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Theory of transport phenomena in coupled low-dimensional superconductors

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Introduction

0.1 Quantum phase slips

Quantum fluctuations have a significant effect on the physics of superconducting nanowires at sufficiently low temperatures, making their behavior dramatically different from the behavior of bulk superconducting samples [ZG19, SZ13, AGZ08, SZ22]. Many of them are directly related to quantum phase slips QPS [ZG19, LMB+01, Hav10] which manifest themselves by local temporal suppression of the superconducting order parameter $\Delta = \Delta_0(x,t)e^{i\varphi(x,t)}$ inside the wire accompanied by jumps of the phase $\pm 2\pi$ Fig. 1.

It is known from the Golubev-Zaikin theory [ZG19, vOGZB99], the primary dynamic variable of a superconducting wire in the low energy limit is the phase of the order parameter. The correspondence effective action for superconducting phase has the next form

$$S_{eff}[\varphi] = \frac{Cv}{8e^2}\int dt \int dx \left(\frac{1}{v}(\partial_t\varphi(x,t))^2 + v(\partial_x\varphi(x,t))^2\right),$$

where C is the geometric capacitance per unit wire length, $v \propto \sqrt{s}$ is the velocity of plasmon excitations in the system. According to the Josephson relation $V = \dot{\varphi}/2e$, the first term in the effective action is related to the capacitive energy of the wire, while the second term corresponds to the magnetic energy of the superconducting condensate. The intensity of phase fluctuations is controlled by the dimensionless parameter $\lambda = \pi Cv/4e^2$. The effective action on the QPS configuration is large, and the amplitude of phase slips per unit wire length γ_{QPS} is exponentially suppressed

$$\gamma_{QPS} \sim \frac{g_\xi \Delta_0}{\xi} e^{-ag_\xi},$$

where $g_\xi = R_q/R_\xi$-dimensionless conductance of a wire section, this the size of the coherence length ξ, $R_q = 2\pi/e^2$-quantum resistance and R_ξ-normal wire resistance, $a \approx 1$-numerical prefactor.

Each QPS process generates plasma excitations with the sound spectrum $\omega = kv$, which propagate along the wire with a speed v and interact with others QPS. Such excitations are called Mooij-Schon plasma modes [MS85]. The presence of such plasma excitations is an important feature of long superconducting nanowires, leading to a number of interesting effects. In particular, the theoretically predicted [RSZ17, RSZ19] and experimentally discovered [ALR+17, ALR+21] smearing of the root singularity in the density of states (DOS) near the superconducting gap, accompanied by a non-vanishing tail in the density of states at subgap energies.

The exchange of Mooij-Schon plasmons gives a logarithmic interaction in the space-time between different quantum phase slips, which at large distances R is proportional to $\sim \lambda \log(R/\xi)$. As it can be seen from the form of interaction, its value is controlled by the parameter λ, and hence the diameter of the wire (cross-sectional area). For sufficiently thick wires, the force of this interaction is sufficiently large, so QPS exist only in bonded pairs [ZG19].

In this case, theory containes a small parameter γ_{QPS} and therefore the influence of QPS phenomena can be studied perturbatively. The effect of interaction between different QPS can significantly

