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Aleksandr M. Kiselev

MAGNETIC FIELDS AND DIFFUSION OF COSMIC RAYS IN GALAXY

PhD Dissertation Summary

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This PhD dissertation was prepared at P.N. Lebedev Physical Institute of the Russian Academy of Sciences

Academic Supervisors: **Yakov N. Istomin**,
Doctor of Science, Professor,
Chief Researcher at LPI,

Vladimir A. Dogiel,
Doctor of Science, Professor,
Chief Researcher at LPI

DISSERTATION TOPIC

The dissertation is devoted to the exploration of magnetic fields in turbulent gas and diffusion of charged particles in a stochastic magnetic fields.

Relevance of the given topic

The standard theory of cosmic rays (CR) formation suggests that primary CR consist mainly of protons and do not contain antimatter. During their propagation in the Galaxy primary CR interact with protons of Galactic gas, resulting in production of secondary CR, including antiprotons and positrons. The secondary particles energy spectrum, calculated in the framework of this theory, falls down with energy by as a power-law. The antiparticles to particles ratio should behave in the same way [1]. However, recent antimatter observations by PAMELA satellite detected an excess of positrons with energies $10 - 100$ GeV in cosmic rays [2]. These results were later extended [3] and confirmed by satellite Fermi [4] and AMS [5]. According to the latest AMS data, an excess of positrons is observed up to energy 500 GeV.

These data attracted much attention. Several theoretical explanations for this effect were proposed, among them the positrons generation in pulsars and in the annihilation of dark matter particles. However another mechanism for the positrons generation in the Galaxy is also possible, that is acceleration of charged particles in giant molecular clouds and secondary CR production there. This mechanism was discussed by Dogiel V. A., Gurevich A. V., Istomin Ya. N. and Zybin K. P. [6, 7, 8] long before the launch of the PAMELA satellite in 2006. However, in these works, a simplified model of particle propagation in the magnetic fields of molecular clouds was used. New data of positrons excess require a thorough study of cosmic rays in molecular clouds.

Molecular clouds are composed of neutral hydrogen. The gas in molecular clouds is weakly ionized, and its motion is turbulent. In such a system, a stochastic magnetic field arises. This process is called the turbulent dynamo. The propagation of charged particles in stochastic magnetic field has the form of diffusion. To calculate the distribution of cosmic rays inside a cloud, it is required to know the diffusion coefficient for particles with different energies. The diffusion coefficient is determined by the spectrum of the magnetic field. Because experimental data are scarce, the magnetic field spectrum in molecular clouds can only be calculated using theoretical models.

One of the popular analytical approaches to the dynamo problem in a turbulent medium is the Kazantsev-Kraichnan model [9], [10] . The mean field model (see for example [11]) and the method of Lagrangian deformations [12] are also used. The equations of single-fluid magnetohydrodynamics are generally used in the literature [13, 14, 15, 16, 17, 18]. Weakly ionized gas in molecular clouds should be described by two-fluid magnetohydrodynamics. The dynamo in such a system has been studied in a few papers, see [19].

Despite many years of research into the magnetic dynamo, many questions in this field remain open. For example, it is known that molecular clouds in the Galactic center are surrounded by a uniform magnetic field. However in theoretical models the role of the mean field is poorly studied. On the other hand, in the Kazantsev-Kraichnan model, the velocity field is considered to be Gaussian random process with zero mean, so all odd-degree correlators are zero. However, this is not true in real turbulence. Kolmogorov's "four-fifths" law states that the third correlator is non-zero. For a better understanding of the magnetic dynamo process, we should take into account the mean magnetic field and the non-Gaussianity of the velocity field in theoretical models.

The structure of the magnetic field determines the diffusion coefficients of charged particles in molecular clouds. However, the calculation of the diffusion coefficient is a non-trivial problem.

The most developed approach for calculation of diffusion coefficients for magnetized particles is the quasilinear theory [20], and its generalizations. In the quasilinear theory magnetic field is assumed to be a sum of the mean homogeneous field and small random fluctuations. It allows one to use the perturbation theory and find the diffusion coefficient analytically. However in many astrophysical objects magnetic fluctuations are comparable with the mean field or even exceed it. So the problem of particle diffusion in the magnetic field with large fluctuations has many applications in CR astrophysics. Among them are problems of propagation of CR in Galactic molecular clouds [21], in the Galaxy as a whole, in clusters of galaxies [22], as well as the problem of CR acceleration on shock waves in supernova remnants [23].

