Some of the results presented in the dissertation were carried out within the framework of research conducted under grants:

- Grant 19-31-90088 ”Stable identification procedures for groups of dependent nodes in networks of random variables”

- Grant 18-07-00524 ”Decision-making methods in problems of identification of graph models”

- Russian Foundation for Humanities Grant 15-32-01052 ”Application of sustainable methods to the analysis of the structural characteristics of stock markets”

Related publications. Based on the dissertation materials 4 papers have been published, 1 certificate of State registration of computer programs was received.

First-tier publications

- Semenov D. A robustness comparison of two market network models/D. Semenov, A. P. Koldanov, P. Koldanov, P. M. Pardalos // IMA Journal Management Mathematics. 2022. Vol. 33. No. 1, pp. 123- 137 (WoS, Scopus).

Second-tier publications

- Koldanov A. P. Construction of a confidence set of related stocks of the stock market/A. P. Koldanov, P. A. Koldanov, D. P. Semenov // Journal of the New Economic Association. 2021. V. 2. No. 50. 12-34 (Scopus).
- Semenov D. Rejection Graph for Multiple Testing of Elliptical Model for Market Network / D. Semenov, P. Koldanov // Computational Aspects and Applications in Large-Scale Networks. Springer Proceedings in Mathematics Statistics Vol. 247. Springer, 2018. P. 221-234 (Scopus).
- Semenov D. P. Homogeneity hypothesis testing for degree distribution in the market graph / D. P. Semenov, P. A. Koldanov // Models, Algorithms, and Technologies for Network Analysis. Springer Proceedings in Mathematics Statistics / Ed. by V. A. Kalyagin, A. I. Nikolaev, P. M. Pardalos, O. Prokopyev. Vol. 197. Springer, 2017. pp. 153-162 (Scopus).
- Koldanov P.A. Estimation of robustness of procedures for identification of network structures / P.A. Koldanov, D.P. Semenov // Certificate of state registration of the computer program. Copyright holder: Federal State Autonomous Educational Institution of

Higher Education "National Research University "Higher School of Economics". Certificate No. 2018660407. Application No. 2018617498. Date of receipt 19.07.2018. Date of state registration in the Register of Computer Programs 22.08.2018.

The author's personal participation in obtaining the results presented in the dissertation. The content of the dissertation and the main provisions submitted for defense reflect the author's personal contribution to the published works. From the works performed in co-authorship to the dissertation results are included that are consistent with the personal involvement of the author.

The structure and scope of the dissertation. The dissertation consists of an introduction, four sections, a conclusion, a bibliographic list of used literature (110 titles) and an appendix. The total amount of work - 116 pages.

The introduction provides an overview of publications on the topic of the dissertation, shows the relevance and significance of the work, defines the goals and objectives of the study, and gives a summary of the content of the dissertation by chapter.

In the first section, the basic concepts are introduced and the problems of identifying network structures are formulated. Subsection 1.1 introduces the concept of a random variables network; various measures of dependence between pairs of random variables are determined, as well as various random variables networks generated by these measures of dependence; considered a network model (complete weighted graph) as a visualization of connections in a network of random variables. In subsection 1.2 various subgraphs of the network model are defined - network structures and their characteristics, which can be used to extract useful information about the network model under study. Subsection 1.3 introduces the concepts of reference and sample network structures and their characteristics. In subsection 1.4, the problem of identifying network structures is formulated as a statistical problem of choosing one of many hypotheses (the problem of choosing one of the adjacency matrices of the graph describing the network structure), the loss function from the erroneous definition of the adjacency matrix of the identified network structure is given, and the risk function is introduced, and the concept robustness of the statistical procedure.

Section 2 proposes a methodology for comparing the robustness of procedures for identifying network structures and characteristics, analyzes the robustness of data on real stock market returns, and shows how the robustness of the market network analysis is related to the chosen network model. In subsection 2.1, examples of true network models and characteristics calculated on real data from the UK market are given, and an algorithm for calculating the risk function of the network structure identification procedure is described. Subsection 2.2 presents the results of a study of the robustness of two types of procedures for identifying network structures and characteristics of network models. Procedures of the first type are based on Pearson's sample correlation, and procedures of the second type are based on the frequency of sign coincidence. The robustness study is based on modeling observations from a mixture distribution of a Gaussian distribution and a Student's distribution. An analysis of the results of studying the robustness of procedures for identifying network structures and their characteristics shows the robustness of procedures based on the frequency of sign coincidence, and the instability of procedures based on Pearson's sample correlation, and this result does not depend on the choice of network structure.

Section 3 proposes procedures for constructing a set of pairs of stocks, which contains all strongly connected pairs of stocks with a probability no less than a given one, the properties of such procedures are studied by the method of statistical modeling. Subsection 3.1 shows the general scheme for constructing confidence sets - sets of pairs of stocks, which, with a probability not less than a given one, contains all pairs of stocks, the value of the measure of connection between which is above a given threshold. Subsection 3.2 presents the mathematical formulation of the problem of constructing a confidence set. Subsection 3.3 shows a general scheme for constructing a confidence set and considers the procedure for constructing a confidence set as a problem of multiple hypothesis testing. Section 3.4 describes the procedures for constructing confidence sets in various random variables networks. Subsection 3.5 shows the results showing the robustness of procedures based on the combination of the individual tests of Kendall and Fechner, and at the same time it is shown that the simultaneous application of hypotheses tests on the value of the classical Pearson correlation coefficient does not lead to robustness when the distribution deviates from normal. Section 3.6 discusses the differences between threshold graphs built on different similarity measures and presents the results of testing the agreement with the elliptical model.

In Section 4, we check the correspondence of real data to the symmetry property of the elliptical model used to describe the joint distribution of stock market stock returns, and also show the results of the practical application of robust procedures to test homogeneity hypotheses on real data from stock markets in different countries. In Subsection 4.1, the issue of the adequacy of using the elliptical distribution as a probabilistic model of stock market returns is examined. There are known results that reject such a model, and at the same time there are results that confirm such a model, and the results obtained relate to the verification of some properties of the elliptical model. In the subsection, one more property of the elliptical model is considered, namely, the property of the symmetry condition for the tails of a two-dimensional distribution. A multiple hypothesis testing procedure is proposed to test the hypothesis of an ellipticalal model for the distribution of stock returns. The conditions of sign symmetry of the distribution of tails are used as individual hypotheses for multiple testing, and uniformly the most powerful tests of the Neyman structure are constructed to test individual hypotheses. For real market data, an appropriate step-by-step procedure for multiple hypothesis testing was applied. The main result is that, under certain conditions, the tail symmetry hypothesis is not rejected. In Subsection 4.2, we study the problem of testing the hypothesis about the homogeneity of the distribution of degrees of vertices in a threshold graph. A procedure for multiple hypothesis testing for the stock markets of China and India has been proposed and applied. The procedure is built using the bootstrap method for individual hypotheses and the Bonferroni correction for multiple testing. It is shown that the hypothesis about the homogeneity of the distribution of degrees for the stock markets for the period 2003-2014 not acceptable. In Subsection 4.3, the Wilcoxon rank sum test is used to detect the dynamics of the vertex degrees of the market graph, since the application of such a test does not require assumptions about the distribution model of the random variables under study and the distribution model of the vertex degrees of the market graph. The main result is the conclusion about the presence of the dynamics of the US stock market, indicating the process of globalization. At the same time, this conclusion is not correct for the Russian market.

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