ABSTRACTS

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НАУКА
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НАЦИОНАЛЬНЫЕ
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РОССИИ

Moscow Institute of Physics and Technology;
Steklov International Mathematical Center;
Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

III International Conference
"MATHEMATICAL PHYSICS, DYNAMICAL SYSTEMS, INFINITE-DIMENSIONAL ANALYSIS"

The conference is supported by the Ministry of Science and Higher Education of the Russian Federation (the grant to the Steklov International Mathematical Center, agreement no. 075-15-2022-265) and integration project "Support for international scientific events and participation of employees and students of the MIPT therein", strategic project no. 2 "Research leadership (from bachelor to Nobel laureate)"

ISBN 978-5-6048945-7-6

III International Conference
"MATHEMATICAL PHYSICS, DYNAMICAL SYSTEMS, INFINITE-DIMENSIONAL ANALYSIS"
dedicated to the 100th anniversary of V.S. Vladimirov, the 100th anniversary of L.D. Kudryavtsev and the 85th anniversary of O.G. Smolyanov

ABSTRACTS

MPDSIDA-2023
Book of Abstracts
July 5 – 13, 2023
Dolgoprudny, Russia

Nonlinear dynamics of a remanufacturing duopoly model and its chaos control

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In this work, we formulated a two-dimensional discrete map that characterizes the interactions between two firms. The first firm is an original equipment manufacturer (OEM) exclusively producing and selling original products. The second firm, known as the third-party remanufacturer, focuses on remanufacturing returned goods to create differentiated products. Firstly, we investigate stability, bifurcations, and chaos. Using Jury's stability criteria, the Cournot-Nash equilibrium's asymptotic stability is examined. The results show that, when consumer willingness to pay and OEM's relative speed of the output adjustment are taken as a bifurcation parameters, the system undergoes flip bifurcations and Neimark-Sacker bifurcations under certain conditions. The Lyapunov exponents demonstrate that the system becomes chaotic through each of the previous bifurcations. Furthermore, we suggest eliminating the chaotic behaviour, which introduces unpredictability into the market, by making slight adjustments to one of the system parameters over a short duration. To achieve this, we employ a controller designed based on the OGY method. Finally, we conduct numerical simulations to illustrate and emphasize the theoretical findings.

Keywords: Duopoly model; Remanufacturing; Competition strategy; Chaos; chaos control.

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Bifurcations of Codimension 2 of Nonlinear Hybrid Dynamical Systems with Constant Discretization Step

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In this paper, we consider a continuous-discrete (a hybrid) system of the form

$$\begin{cases} x'(t) = a_1(\mu)x(t) + b_1(\mu)y(t_k) + a(x(t), y(t_k), \mu), \\ y(t_{k+1}) = a_2(\mu)x(t_{k+1}) + b_2(\mu)y(t_k) + b(x(t_{k+1}), y(t_k), \mu), \end{cases} \tag{1}$$

$x, y \in R, t \in [t_k, t_{k+1}), k = 0, 1, 2, \ldots, \mu \in R$ is a scalar parameter, $|a(x, y, \mu)| = o(|x| + |y|), |b(x, y, \mu)| = o(|x| + |y|)$ at $|x| + |y| \to 0$.

The moments of time t_k set on R a uniform grid with a step $h > 0$:

$$0 = t_0 < t_1 = t_0 + h < t_2 = t_1 + h < \ldots < t_{k+1} = t_k + h < \ldots .$$

Let, for some $\mu = \mu_0$, system (1) have an equilibrium point $x = 0, y = 0$.

The functioning of the system (1) is carried out according to the standard scheme. The system (1) in a certain sense is equivalent to a non-linear discrete dynamical system, the compact representation of which has the form

$$u_{k+1} = A(\mu, h)u_k + \xi(u_k, \mu, h), \tag{2}$$

where

$$u_k = \begin{bmatrix} x(t_k) \\ y(t_k) \end{bmatrix},$$

$$A(\mu, h) = \begin{bmatrix} e^{a_1(\mu)h} & a_1(\mu)^{-1}(e^{a_1(\mu)h} - 1)b_1(\mu) \\ a_2(\mu)e^{a_1(\mu)h} & a_2(\mu)a_1(\mu)^{-1}(e^{a_1(\mu)h} - 1)b_1(\mu) + b_2(\mu) \end{bmatrix},$$

nonlinearity $\xi(u_k, \mu, h)$ is given by the matrix

$$\xi(u_k,\mu,h) = \begin{bmatrix} \varepsilon(x_k,y_k),\mu;h) \\ c(x_k,y_k),\mu;h) \end{bmatrix}, \text{ wherein}$$

$$\varepsilon(x_k,y_k,\mu;h) = e^{(t_k+h)a_1(\mu)} \int\limits_{t_k}^{t_k+h} e^{-sa_1(\mu)} a(x(s,x_k,y_k),y_k,\mu)\, ds\,,$$

$$c(x_k,y_k),\mu;h) = a_2(\mu)\varepsilon(x_k,y_k,\mu;h) + b(x(t_{k+1}),y(t_k),\mu),$$
$$\|\xi(u,\mu,h)\| = o(\|u\|), \|u\| \to 0.$$

The system (2) also at $\mu = \mu_0$ has an equilibrium point $u = 0$, system solutions (1) and (2) are interconnected.

We study the question of bifractions of codimension 2 for system (1) in the vicinity of the equilibrium point $x = 0, y = 0$.

Sufficient signs for the existence of bifurcation points of codimension 2 in system (1) are given in the report, taking into account the connection between the solutions of systems (1) and (2).

Geometric Properties of Planar and Spherical Interception Curves

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In the paper, some geometric properties of the following *plane interception curve* defined by a nonlinear ordinary differential equation are discussed.

Planar Interception Curve. Suppose that two points $P(x, y)$ and Q, initially at $O(0, 0)$ and $A(1, 0)$, respectively, move with constant and equal velocities so that Q is on the line $x = 1$, and P is on the ray OQ. What curve is defined by the point P?

Let us denote $r = |OP|$ and $\angle AOQ = \theta$. Since the speeds of the points P and Q are equal, the length of the curve OP and the length of the line segment AQ, which is $\tan\theta$, are equal for each θ. By using the well-known formula for the length of a curve $r = r(\theta)$, given in polar coordinates, we find that

$$\int_0^\theta \sqrt{r(t)^2 + (r'(t))^2}dt = \tan\theta. \tag{1}$$

By taking the derivative of both sides of (1) and simplifying, we obtain ODE

$$r(\theta)^2 + (r'(\theta))^2 = \frac{1}{\cos^4\theta}, \tag{2}$$

with initial condition $r(0) = 0$. This nonlinear equation appears in problems related to the interception of high-speed targets by beam rider missiles. Note that in the cartesian coordinates, (1) can be written as

$$\int_0^x \sqrt{1 + (y'(t))^2}dt = \frac{y}{x}. \tag{3}$$

By taking the derivative of both sides of (3) and simplifying, we obtain

$$x^2\sqrt{1 + (y'(x))^2} = y'x - y, \tag{4}$$

which in particular agrees with $y(0) = 0$. A parametrization of the solution can be written by solving (4) for y to obtain $y = y'x - x^2\sqrt{(y')^2+1} = px - x^2\sqrt{p^2+1}$, where $y' = p \geq 0$. By noting that $dy = pdx$ and $dy = pdx + xdp - 2x\sqrt{p^2+1}dx - x^2\frac{p}{\sqrt{p^2+1}}dp$, a linear differential equation $\frac{dx}{dp} - \frac{xp}{2\sqrt{p^2+1}} = \frac{1}{2\sqrt{p^2+1}}$ is obtained. By solving this equation, we obtain the parametrization

$$\begin{cases} x(p) = \frac{1}{\sqrt[4]{p^2+1}}\int_0^p \frac{dt}{2\sqrt[4]{t^2+1}}, \\ y(p) = px(p) - (x(p))^2\sqrt{p^2+1}\ (p \geq 0). \end{cases} \tag{5}$$

Theorem 3.1 *Suppose that the tangent line of the curve (4) at the point P, intersects x-axis, y-axis, and the line $x = 1$ at points F, U, and T, respectively. If x is the abscissa of the point $P(x, y)$, then*
1. $|UP| = |OU| + |TQ|$,
2. $(1 - x) \cdot |UP| = |TQ|$,
3. $x \cdot |PT| = |TQ|$,
4. $\sin\angle QPT = |OP| \cdot \sin^2\angle TQP$,
5. *the radius of the circle through O and tangent to the line UT at the point P is equal to the radius of the circle through O and tangent to the line AT at the point Q.*

Theorem 3.2 *The length of the side PQ, and the difference of lengths of the other two sides of $\triangle PQT$ approach to the same limit B^2 as $x \to 1^-$:*

$$\lim_{x\to1^-}|PQ| = \lim_{x\to1^-}(|PT| - |TQ|) = \frac{\Gamma\left(\frac{3}{4}\right)^4}{2\pi} = B^2 \approx 0.3588850048,$$

where B is the second lemniscate constant.

Spherical curve. Suppose that two points P and Q, initially at $B(0, 0, 1)$ and $A(1, 0, 0)$, respectively, move with constant and equal velocity so that Q is on the great circle $z = 0$, $x^2 + y^2 = 1$ of sphere $x^2 + y^2 + z^2 = 1$ with center $O(0, 0, 0)$, and P is on the great circle through B and Q of the sphere.

This spherical curve has similar geometric properties. There are connections with the classical pursuit curve; Mercator projection; Gudermannian function; lemniscate constants; Gauss's constant; Spherical spiral; loxodrom; rhumb line; stereographic projection and logarithmic spiral.

Short-wave asymptotic solutions of a general hyperbolic equation

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In this work considered a hyperbolic equations of the general form:

$$\left(i\frac{\partial}{\partial t}\right)^m u = A\left(x, t, -i\frac{\partial}{\partial x}, i\frac{\partial}{\partial t}\right)u, \quad x \in R^n, \quad u \in C^l, \tag{1}$$

where $A(x, t, p, p_0)$ is a matrix $l \times l$, whose elements are polynomials of degree $\leq m$ by (p, p_0).

Hyperbolicity condition:

let A_m be the highest homogeneous degree of m part of A by (p, p_0), then

1. $A_m(x, t, p, p_0)$ does not contain terms p_0^m;

2. the characteristic equation has the form

$$det(\lambda^m E - A_m(x, t, p, \lambda)) = 0,$$

3. $\lambda_j(x, t, p)$ are smooth at $p \neq 0$, different with $p \neq 0$ and $|\lambda_j - \lambda_k| \geq c_{ij}|p|$.

Task. Let us have a general hyperbolic equation with

$$A(x, t, p, p_0) = A\left(\frac{\Phi(x)}{\varepsilon}, x, t, p, p_0\right),$$

where the parameter $\varepsilon \to 0$, $\Phi(x) : R^n \to R$ is a smooth function, and the equation $\Phi = 0$ defines a smooth regular hypersurface $M \subset R^n$, just as in the above problem.

Conditions on the matrix $A(y, x, t, p, p_0)$.

1. Equation $\det(p_0^m - A_m(y, x, t, p, p_0)) = 0$, where A_m is the highest homogeneous in (p, p_0) part of the matrix A has at $p \neq 0$ exactly ml of real roots $p_0 = H_j(y, x, t, p)$ relative to the variable p_0, and the functions H_j smoothly depend on all their arguments and tend to the limits of $H_j^{\pm}(x, t, p)$ at $y \to \pm\infty$

is faster than any power of y together with all derivatives. Obviously, H_j are positively homogeneous of degree 1 with respect to the variables p.

2. For $x \in M$, the matrix $A_m(y, x, t, \nabla\Phi, 0)$ is invertible,

3. eigenvectors of the matrix $(\lambda^m E - A_m(x, t, p, \lambda))$ smooth vector functions.

Many papers have been devoted to equations with rapidly changing coefficients; the introduction of the fast variable $y = \Phi(x)/\varepsilon$ turns the right part of the wave operator into operator of the form

$$A\left(\frac{\Phi}{\varepsilon}, x, t, -i\frac{\partial}{\partial x} - \frac{i}{\varepsilon}\nabla\Phi\frac{\partial}{\partial y}, -i\frac{\partial}{\partial t}\right).$$

Let's set the initial conditions (Cauchy's problem) of the following form

$$u|_{t=0} = \varphi^0(x)e^{\frac{iS_0(x)}{\varepsilon}}, \quad \frac{\partial^j u}{\partial t^j}|_{t=0} = 0, \quad j = 1, \ldots, m-1. \tag{2}$$

where S_0, φ_0 are smooth functions, and φ_0 is finite, $\nabla S_0|_{supp\varphi^0} \neq 0$,$\Phi|_{suppx_0} < 0$ and $supp\varphi^0 \bigcap M = \emptyset$. Thus, the initial wave packet is located outside the localized inhomogeneity; our goal is to describe the scattering of such a packet when it hits the surface of M. As a result, we get wave packets reflected from this surface and passed through it, which are described using asymptotic series.

The result of this work is an asymptotic series for solving the Cauchy problem; it turns out that geometric optics (i.e. classical trajectories) are determined by the "limiting" Hamiltonians $H_\pm = c_\pm(x)|p|$, and the presence of a jump-like perturbation leads to the restructuring of the complex Maslov sprout at the points of the surface M — new planes describing the past and reflected waves appear.

About singular Klein–Gordon equation

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We will deal with the singular Bessel differential operator B_γ:

$$(B_\gamma)_t = \frac{\partial^2}{\partial t^2} + \frac{\gamma}{t}\frac{\partial}{\partial t} = \frac{1}{t^\gamma}\frac{\partial}{\partial t}t^\gamma\frac{\partial}{\partial t}, \qquad t>0, \qquad \gamma\in\mathbb{R}.$$

We apply Hankel transform method to solve the initial value problem

$$[(\triangle_\gamma)_x - (B_k)_t\,]\,u = c^2u, \qquad c>0, \tag{1}$$

$$u(x,0;k)=f(x), \quad u_t(x,0;k)=0, \quad u=u(x,t;k), \tag{2}$$

where $\gamma_i>0$, $x_i>0$, $i=1,\ldots,n$, $t>0$, $\Delta_\gamma=\sum\limits_{i=1}^n B_{\gamma_i}$ is the Laplace–Bessel operator. We will call (1) the **singular Klein–Gordon equation**. We obtain the distributional solution of (1)–(2) in convenient space. Besides, we give formulas for regular solution of (1)–(2) in particular case of k and of Cauchy the the singular Klein–Gordon equation.

We are looking for the solution $u\in S'_{ev}(\mathbb{R}^n_+)\times C^2_{ev}(0,\infty)$ of (1)–(2), i.e. $u(x,t;k)$ belongs to $S'_{ev}(\mathbb{R}^n_+)$ by variable x and belongs to $C^2_{ev}(0,\infty)$ by variable t. For definitions spaces $S'_{ev}(\mathbb{R}^n_+)$ and $C^2_{ev}(0,\infty)$ see [1].

Theorem. The solution $u\in S'_{ev}(\mathbb{R}^n_+)\times C^2_{ev}(0,\infty)$ of the (1)–(2) for $k\neq -1,-3,-5,\ldots$ is unique and defined by the formula

$$u(x,t;k)=$$

$$=C(n,\gamma,k)\left(t^{1-k}(t^2-|x|^2)_+^{\frac{k-n-|\gamma|-1}{2}}\,j_{\frac{k-n-|\gamma|-1}{2}}\left((t^2-|x|^2)_+^{\frac{1}{2}}\cdot c\right)*f(x)\right)_\gamma,$$

where

$$C(n,\gamma,k)=\frac{2^n\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k-n-|\gamma|+1}{2}\right)\prod\limits_{i=1}^{n}\Gamma\left(\frac{\gamma_i+1}{2}\right)}.$$

In the case when $k<0$ of the (1)–(2) is not unique. When $k<0$ and $k\neq -1,-3,-5,\ldots$ the difference between two arbitrary solutions is always of the form

$$At^{1-k}u(t,x;2-k),\qquad A=const, \tag{3}$$

where $u(t,x;2-k)$ is solution of the Cauchy problem

$$[(\triangle_\gamma)_x-(B_{2-k})_t\,]\,u=c^2u,$$

$$u(x,0;2-k)=\psi(x),\qquad u_t(x,0;2-k)=0,$$

$\psi(x)$ is an arbitrary function or distribution belonging to S'_{ev}, When $k=-1,-3,-5,\ldots$ a nonunique solution of the Cauchy problem (1)–(2) will contain a terms (3) and

$$\frac{e^{\pm\frac{1}{2}\pi ni}\Gamma\left(\frac{n+|\gamma|-k+1}{2}\right)}{2^n\Gamma\left(\frac{1-k}{2}\right)\prod\limits_{i=1}^{n}\Gamma\left(\frac{\gamma_i+1}{2}\right)}\,t^{1-k}\left((t^2-|x|^2-c^2\pm i0)_\gamma^{\frac{k-n-|\gamma|-1}{2}}*f(x)\right)_\gamma.$$

In Theorem

$$(f*g)_\gamma(x)=(f*g)_\gamma=\int\limits_{\mathbb{R}^n_+}f(y)({}^\gamma\mathbf{T}^y_x g)(x)y^\gamma\,dy$$

is the generalized convolution generated by a multidimensional generalized translation ${}^\gamma\mathbf{T}^y_x$ which is given by

$$({}^\gamma\mathbf{T}^y_x f)(x)={}^\gamma\mathbf{T}^y_x f(x)=({}^{\gamma_1}T^{y_1}_{x_1}\ldots{}^{\gamma_n}T^{y_n}_{x_n}f)(x)$$

and each ${}^{\gamma_i}T^{y_i}_{x_i}$ acts for i=$1,\ldots,n$ is

$$({}^{\gamma_i}T^{y_i}_{x_i}f)(x)=\frac{\Gamma\left(\frac{\gamma_i+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{\gamma_i}{2}\right)}\times$$

$$\times \int\limits_0^\pi f(x_1, \ldots, x_{i-1}, \sqrt{x_i^2 + \tau_i^2 - 2x_i y_i \cos\varphi_i}, x_{i+1}, \ldots, x_n) \ \sin^{\gamma_i - 1} \varphi_i \, d\varphi_i.$$

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On calculating the classical capacity of quantum channels generated by probability distributions on finite groups

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One of the most important tasks of quantum information theory is to calculate the upper achievable limit for the number of quantum states used for encoding when transmitting information through several channels used in parallel. The report will tell about new results in this area.

On reverse Faber-Krahn Inequalities

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In 1961, Payne-Weinberger showed that '*among the class of membranes with a given area A, free along the interior boundaries and fixed along the outer boundary of a given length L_0, the concentric annulus has the highest fundamental frequency.*' We discuss the extension of this result to the higher dimension ($N \geq 3$), namely, the reverse Faber-Krahn inequalities for the first eigenvalue of the Laplace operator satisfying the mixed boundary conditions on domains with holes.

A note single-step difference schemes for the solution of stochastic differential equations

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In the present work, the stability of an abstract Cauchy problem for the solution of stochastic differential equation in a Hilbert space with the time-dependent positive operator is proved. In applications, theorems on stability estimates for the solution of four types of the initial boundary value problems for the one dimensional and multidimensional stochastic parabolic equation with dependent coefficients in t and space variables are proved. Single step difference schemes generated by exact difference scheme are presented. The main theorems of the convergence of these difference schemes for the approximate solutions of the time-dependent abstract Cauchy problem for the stochastic parabolic equations are established. In applications, the convergence estimates for the solution of difference schemes for stochastic parabolic differential equations are obtained. Numerical results for the $\frac{1}{2}$ and $\frac{3}{2}$ th order of accuracy difference schemes of the approximate solution of mixed problems for stochastic parabolic equations with Dirichlet, Neumann conditions are provided. Numerical results are given.

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Derivations on Operator Algebras

Shavkat Ayupov

This talk presents a full resolution of the problem stated by Ayupov in 2000, and partly restated in 2014 by Kadison and Liu, concerning derivations on algebras of measurable operators affiliated with von Neumann algebras. First we give preliminaries from the theory of operator algebras, non-commutative integration theory and show the physical background of automorphisms and derivations on operator algebras.

The second part of the talk explains a background of the Ayupov-Kadison-Liu Problem and its connection with general derivation theory in operator algebras starting with fundamental results due to Kaplanski, Kadison, Sakai and others. We shall cite and briefly explain major results concerning derivations on algebras of unbounded operators and list results concerning some special cases of the problem. Finally, the main result yielding the full resolution will be stated.

Dualism of the Theory of Soliton Solutions for Infinite-Dimensional Dynamical Systems and Pointwise Functional Differential Equations

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In the theory of plastic deformation, the following infinite-dimensional dynamical system is studied

$$m\ddot{y}_i = y_{i-1} - 2y_i + y_{i+1} + \phi(y_i), \quad i \in \mathbb{Z}, \quad y_i \in \mathbb{R}, \quad t \in \mathbb{R}, \tag{1}$$

where the potential $\phi(\cdot)$, in particular, is given by a smooth periodic function. The equation (1) is a system with the Frenkel-Kontorova potential [4]. Such a system is a finite difference analog of a nonlinear wave equation, simulates the behavior of a countable number of balls of mass m placed at integer points of a numerical line, where each pair of adjacent balls is connected to each other by an elastic spring, and describes the propagation of longitudinal waves in an infinite homogeneous absolutely elastic rod. The most important class of waves is described by solutions of the traveling wave type (soliton solutions).

This system is associated with the study of the canonical soliton bouquet $\big(Q, d, s, \mathcal{I}, G_\Gamma | Q, g\big)$ with $\Gamma = (Q, d, s, \mathcal{I}, Q, g)$, where: $Q =< \check{q}(t) = t + \tau >$, $\tau > 0$, and, respectively, $Q \cong \mathbb{Z}$; $d = s = 1$, and the selected element q of the group Q coincides with the generator $\check{q}$; phase space $\mathcal{K}^2_{\mathbb{Z}} = \overline{\prod_{i\in\mathbb{Z}}} R^2_i$, $R^2_i = \mathbb{R}^2$ (the superscript 2 is due to the fact that the equations (1) is of the second order) and the corresponding operator G_Γ and the function g.

The presented study demonstrates a fragment of some general approach. For this approach, a formalism has been developed [1], the central element of which is the existence of a one-to-one correspondence between soliton solutions of an infinite-dimensional dynamical system and solutions of a functional differential equation of pointwise type. For the presented finite difference analog of the wave equation with a nonlinear potential of a general form, the presence of a number of additional symmetries is also important. For such a system, the existence of a family of bounded soliton solutions is established [2]. Previously, such a system was studied in the case of a quadratic potential [3].

The research was supported by RSF (project No. 23-11-00080).

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Probabilistic models of fully nonlinear second order PDEs

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We discuss two probabilistic models that allow to construct probabilistic representations of a solution to the Cauchy problem

$$v_s = \Phi(x, v, \nabla v, \nabla^2 v), \quad v(0,x) = v_0(x), (t,x) \in [0,T] \times R^d. \qquad 1$$

From the probabilistic point of view it is suitable to reduce (1) to a problem

$$u_s + \Phi(x, u, \nabla u, \nabla^2 u) = 0, \quad u(T,x) = v_0(x), (t,x) \in [0,T] \times R^d. \qquad 2$$

with respect to a function $u(T-s,x) = v(s,x)$.

The models to be discussed are based on a possibility to construct different probabilistic representations for a solution of (1). To construct the first model assuming that the function $\Phi(x,u,r,q), x \in R^d, u \in R, r \in R^{d\times d}, q \in R^{d\times d\times d}$ has at least 3 bounded continuous derivatives in x we include the equation (1) into a system of semilinear parabolic equations with respect to the function $V = (u, \nabla u, \nabla^u, \nabla^3 u)$ of the form

$$V_s + \frac{1}{2} A^V \nabla^2 V [A^V]^* + a^V \nabla V + B^V \nabla V + c^V V = 0, \quad V(T,x) = h \qquad (3)$$

where $A^V(t,x) = 2\nabla_r F(x,u,r,q)$ and a^V, B^V, c^V are smooth bounded functions depending on V, $a^V(x) = a(x, V(x))$.. To construct a probabilistic representation of a solution to (2) we fix a probability space $(\Omega, \mathcal{F}, P$ and a Wiener process $w(t) \in R^d$ defined on it. Next we consider a stochastic problem of the form

$$d\xi(t) = a^V(\xi(t))dt + A^V(\xi(t))dw(t), \quad \xi(s) = x, \qquad (4)$$

$$d\eta(t) = c^V(\xi(t))\eta(t)dt + C^V(\xi(t))(\eta(t), dw(t)), \quad \eta(s) = h \in R^q \qquad (5)$$

$$\langle h, V(s,x)\rangle = E\langle \eta_{s,h}(T), h(\xi_{s,x}(T))\rangle. \qquad (6)$$

Here $\langle h, V\rangle = \sum_{m=1}^q h_m V_m, q = 1 + d + d^2 + d^3$ and $[C^V]^* A^V = B^V$.

Theorem 1. *Assume that $u(s) \in C^3(R^d)$ satisfies (1). Then it admits a probabilistic representation $u(t,x) = V_1(t,x)$ where V_1 is the first component of the function $V(s,x)$ defined by (5).*

The second approach to construct a probabilistic representation is based on the Pardoux-Peng BSDE theory. Within the framework of this theory we rewrite (2) in the form

$$\frac{\partial u}{\partial t} + \frac{1}{2}\Delta u + \Psi\left(x, \nabla^2 u\right) = 0, \quad u(T,x) = h(x), (t,x) \in [0,T] \times (0,\infty) \quad (7)$$

where $\Psi(x,\Gamma) = \Phi(x,\nabla^2 u) - \frac{1}{2}\Delta u$ and Δ is the Laplace operator. Let u be a classical solution of (2). Consider a process $\xi(t) = x + w(t) - w(s)$ and couple of stochastic processes $y(t) = u(t,\xi(t))$ and $z(t) = \nabla u(t,\xi(t))$. Applying the Ito formula to a smooth function u and a process $\xi(t)$ and keeping in mind (7) we obtain a system

$$dy(t) = -\Psi(\xi(t),\Gamma(t))dt = \langle z(t), dw(t)\rangle, \quad y(T) = h(\xi(T)), \quad (8)$$

$$dz(t) = \alpha(t)dt + \Gamma(t)dw(t). \quad (9)$$

Here $\Gamma(t,x) = \nabla^2 u(t,\xi(t)) \in R^{d\times d}$ and $\alpha(t) = \nabla[u_t(t,\xi(t)) + \frac{1}{2}\Delta u(t,\xi(t))] \in R^d$. Note, that by definition $y(s) = u(s,x)$.

Theorem 2. Let u be a classical solution of (2). Then it admits a representation $u(s,x) = y(s)$ where $y(s)$ satisfies the system (8), (9).

Thus we have reduced the problem (2) either to (4)-(6) or to (8),(9). Note that one can apply the above representations of u to construct its numerical approximation.

The work is supported by grant RSF 22-21-00016.

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Solution of a non-local boundary value problem simulating plasma perturbation by an electric field

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We consider a non-local boundary value problem for the Vlasov–Maxwell system of equations. The equations describe collective phenomena emerging in the plasma layer that occur under an influence of external electric field. The desired functions are the perturbation of the electron density function and the intensity of the self-consistent electric field. In the Vlasov kinetic equation, the collision integral is represented in the Bhatnagar–Gross–Krook form, see [2]. We use the Maxwell or Fermi-Dirac distribution function as an unperturbed one, depending on the properties of the medium. Similar problems have been studied in a number of papers, see, for example, [5],[3]. Under the assumption of small values of the external field, the initial statement reduces to the following system of integral–differential equations [6]:

$$v f_x(x,v) + \alpha f(x,v) = v g(x) + \int_{-\infty}^{\infty} k(\xi) f(x,\xi) d\xi, \tag{1}$$

$$g_x(x) = \beta \int_{-\infty}^{\infty} k(\xi) f(x,\xi) d\xi, \tag{2}$$

where the unknown functions $f(x,v)$ and $g(x)$ express the perturbations of the initial distribution of the electrons and the electric field intensity in plasma, respectively; the phase variables (x,v), which have the sense of dimensionless coordinates and velocities, belong to the strip $\Pi = \{x \in (-l,l), v \in (-\infty,+\infty)\}$. The complex parameter α and real parameter β characterize the properties of plasma and the applied external field, while the even real-valued function $k(\xi)$, is expressed in terms of unperturbed electron density function and $k(\xi)$ satisfies the normalization condition $\int_{-\infty}^{\infty} k(\xi) d\xi = 1$. For example, if the distribution of electrons in the absence of an external field obeys the Fermi-Dirac statistics, then $k(\xi)$ has the form

$$k(\xi) = \frac{C(\mu)}{1 + e^{\xi^2 - \mu}}, \qquad C(\mu) := \left(\int_{-\infty}^{\infty} \frac{d\xi}{1 + e^{\xi^2 - \mu}} \right)^{-1},$$

where μ is the dimensionless chemical potential.