Goals and objectives

The goal of this work is to calculate the diffusion coefficient of charged particles in a stochastic magnetic field.

To achieve this goal, the following objectives were gained:

1. Explore the spatial structure of the magnetic field in Galactic molecular clouds. Derive the evolution equation for the pair correlator of the magnetic field, taking into account the mean magnetic field. Investigate the tensor structure of the anisotropic correlators of the magnetic field in this case. Find the relation between mean magnetic field and the amplitude of magnetic fluctuations.

2. Calculate the correlation length of the magnetic field in the molecular cloud Sgr B2.

3. Construct a generalization of the Kazantsev-Kraichnan model for a non-Gaussian fluid velocity field taking into account third velocity correlator. Derive the evolution equation for the magnetic field pair correlator and investigate its exponential modes. Find the conditions when the magnetic field increases.

4. Calculate the diffusion coefficient of charged particles in a stochastic magnetic field with a small mean field. Find a configuration of a regular magnetic field in which the equations of particle motion are integrated analytically. Average particle motion over the spectrum of the magnetic field and find the dependence of the diffusion coefficient on their energy.

KEY RESULTS

The key results of the work are as follows:

1. The problem of the evolution of a magnetic field in a weakly ionized turbulent gas is solved. Evolution equations of the magnetic field pair correlator are obtained. In the isotropic case with zero mean field, this is one equation of the second order in r. Taking into account mean magnetic field a system of anisotropic equations is obtained. This system is solved in two limit cases. For large mean field anisotropic solution is found analytically. For small mean field the equations are solved in the approximation of isotropic correlators. The unique stationary solution is found for any value of mean field. The amplitude of fluctuations and the correlation length of the magnetic field is calculated as a function of mean field.

2. The correlation length of the magnetic field in Sgr B2 molecular cloud is calculated.

3. Generalization of the Kazantsev-Kraichnan model for non-Gaussian velocity field is developed. The contribution of the three-point velocity correlator to the magnetic field evolution equation is obtained. The exponential modes of this equation are studied numerically. There is a continuous transition from our generalized model to the Kazantsev-Kraichnan model. It is shown that taking into account the three-point velocity correlator reduces the growth rate of the magnetic field.

4. The problem of charged particles diffusion in a random force-free magnetic field is solved. The equations of particle motion are reduced to the pendulum equation. The transition from trapped to untrapped particles is continuously traced. For small Larmor radius r_L of the particle the diffusion coefficient turns

out to be proportional to the Larmor radius for all reasonable magnetic field spectra.

Author's personal contribution

All main results of the dissertation are original and received for the first time. All the results presented in the dissertation were obtained by the author personally or with his direct participation.

The evolution equations of the magnetic field correlators in weakly ionized gas with mean field were obtained by the author personally. The analysis of this equations was carried out personally by the author.

The author worked on the problem of a non-Gaussian velocity field as part of the group of K.P. Zybin. Autor derived evolution equation in the inertial interval. Autor perform the numerical study of exponential modes of this equation.

The idea to explore the diffision in a force-free magnetic field belongs to Ya.N. Istomin. The diffusion coefficient was calculated personally by the author.

PUBLICATIONS AND APPROBATION OF RESEARCH

First-tier publications

The main results of the work are published in five papers in peer-reviewed journals included in the Web of Science database:

1. Istomin, Ya N., and A. Kiselev. "Magnetic field generation in Galactic molecular clouds." Monthly Notices of the Royal Astronomical Society 436.3 (2013): 2774-2784

2. Dogiel, V. A., Chernyshov, D.O., Kiselev, A.M. & Cheng, K.-S. "On the origin of the 6.4 keV line in the Galactic Center region." Astroparticle Physics 54 (2014): 33-39.

3. Dogiel, V. A., Chernyshov, D.O., Kiselev, A.M., Cheng, K.-S., Hui, C. Y. , Ko, C. M., Nobukawa, K. K. & Tsuru, T. G. "Spectrum of relativistic and subrelativistic cosmic rays in the 100 pc central region." The Astrophysical Journal 809.1 (2015): 48.

4. Istomin, Ya N., and Kiselev, A. M. "Diffusion of charged particles in a stochastic force-free magnetic field." Physical Review D 98.8 (2018): 083026.