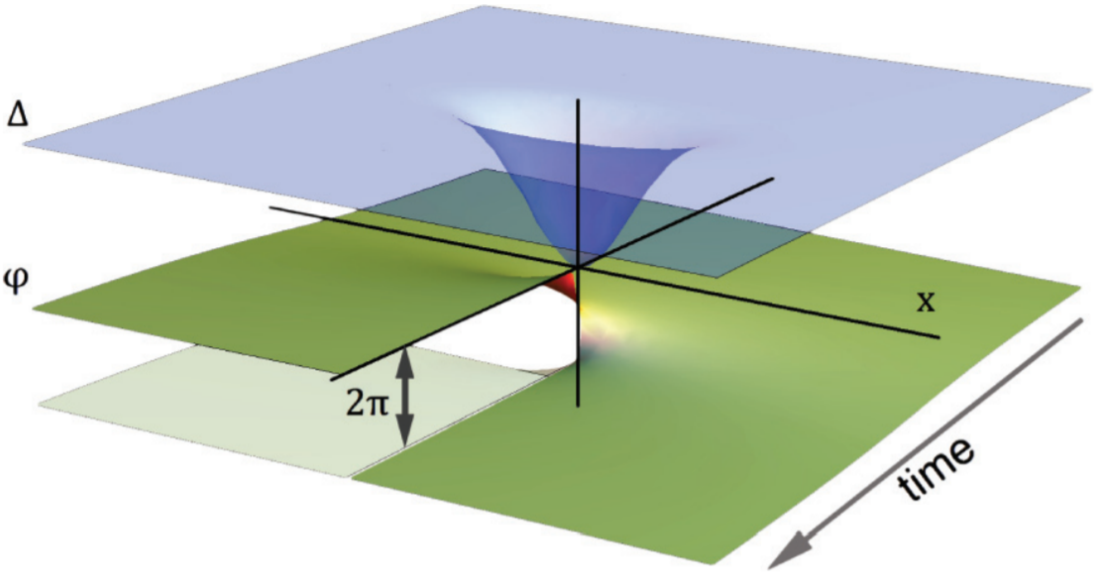

Figure 1: QPS configuration.

affect on the transport properties in a thin superconducting wire. In particular, lead to the appearance of a finite resistance of the superconducting wire $R(T) \propto \gamma^2_{QPS} T^{2\lambda-3}$, which tends to zero in the $T \to 0$ limit, demonstrating superconducting behavior, which agrees with the experimental results [LMB+01, BLT00, ZRTA08, BCA+16].

The investigation of the resistive state in a superconducting wires due to QPS phenomena was carried out using the methods of the imaginary part of free energy in imaginary time in [AGZ08] and aslo within the Keldysh technique in [ZGvOZ97, SZ16]. In addition, it was demonstrated that this two different approaches are totally equivalent [SZ17].

It is also interesting to note that, according to the fluctuation dissipation theorem FDT, phase slips also generate voltage noise in superconducting wires. Of particular interest is the study of nonequilibrium voltage noise, which is described by Poisson statistics in the low-frequency limit. The spectral noise power $S(\omega)$ has a power-law dependence on the flowing external current I. This noise can be caused by both thermal phase slips (TAPS) and quantum tunneling (QPS). At zero frequency limit, the voltage noise due to TAPS and QPS processes has Poisson statistics. An interesting observation is that at finite frequencies (or is the same at short times) the voltage noise induced by the QPS processes no longer satisfies the Poisson statistics and exhibits a rather non-trivial behavior [SZ16].

On the other hand, the interaction between QPS in ultra-thin (this the cross section 10 nm and thinner) wires is weak, thus the phase slips are decoupled and hence the phase of the superconducting order parameter fluctuates strongly along the wire. The parameter γ_{QPS} becomes large, so that the interaction effect between different QPS becomes nonperturbative. In this case, the wire loses long-scale superconducting properties, its total resistance remains finite and increases with temperature decreasing. Thus, in this limit the superconducting wire exhibits insulating behavior even at $T \to 0$. At zero temperature, the transition between these two regimes occurs as a quantum phase transition (QPT) controlled by the wire diameter. Here and below, we will refer to this type of quantum phase transition as a superconductor-insulator transition SIT.

Surprisingly, it turns out that the interplays between QPS phenomena and the transport properties of coupled superconducting systems have hardly been studied in the literature. At the same time, there are a large number of interesting issues in this area, both from a purely theoretical and applied point of view. In particular, in the paper [Ari07] by considering the transport properties of a system of coupled Luttinger liquids, it was shown that in order to have the Coulomb drag effect in such a system it is neccesary to take into account the nonlinearity (curvature) in the spectrum.