In the talk, a new representation is constructed for the general solution of the system of equations (1), (2) in the form of an integral with some density $\psi(\lambda)$. Taking into account the boundary conditions (3) reduces the finding of the function $\psi(\lambda)$ to solving a singular integral equation with the Cauchy kernel on the real axis $\lambda \in (-\infty, +\infty)$. We use the method [4], [7] to construct the solution of this integral equation. The method is based on the reduction this integral equation to the singular Riemann problem. The latter problem is solved with the use of the results of [1] to solve such a problem in the case when data of the Riemann problem are singular. A qualitative study of the dependence of the constructed solution of the problem (1)–(3) on the parameters of the problem determined by the properties of the plasma is carried out.

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Solution of a non-local boundary value problem simulating plasma perturbation by an electric field

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where the unknown functions $f(x,v)$ and $g(x)$ express the perturbations of the initial distribution of the electrons and the electric field intensity in plasma, respectively; the phase variables (x,v), which have the sense of dimensionless coordinates and velocities, belong to the strip $\Pi = \{x \in (-l,l), v \in (-\infty,+\infty)\}$. The complex parameter α and real parameter β characterize the properties of plasma and the applied external field, while the even real-valued function $k(\xi)$, is expressed in terms of unperturbed electron density function and $k(\xi)$ satisfies the normalization condition $\int_{-\infty}^{\infty} k(\xi)\, d\xi = 1$. For example, if the distribution of electrons in the absence of an external field obeys the Fermi-Dirac statistics, then $k(\xi)$ has the form

$$k(\xi) = \frac{C(\mu)}{1 + e^{\xi^2 - \mu}}, \qquad C(\mu) := \left(\int_{-\infty}^{\infty} \frac{d\xi}{1 + e^{\xi^2 - \mu}} \right)^{-1},$$

where μ is the dimensionless chemical potential.

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Average shadowing property

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Due to the inevitable perturbations, ranging from various kinds of errors (in particular, rounding off in computer simulations) to incomplete descriptions of the processes under study, one can observe only approximate realizations of the processes. Therefore, one of the main problems is to answer how the observed trajectories[1], to which we refer as pseudo-trajectories, are connected to the trajectories of the genuine system. One of the possibilities is to find conditions under which there is a real trajectory of the process under study in the vicinity of the obtained realization over the longest possible time interval.

This question becomes especially nontrivial in the case non-autonomous systems, when the system itself changes its behavior over time. At present, there are practically no results in the literature in this direction, and this article fills this gap by proposing a relatively simple test for solving the shadowing problem.

At the level of connections between individual trajectories of a hyperbolic system and the corresponding pseudo-trajectories this property (called the shadowing property) was first posed by D. V. Anosov as a key step of the analysis of structural stability of diffeomorphisms. A similar but much less intuitive approach called "specification" in the same setting was proposed by R. Bowen. Informally, both approaches ensure that errors do not accumulate during the process of modeling. In the systems with the shadowing property each approximate trajectory can be uniformly traced by a true trajectory on the arbitrary long period of time.

Naturally, this is of great importance in chaotic systems, where even an arbitrary small error in the starting position lead to (exponentially in time) large divergence of trajectories.

Further development demonstrated that under very mild assumptions for a diffeomorphism the shadowing property implies the uniform hyperbolicity. To some extent, this limits the theory of uniform shadowing to an important but very special class of hyperbolic dynamical systems. The concept of average shadowing

[1]Approximate trajectories of a system under small perturbations

introduced in [1] about 30 years ago gave a possibility to extend significantly the range of perturbations under consideration in the theory of shadowing, in particular to be able to deal with perturbations which are small only on average but not uniformly.

To realize this idea we developed recently in [2] a fundamentally new construction, consisting in the effective approximation of pseudo-trajectories of autonomous dynamical systems with only a single in time perturbation of the dynamics. The main result is that this single perturbation approximation property with some control of the approximation accuracy implies the shadowing property. In the present talk we extend this approach for non-autonomous systems.

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Szegő-Weinberger type inequalities for symmetric domains with holes

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Let $\mu_2(\Omega)$ be the first positive eigenvalue of the Neumann Laplacian in a bounded domain $\Omega \subset \mathbb{R}^N$. It was proved by Szegő [2] for $N = 2$ and by Weinberger [3] for $N \geq 2$ that $\mu_2(\Omega)$ attains its global maximum among all equimeasurable domains if Ω is a ball. We develop the approach of Weinberger in two directions. Firstly, we refine the Szegő-Weinberger result for a class of domains of the form $\Omega_{\text{out}} \setminus \overline{\Omega}_{\text{in}}$ which are either centrally symmetric or symmetric of order 2 (with respect to any coordinate plane (x_i, x_j)) by showing that $\mu_2(\Omega_{\text{out}} \setminus \overline{\Omega}_{\text{in}}) \leq \mu_2(B_\beta \setminus \overline{B}_\alpha)$, where B_α, B_β are balls centered at the origin such that $B_\alpha \subset \Omega_{\text{in}}$ and $|\Omega_{\text{out}} \setminus \overline{\Omega}_{\text{in}}| = |B_\beta \setminus \overline{B}_\alpha|$. Secondly, we provide Szegő-Weinberger type inequalities for higher eigenvalues by imposing additional symmetry assumptions on the domain. In particular, if $\Omega_{\text{out}} \setminus \overline{\Omega}_{\text{in}}$ is symmetric of order 4, then we prove that $\mu_i(\Omega_{\text{out}} \setminus \overline{\Omega}_{\text{in}}) \leq \mu_i(B_\beta \setminus \overline{B}_\alpha)$ for $i = 3, \dots, N+2$, where we also allow Ω_{in} and B_α to be empty. The talk is based on the work [1].

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Lorentz 2-orbifolds of constant non-zero curvature with the essential isometry group

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Orbifolds can be considered as manifolds with singularities. An n-dimensional orbifold is a connected Hausdorff topological space that can be represented locally as a quotient space $\mathbb{R}^n/\Gamma$ of the arithmetic space $\mathbb{R}^n$ over a finite group of diffeomorphisms Γ; moreover, the group Γ is not fixed and can change when moving from one point to another. Smooth orbifolds naturally form the category $\mathcal{O}rb$.

Lorentz geometry differs significantly from Riemannian geometry. It is known that every smooth orbifold admits a Riemannian metric which is not true for Lorentz metrics. Loretzian orbifolds form the category $\mathcal{L}or$.

The isometry group of a Lorentz orbifold $(\mathcal{N}, g)$ is called inessential if there exists a Riemannian metric h such that the isometry group of the Lorentzian orbifold $(\mathcal{N}, g)$ coincides with the isometry group of the Riemannian orbifold $(\mathcal{N}, h)$. Lorentz orbifolds with inessential full isometry group are referred to be inessential. Otherwise a Lorentz orbifold $(\mathcal{N}, g)$ is said to be essential. Emphasize that an essential Lorentz orbifold $(\mathcal{N}, g)$ is characterized by an improper action of its full isometry group. A compact Lorentz orbifold is essential if and only if its full isometry Lie group is not compact.

In [2] proved that unique compact smooth orbifold "Pillow" admits a complete essential Lorentz metric of zero curvature and all such metric were founded. Among non-compact orbifolds only $\mathbb{Z}_2$-cone admits a complete essential Lorentz metric of zero curvature [1].

In this work we investigate the structure of complete two-dimensional Lorentz orbifolds of constant non-zero curvature with the essential isometry group. At first we prove that every such orbifold may be represented in the form M/Ψ where M is a Lorentz manifold, and Ψ is an isometry group isomorphic either to $\mathbb{Z}_2$ or to $\mathbb{Z}_2 \times \mathbb{Z}_2$. Further we classify two-dimensional essential Lorentz manifolds and apply this classification.

Thus we obtain the classifications of complete two-dimensional essential Lorentz orbifolds of constant non-zero curvature both in the category of smooth orbifolds $\mathcal{O}rb$ and in the category of Lorentz orbifolds $\mathcal{L}or$. These classifications are richer than in the case of zero curvature. In particular, we have proved the existence of three smooth orbifolds with finite fundamental groups admitting essential Lorentzian metrics, while in the case of zero curvature there is only one such orbifold.

The work was supported by the Russian Science Foundation (grant no. 23-71-30008).

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Nonlinear wave in the hypercycle with infinity many members

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An important class of replicator models involves systems of nonlinear ordinary differential equations with dynamics restrained by the standard simplex in the state space and describes macromolecular interactions in various problems of population genetics and evolutionary game theory [2], as well as in theories of the origin of life [4]. Of special interest is the hypercycle model that was proposed by M. Eigen and P. Shuster classical hypercycle is a finite closed network of self-replicating macromolecules (species) which are connected so that each of them catalyzes the replication of the successor, with the last molecule reinforcing the first one. From the sociological perspective, the catalytic support for the replication of other molecules resembles altruistic behavior, in contrast to conventional autocatalysis. However, the actual number of macromolecules in a hypercycle may be huge, and this may significantly complicate the numerical analysis of the associated dynamical system. It may therefore be reasonable to represent the macromolecules as points in some line segment (of cardinality continuum) and to construct an appropriate distributed model of hypercyclic replication. Such a methodology was previously implemented for Crow–Kimura and Eigen quasispecies models, with a single integra-differential equation replacing a large number of ordinary differential equations [1; 4]. Since the model represents an idealized process of replication continuous species in the form of integra-differential equation with space delay in integral simplex. The existence and uniqueness of positive solution are proved. The solutions represent sequence of non-damped nonlinear wave. It is proved existence of Andronov-Hopf bifurcation in steady state position [3]. The results of numerical modelling are presented.

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On symmetries and conservation laws for some differential equations

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Consider the following operator equation:

$$N(u) \equiv P_{2u,t} u_{tt} + P_{1u,t} u_t + P_{3u,t} u_t^2 + Q(t,u) = 0, \tag{1}$$

$$u \in D(N) \subseteq U \subseteq V, \quad t \in [t_0, t_1] \subset \mathbb{R},$$

$$u_t \equiv D_t u \equiv \frac{d}{dt} u, \qquad u_{tt} \equiv \frac{d^2}{dt^2} u.$$

Here $\forall t \in [t_0, t_1]$, $\forall u \in U_1$ operators $P_{iu,t} : U_1 \to V_1$ $(i = \overline{1,3})$ are linear; $Q : [t_0, t_1] \times U_1 \to V_1$ is an arbitrary operator, in general, nonlinear; $D(N)$ is the domain of the operator N, $U = C^2([t_0, t_1]; U_1)$, $V = C([t_0, t_1]; V_1)$, U_1, V_1 are real linear normed spaces, $U_1 \subseteq V_1$.

The operator equation (1) can be an ordinary differential equation, a partial differential equation, an integro-differential equation, a differential-difference equation, etc., and for $P_{3u} \equiv 0$ – a system of such equations.

The following results are obtained.

1. Necessary and sufficient conditions for the representability of the equation (1) in the form of Lagrange equation are obtained.
2. The corresponding Hamiltonian action is constructed.
3. Conditions for the existence of symmetries of the given equation and the constructed functional are obtained.
4. The formulas for finding integrals of the considered equation are obtained.
5. The connection between variational symmetries and symmetries of the given equation and Lie-admissible algebras (including Lie algebras) is established.

The theoretical results are illustrated by some examples.

This talk is based on works[1–7].

This publication has been supported by the RUDN University named after Patrice Lumumba, project 002092-0-000.

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Invariants and integrability of polynomial Liénard and Levinson–Smith differential systems

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We consider the following family of planar differential systems:

$$x_t = y, \quad y_t = -f(x, x_t)x_t - g(x), \tag{1}$$

where the functions $f(x, x_t)$ are $g(x)$ polynomials of their arguments. Systems of the form (1) model nonlinear oscillators with the damping and the restoring force given by $f(x, x_t)$ and $g(x)$, respectively. These systems are called Liénard systems whenever $f(x, x_t)$ is independent of x_t and Levinson–Smith systems in the general case.

The aim of the present talk is to describe the integrability properties [1; 2] of systems (1). We use the theory of invariants and the Darboux theory of integrability. The Darboux theory of integrability provides a collection of methods designed for finding the Darboux and Liouvillian first integrals of planar differential systems [3]. These methods are based on the number and properties of algebraic and exponential invariants of differential systems. A great advantage of the Darboux theory of integrability is that this theory can give the necessary and sufficient conditions of integrability for multi-parameter differential systems. The main difficulty in deriving algebraic invariants lies in the fact that their degrees are not known in advance. We shall describe a method that makes finding the invariants purely algebraic [1; 2].

The research reported in this talk is supported by the Russian Science Foundation grant 19–71–10003, `https://rscf.ru/en/project/19-71-10003/`

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Effective asymptotics of linear (pseudo) differential equations with localized right hand sides

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We discuss a method for constructing semi-classical asymptotic solutions of multidimensional stationary linear inhomogeneous partial differential (and pseudo-differential) equations with localized right-hand sides. These problems are close to the problems on the asymptotics of the Green function for the corresponding operators, in particular, the problems on the asymptotics of the Green function for the Helmholtz equation studied in numerous papers and arise in various fields of physics. The method is based on ideas dating back to V. P. Maslov, V. V. Kucherenko, R. Melrose, G. A. Uhlmann, and allows us to describe asymptotic solutions using constructively defined families of trajectories in the form of WKB functions or the canonical Maslov operator and special functions in the presence of caustics and focal points. The asymptotics contain information about the shape of the wave-generating source. The method is illustrated by various physical examples.

The results were obtained jointly with A. Yu. Anikin, M. Rouleux and A. A. Tolchennikov and supported by the Russian Science Foundation (project 21-11-00341).

Nonlocal de Sitter Gravity and its Exact Cosmological Solutions

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We will present several exact vacuum cosmological solutions of a simple nonlocal de Sitter gravity model. One of these solutions mimics effects that are usually assigned to dark matter and dark energy. Some other solutions are examples of the nonsingular bounce ones in flat, closed and open universe. There are also singular and cyclic solutions. All these cosmological solutions are a result of nonlocality and do not exist in the local de Sitter case. This talk is based on the paper: JHEP 12 (2022) 054.

On the singular trace of the main operations of field theory and the corresponding boundary value problems

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The report is devoted to the formation and study of boundary value problems, the boundary conditions of which contain the basic operations of the field theory of the first order, namely the vector of normal derivatives, curl and divergence.

The formation of the problems is based on the theorem on the trace of a linear combination

$$D_\Gamma u = \frac{\partial u}{\partial n} - [\mathrm{rot} u, n] - \mathrm{div} u \cdot n \tag{1}$$

on the boundary of the domain $G \subset \mathbb{R}^3$ for arbitrary vector fields $u = (u_1, u_2, u_3) \in W_2^1(G)$. Trace of the specified combination on the boundary Γ of domain G is a continuous functional over space $W_2^{1/2}(\Gamma)$, i.e.

$$D_\Gamma u = \left(\frac{\partial u}{\partial n} - [\mathrm{rot} u, n] - \mathrm{div} u \cdot n\right)_\Gamma \in W_2^{-1/2}(\Gamma).$$

This functional is a deviator (difference operator) of bilinear forms corresponding to the Laplace operator, presented in two forms $-\Delta = -\mathrm{div}\nabla$ and $\mathrm{rot}^2 - \nabla\mathrm{div}$. Exactly

$$\langle D_\Gamma u, v\rangle = \int_G (\nabla u, \nabla v)dx - \int_G (\mathrm{rot} u, \mathrm{rot} v)dx - \int_G \mathrm{div} u \cdot \mathrm{div} v dx,$$

where $u \in W_2^1(G)$ and $v \in W_2^1(G)$ are arbitrary fields.

In particular, if $u \in \mathrm{Ker} D_\Gamma u$ (and even more so if $D_\Gamma u = 0$), then

$$||\nabla u||^2_{L_2(G)} = ||\mathrm{rot} u||^2_{L_2(G)} + ||\mathrm{div} u||^2_{L_2(G)}. \tag{$*$}$$

A number of field implications are associated with the functional $D_\Gamma u$:

1. $u|_\Gamma = 0 \Rightarrow D_\Gamma u = 0 \Rightarrow (*)$,

2. $\left\{(u|_\Gamma, n) = 0\& \left(\frac{\partial u}{\partial n}|_\Gamma\right)_{\text{tang}} - [\text{rot}u, n]_\Gamma = 0\right\} \Rightarrow D_\Gamma u = 0 \Rightarrow (*)$,
3. $\left\{[u|_\Gamma, n] = 0\& \left(\frac{\partial u}{\partial n}|_\Gamma\right)_{\text{norm}} - \text{div}u|_\Gamma \cdot n = 0\right\} \Rightarrow D_\Gamma u = 0 \Rightarrow (*)$.

We also note the unconditional connection between the boundary values of the gradient and the curl of vector fields

$$\int_\Gamma ([\nabla p, n], u) d\gamma = \int_\Gamma (\text{rot}u, n) \cdot p d\gamma$$

where $p = p(x)$ is a scalar field.

The report will also indicate the conditions on vector fields, under which not only $D_\Gamma u \in W_2^{-1/2}(\Gamma)$, but also each term in (1), i.e.

$$\frac{\partial u}{\partial n}|_\Gamma \in W_2^{-1/2}(\Gamma),\ [\text{rot}u|_\Gamma, n] \in W_2^{-1/2}(\Gamma),\ \text{div}u|_\Gamma \cdot n \in W_2^{-1/2}(\Gamma).$$

The presence of these traces allows us to form a number of non-standard boundary value problems, examples of which are the following problems:
1) $\text{rot}^2 u - \nabla \text{div} u = h(x), x \in G$,
$-([\text{rot}u \cdot n] + \text{div}u \cdot n)_\Gamma + D_\Gamma u = f(\gamma), \gamma \in \Gamma$
(field version of the Neumann problem);
2) to find a vector function $u \in W_2^1(G)$ and a number $\alpha \in \mathbb{R}$ such that
$-\Delta u = h(x), x \in G$,
$\langle D_\Gamma \Phi, u|_\Gamma \rangle = 0, \frac{\partial u}{\partial n}|_\Gamma = \alpha D_\Gamma \Phi$
(the problem on the kernel of the functional , where a “weight” function is given).

The detailed presentation is available in [1–3].

The work was supported by The Ministry of Science and Higher Education of the Russian Federation (project FSWF-2023-0012).

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Theorem on Number-theory Renormalization of vacuum energy in QFT on the lattice

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We consider a bosonic QFT on a lattice $\mathbb{Z}^d(N)$ with the Hamiltonian

$$\hat{H}_b = \sum_{\mathbf{p}\in\mathbb{Z}^d(N),\mathbf{p}^2\in D} E(\mathbf{p}^2)\,\left(2\hat{b}^{\dagger}_{\mathbf{p}}\hat{b}_{\mathbf{p}} + 1\right), \quad [\hat{b}_{\mathbf{p_1}}, \hat{b}^{\dagger}_{\mathbf{p_2}}] = \delta_{\mathbf{p_1}\mathbf{p_2}}\,\hat{1},$$

$$\mathbf{p}^2 = \sum_{k=1}^{d} p_k^2 \quad \in \mathbb{Z}(N), \quad E : D \to \mathbb{Z}(N), \quad D \subset \mathbb{Z}(N).$$

The bosonic vacuum energy is the sum of the zero oscillation energies for all permissible values of the momentum

$$\mathcal{E} = \mathcal{E}_{vac\ b} = \sum_{\mathbf{p}\in\mathbb{Z}^d(N),\mathbf{p}^2\in D} E(\mathbf{p}^2) = \sum_{k\in D\subset\mathbb{Z}(N)} c_{N\,d}\,(k)\,E(k) \in \mathbb{Z}(N).$$

Here the multiplicity $c_{N\,d}\,(k)$ is

$$c_{N\,d}\,(k) = \Big(\text{the number of nodes } \mathbf{p} \in \mathbb{Z}^d(N) \text{ such that } \sum_{n=1}^{d} p_n^2 \equiv k \pmod{N}\Big). \tag{1}$$

Theorem. For an arbitrary N with $d \geqslant 3$, and for $N = 2^n$ with $d \geqslant 2$

$$\forall k \in \mathbb{Z}(N) \quad c_{N\,d}\,(k) \equiv 0 \pmod{N}. \tag{2}$$

When the conditions of the Theorem are fulfilled for an arbitrary function $E : D \to \mathbb{Z}(N)$ the vacuum energy calculated in the ring of residue classes $\mathbb{Z}(N)$ is zero.

The main idea of the proof. We define the polynomial of τ

$$f_N^d(\tau) = \sum_{k \in \mathbb{Z}(N)} c_{N\,d}(k)\,\tau^k \tag{3}$$

as a generating function for multiplicities $c_{N\,d}(k)$ of $E(k)$ on the lattice $\mathbb{Z}^d(N)$. For one-dimensional generating function we skip the upper index $d = 1$, i.e. $f_N(\tau) \stackrel{\text{def}}{=} f_N^1(\tau)$.

We treat τ as a formal variable with the equivalence relation: $\tau^N = 1$, i.e. the powers of τ in (3) can be considered as elements of $\mathbb{Z}(N)$. The multiplication with such relation we denote as $f \circ g$. In reduction of similar terms so that $\tau^{k+N} = \tau^k$, hence in case $d > 1$ the generating function can be represented as a d times product of one-dimensional generating functions:

$$f_N^d(\tau) = f_N(\tau) \circ \cdots \circ f_N(\tau) = \Big(f_N(\tau)\Big)^d. \tag{4}$$

We define the polynomial $\phi_N(\tau)$ with coefficients all equal to 1:

$$\phi_N(\tau) = \sum_{k \in \mathbb{Z}(N)} \tau^k = 1 + \tau + \tau^2 + \cdots + \tau^{N-1}. \tag{5}$$

Here $\phi_N(\tau) \not\equiv \frac{1-\tau^N}{1-\tau}$ because due to (23) the value 1 of variable τ is possible value.

Next we introduce the polynomial $g_p(\tau)$ using Legendre symbol [2] as character $\chi(k)$:

$$g_p(\tau) = \sum_{k \in \mathbb{Z}(p)} \chi(k) \cdot \tau^k. \tag{6}$$

For $N = p$ prime odd number we have generating function in 3-dimensional case

$$f_p^3(\tau) = f_p \circ f_p \circ f_p = p^2 \cdot \phi_p(\tau) + p \cdot (-1)^{\frac{p-1}{2}} \cdot g_p(\tau). \tag{7}$$

In the expression above one can take the common factor p out of brackets. So each coefficient in polynomial above is divisible by p

$$c_{p\,3}(k) \equiv 0 \pmod p. \tag{8}$$

And for $d > 3$ we have $c_{p\,d}(k) \equiv 0 \pmod p$ is obvious due to factorization (4).

For $N = p^m$ in 3-dimensional case we have recurrent formula as

$$f_{p^m} \circ f_{p^m} \circ f_{p^m} = p^2 \cdot \Phi_{p^m} \circ f^3_{p^{m-1}} + p^{m+\lfloor \frac{m}{2} \rfloor - 1} \cdot \begin{cases} p \cdot g_p(\tau^{p^{m-1}}) & \text{if odd } m, \\ p - \Phi_{p^m}(\tau) & \text{if even } m, \end{cases} \quad (9)$$

where $\Phi_n(\tau)$ is cyclotomic polynomial [1]. Due to (8) and (9) we have

$$c_{p^m\,3}(k) \equiv 0 \pmod{p^m}.$$

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Derivation of the Transport Equation for the Harmonic Crystal Coupled to a Klein–Gordon Field

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The rigorous derivation of macroscopic evolution equations from the microscopic dynamics is one of the central problem of nonequilibrium statistical physics. In the talk, we discuss this problem for the Hamiltonian system consisting of a real scalar Klein–Gordon field $\psi(x)$ and its momentum $\pi(x)$, $x \in \mathbf{R}^d$, coupled to a harmonic crystal described by the deviations $u(k) \in \mathbf{R}^n$ of particles (atoms, molecules, ions, etc) from their equilibrium position and their velocities $v(k) \in \mathbf{R}^n$, $k \in \mathbf{Z}^d$, $d, n \geq 1$. The Hamiltonian functional of the coupled field–crystal system reads

$$\begin{aligned}\mathrm{H}(\psi, u, \pi, v) := \frac{1}{2} \int \Big(|\nabla\psi(x)|^2 + m_0^2 |\psi(x)|^2 + |\pi(x)|^2 \Big)\, dx \\ + \frac{1}{2} \sum_{k \in \mathbf{Z}^d} \Big(\sum_{k' \in \mathbf{Z}^d} u(k) \cdot V(k-k') u(k') + |v(k)|^2 \Big) \\ + \sum_{k \in \mathbf{Z}^d} \int R(x-k) \cdot u(k) \psi(x)\, dx,\end{aligned}$$

where the coupled function $R(x)$ is a $\mathbf{R}^n$-valued smooth function exponentially decaying at infinity. This system can be considered as the description of the motion of Bloch electrons in the periodic medium which is generated by the ionic cores. The derivation of the transport equation is connected with the problem of convergence to an equilibrium measure. Hence, the first step in our investigation is the proof of such convergence. We assume that the initial data is a random function on a phase space with a distribution μ_0. We impose the following conditions on the measure μ_0: μ_0 has a zero mean, it has a finite mean energy density, and it satisfies the mixing condition. We study the distribution μ_t of a random solution at time moments $t \in \mathbf{R}$ and prove that the measures μ_t weakly converge to a limit Gaussian measure as $t \to \infty$ (see [1]).

To derive the transport equation we introduce a small scale parameter $\varepsilon > 0$ and consider a family of the initial measures $\{\mu_0^\varepsilon, \varepsilon > 0\}$ satisfying some conditions. In particular, we assume that the measures μ_0^ε are locally spatially homogeneous (w.r.t. the translations in $\mathbf{Z}^d$) or "slowly vary" under order shifts less than ε^{-1}, and inhomogeneous under shifts of the order ε^{-1}. Given nonzero $\tau \in \mathbf{R}$ and $z \in \mathbf{R}^d$, we study the distribution $\mu^\varepsilon_{\tau/\varepsilon^\kappa, z/\varepsilon}$ of the random solution close to the spatial point $[z/\varepsilon]$ and at time moments τ/ε^κ with an κ, $\kappa \in (0, 1]$. In the case $\kappa < 1$, we prove that the measures $\mu^\varepsilon_{\tau/\varepsilon^\kappa, z/\varepsilon}$ converge to a limit measure as $\varepsilon \to 0$, which is Gaussian and its covariance matrix does not depend on τ. For $\kappa = 1$, $\lim\limits_{\varepsilon\to 0} \mu^\varepsilon_{\tau/\varepsilon, z/\varepsilon} = \mu^G_{\tau,z}$, where $\mu^G_{\tau,z}$ is a Gaussian measure. In particular, we derive the explicit formulas for the covariance matrix of the limit measure. These formulas allow us to conclude that in the Bloch–Floquet–Zak transform the limit covariance matrix evolves according to the following equation:

$$\partial_\tau f_{\tau,z}(\theta) = i\, C(\theta)\nabla\Omega(\theta) \cdot \nabla_z f_{\tau,z}(\theta), \quad C(\theta) = \begin{pmatrix} 0 & \Omega^{-1}(\theta) \\ -\Omega(\theta) & 0 \end{pmatrix}, \tag{1}$$

where $z \in \mathbf{R}^d$, $\tau > 0$, $\theta \in [0, 2\pi]^d$, and, roughly, $\Omega(\theta)$ is the "dispersion relation" of our model. Eqn (1) can be considered as the analog of the Euler equation.

In phonon physics it is standard practice to use the Wigner function $W(t, z, \theta)$ as density of phonons with wave number θ at location z and at specified time t. W evolves according to the semiclassical energy transport equation

$$\partial_t W(t, z, \theta) = -\nabla\Omega(\theta) \cdot \nabla_z W(t, z, \theta), \quad z \in \mathbf{R}^d, \quad t > 0. \tag{2}$$

We show that $W(t, z, \theta)\delta(\theta - \theta')$ at fixed z, t are expressed by the covariance of the limit Gaussian measure $\mu^G_{t,z}$, which is invariant under the dynamics of the problem. Thus, Eqn (2) can be understood as the equation governing the motion of the parameters which characterize the locally stationary measures. For the harmonic crystals, the transport equation (2) was derived in [2; 3].

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Functional of eigenvalues on the manifold of potentials

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We consider the family

$$-y" + p(x)y = \lambda y; \quad y(0) - y(2\pi) = y'(0) - y'(2\pi) = 0 \tag{1}$$

of $p \in P := \{L_2(0, 2\pi) : \int_0^{2\pi} p(x)dx = 0\}$ as a functional parameter. For a fixed potential $p \in P$ the spectrum consists of real eigenvalues with multiplicity at most two: $\lambda_0(p) < \lambda_1^-(p) \leq \lambda_1^+(p) < \ldots < \lambda_n^-(p) \leq \lambda_n^+(p) < \ldots$ Fix a subscript $n \in N$ and let Y_n be the set of eigenfunctions corresponding to eigenvalues with subscript n:

$$Y_n := \{y \in W_2^2(2\pi) : \int_0^{2\pi} y^2 dx = 1 \ \wedge \ \exists (p, \lambda_n^{\mp}) \in P \times R : (1) \, is \, true\}.$$

We are interested in submanyfold of potentials for which the nth lacuna has the same length, $P_n(\Delta\lambda) := \{p \in P : \ \lambda_n^+(p) - \lambda_n^-(p) = \Delta\lambda \geq 0\} \subset P$.