5. Kopyev, A. V., Kiselev, A.M., Il'yn, A.S., Sirota, V.A. & Zybin, K.P. "Non-Gaussian Generalization of the Kazantsev–Kraichnan Model for a Turbulent Dynamo." The Astrophysical Journal 927.2 (2022): 172.

Reports at conferences and seminars

The results of the work were reported by the author at Russian and international scientific conferences:

1. Russian conference on physics and astronomy Physics.A. October 24-25, 2012, St. Petersburg, Ioffe Institute
2. Russian Astronomical Conference «Many Faces of the Universe», September 23-27, 2013, St. Petersburg
3. The ISSI-BJ Meeting «New Approach to Active Processes in Central Regions of Galaxies», June 1-5, 2015, Beijing, ISSI-BJ. «Structure of magnetic fluctuations excited by a turbulence of neutral gas inside molecular clouds»
4. The ISSI-BJ Meeting «New Approach to Active Processes in Central Regions of Galaxies», June 6-8, 2016, Beijing, ISSI-BJ. «Diffusion of charged particles in the turbulent magnetic feld»

The obtained results were also presented by the author at the Astrophysical Seminar in Tamm theoretical department of Lebedev Physical Institute.

CONTENTS

This dissertation contains an introduction, three main chapters, conclusion and two appendices. The volume of dissertation is 123 pages with 20 figures and one table. The reference list contains 63 items.

Chapter 1 is devoted to the study of magnetic field generation by turbulent weakly ionized gas in the presence of mean magnetic field. The motion of a neutral and ionized gas should be described by the two-fluid magnetohydrodynamic equations. The equation for the magnetic field is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \Delta B - a \nabla \times (\mathbf{B} \times (\nabla \times \mathbf{B}) \times \mathbf{B}). \tag{1}$$

where $a = (4\pi\rho_i\mu_{in})^{-1}$. To investigate the dependence of the magnetic field on time, we will study the two-point correlator $\langle \mathbf{B}(\mathbf{r})\mathbf{B}(\mathbf{r}')\rangle$.

In the first chapter, we use the Kazantsev-Kraichnan model and assume that the neutral gas velocity is a delta-correlated Gaussian stochastic process. We also assume the magnetic field to be a Gaussian stochastic process. In the isotropic case, the tensor structure of the magnetic field correlator is described by one scalar function $Q(t,r)$

$$\langle B_i(\mathbf{x},t)B_j(\mathbf{x}+\mathbf{r},t)\rangle = 2Q(t,r)\delta_{ij} + rQ'(t,r)(\delta_{ij} - \frac{r_i r_j}{r^2}). \tag{2}$$

To split the $\langle vB^2\rangle$ correlator, we use the Furutsu-Novikov formula, see [24]. The correlator $\langle B^4\rangle$ is expressed in terms of the product of pair correlators.

In the case of small mean field $H \ll L_0 v_0 \rho_i \mu_{in}$, we obtain the evolution equation

$$\frac{1}{2\tau_c}\frac{\partial Q(r)}{\partial t} = (V(0) - V(r) + \lambda')\,(Q'' + \frac{4Q'}{r}) - V'Q' - \frac{1}{r}(4V' + rV'')(Q + \frac{1}{6}H^2), \tag{3}$$

where we introduce the notation

$$\lambda' = \frac{\eta + 4a(Q(0) + H^2/6)}{\tau_c}. \tag{4}$$

The equation (3) is nonlinear. In this chapter, we search for stationary solutions only. It turns out that there is unique stationary solution for any value of H. We numerically solved this equation for the Kolmogorov spectrum and found both the amplitude b_0 and the correlation length l_{corr} of magnetic field fluctuations as a function of H.

For an arbitrary value of the mean field, we derive a system of anisotropic evolution equations. In the case of large mean field $H^2 \gg L_0 v_0 \rho_i \mu_{in}$, the system of stationary equations has been solved analytically and the anisotropy of the correlators is found.

At the end of the first chapter, I apply the obtained results to molecular cloud Sgr B2, which is located near the Galactic center. I calculate the correlation length of the magnetic field $l_{corr} = 0.4$ pc. From this correlation length we calculate diffusion coefficient of cosmic rays in the cloud [30].