This thesis is devoted to study the interplay between QPS phenomena and transport properties in a system of capacitively coupled superconducting nanowires. In this regard, we have set the following tasks:

- Investigate the dynamics of Mooij-Schon plasma modes in a system of coupled wires
- Investigate non-local voltage fluctuations in such a system. In particular it is important to calculate the first and second voltage cumulants, which correspond to the average voltage and the spectral noise power respectively.
- Provide the renormalization group analysis in the system under consideration and find the BKT phase transition point.
- Investigate the Coulomb drag effect induced by QPS in a system of capacitively coupled nanowires.

0.2 Main results

In the **first** chapter we analyze the effects of quantum fluctuations on the critical and transport properties in a system of coupled superconducting nanowires.

- The quasiclassical dynamics of plasmon excitations in such a system was studied. We demonstrate that in the presence of inter-wire coupling plasma modes in each of the wires get split into two "new" modes propagating with different velocities v_+ v_-

$$v_\pm = \frac{1}{2k}\left[\sqrt{v_1^2+v_2^2+2v_1v_2k} \pm \frac{\sqrt{v_1^2+v_2^2+4C_m^2v_1^2v_2^2}}{\sqrt{v_1^2+v_2^2+2v_1v_2k}}\right],$$

  across the system. These plasma modes form an effective dissipative quantum environment interacting with electrons inside both wires and causing a number of significant implications for the low-temperature behavior of the systems under consideration. A clear picture describing the time evolution of the Mooij-Schon plasma modes and, accordingly, the voltage pulses in the first and second wires is shown in Fig. 2. Our results might have significant implications for the

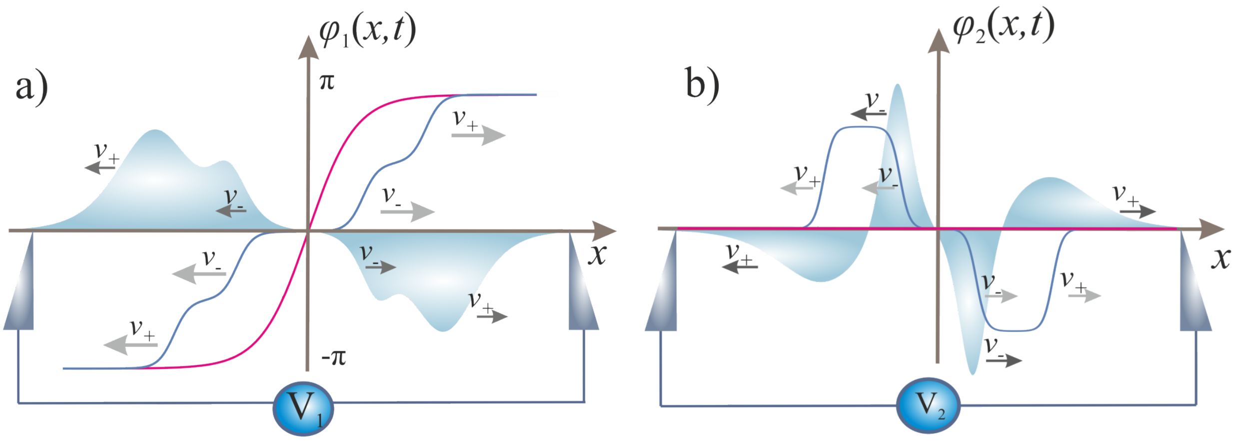

Figure 2: Time-dependent phase configurations in a first (a) and second (b) wires at initial configurations (QPS in the first wire and zero condition in the second), and also corresponding voltage pulses propagating along the wires. Each of these voltage pulses is split into two, with velocities $v_\pm$.

low-temperature behavior of coupled superconducting nanowires. For instance, electron DOS in each of the wires can be affected by fluctuations in a somewhat different manner as compared to the noninteracting case [RSZ17, RSZ19, ALR+17].