Using formulas

$$r(y;x) := \frac{y''(x)}{y(x)} \ (y \in Y_n), \ \lambda(y) = -\frac{1}{2\pi}\int_0^{2\pi} r(y;x)dx, \ p(y;x) = r(y;x) + \lambda(y),$$

we study the properties of the eigenvalue functional $\lambda = \lambda(p)$. It will be shown that on the manifold $P_n(0)$, the functional $\lambda(p)$ is a Morse functional with one critical point, and its Morse index is $2(2n-1)$.

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Ramified continua as global attractors of C^1-smooth self-maps of a cylinder close to skew products

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In the current time the theory of dynamical systems on complicated continua, in particular, on ramified continua, is intensively developing. There are dynamical systems (in particular, ordinary differential equations) that admit continua with a complicated topological structure as their attractors. For example, Plykin attractor is an indecomposable continuum. Indecomposable continua also arise in the study of inverse limits of maps with homoclinic tangencies. Dendrites appear not only in considerations of inverse limits of maps with attractors of Hénon and Lozi types, but also in investigations of limit sets of some Kleinian groups on hyperbolic 3-manifolds.

We consider a C^1-smooth self-map F of a cylinder $M = S^1 \times I_2$, where S^1 is a circle, I_2 is a compact interval of the real line, satisfying

$$F(x,\, y) = (f(x) + \mu(x,\, y),\ g_x(y)), \text{ where } g_x(y) = g(x,\, y), \ (x,\, y) \in M.$$

We study such geometric property of the maps, as existence of C^1-smooth invariant local lamination, and apply this geometric property to the proof of the geometric integrability of maps under consideration. Using obtained results we construct the example of the family of C^1-smooth maps close to skew products so that each map from this family admits the global attractor, which is a one-dimensional ramified continuum with a complicated topological structure. The global attractor of every map from the family under consideration consists of arcs of two types. On the unique circle $S^1 \times \{0\}$ (which is the arc of first type) the map is mixing; on arcs of second type of different lengths homeomorphic to a closed interval (the family of such arcs has continuum cardinality) the map is not mixing. The topological structure of the global attractor and dynamical properties of trajectories on the attractor lead to the property of dense intermittency (in the complement to the attractor) of attraction sets of different ω-limit sets, the union of which coincides with the global attractor.

On ω-limit sets of simplest skew products defined on n-dimensional cells

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We consider here a continuous skew product F with the phase space $I^n = \prod_{i=1}^{n} I_j$ (I_j are closed intervals for all $1 \leq j \leq n$), i. e., a continuous map of the type

$$F(x_1,\, x_2,\, \dots x_n) = (f_1(x_1)), f_2(x_1,\, x_2),\, \dots, f_n(x_1,\, x_2,\, \dots,\, x_n)). \tag{1}$$

We say that a continuous map (1) is *simplest*, if it has a bounded set $\tau(F)$ of (least) periods of its periodic points. It means that there is a nonnegative integer number ν so that $\tau(F) = \{2^i\}_{i=0}^{i=\nu}$ (see [3]). Set $M = 2^\nu$.

We describe the structure of ω-limit sets of continuous simplest skew products on n-dimensional cells ($n \geq 2$). Our results generalize results of the paper [1] obtained for skew products defined on a closed plane rectangle.

Theorem 1. *Let F be a continuous simplest map of type (1) with the phase space I^n ($n \geq 2$). Then for every point $x(\widehat{x}_{n-1},\, x_n) \in I^n$ (here $\widehat{x}_{n-1} = (x_1,\, x_2, \dots, x_{n-1})$) there are a periodic point x_1^0 of a map f_1, and closed intervals $I'_2 \subseteq I_2,\, \dots, I'_n \subseteq I_n$ (possibly, degenerate) such that the ω-limit set $\omega_{F^M}((\widehat{x}_{n-1},\, x_n))$ of the F^M-trajectory $\{F^{Mi}(\widehat{x}_{n-1},\, x_n)\}_{i \geq 0}$ of the point $x(\widehat{x}_{n-1},\, x_n)$ has the form*

$$\omega_{F^M}((\widehat{x}_{n-1},\, x_n)) = \{x_1^0\} \times \prod_{j=2}^{n} I'_j. \tag{2}$$

Moreover, $\omega_{F^M}((\widehat{x}_{n-1},\, x_n))$ consists of F^M-fixed points (compare with [2]).

Denote by $Per(\cdot)$ the set of periodic points of a map and introduce the following concept.

Definition 1. Let $F : I^n \to I^n$ be a continuous simplest skew product. A point $x_1^0 \in Per(f_1)$ is said to be *an exceptional periodic point of the map* f_1, if there are natural numbers $2 \le j_1 < j_2 < \ldots < j_s \le n$ and points $\widehat{x}_2^0(x_1^0, x_2^0) \in Per(\widehat{f}_2)$,..., $\widehat{x}_{j_s-1}^0(x_1^0, x_2^0, \ldots, x_{j_s-1}^0) \in Per\widehat{f}_{j_{s-1}}$ such that for every $1 \le k \le s$ the slice $\bigl(Per(\widehat{f}_{j_k})\bigr)(\widehat{x}_{j_k-1}^0)$ contains a nondegenerate closed interval. Here

$\widehat{f}_{j_k}(x_1, \ldots, x_{j_k}) = (f_1(x_1), f_2(x_1, x_2), \ldots, f_{j_k}(x_1, \ldots, x_{j_k}))$; *the slice* $\bigl(Per(\widehat{f}_{j_k})\bigr)(\widehat{x}_{j_k-1}^0)$ is the set that coincides with

$$\{x_{j_k} : (\widehat{x}_{j_k-1}^0, x_{j_k}) \in Per(\widehat{f}_{j_k})\}.$$

Definition 1 is correct by the properties of natural projections of periodic points sets of skew products on n-dimensional cells ($n \ge 2$).

As a direct corollary of Theorem 1 and Definition 1 we obtain the following claim.

Proposition 1. *Let F be a continuous simplest map of type (1) with the phase space I^n ($n \ge 2$), and let f_1 have no exceptional periodic points. Then the ω-limit set of F-trajectory of every point of I^n is a periodic orbit.*

For the study of simplest skew products with a nonempty set of exceptional periodic points of f_1 divergent series are used.

In formula (1) we set $f_{i,\widehat{x}_{i-1}}(x_i) = f_i(\widehat{x}_{i-1}, x_i)$ for $2 \le i \le n$.

Theorem 2. *Let F be a continuous simplest map of type (1) with the phase space I^n ($n \ge 2$). The following claims are equivalent for a point $(\widehat{x}_{n-1}, x_n) \in I^n$:*

(2.1) *equality (2) holds for some $x_1^0 \in Per(f_1)$ and nondegenerate close intervals $I_{j_1}, I_{j_2}, \ldots, I_{j_s}$, where $2 \le j_1 < \ldots < j_s \le n$;*

(2.2) *for every $1 \le k \le s$ the series*

$$\sum_{r=1}^{+\infty} \varphi_{\widehat{x}_{j_k-1}, Mr}(x_{j_k})$$

is alternating divergent, where

$$\varphi_{\widehat{x}_{j_k-1}, Mr}(x_{j_k}) = f_{j_k,\widehat{x}_{j_k-1},M(r+1)}(x_{j_k}) - f_{j_k,\widehat{x}_{j_k-1},Mr}(x_{j_k}),$$

$$f_{j_k,\widehat{x}_{j_k-1},p}(x_{j_k}) = f_{j_k,\widehat{f}^{p-1}_{j_k-1}(\widehat{x}_{j_k-1})} \circ \ldots \circ f_{j_k,\widehat{x}_{j_k-1}}(x_{j_k}), \;\; p \ge 2.$$

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A Journey into Global Stability: From Monotone to Mixed Monotone and from autonomous to nonautonomous

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The study of global stability of fixed and periodic points of monotone maps and triangular maps in one or higher dimensional spaces have been successful. For general maps, the use of Liapunov functions have had limited success. Recently, Liapunov functions have been successful in obtaining global stability of the disease -free equilibrium of epidemic models.

In this talk, we extend some of these results to mixed monotone maps. A special property of these maps is that they can be embedded in symmetric monotone maps in higher dimension spaces. The aim here is to investigate the global stability of the interior fixed points of mixed monotone autonomous systems.

The study is then extended to non-autonomous systems that are asymptotically autonomous, and to periodic difference equations. For the periodic systems, we show that a periodic cycle is globally asymptotically stable.

These results are then applied to single and multi-species evolutionary competition models such as the Ricker model and the Leslie-Gower model with one trait or multi-traits.

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On measurement-assisted control in a three-level quantum system with dynamical symmetry

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In this talk we consider three-level system with dynamical symmetry: it has conserved quantity under coherent evolution which bounds the transition probability between different eigen states of the free Hamiltonian by $\frac{1}{2}$. However, one could break this symmetry by using incoherent control, i. e. measurement. In [1] it was shown that with the following evolution

$$\rho\left(T_2\right) = U_{f_2}\mathcal{M}_{P_\psi}\left(U_{f_1}\rho\left(-T_1\right)U_{f_1}^\dagger\right)U_{f_2}^\dagger.$$

one could achieve transition probability of approximately 0.678. In this talk, we study kinematic control landscape of the transition probability functional and show that it has the following types of critical points: global maxima, global minima, saddles, second order traps [2].

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Direct and inverse problems for odd-order quasilinear evolution equations

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On the interval $I = (0, R)$ for certain $R > 0$ an initial-boundary value problem for an equation

$$u_t - (-1)^l(\partial_x^{2l+1}u + a_{2l}\partial_x^{2l}u) - \sum_{j=0}^{l-1}(-1)^j\partial_x^j\big[a_{2j+1}(t,x)\partial_x^{j+1}u + a_{2j}(t,x)\partial_x^j u\big]$$

$$+ \sum_{k=0}^{l}(-1)^k\partial_x^k\big[g_k(t,x,u,\ldots,\partial_x^{l-1}u)\big] = f(t,x)$$

for certain natural l with initial and boundary conditions

$$u(0,x) = u_0(x), \quad x \in [0,R],$$

$$\partial_x^j u(t,0) = \partial_x^j u(t,R) = 0,\ j = 0,\ldots,l-1, \quad \partial_x^l u(t,R) = \nu(t), \quad t \geq 0,$$

is considered.

The class of such equations includes the Korteweg–de Vries (KdV) and the Korteweg–de Vries–Burgers equations with their generalizations for higher-order nonlinearity, in particular, the modified KdV equation (mKdV) in the case $l = 1$, the Kawahara, the Benney-Lin equation (also with their generalizations for higher-order nonlinearity), the Kaup–Kupershmidt equation, the third equation of the KdV hierarchy, the second equation of the mKdV hierarchy in the case $l = 2$ and so on.

Weak solutions of these problem are constructed in a class

$$X(Q_T) = C([0,T]; L_2(I)) \cap L_2(0,T; H_0^l(I)),$$

where $Q_T = (0,T) \times I$ for an arbitrary $T > 0$.

The functions a_i are subjected to certain sign and size restrictions (in particular, $a_{2l} \le 0$), the functions g_k — to growth restrictions.

Then for small $u_0 \in L_2(I)$, $\nu \in L_2(0,T)$ and $f \in L_2(Q_T)$ the result on existence and uniqueness of weak solutions is established.

Moreover, large-time exponential decay of these solutions in the norm $L_2(I)$ is also investigated.

Besides this direct problem inverse problems with integral overdetermination

$$\int_0^R u(t,x)\omega(x)\,dx = \varphi(t), \quad t \in [0,T],$$

for certain given functions ω and φ are also studied. We consider two types of inverse problems: in the first one the function f is presented in a form

$$f(t,x) = F(t)g(t,x),$$

where the function g is given and the function F is unknown; in the second one the function ν is unknown. The aim is to find the function F in the first case and the function ν in the second case, such that the corresponding weak solution of the initial-boundary value problem satisfies this additional condition.

From the function ω we always need that

$$\omega \in H^{2l+1}(I), \quad \omega^{(i)}(0) = 0,\ i = 0,\dots,l, \quad \omega^{(i)}(R) = 0,\ i = 0,\dots,l-1.$$

In the first inverse problem it is also assumed that the function $g \in C([0,T];L_2(I))$ verifies the condition

$$\int_0^R g(t,x)\omega(x)\,dx \ne 0 \quad \forall t \in [0,T],$$

in the second problem — that $\omega^{(l)}(R) \ne 0$. In all cases the compatibility condition

$$\varphi(0) = \int_0^R u_0(x)\omega(x)\,dx$$

is also needed.

Under the smallness assumptions on $u_0 \in L_2(I)$, $\varphi \in W_1^1(0,T)$ as well as on $\nu \in L_2(0,T)$ in the first problem and $f \in L_2(Q_T)$ in the second one, the results on existence and uniqueness of the solutions $u \in X(Q_T)$ and $F \in L_1(0,T)$ in the first case, $\nu \in L_2(0, T)$ on the second case are obtained.

The work has been supported by Russian Science Foundation grant 23-21-00101.

On projections of a compact set in R^N

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We apply ideas of geometric measure theory and Baire category theory to topological problems, namely, to topological embeddings of compact sets into Euclidean space.

In 1947, Borsuk constructed a Cantor set in R^N, $N \geq 3$, such that its projection onto any $(N-1)$-plane contains an $(N-1)$-dimensional ball. This can be strengthened: a desired Cantor set can be obtained from an arbitrary Cantor set by an arbitrarily small isotopy of the space R^N.

In the same paper, Borsuk described a knot in R^3 whose orthogonal projection onto any plane contains a 2-dimensional disk. An analysis of Borsuk's work shows that such a knot exists in every equivalence class of knots (both tame and wild). Again, a knot with 2-dimensional projections can be obtained from an arbitrary knot by an arbitrarily small isotopy of the space R^N.

The question arises: how do the dimensions of the projections of a compact set $X \subset R^N$ behave under a typical ambient isotopy or under a typical ambient homeomorphism? (Typical in the sense of the Baire category.)

We solve this problem. Our main result strengthens Väisälä's theorem (1979) connecting Hausdorff dimension and Shtan'ko embedding dimension (denoted by "dem"). In its turn, Väisälä's theorem extends results of Nöbeling (1931) and Szpilrajn (1937) on relationship between Hausdorff dimension and topological dimension.

Theorem 1. *Let $X \subset R^N$ be a compact set, U its bounded open neighborhood, $\varepsilon > 0$. Then for a typical homeomorphism $f \in \mathrm{Homeo}_\varepsilon(\overline{U}, \partial U)$ we have* $\mathrm{dem}\, X = \dim_H(f(X))$.

Here $\mathrm{Homeo}_\varepsilon(\overline{U}, \partial U)$ is the set of all homeomorphisms $f : \overline{U} \cong \overline{U}$ such that $f|_{\overline{U}} = \mathrm{id}$ and $d(f, \mathrm{id}) < \varepsilon$, where d is the standard uniform distance. (Recall that $\mathrm{Homeo}_\varepsilon(\overline{U}, \partial U)$ is a G_δ-subset of $C(\overline{U}, \overline{U})$ since ∂U is closed in $\overline{U}$. Hence $\mathrm{Homeo}_\varepsilon(\overline{U}, \partial U)$ is metrizable by a complete metric, and the notion of a typical element has sense.)

Using Theorem 1, we find out how the projections of a knot "typically" behave. Recall that a typical knot in R^3 is wild (J. Milnor, 1964) and even wild in every point (H.-G. Bothe, 1966).

Corollary 2. *Let* $\Sigma \subset R^3$ *be an arbitrary knot,* U *its bounded open neighborhood,* $\varepsilon > 0$. *Then a typical homeomorphism* $f \in \mathrm{Homeo}_\varepsilon(\overline{U}, \partial U)$ *has the property: the projection of the knot* $f(\Sigma)$ *to any* 2*-plane and to any line is one-dimensional.*

We will also present an isotopic version of Theorem 1 and analogues of Corollary 2 for Cantor sets in R^N.

As a consequence, we get new criteria of tameness and wildness of a Cantor set in terms of its projections.

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Approximation of the C_0-semigroup of the heat equation by iterations of high-order Chernoff functions

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Let $(X, \|\cdot\|)$ be any Banach space and $\mathscr{L}(X)$ denotes the set of all bounded linear operators on X. Next we will use the notions of *strongly continuous one-parameter semigroup* (or just *C_0–semigroup*), *contractive semigroup* and *generator of a strongly continuous semigroup*, definitions of which can be found, for example, in the book of Engel and Nagel [2]. In 1968 Paul Chernoff proved the following theorem:

Theorem 1 (Chernoff [1]). *Let X be a Banach space, $F(t)$ be a strongly continuous function from $[0,\infty)$ to the set of linear contraction operators on X, such that $F(0) = I$. Suppose that the closure A of the strong derivative $F'(0)$ is the generator of contractive C_0–semigroup $\{e^{tA}\}_{t\geq 0}$. Then $[F(t/n)]^n$ converges to e^{tA} in the strong operator topology.*

Let us note that this theorem does not contain an estimate of the rate of convergence. In 2022 was published the theorem that provides such estimate under certain conditions:

Theorem 2 (Galkin, Remizov [3]). *Suppose that:*

1) $T > 0$, $M_1 \geq 1$, $w \geq 0$. $(A, D(A))$ is generator of C_0-semigroup $(e^{tA})_{t\geq 0}$ in a Banach space X, such that $\|e^{tA}\| \leq M_1 e^{wt}$ for $t \in [0,T]$.

2) There are a mapping $F\colon (0,T] \to \mathscr{L}(X)$ and constant $M_2 \geq 1$ such that we have $\|(F(t))^k\| \leq M_2 e^{kwt}$ for all $t \in (0,T]$ and all $k \in \mathbb{N} = \{1,2,3,\ldots\}$.

3) $m \in \mathbb{N}\cup\{0\}$, $p \in \mathbb{N}$, subspace $\mathcal{D} \subset D(A^{m+p})$ is $(e^{tA})_{t\geq 0}$-invariant.

4) There exist such functions $K_j\colon (0,T] \to [0,+\infty)$, $j = 0,1,\ldots,m+p$ that for all $t \in (0,T]$ and all $x \in \mathcal{D}$ we have

$$\left\| F(t)x - \sum_{k=0}^{m} \frac{t^k A^k x}{k!} \right\| \leq t^{m+1} \sum_{j=0}^{m+p} K_j(t) \|A^j x\|.$$

Then for all $t > 0$, all integer $n \geq t/T$ and all $x \in \mathcal{D}$ we have

$$\|(F(t/n))^n x - e^{tA}x\| \leq \frac{M_1 M_2 t^{m+1} e^{wt}}{n^m} \sum_{j=0}^{m+p} C_j(t/n)\|A^j x\|,$$

where $C_{m+1}(t) = K_{m+1}(t)e^{-wt} + M_1/(m+1)!$ *and* $C_j(t) = K_j(t)e^{-wt}$ *for all* $j \neq m+1$.

The mapping $F\colon (0,T] \to \mathscr{L}(X)$ is called a *Chernoff function of order* m *for operator* A iff it satisfies the conditions of theorem 2. Let $UC_b(\mathbb{R})$ be the Banach space of all uniformly continuous bounded functions $f\colon \mathbb{R} \to \mathbb{R}$ with the norm $\|f\| = \sup_{x\in\mathbb{R}} |f(x)|$, and linear operator $L = [f \mapsto f'']$ has domain $D(L) = \{f \in UC_b(\mathbb{R}) \mid f'' \in UC_b(\mathbb{R})\}$. Here we are interesting *how to construct space–shift based Chernoff function* S_m *of any order* m *for operator* L. Previously, the following results were known in this direction:

In 2016 Ivan Remizov [4] found Chernoff function of order 1 containing 3 summands:

$$[S_1(t)f](x) = \frac{1}{2}f(x) + \frac{1}{4}f(x+2\sqrt{t}) + \frac{1}{4}f(x-2\sqrt{t}) = f(x) + tf''(x) + o(t).$$

In 2019 Alexander Vedenin found Chernoff function of order 2 with 3 summands too:

$$[S_2(t)f](x) = \frac{2}{3}f(x) + \frac{1}{6}f(x+\sqrt{6t}) + \frac{1}{6}f(x-\sqrt{6t}) = f(x) + tf''(x) + \frac{t^2}{2}f^{IV}(x) + o(t^2).$$

In general, the following theorem is true:

Theorem 3. *For any natural* m*, there is a unique Chernoff function* S_m *of order* m *for the operator* $L = [f \mapsto f'']$*, having the form* $[S_m(t)f](x) = \sum_{i=1}^{m+1} a_i \cdot f(x + b_i t^{s_i})$.

In this case, the following conditions will be met:

1) $s_1 = \ldots = s_{m+1} = 1/2$*;*

2) the numbers $b_1, \ldots, b_{m+1}$ *are different roots of the orthogonal Chebyshev–Hermite polynomials;*

3) the numbers $a_1, \ldots, a_{m+1}$ *are the Christoffel coefficients corresponding to the quadrature nodes* $b_1, \ldots, b_{m+1}$ *and can be calculated by the formulas*

$$a_i = \frac{2^{m+2}(m+1)!\sqrt{\pi}}{(H'_{m+1}(b_i))^2}, \quad i = 1, \dots, m+1.$$

The authors are partially supported by Laboratory of Dynamical Systems and Applications NRU HSE, grant of the Ministry of science and higher education of the RF, ag. № 075-15-2022-1101

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Error of Chernoff approximations based on Chernoff function with a given t^2-coefficient

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Let $(X, \|\cdot\|)$ be any Banach space and $\mathscr{L}(X)$ denotes the set of all bounded linear operators on X. Next we will use the notions of *strongly continuous one-parameter semigroup* (or just *C_0–semigroup*), *contractive semigroup* and *generator of a strongly continuous semigroup*, definitions of which can be found, for example, in the book of Engel and Nagel [2]. In 1968 Paul Chernoff proved the following theorem:

Theorem 1 (Chernoff [1]). *Let X be a Banach space, $F(t)$ be a strongly continuous function from $[0,\infty)$ to the set of linear contraction operators on X, such that $F(0) = I$. Suppose that the closure A of the strong derivative $F'(0)$ is the generator of contractive C_0–semigroup $\{e^{tA}\}_{t\geq 0}$. Then $[F(t/n)]^n$ converges to e^{tA} in the strong operator topology.*

Let us note that this theorem does not contain an estimate of the rate of convergence. In 2022 was published the theorem that provides such estimate under certain conditions:

Theorem 2 (Galkin, Remizov [3]). *Suppose that:*

1) $T > 0$, $M_1 \geq 1$, $w \geq 0$. $(A, D(A))$ is generator of C_0-semigroup $(e^{tA})_{t\geq 0}$ in a Banach space X, such that $\|e^{tA}\| \leq M_1 e^{wt}$ for $t \in [0, T]$.

2) *There are a mapping $F\colon (0, T] \to \mathscr{L}(X)$ and constant $M_2 \geq 1$ such that we have $\|(F(t))^k\| \leq M_2 e^{kwt}$ for all $t \in (0, T]$ and all $k \in \mathbb{N} = \{1, 2, 3, \ldots\}$.*

3) $m \in \mathbb{N} \cup \{0\}$, $p \in \mathbb{N}$, subspace $\mathcal{D} \subset D(A^{m+p})$ is $(e^{tA})_{t\geq 0}$-invariant.

4) There exist such functions $K_j\colon (0, T] \to [0, +\infty)$, $j = 0, 1, \ldots, m+p$ that for all $t \in (0, T]$ and all $x \in \mathcal{D}$ we have $\left\| F(t)x - \sum_{k=0}^{m} \frac{t^k A^k x}{k!} \right\| \leq t^{m+1} \sum_{j=0}^{m+p} K_j(t) \|A^j x\|$.

Then for all $t > 0$*, all integer* $n \geq t/T$ *and all* $x \in \mathcal{D}$ *we have*

$$\|(F(t/n))^n x - e^{tA}x\| \leq \frac{M_1 M_2 t^{m+1} e^{wt}}{n^m} \sum_{j=0}^{m+p} C_j(t/n)\|A^j x\|,$$

where $C_{m+1}(t) = K_{m+1}(t)e^{-wt} + M_1/(m+1)!$ and $C_j(t) = K_j(t)e^{-wt}$ for all $j \neq m+1$.

So the question arises: *what is the lower estimate of the error* $\|(F(t/n))^n x - e^{tA}x\|$ *?* In 2018, Ivan Remizov formulated the following conjecture:

Conjecture (Remizov [4]). Let $(e^{tA})_{t\geq 0}$ be a C_0-semigroup in a Banach space X, and F is a Chernoff function for operator A (recall that this implies $F(0) = I$ and $F'(0) = A$ but says nothing about $F''(0)$) and number $T > 0$ is fixed. Suppose that vector x is from intersection of domains of operators $F'(t)$, $F''(t)$, $F'''(t)$, $F''''(t)$, $F'(t)F''(t)$, $(F'(t))^2 F''(t)$, $(F''(t))^2$ for each $t \in [0, T]$, and suppose that if $Z(t)$ is any of these operators then function $t \to Z(t)x$ is continuous for each $t \in [0, T]$. Then there exists such a number $C > 0$, that for each $t \in [0, T)$ and each $n \in \mathbb{N}$ the following inequality holds, where $B = F''(0)$:

$$\|(F(t/n))^n x - e^{tA}x - \frac{t^2}{2n}e^{tA}(B - A^2)x\| \leq \frac{C}{n^2}.$$

Although this hypothesis is not true in general, the following theorem holds:

Theorem 3. *Suppose that:*

1) C_0*-semigroup* $(e^{tA})_{t\geq 0}$ *in a Banach space* X *has bounded generator* $A \in \mathscr{L}(X)$.

2) $T > 0$ *and there are a mapping* $F\colon [0, T] \to \mathscr{L}(X)$ *and constants* $M \geq 1$, $w \geq 0$ *such that* $\|(F(t))^k\| \leq Me^{kwt}$ for all $t \in [0, T]$, $k \in \mathbb{N}$.

3) There exist such bounded operator $B \in \mathscr{L}(X)$ *and constant* $K \geq 0$ *that for all* $t \in [0, T]$ *we have* $\left\|F(t) - I - tA - \frac{t^2}{2}B\right\| \leq Kt^3$.

Then there exists such a number $C > 0$*, that for each* $t \in [0, T]$ *and each* $n \in \mathbb{N}$ *the following inequality holds:*

$$\left\|(F(t/n))^n - e^{tA} - \frac{t^2}{2n}\int_0^1 e^{tsA}(B - A^2)e^{t(1-s)A}ds\right\| \leq \frac{C}{n^2}.$$

The authors are partially supported by Laboratory of Dynamical Systems and Applications NRU HSE, grant of the Ministry of science and higher education of the RF, ag. № 075-15-2022-1101.

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Temperature and entropy of self-gravitating dusty atmosphere behind cylindrical shock

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In this work, we have studied the change in temperature and entropy occurring just behind the cylindrical shock front in the e, for this, we have solved the problem with a characteristic approach. The gases of the atmosphere have been considered van der Waals gas, in which some dust particles are considered to be included. In this study these dust particles are completely inert, solid, similar in size and are uniformly distributed throughout the medium. The influence of overtaking disturbances on the parameters, temperature, and entropy production of weak and strong shock propagation with shock implosion, the mass concentration of solid particles in the medium, have been calculated for exponentially changing internal densities incorporating the effect of self-gravitation. The effect of the van der Waals gas in the previously obtained solution for an ideal gas is described by showing it through graphs. In the end, it has been found from this study that the dust particles present in the atmosphere increase the strength of the explosions in it. This increment in values of temperature and entropy become even stronger for the van der Waals gas medium. Here all the work of the computer has been done through MATLAB software.

Ice-water phase transition in flow along small irregularities

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We study the problem of a phase transition in a liquid flow along small irregularities on the plate surface for large Reynolds numbers Re. Namely, we consider the problem of water flow along a small ice irregularity localized at the point x_0, i.e. the streamlined surface has the form $y_s = \varepsilon^{4/3}h(t,(x-x_0)/\varepsilon)$, where $\varepsilon = \text{Re}^{-1/2}$ is a small parameter, $h(t,\xi)$ is an continuous function localized on ξ in a neighborhood of the point x_0, see Fig. 1. Note that the selected scales of irregularity define the double-deck structure of the boundary layer [4]: a thin boundary layer (see I in Fig.1(a)) is formed in addition to the classical Prandtl boundary layer (see II in Fig.1(a)), and the flow in I is described by

$$\varepsilon^{2/3}\frac{\partial u^*}{\partial t} + u^*\left(\frac{\partial u^*}{\partial \xi} - \frac{\partial h}{\partial \xi}\frac{\partial u^*}{\partial \bar\theta}\right) + v^*\frac{\partial u^*}{\partial \bar\theta} = -f''(0)\,v^*\big|_{\bar\theta\to\infty} + \frac{\partial^2 u^*}{\partial \bar\theta^2} + \tag{1}$$
$$+\varepsilon^{2/3}\frac{\partial h}{\partial t}\frac{\partial u^*}{\partial \bar\theta},$$
$$\frac{\partial v^*}{\partial \bar\theta} + \frac{\partial u^*}{\partial \xi} - \frac{\partial h}{\partial \xi}\frac{\partial u^*}{\partial \bar\theta} = 0,$$
$$(u^*,v^*)\big|_{\bar\theta=0} = (0,0),\ (u^*,v^*)\big|_{\xi\to\pm\infty} = (f''(0)\bar\theta,0),\ \frac{\partial u^*}{\partial \bar\theta}\bigg|_{\bar\theta\to\infty} =$$
$$= f''(0),\ u^*\big|_{t=0} = U_0(\xi,\bar\theta),$$

where (u^*,v^*) is the velocity vector, $\xi = (x-x_0)/\varepsilon$, $\bar\theta = (y-y_s)/\varepsilon^{4/3}$ are boundary layer variables, f is the Blasius function, U_0 is some smooth function.