When studying the dynamo, the velocity field $v(r,t)$ is generally considered to be a Gaussian random process. But this is not the case in real turbulence. In **Chapter 2** we takes into account the first non-Gaussian correction, namely the three-point (cubic) velocity field correlator. Thus, a generalization of the turbulent dynamo theory for a non-Guassian velocity field has been constructed.

In the second chapter, I work in the framework of single-fluid MHD. The equation for the magnetic field has the form

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \mathbf{\Delta B}, \tag{5}$$

therefore, the correlator evolution equations will be linear. We do not take into account mean magnetic field. Therefore, the system is isotropic, and the pair correlator of the magnetic field has the form (2). We consider the three-point correlator to be rather small.

To split the correlators, it is required to study the three-point velocity correlator. In general, it is unknown. Suppose it is also delta-correlated

$$\langle v_i(r_1,t_1)v_j(r_2,t_2)v_k(r_3,t_3)\rangle = \beta\tau_c^2 \cdot Y(t_1,t_2,t_3) \cdot \langle v_i(r_1)v_j(r_2)v_k(r_3)\rangle,$$

where the function $Y(t_1,t_2,t_3)$ is normalized to 1. My colleague A. Kopyev obtaine two important results for this correlator in the viscous scale [31] and in the inertial interval [27]. The viscous correlator is defined up to a common factor F. By equating the convolutions of the three-point correlators at $r = r_\nu$, I get a relation between F and the normalization factor β. The sign of β is related to the direction of the energy flow in the turbulent cascade [28], in real three-dimensional turbulence $\beta > 0$.

After rather lengthy calculations, I derive the evolution equation of the pair correlator in the inertial interval $r_\nu < r < L_0$

$$\frac{1}{2\tau_c}\frac{\partial Q(t,r)}{\partial t} = \left(V(0) - V(r) + \frac{\eta}{\tau_c}\right)(Q'' + \frac{4Q'}{r}) - V'Q' - (V'' + \frac{4V'}{r})Q+ \\ + \frac{\beta\varepsilon\tau_c}{30}\left(2rQ''' + 15Q'' + \frac{4Q'}{r} - \frac{24Q}{r^2}\right). \quad (6)$$

The last term is the contribution of the three-point velocity correlator. This equation is one of the main results of this work.

Similar equation for the viscous scale $r < r_\nu$ was derived by A. Kopyev. It should be noted that we use a special program in Wolfram Mathematica. I wrote it for calculations of tensor convolutions. It allows to perform tensor operations symbolically, so these calculations are exact.

In Chapter 2 we investigate exponential modes of the evolution equation

$$Q(t,r) = \mathrm{e}^{\gamma D t}Q(r). \quad (7)$$

I numerically found the maximum positive γ depending on the parameters of the problem, including the value of the cubic correction f (figure 1). In some limit cases this problem was solved analytically [31]. Analytical predictions were confirmed by numerical results.

The most interesting question for astrophysical applications is the generation range, that is, the range of parameters for which there exists solutions with $\gamma > 0$. I found the boundary of the generation range numerically.

It turns out that non-Gaussian term reduces the growth rate and reduces the generation range. Thus, it reduces the generation of the magnetic field.

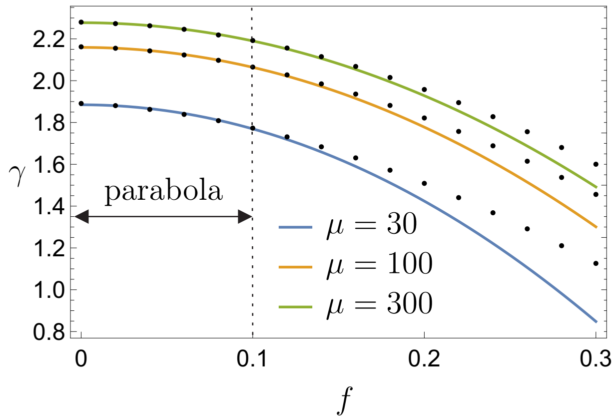

Figure 1 — The growth rate of the magnetic field γ as a function of amplitude of cubic velocity correlator f.

Chapter 3 is dedicated to the motion of charged particles in a stochastic magnetic field with zero mean. It is known that at long times such motion can be described by the diffusion equation.