- We investigated the influence of capacitive coupling on critical properties of system of coupled wires. We derive a set of coupled Berezinskii-Kosterlitz-Thouless-like renormalization group equations

$$\frac{dy_i}{d\log\Lambda} = (2-\lambda_{ii})y_i, \quad i=1,2,$$

where λ_{ii}-diagonal components of matrix

$$\check{\lambda} = \frac{1}{\sqrt{\frac{1}{v_1^2}+\frac{1}{v_2^2}+\frac{2\sqrt{1-\frac{C_m^2}{C_1C_2}}}{v_1v_2}}}\begin{bmatrix} \lambda_1\left(\frac{1}{v_1}+\frac{\sqrt{1-\frac{C_m^2}{C_1C_2}}}{v_2}\right) & R_qC_m/8 \\ R_qC_m/8 & \lambda_2\left(\frac{1}{v_2}+\frac{\sqrt{1-\frac{C_m^2}{C_1C_2}}}{v_1}\right)\end{bmatrix}.$$

demonstrating that interaction between quantum phase slips in one of the wires gets modified due to the effect of plasma modes propagating in another wire. Since the diagonal components λ_{ii} essentially depend on the interaction parameter C_m between the wires, changing the distance between them can significantly affect on corresponding critical properties.

As an example, according to the phase diagrams shown in Fig. 3. superconducting wires can

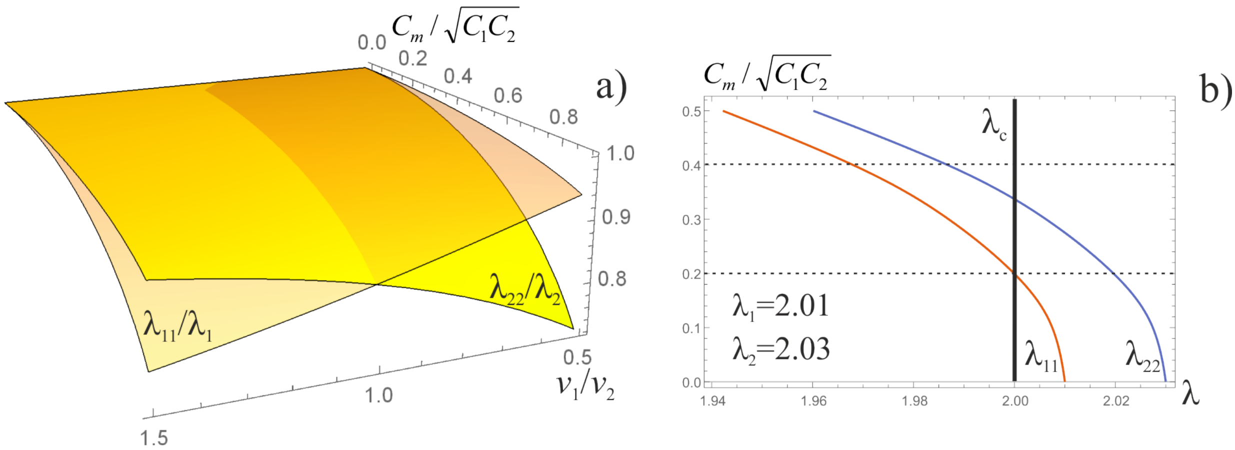

Figure 3:) Critical surfaces corresponding to SIT at $\lambda_{11}=2$ and $\lambda_{22}=2$. b) Phase diagram for two capacitively coupled superconducting nanowires with $\lambda_1=2.01$ and $\lambda_2=2.03$. Both curves $\lambda_{11}(C_m)$ and $\lambda_{22}(C_m)$ crease and cross the critical line $\lambda_c=2$ with increasing mutual capacitance C_m.

become insulators at certain values of C_m. In this chapter, we also considered the generalization of the obtained results to a superconducting wire of more complex geometry.

In the **second** chapter, we investigated non-local voltage fluctuations in the system shown in Fig. 4. The main interest was the study of voltage fluctuations in the second wire, generated by QPS processes in the first one.