We consider two cases of the streamlined surface. First, the entire streamlined surface is ice (i.e., the irregularity is a build-up or notch in the ice) [1], see Fig. 1 (a). Second, the streamlined surface is a substrate of some (non-meltable) material, and the irregularity is a frozen drop [3], see Fig. 1 (b).

The ice-water phase transition is investigated using the phase field system [2] based on the introduction of order function $\varphi = \varphi_\zeta(t, \xi, \theta)$ such that $\varphi = +1$ in the solid phase and $\varphi = -1$ in the liquid phase, and ζ is a regularization parameter (the function φ changes rapidly from -1 to 1 in the ζ-neighborhood of the boundary between the phases). In the scales of the problem under study, the system of phase field equations has the form

$$\varepsilon^{2/3}\frac{\partial T}{\partial t} + A\left[u^*\frac{\partial T}{\partial \xi} + v^*\frac{\partial T}{\partial \theta}\right] - \frac{\lambda_l}{c_l\rho_l}\varepsilon^{-2}\left(\varepsilon^{2/3}\frac{\partial}{\partial \xi}\left(\lambda\frac{\partial T}{\partial \xi}\right) + \frac{\partial}{\partial \theta}\left(\lambda\frac{\partial T}{\partial \theta}\right)\right) = \\ = -\frac{1}{2}\frac{\partial \varphi_\zeta}{\partial t}, \tag{2}$$

$$\zeta^2\alpha\frac{\partial \varphi_\zeta}{\partial t} = \zeta^2\beta\Delta_{\xi,\theta}\varphi_\zeta + \varphi_\zeta(1-\varphi_\zeta^2) + \zeta(1-\varphi_\zeta^2)T/\sqrt{2},$$

where T is the temperature, $\theta = \bar{\theta} + h(t, \xi)$, $A = 1$ in the liquid phase and $A = 0$ in the solid phase (the terms with the coefficient A are continuous at the interface between phases due to the nonslip conditions for velocities, see (1)), λ is the dimensionless heat conduction coefficient, and others parameters are physical constants or their combination, see [1] for details. The boundary between phases at each time moment is defined as zero-level set of function φ, i.e. $h = \{\varphi = 0\}$.

Note that in the problem statement in Fig. 1 (b), there exists points $\xi_1(t)$, $\xi_2(t)$ at which the three media (water, ice and the substrate) contact. System (35) defines the boundary between the phases everywhere except these points. The conditions at the points $\xi_1(t)$, $\xi_2(t)$ should be formulated separately, and we propose two versions [3].

The work was supported by the RSF under grant No. 22-21-00186.

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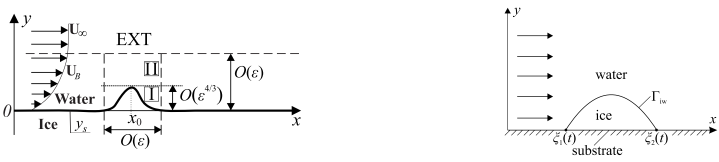

(a) (b)

Figure 1: The geometry of the problem

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Blow-up of solutions of nonlocal parabolic equation under nonlocal boundary condition

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We consider the initial boundary value problem for nonlinear nonlocal parabolic equation

$$u_t = \Delta u + au^p \int_\Omega u^q(y,t)\,dy - bu^m,\ x \in \Omega,\ t > 0, \tag{1}$$

with nonlinear nonlocal boundary condition

$$\frac{\partial u(x,t)}{\partial \nu} = \int_\Omega k(x,y,t)u^l(y,t)\,dy,\ x \in \partial\Omega,\ t > 0, \tag{2}$$

and initial datum

$$u(x,0) = u_0(x),\ x \in \Omega, \tag{3}$$

where a, b, p, q, m, l are positive numbers, Ω is a bounded domain in R^N for $N \geq 1$ with smooth boundary $\partial\Omega$, ν is unit outward normal on $\partial\Omega$.

We suppose that the functions $k(x,y,t)$ and $u_0(x)$ satisfy the following conditions:

$$k(x,y,t) \in C(\partial\Omega \times \overline{\Omega} \times [0,+\infty)),\ k(x,y,t) \geq 0;$$

$$u_0(x) \in C^1(\overline{\Omega}),\ u_0(x) \geq 0 \text{ in } \Omega,\ \frac{\partial u_0(x)}{\partial \nu} = \int_\Omega k(x,y,0)u_0^l(y)\,dy \text{ on } \partial\Omega.$$

Initial boundary value problem for parabolic equation (1) with nonlocal boundary condition

$$u(x,t) = \int_\Omega k(x,y,t)u^l(y,t)\,dy,\ x \in \partial\Omega,\ t > 0$$

was considered in [1; 2].

Let $Q_T = \Omega \times (0,T),\ S_T = \partial\Omega \times (0,T)$, $\Gamma_T = S_T \cup \overline{\Omega} \times \{0\}$, $T > 0$.

Definition. We say that a nonnegative function $u(x,t) \in C^{2,1}(Q_T) \cap C^{1,0}(Q_T \cup \Gamma_T)$ is a supersolution of (1)–(3) in Q_T if

$$u_t \geq \Delta u + au^p \int_\Omega u^q(y,t)\,dy - bu^m,\ (x,t) \in Q_T, \tag{4}$$

$$\frac{\partial u(x,t)}{\partial \nu} \geq \int_\Omega k(x,y,t)u^l(y,t)\,dy,\ (x,t) \in S_T, \tag{5}$$

$$u(x,0) \geq u_0(x),\ x \in \Omega, \tag{6}$$

and $u(x,t) \in C^{2,1}(Q_T) \cap C^{1,0}(Q_T \cup \Gamma_T)$ is a subsolution of (1)–(3) in Q_T if $u \geq 0$ and it satisfies (4)–(6) in the reverse order. We say that $u(x,t)$ is a solution of problem (1)–(3) in Q_T if $u(x,t)$ is both a subsolution and a supersolution of (1)–(3) in Q_T.

Theorem 1. *Let $\overline{u}$ and $\underline{u}$ be a supersolution and a subsolution of problem (1)–(3) in Q_T, respectively. Suppose that $\underline{u}(x,t) > 0$ or $\overline{u}(x,t) > 0$ in $Q_T \cup \Gamma_T$ if* $\min(p,q,l) < 1$. *Then $\overline{u}(x,t) \geq \underline{u}(x,t)$ in $Q_T \cup \Gamma_T$.*

Theorem 2. *Let* $\max(p+q,l) \leq 1$ *or* $1 < \max(p+q,l) < m$. *Then every solution of (1)–(3) is global.*

To formulate finite time blow-up result we suppose that

$$\inf_\Omega \int_{\partial\Omega} k(x,y,0)\,dS_x > 0. \tag{7}$$

Theorem 3. *Let either $r + p >$ $\max(m,1)$ or $l >$ $\max(m,1)$ and (7) hold. Then solutions of (1)–(3) blow up in finite time if initial data are large enough.*

Remark. We improve comparison principle, global existence and blow-up results in [4].

The results of the talk have been published in [3].

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Topological structure of manifolds supporting axiom A systems

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Dynamical systems satisfying an Axiom A (in short, A-systems) were introduced by S. Smale. By definition, a non-wandering set of A-system is the topological closure of periodic orbits endowed with a hyperbolic structure. Due to Smale's Spectral Decomposition Theorem, the non-wandering set of any A-system is a disjoint union of closed, invariant, and topologically transitive sets called *basic sets*.

Let M^n, $n \geq 3$, be a closed orientable n-manifold and $G_k^{diff}(M^n)$ the set of A-diffeomorphisms $f : M^n \to M^n$ satisfying the following conditions:

1. f has $k \geq 0$ nontrivial basic sets each is either an orientable codimension one expanding attractor or an orientable codimension one contracting repeller, and there are no another non-trivial basic sets;
2. the invariant manifolds of isolated saddle periodic orbits are intersected transversally;
3. the codimension one separatrices of isolated saddle periodic orbits are without heteroclinic manifolds.

We prove that if M^n admits $f \in G_k^{diff}(M^n)$, $k \geq 1$, then M^n is the connected sum of k n-tori T^n, and $S^{n-1} \times S^1$'s, and a simply-connected manifold admitting a polar Morse-Smale diffeomorphism with no codimension one saddle periodic orbits where S^l is a l-sphere. For $k = 0$, we show that $G_0^{diff}(M^n)$ consists of Morse-Smale diffeomorphisms. We prove that if M^n admits $f \in G_0^{diff}(M^n)$, then M^n is either S^n or the connected sum of $S^{n-1} \times S^1$'s and simply-connected manifolds N_i^n admitting polar Morse-Smale diffeomorphisms.

Authors thank for a support the Russian Science Foundation (grant 22-11-00027), this work was fulfilled in the Laboratory of Dynamical Systems and Applications of HSE University that being financed by the grant 075-15-2022-1101 of the Ministry of Science and Higher Education of RF.

Automorphisms of limits for inductive sequences of the Toeplitz-Cuntz algebras and generalized means on the P-adic solenoids

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The talk deals with the automorphisms of C^*-algebras and the generalized means on compact groups which are called the P-adic solenoids. The C^*-algebras under our consideration are the inductive limits for the inductive sequences of the Toeplitz-Cuntz algebras whose connecting homomorphisms are defined by tuples consisting of sequences of prime numbers. We consider properties of the limit $*$-endomorphisms of these C^*-algebras. The limit $*$-endomorphisms are induced by morphisms between the copies of the same direct sequences of the Toeplitz-Cuntz algebras.

We discuss the criterion for the limit $*$-endomorphisms to be automorphisms which is formulated in terms of the existence of the generalized means on the P-adic solenoids. This criterion is closely related to the results in [1–3].

The talk is based on the results of the joint work with E. V. Lipacheva (KSPEU, KFU).

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Convex structure of generalized states, effects and ultraproducts

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In the report abstract convex structures of states and generalized states defined on the event space will be considered. The concept of operations on generalized states and related effects and ultraproducts of the corresponding convex structures will also be considered [1; 2].

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Multiplication formulas for Gaussian operators

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The operators of the form exp $\mathcal{P}(\mathbf{R})$, where $\mathcal{P}(\mathbf{R})$ is a polynomial of the order ≤ 2 in the canonical observables $\mathbf{R} = [q_1, p_1, \dots, q_s, p_s]$, constitute a semigroup with respect to the operator multiplication. A number of useful formulas, including square root of a divisible element, were obtained by different authors. Notably, by using expressions $\sqrt{\rho_1}\rho_2\sqrt{\rho_1}$ for Gaussian operators ρ_1, ρ_2 obtained by Lami-Das-Wilde, Banchi-Braunstein-Pirandola and by the author, allow for explicit computation of:

⋆ *Fidelity between two arbitrary Gaussian states*;

⋆ *Accessible information of arbitrary Gaussian state ensemble*;

⋆ *Entropy reduction of arbitrary Gaussian state underlying entanglement-assisted capacity of Gaussian measurement channel.*

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Mat-Balance as the Sewing Machinery Analitic and Numeric Solution

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Let 2-D flow in the porous media domain U to be generated by source(sink) modeled by small area ω inside two dimensional ball-$\tilde{B}$(or radius r_w inside a cell of B_0. Let $U_M(x,y,t)$ to be e discreet solution defined on the grid U_N approximating domain U. Consider connected five-spot cells $\cup_{i=0}^{5} B_i$ with $B_0 \supset \tilde{B}$, and B_1, B_2, B_3, and B_4 surround center cell B_0. This 5-cells characterized by size $\Delta >> r_w$. Assume P_M is a numerical solution of the problem in the discretized domain of the flow. Flow itself is Main goal is: How to accurately interpret numerical value of the p_0, which associates to the box B_0 w.r.t. actual(analytical) value of the pressure on the well $\Gamma_w = \partial B(r_w)$. Note that in the intended application, discretization of the Γ_w is not possible.

To solve this problem, we consider sewing machinery between finite difference and analytical solutions defined at different scale: far away and near the source of the perturbation of the flow. One of the essences of the approach is that coarse problem and boundary value problem in the proxy of the source model, two different flows. We are proposing a method to glue solution via total fluxes, which is predefined on coarse grid. It is important to mention that the coarse solution "does not see" boundary.

From an industrial point of view, our report provides a mathematical tool for analytical interpretation of simulated data for fluid flow around a well in a porous medium. It can be considered as a mathematical "shirt" on famous

Peaceman well-block radius formula for linear (Darcy) radial flow. Note that in known authors literature, rate of the production on the well-q is time independent.

We developed a method to determine Peaceman well block radius R_0 which depends only on stationary parameters, and converges to Peaceman radius R_0 in cylindrical reservoir when external radius of the domain of the flow converges to infinity, for a class of dynamic flows which rottenly is used industry. We enlarge MB equation for three regimes of the Darcy flows:

I. Stationary (SS); II. Pseudo Stationary (PSS); III. Boundary Dominated(BD).

To introduce MB system of equation let first consider the finite set of dependant variables

$$\mathcal{P} = \left\{p_{\pm r_0,0}(s); p_{0,\pm r_0}(s); p_{\pm 1,0}(s); p_{0,\pm 1}(s); q_x^{\pm}(s); q_y^{\pm}(s)\right\}. \tag{1}$$

Let

$$\mathcal{K} = \left\{K_x^{\pm}; K_y^{\pm}\right\} \text{ and } \mathcal{Q} = \left\{Q_x^{\pm}; Q_y^{\pm}\right\}. \tag{2}$$

be inputs, which in this study are considered to be constants. To make discussion more motivated we will highlight intended application.

Major assumption is that process of the transport and changes of the fluid is much "faster" than geological process, and therefore dependents of the **K** and $\mathcal{Q}$ to be ignored. Assume for clarity that process which will be modeled by inputs $\mathcal{P},\ \mathcal{Q}, \mathcal{K}$ are linear, isotropic and one dimensional. In this case variable $q(s)$ and $p_i(s)$ are assumed to be constrained the following MB equation Assume that $i = 0, 1$ then using above arguments, we are considering Algebraic Parametric Structure as a sewing machinery between numerical and "analytical" solutions :

$$\tau \cdot \left(J_{1,0}^p \cdot (p_0(s) - p_1(s)) - I_q \cdot q\right) = L_q^{p_0} \cdot (p_0(s+\tau) - p_0(s)) \tag{3}$$

Value of the coefficients and their dependence on input parameters can vary depending on the intended applications, dimension, geometry and dynamics of the process, discretization, etc.

In a view of the algebraic structure equation 3 q is main input definitive parameter , three others $J_{1,0}^p$, I_q, and $L_q^{p_0}$ are auxiliary but are crucial characteristics of the system.

Let in material balance equation 3

$$I_q(s) = q \cdot \frac{1}{\cdot\Delta}, J_{1,0}^p = 2K \cdot \frac{1}{\Delta_x^2}, L_q^{p_0} = \phi \cdot C_p \frac{V_0}{V}, \frac{V_0}{V} = 1 \tag{4}$$

Then MB equation 3 takes form

$$2K \cdot (p_0(s) - p_1(s)) = -q(s) \cdot \Delta + C^0 \cdot \frac{p_0(s+\tau) - p_0(s)}{\tau} \cdot \Delta^2. \qquad (5)$$

Then symmetrical, isotropic and steady-state steady MB has the form

$$2K \cdot (p_0 - p_1) = q\Delta. \qquad (6)$$

Let $1-D$ domain $(0; R_e)$ is split by grid $[0, \Delta, 2 \cdot \Delta, \cdots N \cdot \Delta]$ were $N\Delta = r_e$. We will say that Peaceman problem is well posed w.r.t. MB 6 for $1-D$ flows if for any given Δ exist R_0 depending on Δ s.t. analytical solution of the 1-D SS problem satisfies equation:

$$-2K \cdot (p_{an}(\Delta) - p_{an}(R_0)) = q \cdot \Delta. \qquad (7)$$

From explicate formula for analytical solution in SS case follows that Peaceman problem to be well posed w.r.t. MB (6) it is necessary and sufficient that $R_0 = \frac{\Delta}{2}$.

Let define the PSS constrain for the solution of algebraic material balance equation assuming in addition that p_0 p_1 and q are conditioned as follows

We will say that for MB satisfies PSS constrains if $q(s) = q$, $p_0(s+\tau) - p_0(s) = q \cdot C_0 \cdot \tau$, and $p_1(s) - p_0(s)$ are s independent,

Then linear 1-D PSS Material Balance will have form

$$2K \cdot (p_1 - p_0) = 2K \cdot = q\Delta \left(1 - \phi c_p \cdot 1 \cdot C_0 \Delta\right) = q\Delta \left(1 - C_1 \Delta\right). \qquad (8)$$

We will say that Peaceman problem for PSS is well posed w.r.t. time dependent MB 5 for $1-D$ flows if for any given Δ, and r_e exist $R_0^{pss}(\Delta, r_e)$ depending on Δ and r_e s.t. analytical solution of the 1-D PSS problem satisfies equation (8) and in constrains for PSS MB . From explicate formula for analytical solution in PSS case follows that Peaceman problem to be well posed w.r.t. MB (8) it is necessary and sufficient that

$$R_0^{pss}(\Delta, r_e) = \frac{\Delta}{1 + \sqrt{1 + \frac{\Delta}{r_e} + 2\frac{\Delta^2}{r_e^2}}} + \frac{2\Delta^2}{r_e}, \text{morover} \lim_{r_e \to \infty} R_0^{pss}(\Delta, r_e) = R_0. \qquad (9)$$

Algebraic MB-constrains for boundary dominated is stated as follows: a. Exist constants $Q_0; \mathbf{P}_1; \mathbf{P}_0$, s.t. for variables p_i and q in MB equation the following ratios : $\frac{q(s)}{p_0(s)} = Q_0(r_e)$, $\frac{p_1(s)}{p_0(s)} = \mathbf{P}_1(\Delta, r_e)$; $\frac{p_0(s+\tau)}{p_0(s)} = \mathbf{P}_0(\Delta, r_e)\frac{e^{-C(K,r_e)\cdot\tau}-1}{\tau}$ are s

independent (Note that in above $\mathbf{P}_0(\Delta, r_e)$ is constant depending on Δ and r_e only). We will say that Peaceman problem is well posed for BD regime of flow if exist analytical solution for which all above constrain and this analytical solution satisfies (5). We proved that exists analytical solution which satisfies MB equation (5) and all above constrains. Moreover correspond R_0^{BD} can be calculated using equation:

$$R_0^{bd} \approx \frac{\frac{\Delta}{2}}{1 - \frac{\phi C_p}{8K}\frac{\pi^2}{r_e^2}\Delta} \text{assuming that } \frac{\phi C_p}{8K}\frac{\pi^2}{r_e^2}\Delta \text{ is small enough.} \tag{10}$$

Article funded by the grant : 122022800272-4.

On Stability of Solitons for a Rotating Charge with a Fixed Center of Mass in the Maxwell Field

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The system for a rotating charge with a fixed center of mass in the Maxwell field reads [1],

$$E(-x,t) = -E(x,t), \;\; B(-x,t) = B(x,t) \tag{1}$$

(symmetry conditions),

$$\dot{E}(x,t) = \nabla \wedge B(x,t) - (\omega(t) \wedge x)\rho(x), \quad \dot{B}(x,t) = -\nabla \wedge E(x,t) \tag{2}$$

(Maxwell equations),

$$\nabla \cdot E(x,t) = \rho(x), \;\; \nabla \cdot B(x,t) = 0 \tag{3}$$

(constraints),

$$I\dot{\omega}(t) = \int x \wedge [E(x,t) + (\omega(t) \wedge x) \wedge B(x,t)]\rho(x)\, dx \tag{4}$$

(Lorentz torque equation).

We consider solutions of finite energy

$$H(\omega, E, B) = \frac{I\omega^2}{2} + \frac{1}{2}\int \Big(|E(x)|^2 + |B(x)|^2\Big)\, dx < \infty. \tag{5}$$

The solitons (stationary solutions) for the system (1)–(4) have the form

$$E(x,t) = E_\omega(x), \;\; B(x,t) = B_\omega(x), \;\; \omega(t) = \omega = const \in \mathbb{R}^3. \tag{6}$$

The solitons satisfy the stationary equations

$$E_\omega(-x) = -E_\omega(x), \;\; B_\omega(-x) = B_\omega(x), \tag{7}$$

$$\nabla \wedge B_\omega(x) - (\omega \wedge x)\rho(x) = 0, \ \ \nabla \wedge E_\omega(x) = 0, \tag{8}$$

$$\nabla \cdot E_\omega(x) = \rho(x), \ \ \nabla \cdot B_\omega(x) = 0, \tag{9}$$

$$\int x \wedge [E_\omega(x) + (\omega \wedge x) \wedge B_\omega(x)]\rho \, dx = 0. \tag{10}$$

Theorem 1. *a) The zero soliton with* $\omega = 0$ *is Lyapunov stable and orbital stable.*

b) Let us fix a non-zero $\omega \in \mathbb{R}^3$ *and* $R > 0$. *Consider solutions to the Cauchy problem for the system (1)–(4) with initial data* $\omega + \Omega_0, \ E_\omega(x) + e_0(x), \ B_\omega(x) + b_0(x)$. *The soliton* $(\omega, E_\omega(x), B_\omega(x))$ *is Lyapunov stable and as well orbital stable with respect to perturbations* $(\Omega_0, e_0(x), b_0(x))$ *such that* $\operatorname{supp} e_0 \subset \{|x| \le R\}$ *and* $\operatorname{supp} b_0 \subset \{|x| \le R\}$.

The angular momentum is defined by

$$M(\omega, E, B) := I\omega + \int x \wedge (E(x) \wedge B(x)) \, dx. \tag{11}$$

Theorem 2. *Let for a non-zero soliton (6)*

$$M_\omega := I\omega + \int x \wedge (E_\omega(x) \wedge B_\omega(x)) \, dx < \infty.$$

Then the soliton is not Lyapunov stable and is not orbital stable.

To prove Theorem 1 we analyze the equation for perturbations of the soliton; the Huygens principle is exploited.

To prove Theorem 2 we construct a special one-parametric family of perturbations of initial data. The perturbations have compact supports which tend to infinity depending on the parameter. Thus, the result on instability is consistent with Theorem 1.

Note that for a close system, under some other special conditions, the stability of solitons is proved in [2].

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On the explicit generating function expression for the invariant measure of critical Galton-Watson Branching Systems

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Let $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$ and $\mathbb{N} = \{1, 2, \ldots\}$. Consider an ordinary Galton-Watson Branching (GWB) system with a state space $\mathcal{S}_0 \subset \mathbb{N}_0$ and an offspring law $\{p_j, j \in \mathcal{S}_0\}$. Let $Z(n)$ be the population size at the moment $n \in \mathbb{N}_0$. The stochastic system $\{Z(n)\}$ forms a reducible, homogeneous and discrete-time Markov chain whose state space consists two classes: $\mathcal{S}_0 = \{0\} \cup \mathcal{S}$, where $\mathcal{S} \subset \mathbb{N}$, therein $\{0\}$ is an absorbing state, and $\mathcal{S}$ is the class of possible essential communicating states. The offspring law $\{p_k, k \in \mathcal{S}\}$ fully defines a structure of the GWB system. In fact, we observe that an appropriate probability generating function (GF) $\mathsf{E}\left[s^{Z(n)} \mid Z(0) = i\right] = \left[f_n(s)\right]^i$ for all $s \in [0, 1)$, where the GF $f_n(s) = \mathsf{E}_1 s^{Z(n)}$ is n-fold functional iteration of GF

$$f(s) := \sum_{k \in \mathcal{S}_0} p_k s^k,$$

i.e. $f_{n+1}(s) = f\left(f_n(s)\right) = f_n\left(f(s)\right)$. Undoubtedly $f(1-) = \sum_{j \in \mathcal{S}_0} p_j = 1$. Denoting q be the smallest root of the equation $f(s) = s$ for $s \in [0, 1]$, we recall that $f_n(s) \to q$ as $n \to \infty$ uniformly in $s \in [0, r]$ for any fixed $r < 1$. So, the GWB system is a discrete dynamic system generated by the GF $f(s)$ and with the fixed point q, which is an extinction probability of a trajectory of the system initiated by one individual; see [1, Ch. I].

Assume that the offspring GF $f(s)$ for $s \in [0, 1)$ has the following form:

$$f(s) = s + (1 - s)^{1+\nu} \mathcal{L}\left(\frac{1}{1 - s}\right), \qquad [f_\nu]$$

where $0 < \nu < 1$ and $\mathcal{L}(*)$ slowly varies ([2]) at infinity. Assumption $[f_\nu]$ implies that the per-capita offspring mean $m := \sum_{j \in \mathcal{S}} j p_j = f'(1-) = 1$ and $f''(1-) = \infty$, so that our system is critical type with infinite variance. In this case Slack [4] has shown that there exists an invariant measure whose GF $U(s)$ has the following local expression:

$$U(s) \sim \frac{1}{\nu(1-s)^{\nu} L(1-s)} \qquad \text{as} \quad s \uparrow 1,$$

where $L(*)$ is a slowly varying function at zero.

In this report we provide an alternative argument against Slack's one and we obtain the global expression for all $s \in [0,1)$ of the function $U(s)$ and its its derivative. Let

$$\mathcal{V}(s) := \frac{1}{\nu\Lambda(1-s)} \qquad \text{and} \qquad J(s) := \frac{1-f'(s)}{\Lambda(1-s)} - 1,$$

where $\Lambda(y) := y^{\nu}\mathcal{L}(1/y)$.

Theorem 43.1 *If $p_0 > 0$ and the condition $[f_\nu]$ holds, then*

(1) *the GF $U(s)$ has the following form:*

$$U(s) = \mathcal{V}(s) - \mathcal{V}(0);$$

(2) *the derivative $U'(s)$ has the following expression:*

$$U'(s) = J(s)\frac{\mathcal{V}(s)}{1-s}.$$

Remark 1 *Undoubtedly, the function $U(s)$ admits the form of a power series expansion $U(s) = \sum_{j\in\mathcal{S}} u_j s^j$, where $u_j = \sum_{k\in\mathcal{S}} u_k P_{kj}(1)$ and $\sum_{k\in\mathcal{S}} u_k p_0^k = 1$; see [4, Lemma 4]. Then it immediately follows that*

$$u_1 = U'(0) = \frac{J(0)}{\nu p_0} = \frac{1-p_0-p_1}{\nu p_0^2}.$$

Assertions of Theorem improve the corresponding results in [3].

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Lev Dmitrievich Kudryavtsev in the MIPT

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The report will talk about the work of Lev Dmitrievich in the MIPT from 1947 to 2012.

Number-theory renormalization

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In most models of quantum field theory (QFT), the problem of renormalization arises. We assume that the problem may be related to the uncritical use of real numbers. The possibility of using other numerical systems was raised in the monograph [4]. The role of number theory in physics was discussed in the paper [5].

We build a lattice QFT using arithmetics in the ring $\mathbb{Z}(N)$ of residue classes modulo N. This arithmetic is native for lattice. The renormalization is $N \equiv 0 \pmod N$ and the transition from the representation of $\mathbb{Z}(N)$ as $\{0, \dots, N-1\}$ to $\{-n, \dots, N-n-1\}$. The renormalization and its generalization from lattice to continuous space arises in consideration of digital representation of quantum coordinates and momenta [1], [3], [2].