We consider a motion of a particle with charge q and velocity v_0 in a stationary magnetic $\mathbf{B}(\mathbf{r})$. We assume that there is no electric field in the system, then the absolute value of the particle's velocity is conserved. We study the case when the mean magnetic field is equal to zero $\langle \mathbf{B} \rangle = \mathbf{0}$. Let us denote by L_0 the correlation length of the magnetic field, by B_{LS} the magnitude of the large-scale field. Now we consider L_0 to be a known parameter of the problem. We introduce the notation

$$r_L = \frac{mc\gamma v_0}{qB_{LS}}, \qquad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - v_0^2/c^2}} \tag{8}$$

for the Larmor radius in a large-scale magnetic field B_{LS}. Depending on the energy, particles can be divided into magnetized $r_L < L_0$ and non-magnetized $r_L > L_0$. The diffusion coefficient of non-magnetized particles is quite easy to calculate, see, for example, [29], $\langle D \rangle \sim \frac{v_0 r_L^2}{L_0}$.

To describe the motion of magnetized particles is much more difficult. We assume that the particle motion has a diffusive character at large times, so $\langle r^2(t) \rangle = 2Dt$. The angle brackets denote averaging over realizations of a random magnetic field. Consequently,

$$\langle D \rangle = \frac{1}{2} \frac{d\langle r^2 \rangle}{dt} = \langle r_i v_i \rangle. \tag{9}$$

It is known that the electric current in plasma is created by electrons. The mass of an electron is small. Then, in the absence of an electric field, the Ampere force acting on electrons must be zero, $\mathbf{j} \times \mathbf{B} = 0$. In MHD approximation $\mathbf{j} \sim \operatorname{rot} \mathbf{B}$. Therefore, the magnetic field satisfies the relation

$$\operatorname{rot} \mathbf{B} = k\mathbf{B}. \tag{10}$$

We name such field "force-free". From the equation div$\mathbf{B} = 0$, one can see that $k(\mathbf{r})$ is constant along the magnetic field lines.

We assume that the magnetic field in the whole system is a superposition of individual magnetic cells, in each cell k is constant. Different cells have different values of k, various directions ∇k and different values of the magnetic field magnitude B_0. First we determine the motion of the charged particle in one magnetic cell. We choose the coordinate axis z so that $B_z = 0$. The magnetic field has the following configuration

$$\mathbf{B}(\mathbf{r}) = B_0 \left(\sin kz, \cos kz, 0\right). \tag{11}$$

In such magnetic field, two equations of motion of the particle are integrated and the system is reduces to one equation on $z(t)$. By substitution, this equation is reduced to the equation of a mathematical pendulum

$$\frac{d^2\psi}{dt^2} = -\omega^2 \sin\psi, \tag{12}$$

where the value of ω depends on the initial velocity of the particle. Now, using (9), the diffusion coefficient can be represented as a sum of elliptic integrals. By averaging over the initial velocities and coordinates of the particle, I calculated diffusion coefficient in one cell. It depends on parameters k, B_0 of the magnetic field.

Next, I averaged the diffusion coefficient $\langle D\rangle$ over the spectrum of magnetic fluctuations $B_0(r)$, assuming the spectrum to be a power law with exponent α. For $r_L \ll L_0$ I found the asymptotics analytically

$$\langle D(A)\rangle \simeq \frac{2\pi}{3(2-\alpha)} v_0 r_L. \tag{13}$$

I also computed $\langle D(A)\rangle$ numerically for arbitrary $A < 1$ and different values of α. The mean diffusion coefficient weakly depends on the exponent α.

Particles with $A > 1$ are non-magnetized, the diffusion coefficient for them is given above. So, for the Kolmogorov spectrum $\alpha = 2/3$ the diffusion coefficient is approximately equal to

$$\langle D\rangle \simeq \begin{cases} (\pi/2) v_0 r_L, & r_L < L_0/2\pi \\ (5\pi^2/8)(v_0 r_L^2/L_0), & r_L > L_0/2\pi. \end{cases} \tag{14}$$

Note that the two asymptotics practically equial at the point $r_L = L_0/2\pi$. The diffusion coefficient weakly depends on the spectrum of magnetic fluctuations.

For relativistic particles, the Larmor radius is proportional to the energy. Therefore, at low energies the diffusion coefficient is proportional to the energy, and at high energies energies – to the the particle's energy squared.

At the end of Chapter 3 we study the application of our results to the cosmic rays diffusion in the Galaxy.

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