- We calculated the average value of voltage for both wires in such a systems. Corresponding expression takes the from

$$<\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}>=\begin{bmatrix} \Phi_0\left(\Gamma_{QPS}(I\Phi_0)-\Gamma_{QPS}(-I\Phi_0)\right) \\ 0 \end{bmatrix},$$

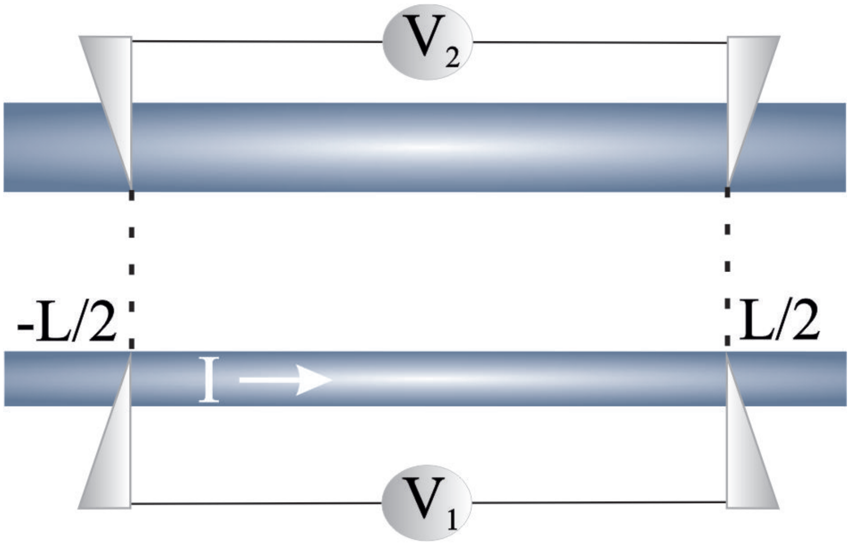

Figure 4: Sustem of capacitively coupled superconducting wires. The first wire is thin enough so that quantum phase slip processes can occur with the amplitude γ_{QPS}. The second wire is thicker, implying that one can completely disregard QPS effects in this wire. An external current bias I is applied to the first wire, whereas no transport current flows across the second wire

where

$$\Gamma_{QPS}(\omega) = \gamma_{QPS}^2(2\pi T\tau_0)^{2\lambda_{11}} e^{\frac{\omega}{2T}} \frac{|\Gamma(\lambda_{11} + \frac{i\omega}{2\pi T})|}{16\pi T\Gamma(2\lambda_{11})}$$

defines the quantum decay rate of the current state in the first wire due to QPS. It has also been shown that the addition of a linear dissipative element to the considering system gives the same result. In particular, in a system with a resistor, the average voltage in the second wire also vanishes.

- We have obtained an expression for the spectral noise power $S_2(\omega)$ for the second wire at finite frequency

$$S_2(\omega) = iG^K_{V_2V_2}(\omega) + \Big(\frac{1}{2}G^R_{V_2\chi_1}(x_{qps},\omega)G^R_{V_2\chi_1}(x_{qps},-\omega) \times \sum_{\pm}[\Gamma_{QPS}(\omega \pm I\Phi_0)] + G^R_{V_2\chi_1}(x_{qps},\omega)G^K_{V_2\chi_1}(x_{qps},\omega) \times \sum_{\pm}[\Gamma^R_{QPS}(\omega \pm I\Phi_0) - \Gamma^R_{QPS}(\pm I\Phi_0)] + (\omega \to -\omega)\Big),$$

which contains contributions from both the equilibrium Johnson-Nyquist noise and the non-equilibrium contribution induced by QPS processes in the first wire.In the limit, when the time during which the voltage pulses reach the voltage contacts is small, the contribution induced by QPS processes takes a simple form

$$S_2^{QPS}(\omega) = \tau_{21}^2\Phi_0^2\omega^2 \sum_{\pm}\left[\left(2n_B + \frac{3}{2}\right)\Gamma_{QPS}(-\omega \pm I\Phi_0) - \left(2n_B + \frac{1}{2}\right)\Gamma_{QPS}(\omega \pm I\Phi_0)\right],$$

whose graph is shown in Fig. 5. We demonstrate that quantum phase slips in one of these nanowires induce voltage fluctuations in another one. These fluctuations are characterized by