We consider a bosonic and fermionuc QFT on a lattice $\mathbb{Z}^d(N)$ with the Hamiltonians

$$\hat{H}_b = \sum_{\mathbf{p}\in\mathbb{Z}^d(N),\mathbf{p}^2\in D} E(\mathbf{p}^2)\,\left(2\hat{b}^\dagger_{\mathbf{p}}\hat{b}_{\mathbf{p}} + 1\right), \quad [\hat{b}_{\mathbf{p_1}}, \hat{b}^\dagger_{\mathbf{p_2}}] = \delta_{\mathbf{p_1}\mathbf{p_2}}\,\hat{1},$$

$$\hat{H}_f = \sum_{\mathbf{p}\in\mathbb{Z}^d(N),\mathbf{p}^2\in D} E(\mathbf{p}^2)\,\left(\hat{a}^\dagger_{\mathbf{p}+}\hat{a}_{\mathbf{p}+} - \hat{a}^\dagger_{\mathbf{p}-}\hat{a}_{\mathbf{p}-}\right), \quad [\hat{a}_{\mathbf{p_1}\sigma_1}, \hat{a}^\dagger_{\mathbf{p_2}\sigma_2}]_+ =$$

$$= \delta_{\mathbf{p_1}\mathbf{p_2}}\,\delta_{\sigma_1\sigma_2}\,\hat{1},$$

$$\mathbf{p}^2 = \sum_{k=1}^{d} p_k^2 \in \mathbb{Z}(N), \quad E : D \to \mathbb{Z}(N), \quad D \subset \mathbb{Z}(N).$$

The bosonic vacuum energy is the sum of the zero oscillation energies for all permissible values of the momentum and fermionic vacumm energy is the sum of all negative energies

$$\mathcal{E} = \mathcal{E}_{vac\ b} = -\mathcal{E}_{vac\ f} = \sum_{\mathbf{p}\in\mathbb{Z}^d(N),\mathbf{p}^2\in D} E(\mathbf{p}^2) = \sum_{k\in D\subset\mathbb{Z}(N)} c_{Nd2}(k)\,E(k) \in \mathbb{Z}(N).$$

Here the multiplicity $c_{Ndm}(k)$ is

$$c_{Ndm}(k) = \Big(\text{the number of nodes } \mathbf{p} \in \mathbb{Z}^d(N) \text{ such that } \sum_{n=1}^{d} p_n^m \equiv k \pmod N\Big). \tag{1}$$

Theorem. For an arbitrary N with $d \geqslant 3$, and for $N = 2^n$ with $d \geqslant 2$

$$\forall k \in \mathbb{Z}(N) \quad c_{Nd2}(k) \equiv 0 \pmod N. \tag{2}$$

When the conditions of the Theorem are fulfilled for an arbitrary function $E : D \to \mathbb{Z}(N)$ the vacuum energy calculated in the ring of residue classes $\mathbb{Z}(N)$ is zero.

The renormalization of vacuum energy is the first step of our research programm. We plan to consider other QFT effect in $\mathbb{Z}(N)$ arithmetics.

The problem is also interesting from a number-theoretic point of view. One of us (V. Naumov) formulated and numerically tested the following hypothesis:

1) For arbitrary integers $d-1 \geqslant m \geqslant 0$ *and* $N \geqslant 1$, $\forall k \in \mathbb{Z}(N)$ $c_{Ndm}(k) \equiv 0 \pmod N$.

2) If $d < m+1$, *then* $\exists N \in \mathbb{N}: \ c_{Ndm} \not\equiv 0 \pmod N$.

The hypothesis is obvious for $m = 0$ and $m = 1$. For $m = 2$, the hypothesis follows from the Theorem. The first statement of the hypothesis is verified numerically for all cases $m \leqslant 8$ and $N \leqslant 1000$; $m \leqslant 35$ and $N \leqslant 300$; $m \leqslant 100$ and $N \leqslant 37$. The second statement of the hypothesis is verified for all $m \leqslant 100$.

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Control problem for a parabolic system with disturbances and a convex goal

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The distribution of the temperature $T_i(x,t)$ in the i–th ($i = \overline{1,m}$) homogeneous rod of unit length as a function of time t is described by the heat equation

$$\frac{\partial T_i(x,t)}{\partial t} = \frac{\partial^2 T_i(x,t)}{\partial x^2} + f_i(x,t), \quad 0 \le t \le p, \quad 0 \le x \le 1, \quad i = \overline{1,m}. \quad (1)$$

We know the estimate for continuous functions $f_i(x,t)$, which are the densities of heat sources,

$$f^{(1)}(x,t) \le f_i(x,t) \le f^{(2)}(x,t), \quad 0 \le t \le p, \quad 0 \le x \le 1, \quad i = \overline{1,m}. \quad (2)$$

Here, the functions $f^{(1)}$ and $f^{(2)}$ are continuous.

At the initial time $t = 0$, the temperature distributions $T_i(x,0) = g_i(x)$, $i = \overline{1,m}$ are given, where $g_i(x)$ are continuous functions, and $g_i(1) = 0$, $i = \overline{1,m}$. We assume that the controlled temperatures $T_i(0,t)$ and $T_i(1,t)$ at the ends of each i-th rod vary according to equations

$$\frac{dT_i(0,t)}{dt} = a^{(1)}(t) + a^{(2)}(t)\xi_i(t), \quad |\xi_i(t)| \le 1, \quad (3)$$

$$\frac{dT_i(1,t)}{dt} = b^{(1)}(t) + b^{(2)}(t)\eta_i(t), \quad |\eta_i(t)| \le 1. \quad (4)$$

Here the functions $a^{(k)}(t)$, $b^{(k)}(t)$, $k = 1,2$ are continuous for $0 \le t \le p$, and $a^{(2)}(t) \ge 0$, $b^{(2)}(t) \ge 0$. The functions $\xi_i(t)$ are controls, and the functions $\eta_i(t)$ are disturbances.

Given a continuous function $\sigma : [0,1] \to R$ that satisfies the conditions $\sigma(0) = \sigma(1) = 0$. The function $\sigma(x)$ is used to determine the mean value of temperature

$$\int_0^1 T_i(x,t)\sigma(x)dx, \quad 0 \le t \le p, \quad i = \overline{1,m},$$

realized at time t in i-th rod.

Let number $\varepsilon \ge 0$ is given. The goal of choosing controls $\xi_i(t)$ (3) is the fulfilment of the inequality

$$\max_{i,j=\overline{1,m}} \left| \int_0^1 T_i(x,p)\sigma(x)dx - \int_0^1 T_j(x,p)\sigma(x)dx \right| \le \varepsilon \tag{5}$$

for any realized disturbances $\eta_i(t)$ (4), $i = \overline{1,m}$ and for any continuous functions $f_i(x,t)$ (2), $i = \overline{1,m}$.

After the change of variables, we obtain the following single-type differential game

$$\dot{z}(t) = -a(t)u + b(t)v, \quad z \in R^m, \quad u \in Q, \quad v \in Q, \tag{6}$$

with terminal set

$$Z(\varepsilon) = \{z \in R^m : f(z) \le \varepsilon\}. \tag{7}$$

Here, Q is a convex compact, f is a convex function.

Using [2], we construct the alternating integral [1] in the game (6), (7)

$$W(t,\varepsilon) = Z(\varepsilon) \dot{-} \beta(t)Q + \alpha(t)Q.$$

Here, $A \dot{-} B = \{z \in R^m : z + B \subset A\}$ denotes the geometric difference [1] for two sets A and B from R^m;

$$\beta(t) = \max_{t \le \tau \le p} \int_t^p (b(r) - a(r))dr, \quad \alpha(t) = \max_{t \le \tau \le p} \int_t^\tau (a(r) - b(r))dr.$$

For the initial positions $z(0) \in W(0,\varepsilon)$, we have construct a control $u(t,z)$ that solves problem (6), (7). In addition, controls ξ_i (3) are constructed that solve problem (1), (5).

The research was supported by a grant from the Russian Science Foundation no. 19-11-00105, https://rscf.ru/project/19-11-00105/.

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Displacement operators approximations for noncanonical commutation relations

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In this talk we consider the problem of approximating displacement operators acting in optical phase space. We study cases of noncanonical commutation relations and introduce the concept of displacement duality for ladder operators. We propose an example of a parametric family of noncanonical commutation relations for which one can construct unitary displacement operators that satisfy the semigroup property on lines passing through the origin in the phase space.

Irreversible dynamics of composite open quantum systems

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The usual setup of open quantum systems theory assumes unitary dynamics of a system and a heat bath. In this work we consider dynamics, where the system of interest and the reservoir initially constitute a composite open quantum system instead. This approach seems to be natural when a system in a reservoir is composite, i.e. consists of subsystems, and we are only interested in one of the subsystems.

We use the Nakajima-Zwanzig projection approach in our work, both because it is widely used in open quantum systems theory, and because it allows using different projection, i.e. different approaches to identification of the system of interest inside the composite system, in a uniform manner. We assume a small coupling between the system of interest and the reservoir. We are looking for the approach which in principle can work in all the orders of perturbation theory, so we derive time-convolutionless master equations. We also use an inhomogenious term to take into account the non-factorizible initial conditions.

To illustrate our results we consider the Jaynes–Cummings model with dissipation. In our opinion, the most interesting feature of this example is that inhomogeneous terms in time-convolutionless master equations vanish in the Bogolubov-van Hove perturbation theory, but lead to renormalization of initial conditions.

Ordered Exponential and Its Features in Yang–Mills Effective Action

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Ordered exponential often occurs in mathematical and theoretical physics. For example, it is widely used in differential geometry as a solution of the parallel transport equation on the principle bundles. The ordered exponential also appears in the field theory, since it is used to make a transition to the Fock–Schwinger gauge in the non-Abelian gauge theories. In addition, it arises in the theory of integrable models, in the heat kernel method, and in other areas.

Formally, the ordered exponential is represented as a product of integrals. In our work we consider a rather general case of a smooth Riemannian manifold with an arbitrary integer dimension. Several new representations were obtained for the ordered exponential as an infinite sum with covariant derivatives and the limit of the operator exponent. Also, non-trivial properties of the product of several ordered exponentials are proved.

The application of the obtained results to the four-dimensional quantum Yang–Mills theory is discussed separately. As is known, the form of the effective action is important in loop calculations. Using the obtained properties, it was shown that the effective action of the Yang–Mills theory depends on the field strength and its covariant derivatives.

This research is supported by the Ministry of Science and Higher Education of the Russian Federation, agreement 075-15-2022-289, and by the "BASIS" foundation grant "Young Russian Mathematics".

On generating phase-damping channels

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The report consists of two parts. The first part deals with the quantum channels on composite quantum systems. The quantum channels on composite systems were considered by a number of authors (see [2] and the references therein). The main objects of our study are the channels on composite systems that uniquely determine the channels on subsystems.

In the sequel, all Hilbert spaces are assumed to be finite-dimensional. For a Hilbert space H, we denote by $L(H)$ the linear space of all linear operators on H. Positive semidefinite linear operators in $L(H)$ having trace equal to 1 are called *quantum states*. The set of all quantum states acting on H is denoted by $S(H)$. We recall that, for Hilbert spaces H_1 and H_2, a linear completely positive trace-preserving operator $\Phi: L(H_1) \to L(H_2)$ is called *a quantum channel*. *The partial trace* over a Hilbert space E is the quantum channel defined by the formula

$$tr_E : L(H \otimes E) \longrightarrow L(H), \quad tr_E(A \otimes B) = tr(B) \cdot A,$$

where $A \in L(H)$, $B \in L(E)$ and $tr(B)$ is the trace of B. For a quantum state $\sigma \in S(E)$, by *the preparation $\mathcal{P}_\sigma$ of composite system* we call the quantum channel defined as follows:

$$\mathcal{P}_\sigma : L(H) \longrightarrow L(H \otimes E), \quad \mathcal{P}_\sigma(A) = A \otimes \sigma,$$

whenever $A \in L(H)$.

To introduce the notion of a generating cannel, we consider a quantum channel $\Phi : L(H \otimes E) \longrightarrow L(H \otimes E)$ on a composite system. Then, for each quantum state $\sigma \in S(E)$, we define the channel $G_{\Phi,\sigma}$ by

$$G_{\Phi,\sigma} = tr_E \circ \Phi \circ \mathcal{P}_\sigma : L(H) \longrightarrow L(H).$$

The quantum channel Φ on a composite system is called *a generating channel* if the following equality holds

$$G_{\Phi,\sigma_1} = G_{\Phi,\sigma_2}$$

whenever $\sigma_1, \sigma_2 \in S(E)$. In this case, the quantum channel $G_{\Phi,\sigma}$, $\sigma \in S(E)$, is denoted by G_Φ and is called *the channel generated by* Φ.

We discuss properties of generating and generated channels. To this end, we give several examples of *the phase-damping channels*. The properties of the phase-damping channels were studied in [1; 3].

The second part of the report is concerned with one-parameter families of quantum channels. Such families are also called the quantum processes.

We consider the quantum processes consisting of the phase-damping channels. The divisibility properties of the quantum processes are discussed in our report.

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Integrable billiard systems on multidimensional CW-complexes

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Hamiltonian system with n degrees is called an integrable one if it has a set of n independent involutive first integrals. In the case of two degrees of freedom, one usually requires the presence of a first additional integral independent on the Hamiltonian. The phase space of such a system is stratified into common level surfaces of the first integrals of the system, i.e. a Liouville foliation with singularities is given on it. A typical regular fiber is homeomorphic to an n-dimensional torus, and non-degenerate singularities are described in terms of direct products factorized by the action of a finite group.

In recent years, interesting results have been obtained on integrable billiards. The proof of several analogues of the classical Birkhoff billiard conjecture showed that the class of flat integrable billiards "almost coincides" with the class of confocal billiards. The domains (tables) of these billiards are bounded by a piecewise-smooth curve composed of smooth arcs of ellipses and hyperbolas with common foci.

Nevertheless, from the topological point of view (to which the Liouville foliations are fiberwise homeomorphic), this class of billiards with 2 d.o.f. has been substantially extended preserving integrability. V.Vedyushkina introduced billiards on piecewise-flat CW-complexes with permutations on its 1-edges (acting on flat 2-regions glued along them), called *billiard books*. The class of Liouville foliations realized by them turned out to be very wide, and according to the conjecture of A. Fomenko a wide class of such integrable systems which appear in dynamics, geometry and mathematical physics and be realized by such billiards. The possibility to realize an arbitrary non-degenerate singularity and an arbitrary base of the Liouville foliation (in the non-singular energy zone) of a system with such singularities has already been proved by V. Vedyushkina and I. Kharcheva. The classification of billiard books and their topological invariants is of particular interest: the nontriviality of the foliation turns out to be encoded in a sophisticated way by systems of permutations on the 1-edges of the table-complex commuting at its 0-vertices.

The report is devoted to the development of this approach in the case of higher dimension: n-dimensional cells of CW-complex are glued together from regions of R^n bounded by quadrics from a family of confocal quadrics

$$\frac{x_1^2}{a_1-\lambda}+\cdots+\frac{x_n^2}{a_n-\lambda}=1, \qquad 0<a_n<\cdots<a_1; \quad \lambda\in[0,a_1]\,.$$

The $n-1$-dimensional facets of this complex are equipped with cyclic permutations acting on the regions, and the condition of their commutation is assigned to cells of dimension $n-2$. The notion of a multidimensional billiard book will be introduced, basic properties of them and billiard systems owill be discussed with an illustration in the case of a three-dimensional space and a family of confocal quadrics (containing ellipsoids, one-sheeted and two-sheeted hyperboloids). This construction will be used to model non-degenerate singularities. systems with 3 degrees of freedom.

This work was supported by the Russian Science Foundation grant 22-71-10106 and done at Lomonosov Moscow State University.

Subordination principle and Feynman-Kac formulae for generalized time fractional evolution equations

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We consider generalized time-fractional evolution equations of the form

$$u(t) = u_0 + \int_0^t k(t,s)Lu(s)ds$$

with a fairly general memory kernel k and an operator L being the generator of a strongly continuous semigroup. In particular, L may be the generator L_0 of a Markov process ξ on some state space Q, or $L := L_0 + b\nabla + V$ for a suitable potential V and drift b, or L generating subordinate semigroups or Schrödinger type groups. This class of evolution equations includes in particular time- and space- fractional heat and Schrödinger type equations. Such equations are used in models of anomalous diffusion.

We show that the subordination principle holds for such evolution equations and obtain stochastic solutions and Feynman-Kac formulae for solutions of these equations with the use of different stochastic processes, such as subordinate Markov processes and randomely scaled Gaussian processes. In particular, we obtain some Feynman-Kac formulae with generalized grey Brownian motion and other related self-similar processes with stationary increments.

The talk is based on the joint work with Ch. Bender and M. Bormann.

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On applications of real analysis methods to steady-state Navier-Stokes system

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In recent years, using the geometric and real analysis methods, essential progress has been achieved in some classical Leray's problems on stationary motions of viscous incompressible fluid: the existence of solutions to a boundary value problem in a bounded plane and three-dimensional axisymmetric domains under the necessary and sufficient condition of zero total flux; the uniqueness of the solutions to the plane flow around an obstacle problem in the class of all D-solutions, the nontriviality of the Leray solutions (obtained by the "invading domains" method) and their convergence to a given limit at low Reynolds numbers; and, more generally, the existence and properties of D-solutions to the boundary value problem in exterior domains in the plane and three-dimensional axisymmetric case, etc. A review of these advances and methods will be the focus of the talk. Most of the reviewed results were obtained in our joint articles with Konstantin Pileckas, Remigio Russo, Xiao Ren, and Julien Guillod, see, e.g., the recent survey paper J. Math. Fluid Mech. **25** (55) (2023).

Limits of Constrained Minimum Problems in Variable Domains

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We consider a sequence of integral functionals $F_s \colon W^{1,p}(\Omega_s) \to \mathbf{R}$ and a sequence of generally non-integral functionals $G_s \colon W^{1,p}(\Omega_s) \to \mathbf{R}$, where $\{\Omega_s\}$ is a sequence of domains in $\mathbf{R}^n$ contained in a bounded domain $\Omega \subset \mathbf{R}^n$ $(n \geq 2)$ and $p > 1$. We assume that, for every $s \in \mathbf{N}$, the integrand $f_s \colon \Omega_s \times \mathbf{R}^n \to \mathbf{R}$ of the functional F_s satisfies a convexity condition and the inequality

$$c_1|\xi|^p - \mu_s(x) \leq f_s(x,\xi) \leq c_2|\xi|^p + \mu_s(x)$$

for almost every $x \in \Omega_s$ and every $\xi \in \mathbf{R}^n$, where c_1 and c_2 are preassigned positive constants and μ_s is a nonnegative function in $L^1(\Omega_s)$. In addition, we assume that the sequence of norms $\|\mu_s\|_{L^1(\Omega_s)}$ is bounded. The functionals G_s are assumed to be weakly lower semicontinuous and coercive with respect to L^p-norms of functions in $W^{1,p}(\Omega_s)$.

Along with the functionals F_s and G_s, we consider sequences $V_s \subset W^{1,p}(\Omega_s)$ of the following forms:

$$V_s = \{v \in W^{1,p}(\Omega_s) : \varphi \leq v \leq \psi \text{ a.e. in } \Omega_s\}, \tag{1}$$

$$V_s = \{v \in W^{1,p}(\Omega_s) : h_s(v) \leq 0 \text{ a.e. in } \Omega_s\}, \tag{2}$$

$$V_s = \{v \in W^{1,p}(\Omega_s) : M_s(v) \leq 0 \text{ a.e. in } \Omega_s\}. \tag{3}$$

Here, $\varphi, \psi \colon \Omega \to \overline{\mathbf{R}}$ are measurable functions, $\{h_s\}$ is a sequence of functions on $\mathbf{R}$, and M_s is a mapping from the space $W^{1,p}(\Omega_s)$ to the set of all functions defined on Ω_s. Obviously, sets of the form (2) can be written in the form (3). However, it is more convenient to study the particular case (2) separately.

In the talk, we describe conditions on the involved domains and mappings which ensure the convergence of minimizers and minimum values of the functionals $F_s + G_s$ on the sets V_s to a minimizer and the minimum value of a functional

$J\colon W^{1,p}(\Omega) \to \mathbf{R}$ on the corresponding limit set $V \subset W^{1,p}(\Omega)$. In particular, in each of cases (1)–(3), we assume that the sequence of spaces $W^{1,p}(\Omega_s)$ is strongly connected with the space $W^{1,p}(\Omega)$ and the sequence of functionals F_s -converges to a functional $F\colon W^{1,p}(\Omega) \to \mathbf{R}$ (see, e.g., [3] for the related notions).

As for the constraint sets V_s, in case (1), we assume that there exist functions $\bar{\varphi}, \bar{\psi} \in W^{1,p}(\Omega)$ such that $\varphi \leq \bar{\varphi} < \bar{\psi} \leq \psi$ a.e. in Ω.

In case (2), defining, for any $a\colon \mathbf{R} \to \mathbf{R}$, $\Phi(a) = \{t \in \mathbf{R} : a(t) \leq 0\}$, we require that the sets $\Phi(h_s)$ be nonempty and closed and there exist a function $h\colon \mathbf{R} \to \mathbf{R}$ such that the set $\Phi(h)$ is nonempty and closed and the following conditions are satisfied:

(a) if $t \in \Phi(h)$, then there exist $t_1, t_2 \in \mathbf{R}$ such that $t_1 < t_2$ and $t \in [t_1, t_2] \subset \Phi(h)$;

(b) if $t_1, t_2 \in \mathbf{R}$, $t_1 < t_2$, $(t_1, t_2) \subset \Phi(h)$, and $0 < \sigma < (t_2 - t_1)/2$, then there exists

$\bar{s} \in \mathbf{N}$ such that, for any $s \in \mathbf{N}$, $s \geq \bar{s}$, we have $[t_1 + \sigma, t_2 - \sigma] \subset \Phi(h_s)$;

(c) if $t_s \to t$ in $\mathbf{R}$, $\{\tilde{s}_j\}$ is an increasing sequence in $\mathbf{N}$, and, for any $j \in \mathbf{N}$, we have

$t_{\tilde{s}_j} \in \Phi(h_{\tilde{s}_j})$, then $t \in \Phi(h)$.

Finally, in case (3), we assume some conditions characterizing both the internal properties of the mappings M_s and their relation to a mapping M from the space $W^{1,p}(\Omega)$ to the set of all functions defined on Ω. These conditions make it possible to study the convergence of solutions of variational problems with variable unilateral constraints combining the pointwise and functional dependence.

For the strict statements and proofs of the results presented in the talk, see [1–3].

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Scenario of the mildly stable transition from a structurally stable 3-diffeomorphism with a two-dimensional expanding attractor to a hyperbolic automorphism

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The results presented below were obtained in collaboration with V.Z. Grines and O.V. Pochinka.

Let M^3 is a closed three-dimensional manifold and G is the class of the structurally stable A-diffeomorphisms of the manifold M^3 such that if diffeomorphism $f \in G$ than non-wandering set of f $NW(f)$ contains a two-dimensional expanding attractor. V. Z. Grines and E. V. Zhuzhoma [3] proved that other basic sets of $NW(f)$ are trivial and the supporting manifold M^3 is diffeomorphic to the three-dimensional torus.

For any matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \in GL(3, Z)$ we denote by $\widehat{A} : T^3 \to T^3$ the diffeomorphism defined by formula

$$\widehat{A}(x, y, z) = (a_{11}x+a_{12}y+a_{13}z, a_{21}x+a_{22}y+a_{12}z, a_{31}x+a_{32}y+a_{33}z) \pmod 1.$$

According to J. Franks among the homotopy to identity continuous maps of the torus T^3 there is a unique map h_f which semi-conjugates the diffeomorphism f with the diffeomorphism $\widehat{A}_f$.

V. Z. Grines and E. V. Zhuzhoma [2] proved that the image $h_f(\Lambda)$ of the set Λ is the whole torus T^3, and the set $B_f = \{x \in T^3 : h_f^{-1}(x)$ consists of more than one point$\}$ is the union of the finitely many periodic points $P_f = \{\varrho_1, \varrho_2, \ldots, \varrho_k\}$ of the diffeomorphisms $\widehat{A}_f$ and their unstable manifolds. In this case $h_f^{-1}(\varrho_i) \cap \Lambda$, $i \in \{1, 2, \ldots, k\}$ consists of a pair of boundary associated points p_i, q_i of the basic set Λ and $h_f(L_{p_i q_i}) = \varrho_i$.

Connecting an arbitrary diffeomorphism from class G and a diffeomorphism of the same class whose non-wandering set does not contain isolated saddle points a smooth arc was constructed [1].

There is discussed the following result in the report.

Theorem. For any diffeomorphism $f \in G$, there exists a mildly stable arc $\xi_t : T^3 \to T^3$, $t \in [0, 1]$ connecting $\xi_0 = f$ with the hyperbolic automorphism $\xi_1 = \widehat{A}_f$.

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Metrics of space-time: the holonomic Kaluza–Klein theory and the nonholonomic model of sub-Lorentzian geometry

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The Kaluza-Klein theory is a geometric model of space-time where the base manifold is 5-dimensional. The equations of geodesics are equations of motion of a charged particle in the electromagnetic and gravitational fields. The Einstein and Maxwell equations are solutions of a variational problem for the scalar curvature field. Yet the problem of the physical meaning of the "fifth coordinate" is solved by different authors in different ways. Yu.B. Rumer considered this coordinate as the physical action. Classically it is considered as a length. Since the Kaluza-Klein theory can include an arbitrary conversion constant in the metric tensor the physical dimension of the "fifth coordinate" can also be arbitrary. However if we do not include the conversion constant in the metric tensor the theory remains correct and the physical dimension of the "fifth coordinate" is uniquely defined (volt·second).

A similar theory can be constructed using nonholonomic constraints on the velocity field of admissible paths. In this model the base manifold is 5-dimensional and the distribution defining nonholonomic constraints is 4-dimensional. The metric tensor of the distribution has the Lorentz signature. Therefore the causal structure in this model is nearly the same as in the general relativity theory. We consider the extension of the Schouten curvature tensor of sub-Riemannian geometry. We also consider geometric questions related to the invariance of some objects necessary for both models of space-time.

Local attractors of the Cahn-Hilliard-Oono equation

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One of the most famous nonlinear equations of mathematical physics can be considered the Cahn-Hilliard equation [1]. Accounting for convection led to one of its modified versions [3] (see also [2])

$$u_t + u_{xxxx} + bu_{xx} + au + b_1(u^3)_{xx} + a_2(u^2)_x = 0. \tag{1}$$

Here $u = u(t,x), b, b_1, a, a_2$ are real constants. Moreover, $a \geq 0$, and of particular interest is the case when $a = 0$, but $a_2 \neq 0$. Usually, it is studied together with periodic boundary conditions

$$u(t, x + 2\pi) = u(t, x). \tag{2}$$

Another choice of boundary conditions is also possible. For example, the replacement of boundary conditions (2) by the Neumann conditions. From a physical point of view, the case when $u = u(t,x,y)$, but within the framework of the report we confine ourselves to the case of considering the boundary value problem (BVP) (1),(2).

First let $a > 0$ and parameters a, b are chosen as follows

$$b = (m^2+(m+\delta)^2)(1+\nu\varepsilon),\ a = m^2(m+\delta)^2,\ \nu = \pm 1,\ \delta \in (-1,1),\ \varepsilon \in (0,\varepsilon_0),\ m \in \mathbb{N}.$$

Then BVP (1),(2) in the vicinity of the zero solution has a one-dimensional invariant manifold formed by spatially inhomogeneous solutions of BVP (1), (2). The question of the stability of this manifold is investigated.

But if $m \neq 1$ and, moreover

$$b = (m^2+(m+1)^2)(1+\nu_1\varepsilon),\ a = m^2(m+1)^2(1+\nu_2\varepsilon),\ \nu_1^2+\nu_2^2 = 1,\ \varepsilon \in (0,\varepsilon_0),\ \nu_1, \nu_2 \in \mathbb{R},$$

then in this case the BVP (1),(2) already has a two-dimensional invariant manifold, which is also formed by spatially inhomogeneous solutions. An answer is given about the stability of solutions. In all such cases, asymptotic formulas are obtained in power expansions $\varepsilon^{1/2}$. Let $b = 5(1 + \nu_1\varepsilon), a = 4(1 + \nu_2\varepsilon)$. In this case, asymptotic formulas are also obtained for solutions that form invariant manifolds, but the asymptotic formulas for them differ from those obtained for $m \neq 1$.

A special version of the problem arises if $a = 0$, but $a_2 \neq 0$ and $b_1 \neq 0$. Then for

$$\alpha = \alpha_\pm(1 + \nu\varepsilon),\ \nu = \pm 1,\ \alpha_\pm = \pm\sqrt{(1-b)/(3b_1)},\ \varepsilon \in (0, \varepsilon_0)$$

the BVP (1),(2) has a two-dimensional invariant manifold formed by solutions periodic in t and inhomogeneous in the space variable. Conditions are obtained under which the invariant manifold will be a local attractor. At the same time, the solutions belonging to such an attractor are unstable in the sense of Lyapunov's definition, but are orbitally stable.

The main results can be extended to the case of two space variables.

This work was carried out within the framework of a development programme for the Regional Scientific and Educational Mathematical Center of the Yaroslavl State University with financial support from the Ministry of Science and Higher Education of the Russian Federation (Agreement on provision of subsidy from the federal budget No. 075-02-2023-948).

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On strict monotonicity of the p-torsional rigidity over annuli

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In this talk, for $d \geq 2$ and $\frac{2d+2}{d+2} < p < \infty$, we analyze the monotonicity behavior of the p-torsional rigidity of annular domains in $\mathbb{R}^d$, when the inner ball moves towards the outer boundary. We employ the polarization method (for the Dirichlet boundary conditions), the finer geometry of the torsion function, and the shape calculus (for the mixed boundary conditions) to get the required strict monotonicity.

On complete controllability of some three- and four-level closed quantum systems

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The presented work approaches complete controllability problem [1] for several types of three-level and four-level quantum systems which are relevant to up-to-date pieces of theoretical control landscapes analysis [2; 5]. In details, it gains complete controllability conditions for three-level systems with two allowed transitions between energy levels (e.g., Λ- and V-atoms), three-level systems with three allowed transitions and both level and level-spacing degeneracies, and some four-level systems with "chained" interaction Hamiltonian which corresponds to the dipole type of interaction. These systems have been addressed before for either exact or real-valued matrix elements in free and interaction Hamiltonians only [3–5], while the presented research maintains mathematical generality by considering arbitrary complex valued matrix elements, which is crucial for both obtaining fundamental properties of the considered systems and exploring applied quantum control problems related with control landscape analysis.

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Sensitivity to initial conditions of some class of topological foliations

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The sensitivity of topological foliations with topological integrable Ehresmann connection is investigated. The study of the dynamical properties of topological foliations was started by us with N.S. Tonysheva in [4].

R. L. Devaney [3] by a dynamical system means a cascade, and he says that a dynamical system is chaotic if it has the following properties: 1) topologically transitivity; 2) density of periodic points and 3) sensitivity to initial conditions. Banks and others showed that sensitivity of a dynamical system follows from the conditions 1) and 2). Taking this in account, R. C. Churchill in [2] introduced a notion of chaotic foliations. We use a more general approach which was given in [1], where a foliation (M, F) is called chaotic if (M, F) has a dense leaf and the union of closed leaves is dense in M. Here a leaf L is closed if L forms a closed subset in M, and L may be not compact. As we see, the concept of sensitivity is inextricably linked with the concept of chaos, but in the works [4], [1] and others this problem for foliations is not considered.

The object of our investigation is topological foliations (M, F) with a topological integrable Ehresmann connection on n-dimensional manifolds M. A foliation (M, F) has an arbitrary codimension q where $0 < q < n$. The integrable Ehresmann connection is a q-dimensional topological foliation on M with some additional properties. The notion of an topological integrable Ehresmann connection for the foliations (M, F) was introduced by us in [4]. Using the Ehresmann connection we define a vertical-horizontal homotopy and prove its uniqueness. The existence of vertical-horizontal homotopy allows us to determine a transfer of the horizontal paths along the vertical ones. Applying vertical-horizontal homotopy, we introduce the notion of sensitivity to initial conditions for the investigated class of topological foliations.

One of the main results of this work is the following theorem.

Theorem. *Let (M, F) be a topological foliation of codimension q with topological integrable Ehresmann connection on n-dimensional manifold M, $0 < q < n$. If the foliation (M, F) is chaotic, then (M, F) is sensitive to initial conditions.*

As examples we construct a countable family of chaotic topological foliations on a non-compact 3-manifolds M using the suspension method.

The work was partially supported by the Russian Science Foundation (grant no. 22-21-00304).

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Automorphisms of the semigroup C^*-algebra for the free product of semigroups of rational numbers

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The report deals with the reduced semigroup C^*-algebra $C_r^*(S)$ for the free product S of a finite family of semigroups.

First we take a tuple of infinite sequences

$$P_1 = (p_{11}, p_{21}, \ldots), \ldots, P_n = (p_{1n}, p_{2n}, \ldots) \tag{1}$$

consisting of arbitrary prime integers p_{ki}, where $k = 1, 2, 3, \ldots$ and $i = 1, \ldots, n$.

Then, we take the additive semigroups of positive rational numbers $\mathbb{Q}^+_{P_k}$, $1 \leq k \leq n$, corresponding to the sequences of prime numbers in (1). These semigroups are defined by

$$\mathbb{Q}^+_{P_k} = \left\{ \frac{m}{p_{1k} \ldots p_{sk}} \mid m \in \mathbb{N}, s \in \mathbb{N} \right\}.$$

Next, we consider the free product of the semigroups $\mathbb{Q}^+_{P_k} \times \{k\}$, $1 \leq k \leq n$ which is denoted by

$$S := (\mathbb{Q}^+_{P_1} \times \{1\}) * \ldots * (\mathbb{Q}^+_{P_n} \times \{n\}) \sqcup \{0\},$$

where 0 is the neutral element. Note that S is a non-abelian semigroup with the cancellation property.

It was proved in [1] that the reduced semigroup C^*-algebra $C_r^*(S)$ is an inductive limit for the direct sequence of the Toeplitz-Cuntz algebras associated with the tuple of sequences of prime numbers (1).

The Toeplitz-Cuntz algebra $\mathcal{TO}_n$ is the universal C^*-algebra on generators $U_1, U_2, \ldots, U_n$ subject to the following relations:

i) $U_k^* U_k = 1$ for $k = 1, 2, \ldots, n$;

ii) $U_k^* U_l = 0$ whenever $k \neq l$;

iii) $U_1U_1^* + U_2U_2^* + \cdots + U_nU_n^* < 1$.

Thus, we have the direct sequence of the Toeplitz-Cuntz algebras and its direct limit

$$\mathcal{TO}_n \xrightarrow{\varphi_1} \mathcal{TO}_n \xrightarrow{\varphi_2} \mathcal{TO}_n \xrightarrow{\varphi_3} \cdots \qquad C_r^*(S),$$

where $\varphi_k : \mathcal{TO}_n \longrightarrow \mathcal{TO}_n : U_i \longmapsto U_i^{p_{ki}}$ for every $k \in \mathbb{N}$.

Let $L = (l_1, \ldots, l_n)$ be a multi-index consisting of positive integers. We construct the following diagram

$$\begin{array}{ccccccc}
\mathcal{TO}_n & \xrightarrow{\varphi_1} & \mathcal{TO}_n & \xrightarrow{\varphi_2} & \cdots & & C_r^*(S) \\
\theta_1^L \downarrow & & \downarrow \theta_2^L & & & & \downarrow \theta^L \\
\mathcal{TO}_n & \xrightarrow{\varphi_1} & \mathcal{TO}_n & \xrightarrow{\varphi_2} & \cdots & & C_r^*(S),
\end{array}$$

where, for each $k \in \mathbb{N}$, the morphism $\theta_k^L : \mathcal{TO}_n \longrightarrow \mathcal{TO}_n : U_i \longmapsto U_i^{l_i}$ is the $*$-endomorphism associated with the multi-index L, and θ^L is the limit $*$-endomorphism of the C^*-algebra $C_r^*(S)$.

In the talk we shall discuss the following criterion formulated in number-theoretic terms [2].

The limit endomorphism

$$\theta^L : C_r^*(S) \longrightarrow C_r^*(S)$$

is an automorphism of the C^**-algebra* $C_r^*(S)$ *if and only if, for each* $i = 1, \ldots, n$*, either* $l_i = 1$ *or every prime divisor of the integer* l_i *occurs infinitely often in the sequence* P_i.

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Mathematical modeling of stochastic vibrations of a string with moving border

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The study of viscoelasticity includes the analysis of the stochastic stability of stochastic viscoelastic systems, their reliability, etc. The paper considers stochastic linear longitudinal oscillations of a string with moving boundaries. The case of a difference kernel makes it possible to reduce the problem of analyzing a system of stochastic integro-differential equations to the study of a system of stochastic differential equations. To estimate the expansion coefficients, it is proposed to apply the statistical numerical Monte Carlo method.

At present, reliability issues in the design of machines and mechanisms require more and more complete consideration of the dynamic phenomena that take place in the designed objects. The widespread use in technology of mechanical objects with moving boundaries necessitates the development of methods for their calculation. The problem of oscillations of systems with moving boundaries is related to obtaining solutions to integro-differential and partial differential equations in time-variable domains [1–17]. Such tasks are currently not well understood. Their peculiarity is the difficulty in using the known methods of mathematical physics, suitable for problems with fixed boundaries. The complexity of the solutions obtained is explained by the fact that up to now there has not been a sufficiently general approach to the analysis of the features of the dynamics of such systems. In connection with the danger of resonance, the study of forced oscillations is of great importance here. Attempts to investigate this process have been made, but the results obtained are limited mainly by a qualitative description of dynamic phenomena [5; 14; 15; 17]. In addition, it is recognized that deterministic modeling of systems cannot be adequate for some

types of problems, so it is necessary to switch to probabilistic-statistical, where there are random variables, stochastic fluctuations. When solving here, mainly approximate methods are used [1; 2; 6; 8; 10], since obtaining exact solutions is possible only in the simplest cases [9].

If the damping of transverse vibrations is mainly due to the action of external damping forces, then in the case of longitudinal vibrations, the damping is mainly affected by elastic imperfections in the material of the vibrating object [1; 2; 6; 8–10]. The study of viscoelasticity includes the analysis of the stochastic stability of stochastic viscoelastic systems, their reliability, etc. The paper considers stochastic linear longitudinal oscillations of a string with moving boundaries. The case of a difference kernel makes it possible to reduce the problem of analyzing a system of stochastic integro-differential equations to the study of a system of stochastic differential equations. To estimate the expansion coefficients, it is proposed to apply the statistical numerical Monte Carlo method [16].

Keywords: stochastic longitudinal oscillations, vibrations of a string, moving boundaries.

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Operator approach for solving stochastic equations

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The paper deals how with using the operator approach get an idea stochastic differential solution equations using the functional integral.

Stochastic heat equation. We consider the following Cauchy problem for stochastic differential equation of heat type:

$$d\Psi_\omega(t)(q) = \alpha(\Psi_\omega(t))''(q)dt + (\alpha V(q) - \frac{\lambda}{4}q^2)\Psi_\omega(t)(q)dt+$$

$$+\sqrt{\frac{\lambda}{2}}q\Psi_\omega(t)(q)dw(t), \Psi_\omega(0,q) = \varphi_0(q),$$

where $\varphi_0 \in C(\mathbf{R}^1)$ and φ_0 is bounded. $\Psi_\omega(t)(q)$ is a random function, defined for all positive t, q is an element of the configuration space, and w is Wiener's measure on a probability space; the elements of the probability space are denoted by the symbol ω, i.e ω is continuous functions on $[0,t]$.

The following theorem is true, and it gives the presentation of the solution to this problem with using the functional integral (Feynman-Katz formula).

Theorem 1 *Let the function V is continuous and bounded, function φ_0 is bounded and two times differential and its first and second derivatives are also bounded. Then when $\alpha = 1$ the solution of the Cauchy problem for the stochastic heat equation exists and can be represented as follows:*

$$\Psi(t,\omega)(q) = \int_{C_0[0,t]} \exp\left\{\int_0^t V(q+\xi(\tau))d\tau - \int_0^t \frac{\lambda}{2}(q+\xi(\tau))^2 d\tau\right\} \times$$

$$\times \exp\left\{\sqrt{\frac{\lambda}{2}}\int_0^t (q+\xi(\tau))dB_\omega(\tau)\right\}\varphi_0(q+\xi(t))w_{0,t}(d\xi).$$

Here $C_0[0,t]$ is the space of continuous functions, vanishes in zero, $w_{0,t}$ is the Wiener's measure on this space.

The proof of this theorem is based on the application of the $\widehat{I}$to formula to the integrand.

The resulting solution can be analytically continued to the appropriate area in such a way that we obtain solutions of the Belavkin equation.

Operator approach. Interesting is the method of obtaining the same representation of the solution based on considering the right side of the equation from the Cauchy problem (1) as a random operator.

We consider a stochastic heat equation with respect to the random functions φ of real variables taking values in space $L_2(\mathbf{R}^1)$:

$$d\varphi(t) = (\varphi(t))'' \, dt + V(q)\varphi(t)dt - \frac{\lambda}{4}q^2\varphi(t)dt + \sqrt{\frac{\lambda}{2}}q\varphi(t)dw(t).$$

The random operator on the right hand side of the equation, applied to the function $\varphi(t)$, will be denoted by the symbol $A(t)$.

The solution to the Cauchy problem is given by the two-parameter family of random operators $F(t_1, t_2)$, $t_1, t_2 \in \mathbf{R}^1$, $t_1 \leq t_2$, acting in space $L_2(\mathbf{R}^1)$.

$$(F(t_1, t_2)h)\,(q) = \int\limits_{C_0([t_1,t_2]} (h(q + \xi(t_2)))\times$$

$$\times \exp\Big\{ \int_{t_1}^{t_2} V(q + \xi(\tau))d\tau - \int_{t_1}^{t_2} \frac{\lambda}{2}(q + \xi(\tau))^2 d\tau \Big\}\times$$

$$\times \exp\Big\{ \int_{t_1}^{t_2} \sqrt{\frac{\lambda}{2}}(q + \xi(\tau))dB(\tau) \Big\} w_{t_1,t_2}(d\xi).$$

h is an element of the space $L_2(\mathbf{R}^1)$.

In the case when these operators are selfadjoint, the following theorem holds.

Theorem 2 *For any* $t_1 \in [0, +\infty)$ *and any twice differentiable function* h *with the bounded derivatives, the function* ψ, *defined by equality* $\psi(t_2) = F(t_1, t_2)h$ *(*$t_2 > t_1$*), is a solution to the Cauchy problem for the stochastic differential equation with initial data* (t_1, h).

Since the equation is stochastic, there are difficulties that are not present in the non-stochastic case. These difficulties will be discussed in this paper.

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Analysis of a SEIT compartmental model of transmission of Tuberculosis with treatment

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We investigate the global stability and bifurcation of discrete-time mathematical models of Tuberculosis. First we study a SEI mathematical model of TB with endogenous or exogenous infection without treatment. Then we extend our study to SEIT mathematical model with treatment with endogenous or exogenous infection.

Tuberculosis (TB) is an airborne infectious disease that transmits between individuals via droplets with TB bacilli, that is, *Mycobacterium tuberculosis*. The bacteria are put into the air when a person with active TB infection in the lungs or throat coughs, speaks, or sings, and nearby people may breathe in the bacteria, thus becoming infected. Once someone is exposed, TB bacteria can live in the body, human or animal, for years if not decades without any symptoms, called latent TB infection. In fact, many people who have latent TB never develop the infectious disease, but they still test positive, though not infectious, meaning they cannot spread TB bacteria to others. TB bacteria turns into active infection if the immune system cannot stem the growth rate.

Common symptoms of tuberculosis are chest pain, weight loss, fever, a persistent cough that may contain blood, etc. Nevertheless, active TB infection can be treated by prolonged use of antibiotics. We conclude the SEIT model is sufficient to describe the transmission pattern of tuberculosis. However, the reality is that while models assume we have access to complete data on TB cases, every active infection is not reported. Moreover, since latent TB carriers exhibit no symptoms, their exact number is far more difficult to estimate, and treatment often goes half-done because of its length or duration. For decades it has been assumed that postprimary tuberculosis is usually caused by reactivation of endogenous infection rather than by a new, exogenous infection. However, Exogenous reinfection appears to be a major cause of postprimary tuberculosis after a previous cure in an area with a high incidence of this disease. This finding emphasizes the importance of achieving cures and of preventing anyone with infectious tuberculosis from exposing others to the disease.

The SEI Compartmental Model (with no treatment) The host population is divided into the following epidemiological classes or subgroups: susceptibles (S), exposed (E, infected but not infectious), infectious (I).

$N(t) = S(t) + E(t) + I(t)$ denotes the total population. Let Λ be the recruitment rate of the population, d be the natural death rate, γ be the death rate caused by the disease, and the mean exposed period is $\frac{1}{\alpha}$ where $\alpha > 0$ is the rate of loss of latency. The parameters α, γ and d, verify $0 \leq \alpha \leq 1$, $0 \leq \gamma \leq 1$, $0 \leq d \leq 1$.

Assuming there is exogenous reinfection, the disease dynamics may be represented by the following system of difference equations

$$\begin{cases} S(t+1) = \Lambda + (1-\mu_1-d)S(t) + \mu_1\varphi_1(I(t)/N(t))S(t) + r_1E(t) \\ \qquad\qquad\quad +r_2I(t) \\ E(t+1) = \mu_1(1-\varphi_1(I(t)/N(t)))S(t) + \mu_2\varphi_2(I(t)/N(t))E(t) \\ \qquad\qquad\quad +(1-\mu_2-d-\alpha-r_1)E(t) \\ I(t+1) = \alpha E(t) + \mu_2(1-\varphi_2(I(t)/N(t)))E(t) \\ \qquad\qquad\quad +(1-d-\gamma-r_2)I(t) \end{cases} \tag{1}$$

where r_1 and r_2 are the rate of recovering people from Exposed and Infectious. The fraction of susceptibles that escapes the infection at time t is $\mu_1\varphi_1(I(t)/N)$ where $\varphi_1(I/N)$ is the escape function, and μ_1, $0 \leq \mu_1 \leq 1$ is the level of infection. The contact between susceptibles and infected individuals is assumed to be a Poisson process given by $\varphi_i(I(t)/N) = e^{-\beta_i I(t)/N}$, $\beta_i > 0$, $i = 1, 2$, where β_i is called the transmission coefficient which will be used here. The term $\mu_2\varphi_2(I(t)/N(t))$ models the exogenous reinfection rates with μ_2 representing the level of reinfection, $0 \leq \mu_2 \leq 1$.

We study the existence and unicity of both Disease Free Equilibrium (DFE) and Endemic Equilibrium (EE), their local and global stability. Next we consider the SEIT model in which Infectious people are treated. We proof the same properties of DFE and EE. Numerical examples are given to illustrate these theoretical results.

Topological entropy and minimal sets of continuous maps on ramified continua

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By *continuum* we mean a compact connected metric space. *A dendrite* is a locally connected continuum that does not contain subsets homeomorphic to a circle. *A finite graph* is a continuum that can be presented as the union of finitely many arcs any two of which are either disjoint or intersect only in one or both of their endpoints.

Let X be a dendrite or a finite graph, $f : X \to X$ be a continuous map.

A nonempty set M in X is called to be *minimal under* f if it is closed, f-invariant and does not contain any proper subset satisfied these conditions. A periodic orbit gives the simplest example of a minimal set. It is known that a compact f-invariant set Y in X contains some minimal set (see, e.g., [3]).

We say that M is *totally minimal under* f, if M is minimal under f^n for all $n \geq 1$.

If M is not minimal for some $n \geq 2$, then there are pairwise disjoint compact subsets $M_i \subset M$, uniquely defined up to the order, with $M = M_0 \cup M_1 \cup \ldots \cup M_{k-1}$, such that $k \geq 2$ is a divisor of n, $f(M_i) = M_{i+1} (mod\, k)$ and M_i is minimal under f^k for each $0 \leq i \leq k-1$ (see, e.g., [1; 3; 4]). In this case we say that M has a periodic decomposition. A number k is called a length of a periodic decomposition of M. We note that M has at most one periodic decomposition of a length k.

We say that a minimal set M is *a relatively totally minimal under* f if there is a periodic decomposition $\{M_0, M_1, \ldots, M_{k-1}\}$ of M such that M_i is a totally minimal set under f^k for every $0 \leq i \leq k-1$.

We note that there are minimal sets that are neither totally minimal under f nor relatively totally minimal under f. Adding machines on the set of all one-sided infinite sequences gives us the fundamental examples of such minimal sets (see, e.g., [1; 2]).

In the report we study a relation between the structure of infinite minimal sets and topological entropy of continuous maps on finite graphs and dendrites.

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Nonlinear Acceleration in Nonholonomic Mechanics

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This talk summarizes a series of studies [1–7] of the dynamics of nonholonomic systems with varying mass distribution due to the prescribed periodic motion of some structural components (rotor, point masses etc.). Depending on the choice of the law of variation of mass distribution, such systems generally exhibit a large variety of behavior, both regular and chaotic. In addition, it turns out that nonholonomic systems are one of the simplest mathematical models of mechanical systems exhibiting a phenomenon known as unbounded speedup, which is due to redistribution of internal masses. In this paper, for a certain class of nonholonomic systems (including the Chaplygin sleigh, the Suslov system and Roller Racer) we find a criterion which must be satisfied by the periodic variation of mass distribution for the existence of speeding-up trajectories.

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Uniqueness and Existence of solutions of the Cauchy problem for a hyperbolic equation with periodic coefficients

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The main goal of this note is to establish the uniqueness and existence of a solution to the Cauchy problem for a second-order hyperbolic equation with periodic coefficients.

Keywords: hyperbolic equation, Cauchy problem, periodic coefficients, uniqueness and existence.

Let us consider a triangular region $G := \triangle APB$ on the plane (x, t), bounded by the base $AB := \{t = 0\}$ and two characteristics $PA := \{x + t = 0\}$ and $PB := \{x - t = 0\}$.

Consider as $t \to \infty$ the following Cauchy value problem:

$$u_{tt}(x,t) - (p(x)\, u_x(x,t))_x + q(x)\, u(x,t) = 0, \quad x \in \mathbb{R},\ t > 0, \tag{1}$$

$$u(x,t)|_{t=0} = 0, \quad u_t(x,t)|_{t=0} = 0, \tag{2}$$

where the functions $p(x)$ and $q(x)$ are periodic with period 1, and

$$p(x+1) = p(x) \geq \text{const} > 0, \quad q(x+1) = q(x) \geq 0.$$

In addition, the functions $p(x)$, $q(x)$ are continuous or have a finite number of discontinuities of the first kind on the period.

Theorem 67.1 *If the functions $u(x,t)$ and $u_t(x,t)$ vanish along the base $AB := \{t = 0\}$ of a triangle $G := \triangle APB$ with sides $PA := \{x + t = 0\}$ and $PB := \{x - t = 0\}$, then the solution $u(x,t)$ of the Cauchy problem (1), (2) exists and $u(x,t) \equiv 0$ throughout the triangle $G := \triangle APB$.*

We note that the first results on the asymptotic expansion of solutions of the Cauchy problem for a hyperbolic equation with periodic coefficients were announced in [5] in the form of brief communications, and full proofs are given in papers [1] and [2], as in the case of a positive operator Hill $H_0 > 0$, and in the case when the left end of the spectrum $\sigma(H_0)$ of the Hill operator H_0 is non-positive.

Similar questions for solutions of the initial-boundary value problem on the semi-axis were announced in the form of short communications in [3], and full proofs are given in [4].

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Behavior as $t \to \infty$ of solutions of a mixed problem for a hyperbolic equation with periodic coefficients on the semi-axis

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We consider the asymptotic behavior (as $t \to \infty$) of solutions to an initial boundary value problem for a second-order hyperbolic equation with periodic coefficients on the semi-axis. The main approach to studying the problem under consideration is based on the spectral theory of differential operators, as well as on the properties of the spectrum $\sigma(H_0)$ of the one-dimensional Schrödinger operator

$$H_0 := -\frac{d}{dx}\left(p(x)\frac{d}{dx}\right) + q(x)$$

with periodic coefficients $p(x)$ and $q(x)$.

Keywords: asymptotic behavior, hyperbolic equation, initial-boundary value problem, periodic coefficients.

Consider as $t \to \infty$ the following initial-boundary value problem

$$u_{tt}(x,t) - (p(x)\, u_x(x,t))_x + q(x)\, u(x,t) = 0, \quad x > 0,\, t > 0, \tag{1}$$

$$u(x,t)|_{t=0} = 0, \quad u_t(x,t)|_{t=0} = f(x), \quad x \geq 0, \tag{2}$$

$$u(x,t)|_{x=0} = 0, \quad t \geq 0, \tag{3}$$

where $p(x)$ and $q(x)$ are 1-periodic functions, $p(x+1) = p(x) \geq C > 0, \quad q(x+1) = q(x) \geq 0$.

Here, we assume that the functions $p(x)$ and $q(x)$ are continuous or have a finite number of discontinuities of the first kind on the period, $f \in C_0^\infty(\mathbb{R})$, $\operatorname{supp} f \subset [0,1]$.

The present note is devoted to the asymptotic behavior as $t \to \infty$ of solutions to the initial-boundary (mixed) problem for a one-dimensional second-order hyperbolic equation with periodic coefficients $p(x)$ and $q(x)$ on the semi-axis $x > 0$.

Theorem 68.1 *If the one-dimensional Schrödinger operator (Hill operator)* H_0 *is positive,* $p(x) \geq C > 0,\ q(x) \geq 0$*, then there is a compact operator*

$$M : L^2[0,1] \longmapsto L^2[0,1]$$

such that for $x \in [0,1]$ *and* $t > 0$ *the solution to the initial boundary value problem (1)–(3) has the form*

$$u(x,t) = u_1(x,t) + v(x,t),$$

where $u_1(x,t)$ *is the solution to the following mixed problem*

$$\begin{cases} u_{tt}(x,t) - (p(x)\,u_x(x,t))_x + q(x)\,u(x,t) = 0, \quad x \in [0,1],\ t > 0, \\ u(x,t)|_{t=0} = 0, \quad u_t(x,t)|_{t=0} = M[f(x)], \quad x \in [0,1], \\ u(x,t)|_{x=0} = u(x,t)|_{x=1} = 0, \quad t \geq 0, \end{cases}$$

while the function $v(x,t)$ *for* $x \in [0,1],\ t > 0$ *satisfies the estimate*

$$|v(x,t)| \leq \frac{C}{t}||f; L^2(\mathbb{R})||;$$

the function $u_1(x,t)$ *has the form*

$$u_1(x,t) = \sum_{\nu=1}^{\infty} b_\nu\, f_\nu\, v(x,n_\nu)\sin(\gamma_\nu t);$$

here $v(x,n_\nu)$ *is the normalized eigenfunction of the following eigenvalue problem:*

$$\begin{cases} -(p(x)y')' + q(x)y = k^2 y, \quad 0 < x < 1, \\ y(0) = y(1), \end{cases}$$

corresponding to the eigenvalue γ_ν^2*,*

$$f_\nu = \int_0^1 v(x, n_\nu) f(\xi)\, d\,\xi,\ \nu = 1, 2, \ldots,$$

f_ν *are the coefficients of the expansion of the function* $f(x)$ *in the Fourier series in the system* $\{\hat{v}(x, n_\nu)\}_{n=1}^{\infty}$*,* b_ν *are some constants of order* $o(\frac{1}{\nu})$ *as* $\nu \to \infty$*.*

In the form of brief communications, the main results of this note have been partly presented in [3], and the full proofs are given in [4].

In the papers [1] and [2], similar questions were considered for the Cauchy problem with initial conditions, as in the case of a positive Hill operator $H_0 > 0$, and also in the case when the left end of the spectrum $\sigma(H_0)$ of the operator Hill H_0 is non-positive.

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On dynamics of large particle systems with interaction in different fields

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We consider countable systems of point particles under various random and deterministic external influences on real line. Particles are of unit masses with coordinates $\{x_k\}_{k\in\mathbf{Z}}$ and velocities $\{v_k\}_{k\in\mathbf{Z}}$. The dynamics of the system is determined by the formal Hamiltonian (the total energy of the system):

$$H(x(t),v(t)) = \sum_{k\in\mathbf{Z}} \frac{v_k^2}{2} + \sum_{k\in\mathbf{Z}} \frac{a_{kk}}{2}(x_k(t)-ka)^2 + \sum_{k,j\in\mathbf{Z},k\neq j} \frac{a_{kj}}{2}(x_k(t)-x_j(t)-(k-j)a)^2,$$

where parameters $a>0,\ a_{kk}\geq 0,\ (V)_{kj}=a_{kj}$ – linear operator in some linear space (in each model the conditions will be dealt with separately). The equilibrium position (minimum energy) will be $x_k = ka, \quad v_k = 0, \quad k\in\mathbf{Z}$. In this case it will be convenient to move on to new variables – deviations: $q_k(t) = x_k - ka, \quad p_k(t) = \dot{q}_k(t) = v_k(t)$.

One of the models under consideration is determined by hamiltonian:

$$H(q(t),p(t)) = \frac{1}{2}\sum_{k\in\mathbf{Z}} p_k^2(t) + \frac{1}{2}\sum_{k,j\in\mathbf{Z}} a(k-j)q_k(t)q_j(t),$$

where the real-valued function $a(k)$ satisfies three natural conditions (see [2] for details).

We assume also that the initial conditions $\{q_j(0)\}_j, \{p_j(0)\}_j$ lie in Hilbert space L:

$$L = \{\psi = (q,p):\ q\in l_2(\mathbf{Z}),\ p\in l_2(\mathbf{Z})\}.$$

Suppose, moreover, that on particle with a fixed number $n \in \mathbf{Z}$ external force $f(t)$ acts. Then the motion of the system is described by the following infinite ODE system:

$$\ddot{q}_j = -\sum_k a(k-j)q_k + f(t)\delta_{j,n},\ j \in \mathbf{Z}. \tag{1}$$

We assume that $f(t)$ is a stochastic process satisfying the following conditions:

A1) real-valued centered stationary in a wide sense process with continuous covariance function.

A2) the support of the spectral measure of the process $f(t)$ μ is isolated from the plus or minus "of the root" of the set $E = [e_1; e_2]$, where $e_1 > 0$ (see [2] for details), i. e. there is an open set U containing $\pm[\sqrt{e_1}, \sqrt{e_2}]$ such that $\mu(U) = 0$.

Theorem. Consider conditions A1) and A2) and $\psi(0) = (q(0), p(0)) \in L$ hold. Then there is random process $\eta(t) = (q^\infty(t), p^\infty(t))$ such that the following conditions hold:

1. $\eta(t)$ is a solution to the system (1) with some initial conditions;
2. the difference $\psi(t) - \eta(t)$ converges to zero as $t \to +\infty$ component-by-component with probability one, and the trajectories of the process are continuous and infinitely differentiable a.s.;
3. each component of $\eta(t)$ is a stationary process, satisfying condition A1) and $P(\eta(t) \in L) = 1$ for all $t \geq 0$;
4. there exist positive constants c_1, c_2 and $0 < r < 1$ such that

$$Dq_k^\infty(0) \leq c_1 r^{|n-k|},\ Dp_k^\infty(0) \leq c_2 r^{|n-k|}.$$

5. $\lim_{t\to+\infty} \mathbf{E}H(\psi(t)) = \mathbf{E}H(\eta(0)) + H(\psi(0))$.