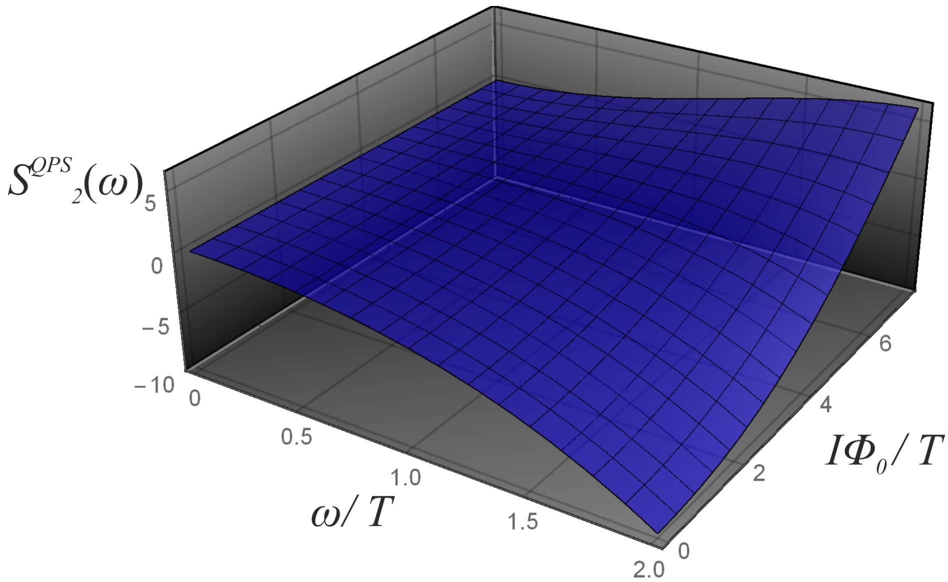

Figure 5: QPS-induced voltage noise frequency spectrum in the second wire (arbitrary units)

zero average voltage and non-vanishing voltage noise which exhibits a non-trivial behavior as a function of frequency and bias current. However, these fluctuations persist even in the limit $I \to 0$, i.e., provided our structure remains in equilibrium. It is interesting to note that for some values of the external current, $S_2^{QPS}(\omega)$ can take negative values, as can be seen from Fig. 5. At the same time, it was also shown that the equilibrium contribution $G^K_{V_2V_2}$ is parametrically larger than $S_2^{QPS}(\omega)$, so that the noise spectral power $S_2(\omega)$ remains positive, which is in a good agreement with the general principles.

The **third** chapter is devoted to study the Coulomb drag effect in a system of capacitively coupled superconducting nanowires. The main interest is to obtain a non-zero average voltage in the second wire, while the external bias current flowing through the first wire.

- Based on the analysis of the dynamics of plasma excitation in the observed system, it was demonstrated that in order to obtain the non-vanishing induced average voltage in the second wire it is necessary to add the nonlinear element to the system. Which agrees with the results of set of works devoted to study the Coulomb drag effects in various systems [BSS19], [Ari07].

  It was demonstrated that the role of non-linear element in considering system may plays the corresponding QPS phenomena in the second wire. When the voltage pulses in the second wire, generated by the QPS processes in the first will interact in a non-linear manner with the QPS in the second wire, which will lead to asymmetry of the contributions at the voltage contacts and, accordingly, to a non-zero average voltage

- Since the interaction Hamiltonian describing QPS processes can be represented as a sum of vertex operators $\hat{L}_i^\sigma(x,\tau) = e^{i\sigma\hat{\chi}_i(x,\tau)}$ of flux quantum tunneling Φ_0

$$H_{int}(t) = -\frac{\gamma_1}{2}\sum_{\sigma=\pm}\int_{-L/2}^{L/2} dx_1 \hat{L}_1^\sigma(x_1,t)e^{i\sigma I\Phi_0 t} - \frac{\gamma_2}{2}\sum_{\sigma=\pm}\int_{-L/2}^{L/2} dx_1 \hat{L}_2^\sigma(x_1,t),$$

  then in order to calculate the induced voltage in the second wire, it is convinient to use the operator perturbation theory, where the average over the ensemble provided as

$$\langle \hat{V}_2(t) \rangle = \sum_{k=1}^{N} (-i)^k \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_{k-1}} dt_k \times \langle [[\hat{V}_2^0(t), \hat{H}_{QPS}(t_1)], \hat{H}_{QPS}(t_2)], \cdots, \hat{H}_{QPS}(t_k)] \rangle.$$