Generally speaking, this assertion does not imply the weak convergence of $\psi(t)$ components to the corresponding $\eta(0)$ components. But additional strict stationarity of process $f(t)$ is sufficient condition for that.

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Semigroups of operators associated to stochastic processes and their generators

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A wide class of processes arising in various areas of natural science, economics and social phenomena can be mathematically described by stochastic differential equations (SDE). The most studied is the class of diffusion SDEs with Wiener processes being the randomness sources. The solutions of such equations, due to the continuity properties of Wiener processes have continuous trajectories. Therefore, modeling based on diffusion-type equations is most suitable for describing processes that do not have jumps. Simulation based on Levy processes allows one to study along with continuous, jump processes.

Levy processes are an important class of homogeneous Markov processes. The Markov and homogeneous properties lead to the fact that, along with transition probability P, the key characteristic of Levy processes is the semigroup of operators.

The talk is devoted to the study of properties of semigroup operators

$$U(t)f(x) = \int_{\mathbb{R}^n} f(y)P(0,x;t,dy), \quad x \in \mathbb{R}^n, t \geq 0,$$

acting in the space $C_0(\mathbb{R}^n)$ or $L_2(\mathbb{R}^n)$.

The semigroups associated with basic Levy processes, shift processes, Wiener, Poisson, compound Poisson and symmetric stable processes are considered in the form

$$U(t)f(x) = \langle f(\cdot),\, p(0,x;t,\cdot)\rangle,$$

where transition density p is the generalized function on space $C_0(\mathbb{R})$. It is shown that the generators of the Levy semigroups are operators with kernels from the space $\mathcal{S}'(\mathbb{R}^{2n})$. The proof of this fact is based on the property of Levy process generators to be pseudo-differential operators [1].

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Methods for description of open quantum system dynamics based on the thermodynamical approach

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Because of the correlations between the system and the environment in open quantum systems, when solving the evolution equation we obtain information related not only to the system, but to the environment as well. In order to separate information related to the system from information related to the environment, we use the method of projective operators, in particular the Nakajima-Zwanzig method.

We will approximate the exact density matrix by its quasi-equilibrium value. Assuming that the system is described by a set of variables $\{P_m\}$ we apply the maximum entropy principle from thermodynamics of nonequilibrium systems and obtain the form ρ_{rel}. We consider two kinds of entropies: Gibbs entropy and Renyi entropy.

We consider a two-level system in an external field, which interacts with an external reservoir. In the resulting evolution equation we make a "dissipative Wick's turn" when performing the Bogolyubov-van Hov overstretch $t \to \lambda^{-2}t$. This turn is done so that the growing exponents in the evolution equation become delta functions in the Bogolyubov-van Hov limit $\lambda \to 0$.

Our results converge to the results obtained strictly in the same limit.

The effect of heat generation on the stream in the pipe and locking the flow

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The motion of a liquid in a three-dimensional vertical pipe is considered. Based on Euler's equations, Darcy's law is valid for some modes. Then the problem is reduced to a one-dimensional system for density, pressure, and longitudinal velocity (averaged in the cross-section). We consider boundary problem with known pressure values at the beginning and the end of the pipe, and temperature value at the beginning of the pipe. This problem simulates the effect of overheating of the liquid at the end of the pipe: when the heat release increases above a certain threshold, the density turns to zero, and the temperature goes to infinity, as well as the effect of locking the flow in the end of the pipe (the velocity is zero). These effects occur both in the ideal gas approximation and for the Van der Waals model. Similar effects in the case of an ideal gas was observed in the work [2], where the system was solved for pressure, velocity, density and entropy. Obtained analytical and numerical results fit the experimental data from [1].

The authors are grateful to V. G. Danilov and A. A. Kovalishin for valuable discussions. The work was carried out in the frame of the government program (state registration No. 123021700044-0).

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Attractors with non-invariant interior

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The properties of generic endomorphisms are somewhat different from the properties of generic diffeomorphisms. It is conjectured that C^1-generic diffeomorphisms (of a connected manifold) whose non-wandering set has a non-empty interior are transitive. In contrast, for endomorphisms there are known open examples of attractors with non-empty interior for non-transitive maps.

We build on the ideas of these examples to show that the interior of the nonwandering set or attractor can be not only non-empty, but also non-invariant, and in a persistent way. That is, we construct an open set of maps that take a point in the interior of the attractor to the boundary of it. This is another contrast with diffeomorphisms, as for a diffeomorphism the interior of an invariant compact set is always invariant. In the known examples of attractors with non-empty interior the interior also is invariant.

We focus on the smallest dimension where a robust example of this type is possible. However, the main result is also valid for any manifold of dimension higher than 2.

Our initial approach was to first construct a skew product over a circle extension with the required properties, and then perturb it in the class of endomorphisms and use the technique of Ilyashenko-Negut to regain the structure of a skew product and prove that the non-invariance properties are persistent. Later we came up with a simpler geometric argument. Nevertheless, the "main" map in the proof is a skew product.

On Optimization of Coherent and Incoherent Controls in One- and Two-Qubit Open Systems

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Control of quantum systems, e. g., individual atoms, molecules is an important direction in modern quantum technologies [8]. Often open quantum systems with Markovian dynamics are described via the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master-equation, and controlling such a system is modelled in terms of coherent control entering in the system's Hamiltonian. However, there is known the approach (see the fundamental works [5; 6] and the subsequent works, e. g., [1; 3; 4; 7]), where such a system's environment can be considered as a resource via introducing incoherent control in the superoperator of dissipation and also in the effective Hamiltonian. The talk considers some one- and two-qubit open quantum systems whose dynamics is described via the GKSL master equation in the weak coupling limit (WCL) approach, and coherent u and incoherent n controls are used:

$$\frac{d\rho(t)}{dt} = -i[H_0 + H_{\mathrm{eff},n(t)} + H_{u(t)}, \rho(t)] + \mathcal{L}_{n(t)}(\rho(t)), \quad \rho(0) = \rho_0, \tag{1}$$

where H_0, $H_{\mathrm{eff},n(t)}$, and $H_{u(t)}$ are, correspondingly, some free, effective, and interaction Hamiltonians; $\mathcal{L}_{n(t)}(\rho(t))$ is the WCL type's superoperator of dissipation acting on $\rho(t)$. Consider $N = 2$ and $N = 4$, i. e., correspondingly, for one- and two-qubit cases.

Based on the approach considering both coherent and incoherent controls in the GKSL type's quantum systems, some one- and two-qubit systems of this type and various control objectives are considered, the Krotov method, one- and two-step gradient projection methods, etc. are adapted [2–4]. In regard to using incoherent control as a recourse in addition to coherent control, the talk shows the different cases of the numerically optimized controls.

Partial support from the RSF No 22-11-00330, Minobrnauki No 075-15-2020-788, and "Priority 2030".

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On the asymptotic stability of the hybrid systems

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A continuous-discrete system of functional differential equations (also called hybrid) is a system whose state is described by two groups of interrelated variables: some variables that are being functions of continuous time satisfy differential equations; others are being functions of discrete time satisfy difference equations.

Hybrid systems are applied in studying technical objects with impulse and digital control, as well as in problems of economic dynamics models [4].

It is natural to search for the solution of hybrid systems step by step, integrating the system on each interval, but one can't study the solution asymptotic properties by this method, so the problem of stability for such systems is actual.

Various methods are used to study the stability of hybrid systems. The approaches based on the Lyapunov method are applied in papers [3; 4] the fixed point principle is applied in paper [2], the Azbelev's W-method is applied in papers [1].

Exact effective coefficient criteria for asymptotic stability can be obtained for hybrid systems in which a continuous subsystem with continuous time is a system of ordinary differential equations [5; 6].

As far as the author of the current paper knows, there are no exact effective coefficient stability criteria for hybrid systems in which the subsystem with continuous time is a system of delay differential equations with. Consider the Cauchy problem for an example of the hybrid system of this class.

$$\begin{cases} \dot{x}(t) + ax(t-1) = y(n), \quad t \in [n, n+1), \\ x(t) = \psi(t), \quad t \in [-1, 0), \\ y(n) = -bx(n), \\ x(0) = x_0. \end{cases} \qquad n \in \mathbb{N}_0, \tag{1}$$

where $a, b, x_0 \in \mathbb{R}$, $\mathbb{N}_0 = \{0, 1, 2, \dots\}$, the initial function ψ is assumed to be summable.

System (1) is called asymptotically stable if $\lim_{t\to\infty} x(t) = 0$ for any ψ and x_0.

Let's introduce the operator S that acts in space $C[0,1]$:

$$(Sx)(\tau) = x(1)(1 - b\tau) - a \int_0^\tau x(s)\,ds.$$

Consider the equation

$$\mu - b + (a+b)e^{-\mu} = 0. \qquad (2)$$

for the variable μ.

Theorem. *Suppose* $a \neq 0$. *Then the following statements are equivalent:*

- *system* (1) *is asymptotically stable,*
- *all eigenvalues of the operator* S *lie inside the unit circle,*
- *the inequality* $|\mu| > |a|$ *holds for any root of equation* (2).

This work is supported by the Russian Science Foundation, Project no. 22-21-00517.

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Half-space Dirichlet problem with summable boundary-value functions for elliptic equations with general-kind nonlocal potentials

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The Dirichlet problem with a summable boundary-value function for the equation

$$\sum_{j=1}^{n}\frac{\partial^2 u}{\partial x_j^2}(x,y)-\sum_{k=1}^{m}a_k u(x+h_k,y)+\frac{\partial^2 u}{\partial y^2}(x,y)=0$$

is considered in the half-space $\mathbf{R}^n\times(0,\infty)$, where m and n are positive integers, $a_1,\dots,a_m$ are nonnegative constants, and $h_k:=(h_{k1},\dots,h_{kn})$, $k\in\overline{1,m}$, are vectors from $\mathbf{R}^n$ with real coordinates.

Under the assumption that

$$\max_{k\in\overline{1,m}}|h_k|\max\left\{\sum_{k=1}^{m}a_k,\sqrt{\sum_{k=1}^{m}a_k}\right\}<\frac{\pi}{2},$$

we construct a solution, express it by a Poisson-like integral representation, prove its infinite smoothness outside the hyperplane $\{y=0\}$, show that it uniformly decays (with all its partial derivatives with respect to all independent variables) as $y\to 0$, and estimate the rate of this uniform decay.

This work is supported by the Ministry of Science and Higher Education of the Russian Federation (project number FSSF-2023-0016).

On classes of holomorphic functions in tube domains

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Let C be a proper open convex cone in the n-dimensional real space R^n with vertex at the origin, b be a convex continuous positively homogeneous function of degree 1 on $\overline{C}$. In the talk there will be presented new and recent results devoted to description of the strong dual to some spaces of functions on the unbounded closed convex set

$$U(b,C) = \{\xi \in R^n : -\langle \xi, y \rangle \le b(y), \forall y \in C\}$$

in terms of the Fourier-Laplace transform of functionals. Some of them develop and complement well-known results of V. S. Vladimirov and J. W. De Roever on this topic.

Here we present a small extension of the result of V.S. Vladimirov from [1, §10.5]. Let $k \in C(R^n)$ be such that for $\xi, \eta \in R^n$

$$k(\xi + \eta) \le C(1 + \|\xi\|)^N k(\eta),$$

where C and N are positive constants. Let $L^2_k(U(b,C))$ be a space of measurable functions f with support in $U(b,C)$ such that

$$\|f\|^2_{2,k} = \int\limits_{U(b,C)} |k(x)f(x)|^2 \, dx < \infty.$$

Let $B_{2,k}$ be the space of all distributions $u \in S'(R^n)$ which are Fourier transforms of functions belonging to $L^2_k(U(b,C))$, i.e., $u \in S'(R^n)$ is in $B_{2,k}$ if there exists $g \in L^2_k(U(b,C))$ such that $u = F[g]$ (we use a notation from [1]). The space $B_{2,k}$ is endowed with the norm $\|u\|_{(k)} = \|F^{-1}\|_{2,k} = \|g\|_{2,k}$. Let $H_{2,k}(T_C)$ be a space of functions f holomorphic in $T_C = R^n + iC$ with the norm

$$p_k(f) = \sup_{y \in C} \frac{\|f(x+iy)\|_{(k)}}{e^{b(y)}}.$$

Theorem. *$f \in H_{2,k}(T_C)$ iff $f(z) = \int\limits_{U(b,C)} g(\xi)e^{i\langle \xi,z\rangle}\,d\xi,\; z \in T_C$, where $g \in B_{2,k}$.*

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The problem of interpolation in the preimage of a convolution operator

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The conditions for the multipoint de la Vallee Poussin problem to have a solution are found at the preimage of a convolution operator when the zeros of the characteristic function and nodal points that are zeros of an entire function, are inside corners of the complex plane.

Hardy type inequalities with remainders

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The report is devoted to Hardy type inequalities with remainders. The proofs of these inequalities are based on one-dimensional inequalities. One-dimensional inequalities are the analytical basis for solving geometric problems. We will provide a brief overview of the results in this direction.

More precisely, Hardy type inequalities with an additional term are considered for compactly supported smooth functions in convex domains. Constants in these multidimensional inequalities will depend on the geometric characteristics of the regions, for example, such as the volume, diameter or inner radius. The Bessel functions and their properties are used.

Reconstruction by modules of measurements of a vector-signal in finite-dimensional and infinite-dimensional spaces

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There is a situation in many applied researches: we have a system of so-called measuring vectors $\Phi = \{\varphi_i\}_{i=1}^N$ in Euclidean or unitary space $\mathbf{H}^D$. The researcher has access to the measurement results of the unknown vector-signal $\mathbf{x}$ in the form of modules of scalar products $|\langle \mathbf{x}, \varphi_i \rangle|$, the phases (or signs) of these products are unknown. Is it possible to restore the vector $\mathbf{x}$? Since the modules of the scalar products do not change by passing from the vector $\mathbf{x}$ to the vector $h\,\mathbf{x}$ with $|h| = 1$, a factorization is performed beforehand for a neat formulation of the problem . Let $\mathbf{T} = \{h \in \mathbf{H} : |h| = 1\}$. The factor space $\mathbf{H}^D/\mathbf{T}$ is introduced as a set of equivalence classes: $\mathbf{x} \sim \mathbf{y}$, if there exists $h \in \mathbf{T} : \mathbf{x} = h\,\mathbf{y}$. Thus, the problem arises of vector reconstruction from modules of measurements. Firstly we formulate the reconstruction problem as a problem about the property of a system of measurement vectors.

Definition. A family of vectors $\Phi = \{\varphi_i\}_{i=1}^N$ in $\mathbf{R}^D$ (or $\mathbf{C}^D$) does the reconstruction by modules of measurements (RMM), if for any $\mathbf{x}, \mathbf{y} \in \mathbf{R}^D$ (or $\mathbf{C}^D$), satisfying $|\langle \mathbf{x}, \varphi_i \rangle| = |\langle \mathbf{y}, \varphi_i \rangle|$ for all $i = 1, \ldots, N$, we have $\mathbf{x} = c\mathbf{y}$, where $c = \pm 1$ for $\mathbf{R}^D$ (and $c \in \mathbf{T}$ for $\mathbf{C}^D$.)

This definition can be formulated as a property of injectivity of the nonlinear operator:

$$\mathcal{A} : \mathbf{H}^D/\mathbf{T} \to \mathbf{R}^N,\ (\mathcal{A}(\mathbf{x}))(n) := |\langle \mathbf{x}, \varphi_n \rangle|^2,\ n = 1, \ldots, N.$$

It's known that there are $2D - 1$ vectors in $\mathbf{R}^D$, which ensure such reconsruction. These vectors may be received by using Vandermonde matrices in order to construct systems of vectors $\Phi = \{\varphi_i\}_{i=1}^N$ in $\mathbf{R}^D$ with full spark for any $N \geq D$. We recall that *spark* of the system Φ is the smallest number of linearly dependent vectors of the system. For the system with full spark spark $() = D + 1$, i. e. each subsystem of D vectors of the system Φ consists of linearly independent vectors.

The question about the minimal number of vectors to ensure the reconstruction in $\mathbf{C}^D$ is still open.

Some applied problems required investigation of the possibility of signal reconstruction by norms of its projections on subspaces, for example, the twinning problem known in crystallography.

Definition. The family of subspaces $\{W_j\}_{j=1}^N$ in $\mathbf{H}^D$ with the corresponding orthogonal projectors $\{P_j\}_{j=1}^N$ *does reconstruction by projections* (RP), if for arbitrary $\mathbf{x}$, $\mathbf{y} \in \mathbf{H}^D$ such that $\|P_j\mathbf{x}\| = \|P_j\mathbf{y}\|$ for all $j = 1, \ldots, N$ we have $\mathbf{x} = c\mathbf{y}$ for some $|c| = 1$.

Some results in this area will be represented in the talk.

In the third part of the talk an infinite dimensional version of the reconstruction will be represented. We'll consider real space ℓ_2.

Definition. The family of vectors $\Phi = \{\varphi_j\}_{j\in J}$ does (RMM), if equalities

$$|\langle\varphi_j, \mathbf{x}\rangle| = |\langle\varphi_j, \mathbf{y}\rangle| \text{ for all } j,$$

for arbitrary $\mathbf{x}, \mathbf{y} \in \ell^2$ provide equality $\mathbf{x} = \pm\mathbf{y}$.

Some results from the finite dimensional space have analogous also in infinite dimensional space, but at the same time there are many differences.

The work performed under the development program of Volga Region Mathematical Center (agreement no. 075-02-2023-878).

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Miura-like transformations of nonlinear lattices and inverse spectral problems for band operators.

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Since the classical works of Kac, van Moerbeke and Moser, the inverse spectral problems for operators generated by possibly infinite band matrices (we call them band operators; in particular, the Jacobi operators belong to this class) have been widely applied to the integration via Lax pair formalism of certain nonlinear dynamical systems (nonlinear lattices). Here we consider a version of the inverse spectral problem method for band operators, where a key role is played by the moments of the Weyl function (or Weyl matrix) of a given band operator, which are used for unique reconstruction of the latter, see [1–3].

In the study of nonlinear integrable equations, an important role is played by various Bäcklund-Miura type transformations between them. In [2; 3] we found that a discrete Miura transformation, which relates the Volterra and Toda nonlinear lattices to each other, can be easily described in terms of the above mentioned moments (in particular, such description allows one to establish a bijection between the classical semi-infinite Volterra lattices and the semi-infinite Toda lattices characterized by positivity of Jacobi operators in their Lax representation).

Here we discuss similar issues for the semi-infinite Bogoyavlensky lattices BL1(p):

$$\frac{da_i}{dt} = a_i \left(\prod_{j=1}^{p} a_{i+j} - \prod_{j=1}^{p} a_{i-j} \right);$$

and BL2(p):

$$\frac{db_i}{dt} = b_i \left(\sum_{j=1}^{p} b_{i+j} - \sum_{j=1}^{p} b_{i-j} \right);$$

for some fixed $p \geq 1$.

The Lax operators for both these lattices are band operators which have a special structure: they contain only two nonzero diagonals, see [3] for details. Also, in the finite lattice case, due to the special structure of the moments corresponding to the finite Lax operators, some non-standard first integrals for $BL1(p)$ and $BL2(p)$ were found.

Also we consider the non-Abelian Volterra and Toda lattices (i. e. the lattices with matrix or operator elements) and discuss a similar description of a discrete Miura transformation between them. As in the classical case, such description allows one to establish a bijection between these two classes of non-Abelian systems.

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Periodicity and different systems of co-ordinates

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The $z = f(p)$ function is regular in some open G area, $(A,0) \in G, (0,0) \in G, A > 0$. We define the C_f set from the $C_f = \{(p,z) : z = f(p)\}$ equality. We can consider a new center of co-ordinates in the $(A,0)$ point (with the new w complex variable). In the first fact by definition $z = f_2(w) = f(w + A)$; the $z = f_2(w)$ equation is the equation of the same C_f set in the new co-ordinates. It is obviously, for the reverse $f_2^{-1}(z) = w, f^{-1}(z) = p$ functions we obtain the $f_2^{-1}(z) + A = f^{-1}(z)$ equality, (with help $p - w = A$). We get $df_2^{-1}(z)/dz = df^{-1}(z)/dz$ for wide class of the $f(p)$ functions. For the $C_f^1 = \{(p,z) : z = df(p)/dp\}$ set we obtain the $df_2^{-1}(z)/dz = df^{-1}(z)/dz$ equality.

We use, that the same analytical $df_2^{-1}(z)/dz, df^{-1}(z)/dz$ expressions take place only for the same analytical $df_2(z_1)/dz_1, df(z_1)/dz_1$ expressions of the $z = df_2(w)/dw, z = df(p)/dp$ functions, if $p = w$, [1,2,3], (the fact is obvious, if the p, w variables are marked by the single $p = w = z_1$ letter). We can use, that $z = df(p)/dp = df_2(w)/dw$ for the $(\cdot)p = (\cdot)w$ points, when $p = w + A$. The same analytical $df_2(z_1)/dz_1, df(z_1)/dz_1$ expressions take place only for the periodic $df(p + A)/dp = df(p)/dp$ function, if, for instance, $df(p)/dp \neq 0, p \in G, G = \{p : Re\, p > a > 0, A > a, f(p) = 1/p$.

In the second fact the $z = f_2(w)$ equation of the C_f set is equal to the $z = f_2(p - A)$ equation, $w = w_0$, (we use $p - A = w$ for the w variable in the new co-ordinates with the $(A,0)$ center). The $z = f_2(p - A)$ equation is defined in the primary system of co-ordinates with the p complex variable (p is the radius-vector in the primary system of co-ordinates with the $(0,0)$ center). The same $z = f_2(R - A)$ equality takes place in the second system of co-ordinates with the $(A,0)$ center too, (for all $z, R = p$); in the situation we can consider the new $R - A$ vector with the R radius-vector in the second system of co-ordinates wit the $(A,0)$ center. We obtain $R - A = w_0$ too, and the both $z = f_2(R - A), z = f_2(p - A)$ equalities are as the equality in the primary system so as in the second system of co-ordinates for all $p = A$, (the equalities of the C_f set). The same equality in the two system of co-ordinates is equivalent to the periodicity of the $f_2(w)$ function with the A period.

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On estimates for the energy levels of a quantum billiard

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In the present talk we deal with estimates for the energy levels of a quantum billiard in a large class of domains (domains with Hölder singularities, Ahlfors type domains). The Hausdorff dimension of the Ahlfors type domains boundary can be any number in $[1, 2)$.

Our method is based on composition operators on Sobolev spaces generated by (quasi)-conformal mappings. On this way, we obtain lower and upper estimates for the energy levels of a quantum billiard.

The research was supported by RSF (Grant No. 23-21-00080).

Quantum control by the environment and by quantum measurements

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Control of quantum systems is an important branch of quantum physics related both with fundamental interest and with existing and prospective applications to quantum technologies [1; 3; 12]. Often controlled quantum systems are open, i.e. interacting with the environment, which is considered as having deleterious effect on the ability to control the system. However, in some cases it can be exploited as a useful resource.

Various approaches for using the environment as a resource exist. We will discuss some recent results based on the *incoherent control* approach [2]. In this approach, density matrix ρ_t of the quantum system evolves under the action of coherent and incoherent controls according to the master equation with time dependent decoherence rates $\gamma_k(t)$,

$$\frac{d\rho_t}{dt} = \mathcal{L}_{u,n(t)}(\rho_t) = -i[H_u(t), \rho_t] + \sum_k \gamma_k(t)\mathcal{D}_k(\rho_t)\,.$$

Here $H_u(t)$ is the Hamiltonian describing free system dynamics and its interaction with coherent control $u(t)$ (e.g., a laser field), $n(t) \geq 0$ is generally time-dependent incoherent control (e.g., spectral density of incoherent photons), k denotes pairs of energy levels in the controlled system and $\mathcal{D}_k$ is a dissipator, for which two physical classes were exploited — incoherent photons and quantum gas, with two explicit forms of $\mathcal{D}_k$ derived in the weak coupling (describing atom interacting with photons) and low density (describing quantum system interacting with a quantum gas) limits [8]. Generally, coherent control can also enter in the dissipator, and in opposite, incoherent control also modifies the Hamiltonian via Lamb shift. Non-Markovian master equations can be considered for incoherent control as well. This method was shown to provide approximate density marix controllability of generic open quantum systems [7], it was applied to control of two-qubit systems [6], description of reachable state for a qubit interacting with the environment [5], development of speed gradient approach for energy manipulation in quantum oscillator interacting with the environment [9], gradient based optimization of coherent and incoherent controls in open quantum systems [10], etc.

Related to control by the environmen is control by back-action of quantum measurements. Often performing measurement on quantum system, even without reading the measurement result, leads to change of its state, known as quantum state collapse. This change can be used for controlling quantum systems [11]. Recently, quantum measurements with feedback were applied to transport in photosynthetic systems [4] and to analysis of quantum control landscapes in uncontrollable quantum systems with degenerate transitions [2].

RSF 22-11-00330, Minobrnauki 075-15-2020-788, K2-2022-025, "Priority 2030".

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Dynamics and gradient optimization for a two-level open quantum system

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Quantum control which studies methods for manipulation of individual quantum systems is an important tool necessary for development of quantum technologies [7]. Often in experimental circumstances controlled systems can not be isolated from the environment, so that they are open quantum systems. Moreover, in some cases the environment can be used for actively controlling quantum systems, as for example in incoherent control [2; 3]. While in some cases the solution for the optimal shape of the control can be obtained analytically, often it is not the case and various numerical optimization methods are needed. A large class of methods are gradient-based numerical optimization algorithms, one of which is GRadient Ascent Pulse Engineering (GRAPE) developed originally for design of NMR pulse sequences [1] and later applied to various problems, e.g. [4; 9].

In this talk, we consider the state-to-state transfer control problem for an open two-level quantum system (qubit) whose evolution is governed by master equation with GKSL-type dissipative terms driven by coherent and incoherent controls [5; 6]. General form of the GKSL master equation in the absence of controls was derived in particular in the weak coupling limit and in the stochastic limit of quantum theory. We consider the specific model of such master equation which includes coherent and incoherent controls.

The state of the system is represented by a vector in the Bloch ball. We consider piecewise constant control as they are commonly used for gradient optimization methods. Then we derive expressions for the dynamics and objective functional gradient using matrix exponentials. Due to low dimension of the system, the corresponding 3×3 matrix exponentials can be analytically diagonalized. For that we find eigenvalues and eigenvectors of the right-hand side matrix of the system evolution equation. Roots of the third order characteristic equation are analytically found using the Cardano's formula. This enables obtaining exact form of matrix exponentials included in the dynamics and functional gradient expressions necessary for control landscape analysis. We analyzed the surface of stationary states in Bloch ball for different constant controls (u, n). We estimated first non-zero term of the objective functional important for robustness of optimal control.

Partial support by Minobrnauki № 075-15-2020-788, Priority 2030, and RSF № 22-11-00330.