Thus, the perturbative calculation of the average voltage in the second wire was reduced to the analysis of multiple commutators of $\hat{L}_i^\sigma$ operators.

- We obtain an expression for the induced voltage in the second wire perturbatively with respect to the QPS amplitudes γ_1,γ_2 of the first and second wires, respectively. It has also been demonstrated that the induced voltage can be represented as the sum of three contributions, each of which is proportional to an operator of the form

$$\langle \hat{L}_{2ijk}^{\sigma_1 \ldots \sigma_4}(t_1, x_1; \ldots t_4, x_4) \rangle = \langle e^{i\sigma_1 \hat{\chi}_2(x_1,t_1) + i\sigma_2 \hat{\chi}_i(x_2,t_2) + i\sigma_3 \hat{\chi}_j(x_3,t_3) + i\sigma_4 \hat{\chi}_k(x_4,t_4)} \rangle, \ i, j, k = 1, 2.$$

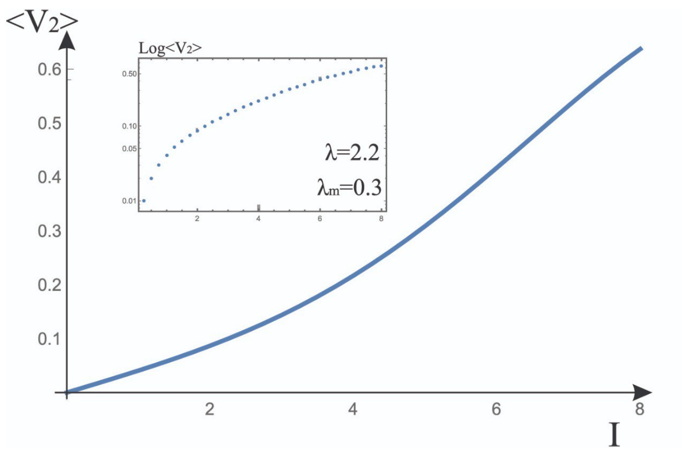

Figure 6: I-V curve for the induced voltage $\langle V_2(t) \rangle$ in the limnit $I\Phi_0 \gg T$.

- We also obtained the corresponding I-V dependence for the induced voltage in the second wire in the $I\Phi_0 \gg T$ limit, shown in Fig. 6.

- It was also demostrated that in the low temperature limit the cross resistace of the second wire as a function of external bias current flowing in the first wire demostrate a power law behavior as $R_{21} \sim \gamma_1^2 \gamma_2^2 I^{4\lambda - 7}$. In the opposite limit $I\Phi_0 \ll T$, the cross resistance is a power function of temperature, showing the universal behavior as $R_{21} \sim \gamma_1^2 \gamma_2^2 T^{4\lambda - 7}$.

The thesis is based on three publications.

1. Latyshev A., Semenov A. G., Zaikin A. D. *Voltage fluctuations in a system of capacitively coupled superconducting nanowires* //Journal of Superconductivity and Novel Magnetism. – 2020. – . 33. – №. 8. – . 2329-2334.

2. Latyshev A., Semenov A. G., Zaikin A. D. *Superconductor–insulator transition in capacitively coupled superconducting nanowires* //Beilstein journal of nanotechnology. – 2020. – . 11. – №. 1. – . 1402-1408.

3. Latyshev A., Semenov A. G., Zaikin A. D. *Plasma modes in capacitively coupled superconducting nanowires* //Beilstein Journal of Nanotechnology. – 2022. – . 13. – №. 1. – . 292-297.

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