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Rate of convergence of ergodic averages for $\mathbb{Z}^2$ and $\mathbb{R}^2$ actions via estimates of spectral measure

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Let (Ω, μ) be a measure space and $A_n f$ or $A_t f$ be ergodic averages generated by one automorphism T or one flow $\{T^t\}_{t\in\mathbb{R}}$ and $f \in L_2^0(\Omega,\mu)$. It is known (see [1] for example) that if the spectral measure $\sigma_f((-\delta,\delta]) \le A\delta^\alpha$ for some $A > 0$, $\alpha \ge 0$ and any $\delta > 0$ then

	$\alpha \in [0,2)$	$\alpha = 2$	$\alpha > 2$
$\|A_t f\|_2^2$	$\mathcal{O}(t^{-\alpha})$	$\mathcal{O}(t^{-2}\ln t)$	$\mathcal{O}(t^{-2})$
$\|A_n f\|_2^2$	$\mathcal{O}(n^{-\alpha})$	$\mathcal{O}(n^{-2}\ln n)$	$\mathcal{O}(n^{-2})$

In the talk we present the similar result for $\mathbb{Z}^2$ and $\mathbb{R}^2$ actions and discuss the cases $d > 2$. Namely, denote the ergodic averages

$$A_n f(\omega) = \frac{1}{n_1 n_2}\sum_{i=0}^{n_1-1}\sum_{j=0}^{n_2-1} f(T_1^i T_2^j \omega),\ A_t f(\omega) = \frac{1}{t_1 t_2}\int_0^{t_1}\int_0^{t_2} f(T_1^{s_1}T_2^{s_2}\omega)\, ds_1 ds_2.$$

Let for the spectral measure

$$\sigma_f((-\delta_1,\delta_1]\times(-\delta_2,\delta_2]) \le A\delta_1^\alpha \delta_2^\beta$$

for some $A > 0$, $\alpha, \beta \ge 0$ and any $\delta_1, \delta_2 > 0$, and

$$\alpha^2 + \alpha\beta + \beta^2 = \kappa(\alpha+\beta),\ \kappa \ge 0.$$

Then

	$\kappa \in [0,2)$	$\kappa = 2$	$\kappa > 2$
$\|A_t f\|_2^2$	$\mathcal{O}(t_1^{-\alpha}t_2^{-\beta})$	$\mathcal{O}(t_1^{-\alpha}t_2^{-\beta}\ln t_1^\alpha t_2^\beta)$	$\mathcal{O}\left(t_1^{-\frac{2\alpha}{\kappa}} t_2^{-\frac{2\beta}{\kappa}}\right)$
$\|A_n f\|_2^2$	$\mathcal{O}(n_1^{-\alpha}n_2^{-\beta})$	$\mathcal{O}(n_1^{-\alpha}n_2^{-\beta}\ln n_1^\alpha n_2^\beta)$	$\mathcal{O}\left(n_1^{-\frac{2\alpha}{\kappa}} n_2^{-\frac{2\beta}{\kappa}}\right)$

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Renormalizations in digital representation of continuous observables

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Let's consider the binary system (base $q = 2$) with the set of digits is $\{0, 1\}$. Then the plot of digits number 0 and 1 has the following view:

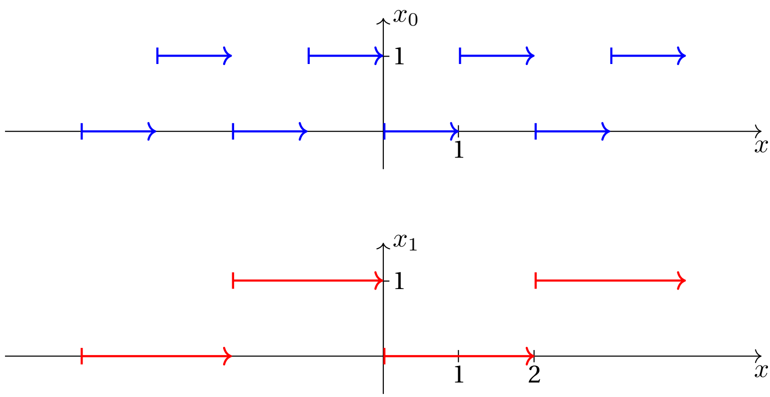

Figure 1: Plot of the digits number 0 and 1 for the binary non-symmetric system

As we define the digit number s in the following way ($x_s = \mathbf{d}(s, x)$ is the s-th digit in the digital expansion of x): $\mathbf{d}(s, x) = \mathbf{d}(0, q^{-s}x) = \mathbf{D}(q^{-s}x)$, which means that the digit number s is the digit number 0 with the changed scale over x-axis, it becomes obvious, that for the negative numbers, any series in binary non-symmetric system does not converge and hence we need a renormalization procedure.

To solve the emerging problem, let's consider the arbitrary positional base-q system with digits x_s. As we have seen, for such a system the series

$$\sum_{s=-\infty}^{\infty} x_s q^s, \tag{1}$$

does not converge in general case, but we are always able to recover the number x by the row of its digits $(x_s)_{s=-\infty}^{\infty}$. This enables us to associate any finite x with the series (1). All the divergences, which emerge in the discussed case have the form:

$$C \cdot \sum_{s=s_0}^{\infty} q^s, \tag{2}$$

where C is a constant and q is the base of the numeral system. For such a case we can introduce the renormalization rule, which can be represented by the following formal calculation (which is one of the possible ways to renormalize the divergent digital sums; the alternative ways were described in [2]):

$$x = \sum_{s=-\infty}^{\infty\,\prime} x_s q^s = \frac{1}{q-1} \sum_{s\in\mathbb{Z}} (x_{s-1} - x_s) q^s. \tag{3}$$

The renormalization method can be generalized from the line to the case of the finite lattice, on which all observables are finite. Let us consider the lattice $\Delta x \cdot \mathbb{Z}_N$. It does not have any infinite nodes and therefore the motivation for the renormalization is no longer the convergence of the divergent series.

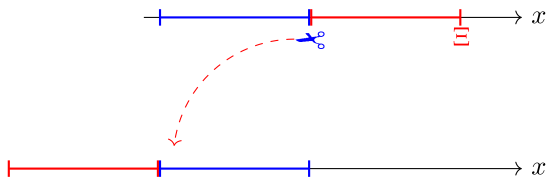

Figure 2: Renormalization on the finite lattice changes the representation of the lattice

Its meaning is now to switch the representation of the lattice for $\mathbb{Z}_N$ from $\{0, 1, \ldots, N-1\}$ to $\{-k, -k+1, \ldots, -2, -1, 0, 1, 2, \ldots, N-k-1\}$. We can see, that the renormalization on the lattice enables us to renormalize big positive value to the small negative one. In [1] was shown that the renormalization, induced by the lattice itself enables to nullify the "quasienergy" and more sophisticated renormalizations are not needed in this case, but this renormalization may be applicable to the description of Casimir effect on the lattice.

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Kantorovich optimal transportation problems with a parameter

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Let us recall that the Kantorovich optimal transportation problem deals with a triple (μ, ν, h), where μ and ν are Borel probability measures on topological spaces X and Y respectively, and $h \geq 0$ is a Borel function on $X \times Y$. The measures μ and ν are called marginal distributions or marginals, and h is called a cost function. The Kantorovich problem consists in minimization of the integral

$$\int_{X\times Y} h(x,y)\,\sigma(dx\,dy)$$

over the set $\Pi(\mu,\nu)$ of all Borel probability measures on the product $X \times Y$ with projections μ and ν on the factors, that is, $\sigma(A\times Y) = \mu(A)$ and $\sigma(X\times B) = \nu(B)$ for all Borel sets $A \subset X$ and $B \subset Y$.

In general, there is only infimum

$$K_h(\mu,\nu) = \inf_{\sigma\in\Pi(\mu,\nu)} \int_{X\times Y} h(x,y)\,\sigma(dx\,dy),$$

which may be infiite. If the cost function h is continuous (or at least lower semicontinuous) and bounded and the measures μ and ν are Radon, then the minimum is attained and measures on which it is attained are called optimal measures or optimal Kantorovich plans.

We consider optimal transportation of measures on metric and topological spaces in the case where the cost function and marginal distributions depend on a parameter with values in a metric space. Here the questions naturally arise about the continuity with respect to t of the optimal cost $K_{h_t}(\mu_t,\nu_t)$ and also about the possibility to select an optimal plan in $\Pi(\mu_t,\nu_t)$ continuous with respect to the parameter. In addition, the set of all transport plans $\Pi(\mu_t,\nu_t)$ also depends on the parameter, so that one can ask about its continuity when the space of sets of measures is equipped with the Hausdorff metric generated by some metric on the space of measures.

We obtain an estimate for the Hausdorff distance between the sets of probability measures with given marginals via the distances between the marginals themselves. This estimate is used to prove the continuity of the cost of optimal transportation with respect to the parameter in the case of continuous dependence of the cost function and marginal distributions on this parameter. Another application of the estimate for the Hausdorff distance concerns discrete approximations of the transportation problem.

Probability measure on conformal mappings

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Let us consider the set of conformal mappings from the unit disk D onto all possible simply connected locally univalent domains with conditions $f(0) = 0, f'(0) > 0$. Each such mapping is determinate by a pair c, λ, where $c -$ positive number responsible for scale, $\lambda -$ periodic function on $[0, 2]$ with integral

$$\int_0^2 \lambda(t)dt = 2. \tag{1}$$

Using this correspondence, it is proposed to introduce a probability measure on the set of mappings, given as a functional of λ. The paper considers various issues related to the construction of the theory and its justification.

Moments method in optimal control investigation and state estimation for fractional-order systems

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The moments method, widely used in control theory for linear integer-order systems, allows us to reduce the optimal control problem to the l-problem of moments. Optimality in this case can be understood as the minimality of the control norm for a given control time, or as time-optimality for a given restriction on the control norm. It is also known the Krasovskii problem of the state estimation of a dynamic integer-order system under the action of external disturbances, which where inestigated using the method of moments.

In this work the possibility of a similar application of the method of moments in optimal control and state estimation problems analyzed for dynamical systems that are described by linear equations of fractional order:

$$ {}_0D_t^{\alpha_i} q_i(t) = a_{ik} q_k(t) + b_{ik} u_k(t) + f_i(t), i = 1, \dots, N. \quad (1)$$

In equation (1) the fractional differentiation operator can be understood in the sense of Hilfer, Hadamard, Erdei-Kober, Caputo-Fabrizio, Atangana-Baleanu etc. The functions $q_i(t)$, $u_i(t)$ and $f_i(t)$ describe, respectively, the state of the system, control and external disturbance. The control is the p-integrable function $p > 1$.

When studying the optimal control problem of the system (1), conditions are obtained under which the corresponding l-problem of moments is correct and solvable. These conditions relate the parameters of the fractional differentiation operator to the parameter p. Some explicit solutions to the optimal control problem are also obtained. When studying the problem of the state estimation of the system (1), similar conditions for the correctness and solvability of the resulting moment problem were also obtained and its explicit solution was constructed.

In addition, the possibility of using the method of moments for an approximate solution of the optimal control problem for systems with distributed parameters described by the diffusion-wave equation is investigated:

$$ r(x) \ {}_0D_t^{\alpha} Q(x,t) = \frac{\partial}{\partial x}\left[w(x) \frac{\partial Q(x,t)}{\partial x} \right] - q(x)Q(x,t) + +f(x,t) + u(x,t), \quad (2)$$

where $Q(x,t)$ — state of the system, $\alpha \in (0,1)$or$(1,2)$, $t \geq 0$, $x \in [0,L]$, $(x,t) \in \Omega = [0,L] \times [0,\infty)$. In this case the distributed and boundary controls have been considered. The approximate solution derived for the optimal control problem for system (2), based on the corresponding moment problem, which correctness and solvability analyzed.

Remark on boundary conditions for the KPZ equation

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The Kardar-Parisi-Zhang (KPZ) equation is known to be very popular model of the growth of the solid state surface (see [2] and references therein). As a rule this equation is solved on the whole two-dimensional plane without any boundary conditions [1; 3]. Meanwhile, technologically, the process of crystal surface growth always occurs in a bounded domain. Thus, there is always a question: under what conditions on this bounded domain can the solution of the KPZ-equation on the whole plane be applied?

To clarify this situation let one consider the following model problem:

$$\frac{\partial h}{\partial t} = \frac{1}{2}\left(\frac{\partial h}{\partial x}\right)^2 + \frac{\partial^2 h}{\partial x^2}\,, \tag{1}$$

where $h(x,t)$ is a shape of growing surface with cylindrical generatrix.

Further let us provide the one-dimensional KPZ equation (1) by the next initial condition:

$$h(x,0) = 2\,\ln[\,1 + m_0\,\theta(1-|x|)\,]\,, \qquad |m_0| < 1\,, \tag{2}$$

where $\theta(x)$ is the Heaviside step function, and parameter m_0 plays the role completely analogous to one of modulation depth in radio engineering.

If somebody considers equation (1) and initial condition (2) on the whole straight line $-\infty < x < +\infty$, then exact solution of the Cauchy problem (1)-(2) is equal to:

$$h(x,t) = 2\,\ln\left(1 + \frac{m_0}{2}\,\left[erf\left(\frac{x+1}{2\sqrt{t}}\right) - erf\left(\frac{x-1}{2\sqrt{t}}\right)\right]\right)\,, \tag{3}$$

where $erf(x)$ is the well-known Gauss error function.

On the other side if equation (1) is considered on the segment $x \in [-L\,,L]$ then it ought to be endowed by some boundary conditions (of course it is assumed that $L > 1$).

The simplest boundary conditions express the absence of surface diffusion via the ends of the segment:

$$\frac{\partial h}{\partial x}|_{x=-L} = \frac{\partial h}{\partial x}|_{x=+L} = 0\,. \tag{4}$$

Exact solution of the initial-boundary value problem (2) and (4) for equation (1) is equal to:

$$h(x,t) = 2\,\ln\left(1 + \frac{m_0}{L}\,\left[1 + 2\,\sum_{n=1}^{\infty} \exp\left(-\frac{\pi^2 n^2 t}{L^2}\right)\,\frac{\sin(\pi\,n/L)}{\pi\,n/L}\,\cos\frac{\pi\,n\,x}{L}\right]\right). \tag{5}$$

In this report numerical comparison of expressions (3) and (5) has been done. In particular the KPZ equation (1) proves to forget about the presence of boundary conditions (4) under quite large times.

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Semigroup approach to studying Volterra integro-differential equations arising in viscoelasticity theory

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Abstract Volterra integro-differential equations with integral operator kernels representable by Stieltjes integrals are studied. These integro-differential equations can be realized as partial integro-differential equations arising in the theory of viscoelasticity and the theory of heat propagation in media with memory and have many of other important applications.The approach is based on the study of one-parameter semigroups for linear evolution equations.

Results on the existence of a strongly continuous contraction semigroup generated by a Volterra integro-differential equation with operator coefficients in a Hilbert space are stated. The statement of the corresponding Cauchy problem for a first-order differential equation is given, and a theorem on the well-posed solvability of this problem is stated. The properties of the generator of the semigroup and the properties of the operator function associated with it (the symbol of the original integro-differential equation) are studied. (See [1; 2]).

This work is supported by the Ministry of Science and Higher Education of the Russian Federation (project number FSSF-2023-0016).

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On one integral inequality for logarithmic derivatives of measures

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Let ρ be an absolutely continuous probability density on the real line. The quantity

$$J(\rho) = \int\limits_{-\infty}^{+\infty} \frac{|\rho'(x)|^2}{\rho(x)} dt$$

is called the *Fisher Information* of the density ρ. Since the Fisher information appears naturally in many mathematical problems, it is useful to know general conditions which ensure that $J(\rho)$ is finite.

In [4] A. V. Uglanov suggested the next elementary and useful lemma which states that, roughly speaking, the Fisher Information of every smooth enough density φ can be estimated via the sum of L_1-norms of derivatives of this density.

Lemma 1. There exists a number C such that for every non-negative twice differentiable function $\varphi : R^1 \to R^1$ with an absolutely continuous second derivative φ the following inequality holds, where we set $0/0 = 0$:

$$J(\varphi) = \int\limits_{-\infty}^{+\infty} \frac{|\varphi'(t)|^2}{\varphi(t)} dt \le C \int\limits_{-\infty}^{+\infty} [|\varphi'(t)| + |\varphi''(t)| + |\varphi'''(t)|]\, dt.$$

This result was reinforced in Bogachev [2; 3] in the following way.

Lemma 2. If $\varphi : R^1 \to [0, +\infty)$ is twice differentiable and φ'' has bounded variation, then

$$J(\varphi) = \int\limits_{-\infty}^{+\infty} \frac{|\varphi'(t)|^2}{\varphi(t)} dt \le 8||\varphi'||_{L_1} + 6||\varphi''||_{L_1} + 2Var\varphi''.$$

We denote by $AC^l(a, b)$ collection of all continuous functions u on the interval (a, b) having continuous derivatives up to order $l - 1$ such that the derivative u^{l-1}

is locally absolutely continuous. In the recent work Bobkov [1], the previous inequality was slightly sharpened.

Lemma 3. For any non-negative function p of class $AC^3(R^1)$

$$J(p) = \int\limits_{-\infty}^{+\infty} \frac{|p'(x)|^2}{p(x)} dx \leq 2 \int\limits_{-\infty}^{+\infty} |p'(x)| dx + 4 \int\limits_{-\infty}^{+\infty} |p''(x)| dx + 2 \int\limits_{-\infty}^{+\infty} |p'''(x)| dx.$$

Taking $\varphi(t) = |t| \left|\ln |t|\right|^{-1}$ in $(-\delta, \delta)$, we can see that $J(\varphi)$ cannot be estimated via $||\varphi'||_{L_1}$ and $||\varphi''||_{L_1}$. It seems interesting, how much we can weaken the assumptions in these lemmas still preserving the finite Fisher information. It turned out that even in $AC^3(R^1)$ we can find a function φ with a bounded second derivative that $\varphi', \varphi'' \in L_1(R^1)$, but $J(\varphi) = +\infty$.

It also turned out that the inequality in Lemma 3 can be strengthened the next way.

For any non-negative function p of class $AC^3(R^1)$

$$J(p) = \int\limits_{-\infty}^{+\infty} \frac{|p'(x)|^2}{p(x)} dx \leq \int\limits_{-\infty}^{+\infty} |p'(x)| dx + 2 \int\limits_{-\infty}^{+\infty} |p''(x)| dx + 2 \int\limits_{-\infty}^{+\infty} |p'''(x)| dx.$$

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The interaction of shocks in 2D pressureless medium

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Let $\mathbf{x} \equiv (x, y)$, $(t, \mathbf{x}) \in R_+ \times R^2$, $\nabla = (\partial/\partial x, \partial/\partial y)$, then 2D system of pressureless gas dynamics can be written as follows

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot \varrho \mathbf{u} = 0 \quad , \quad \frac{\partial (\varrho \mathbf{u})}{\partial t} + \nabla \cdot (\varrho \mathbf{u} \otimes \mathbf{u}) = 0 \,, \tag{1}$$

where $\varrho \geq 0$ is the density of matter, $\mathbf{u} \equiv (u, v)$ – velocity vector and $\otimes$ denotes the tensor product. Suppose $\varrho(0, \mathbf{x}) = \varrho_0(\mathbf{x})$, $\mathbf{u}(0, \mathbf{x}) = \mathbf{u}_0(\mathbf{x})$.

The system (1) has a single eigenvalue and incomplete system of eigenvectors; thus, strictly speaking, it is not hyperbolic. But nevertheless it has many evolution properties similar to standard theory of systems of hyperbolic conservation laws. Thus it can be rendered as degenerate nonstrictly hyperbolic system. Because of such degeneracy the generalized solutions of (1) are considered in the spaces of measures, see e. g. [4]. As in conservation laws framework the system (1) permits the formation of singularities. But these singularities are the concentrations of corresponding measures on the manifolds of different dimensions. In 2D case, as it is shown in [1; 3], there exist the moving curves in the space $\mathbf{x}$ with the mass and momentum measures along the curve, having the densities P and $\mathbf{I}$ respectively and evolving with respect to the equations

$$\begin{array}{l} \frac{\partial P}{\partial t} = \frac{\partial \chi}{\partial l} \left\{ V[\varrho] - [\varrho v] \right\} - \frac{\partial \gamma}{\partial l} \left\{ U[\varrho] - [\varrho u] \right\} \\ \frac{\partial \mathbf{I}}{\partial t} = \frac{\partial \chi}{\partial l} \left\{ V[\varrho \mathbf{u}] - [\varrho v \mathbf{u}] \right\} - \frac{\partial \gamma}{\partial l} \left\{ U[\varrho \mathbf{u}] - [\varrho u \mathbf{u}] \right\} \end{array} , \tag{2}$$

where $\mathbf{X} \equiv (\chi(t, l), \gamma(t, l))$ are parametric representation of moving singularity curve, $(U, V) \equiv (\partial \chi/\partial t, \partial \gamma/\partial t)$, and for any value f it is denoted $[f] \equiv f(t, \mathbf{X} + 0) - f(t, \mathbf{X} - 0)$. Let us note that the system (2) actually is the Renkine-Hugoniot condition for (1) in case when the singularities have the form of moving curves in the space $\mathbf{x}$. It is also shown in [2] that the collision of moving singularity curves leads to the formation of similar singularity curve with larger mass.

But in this talk we claim that there exists another form of collision. Consider the Riemann initial data for (1), i.e. the initial data that are constant for each quadrant in $\mathbf{x} \in R^2$ as $t = 0$. Let enumerate quadrants in the standard manner by the index $i = 1, 2, 3, 4$; the initial constants will be enumerated by the same index. Then the following theorem is true.

Theorem 1. *Consider the Riemann initial data for the system (1):* $u_1 = u_4 = -u$, $u_2 = u_3 = u$, $v_1 = v_2 = -v$, $v_3 = v_4 = v$, $\varrho_i = \varrho, i \neq 4$, $\varrho_4 = R$, *where* $u > 0, v > 0$ *and* $0 < \varrho < R$ *are constants. Then in the solution of corresponding Riemann problem there exist two solutions of (2) that interact with the formation of* δ*-functions in* $P, \mathbf{I}$ *in one point of the space* $\mathbf{x}$.

Such situation was anticipated in [2], where the corresponding addition to Renkine-Hugoniot conditions was given. We also would like to mention that the family of singularities beyond the surfaces of codimension one should have much more reach structure in higher space dimensions.

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Finitely generated weak closures of ergodic actions

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Weak closures of group actions in unitary representations generate various semigroup structures. In ergodic theory, such closures have found many useful applications. However, explicit descriptions of nontrivial weak closures of dynamical systems have been obtained for a narrow range of actions. We give a large class of new examples.

A function P of an operator T acting in a Hilbert space is admissible if $P(T) = \sum_{i=0}^{\infty} c_i T^i$, $c_i \geq 0$, $0 < c_0 < \sum_{i=0}^{\infty} c_i \leq 1$. For any set of admissible functions $P_1, \ldots, P_k$, we prove the existence of unitary operator T with continuous spectrum such that the weak closure of the action induced by T is a semigroup, generated by 0, T, $P_1(T), \ldots, P_k(T)$ and by the operators adjoint to them.

The action we need has been induced by a specially constructed ergodic transformation of an infinite measure space. The corresponding construction combines the technique of rank one transformation with random parameters with the so-called Sidon ones. The formers ensure the presence of the necessary operators in the weak closure of the action, while the latters eliminate the presence of unnecessary operators in it.

The Collet-Eckmann set for a family of one-dimensional non-smooth Lorenz maps

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In the talk, we consider a two-parameter family of one-dimensional maps with parameters c and ν given on the compact interval $[-1, 1]$ by

$$\bar{x} = T_{c,\nu}(x) = (-1 + |x|^{\nu} \cdot (c + \phi(x; c, \nu)) \cdot sign(x),$$

where $c > 0$, $\nu > 1$ and $\phi(x; c, \nu)$ is a $C^{1+\rho}$-function whose derivatives with respect to x and c are small quantities of order $\nu - 1$. The map $T_{c,\nu}$ appears as a one-dimensional models for dynamical systems with Lorenz-like attractors [1] or Rovella attractors [6]. For those systems, $T_{c,\nu}$ can be regarded as a factor map related to an appropriate strong stable invariant foliation (see [1; 5; 6]). In general, this foliation has low smoothness, therefore we assume that the small term $\phi(x; c, \nu)$ is only a Holder function.

We consider the map $T_{c,\nu}$ when ν is arbitarilly close to 1 and study the problem of the description of the set of parameters such that $T_{c,\nu}$ satisfies Collet-Eckman condition (see [3] and item (1) in Theorem below). We generalise the classical result by Benedicks and Carleson [2] which states the positiveness of the Lebesgue measure of the parameters satisfying Collet-Eckman condition for one-parameter family of quadratic maps $\bar{x} = -1 + cx^2$ (see also [4; 7] for further generalisations). Our approach is based mainly on ideas from [2]. However, for $\nu < 2$ the map $T_{c,\nu}$ has an unbounded second derivative, which does not allow to use directly the ideas of Benedicks and Carleson. Our main result is presented in the following theorem.

Theorem. *In the* (c, ν)*-plane there is a set* E *of positive Lebesgue measure* $\lambda(E)$ *such that for the family* $T_{c,\nu}(x)$ *the following holds:*

(1) *there exists a constant* $\gamma > 0$ *such that for any* $(c, \nu) \in E$ *one has*

$$DT^n_{c,\nu}(1) = DT^n_{c,\nu}(-1) > e^{\gamma n}, \ \forall n \in \mathbf{N};$$

(2) *there exists a constant* $\tilde{\gamma} > 0$ *such that for any* $(c, \nu) \in E$ *and* $x \in [-1, 1]$

$$\limsup_{n\to\infty} \frac{\ln DT^n_{c,\nu}(x)}{n} > \tilde{\gamma}, \;\; \textit{if } T^n_{c,\nu}(x) \neq 0, \;\; \forall n \in \mathbf{N};$$

(3) *for any* $\nu \in (1, 1+\delta)$, *the point* $c = 2$ *is a density point of the set* $E^\nu = E \cap \{\nu = const\}$, *i.e.,*

$$\lim_{\Delta\to+0} \frac{\lambda(E^\nu \cap [2, 2-\Delta))}{\Delta} = 1.$$

(4) *the topological limit of* E^ν *as* $\nu \to 1$ *contains a non-trivial interval of the form* $[c_0, 2]$ *for some constant* $c_0 < 2$.

We also discuss in the talk, the application of the above results to homoclinic bifurcations of the separatrix figure-eight with neutral saddle equilibrium which may create chaotic Lorenz and Rovella attractors.

This work was supported by the Russian Science Foundation grant No. 19-71-10048.

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Unitary representation of random transformations of a Hilbert space and limit theorems

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To obtain a unitary representation of groups of self-mappings of a Hilbert space we introduce a measure on a Hilbert space such that this measure is invariant with respect to the group of transformation. We describe a measures with the above invariance property such that these measures are nonnegative, complete, locally finite, σ-finite. But they are not countable additive Borel measures.

In the space of functions that are quadratically integrable with respect to an invariant measure we obtain representations of the group of isometries of the Hilbert space and the group of symplectomorphisms. We study the continuity in the strong operator topology of the obtained representation. Subgroups having a continuous representation are obtained. Also the strongly continuous unitary representation of above groups are obtained in invariant subspaces of the space of quadratically integrable functions.

A random unitary group are studied as the unitary representation of a random group of self-mappings of a Hilbert space. Limit distribution for compositions of independent identically distributed random transformation and compositions of its unitary representations are analyzed in the form of convergence in measure and convergence in distribution.

Noise Effects on Chaotic Hamiltonian Dynamics

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We analyze some noise effects on Hamiltonian Dynamics. In particular, we focus in the paradigmatic Hénon-Heiles Hamiltonian system where we study the effect of noise in the escapes from the potential. Furthermore, we develop some techniques such as probability basins aiming to quantify the unpredictability of the noisy Hamiltonians. This is a joint work with A. R. Nieto and J. M. Seoane from URJC, Spain.

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Eta-invariants for families of operators with shifts

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Melrose defined eta-invariants for parameter-dependent elliptic operators in the sense of Agranovich and Vishik. In this approach, the eta-invariant is defined as a regularization of the winding number of the family.

In noncommutative geometry and in the theory of nonlocal problems, an important role is played by a class of operators associated with group actions on manifolds. More precisely, for an action of a group on a manifold, we consider operators equal to linear combinations of shift operators induced by the action of the group, where the coefficients are pseudodifferential operators.

We study eta-invariants for such operators, where the coefficients are parameter-dependent operators. This problem was solved at least in the situation where 1) the group is of polynomial growth; 2) it acts isometrically on the manifold; 3) the coefficients can be elements of the algebra of parameter-dependent operators generated by the classical parameter-dependent pseudodifferential operators and the operators of multiplication by periodic functions.

Note that parameter-dependent operators of this type arise in the study of nonlocal problems on manifolds with conical points and on manifolds with cylindrical ends. In the case of the trivial group, this problem was solved in our recent paper [1]. We apply methods developed in the cited paper.

The talk is based on joint work with Konstantin N. Zhuikov, see [2].

This work was supported by the Ministry of Science and Higher Education of the Russian Federation (project number FSSF-2023-0016).

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On the strange nonlocal homogenized problem arising as the limit of a Poisson Equation with dynamical Signoroni conditions on the boundary of perforations of critical size and arbitrary shape

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The talk addresses the homogenization of the problem for the Poisson equation in a domain perforated by arbitrary shaped perforations. A dynamic Signorini condition with a large coefficient is specified on the boundary to these inclusions. We are concerned with the critical relation between this coefficient,the period of the structure, and the size of the holes. We show that in this case homogenized problem contains a new nonlocal "strange" term and proves the convergence theorem.

Feynman Integrals in Quantum Gravity

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The enormous popularity of gravity in the last several decades motivated by its role in string theory and studies of BH physics in the dimensional reduction approach has grown after realizing the Schwarzian nature of the JT dilaton gravity and the relation of this theory to SYK model.

The general form of the 2D gravity action up to the terms quadratic in curvature K is

$$\tilde{\mathcal{A}} = c_0 \int \sqrt{\mathcal{G}}\, d^2x + c_1 \int K \sqrt{\mathcal{G}}\, d^2x + c_2 \int K^2 \sqrt{\mathcal{G}}\, d^2x \,. \tag{1}$$

The first two terms do not determine the dynamics of 2D gravity. While the part of the action quadratic in the Gaussian curvature does.

Commonly it is transformed to the dilaton gravity action. An alternative way is to deal only with the geometric structures of the surface.

The action (1) is invariant under general coordinate transformations. Here, we reduce the set of coordinate transformations and consider the action restricted to the conformal gauge, where the metric of the 2D surface looks like

$$dl^2 = g(u,\, v)\, \left(du^2 + dv^2\right) = g(z,\, \bar{z})\, dz\, d\bar{z} \qquad \sqrt{\mathcal{G}} = g\,. \tag{2}$$

$$K = -\frac{1}{2g}\, \Delta\, \log g\,, \tag{3}$$

where Δ stands for the Laplacian.

We consider the specific form of the action (1)

$$A = \frac{\lambda^2}{2} \int\limits_d \left(K + 4\right)^2\, g(z,\, \bar{z})\, dz\, d\bar{z} = \frac{\lambda^2}{2} \int\limits_d \left(\Delta\psi\right)^2\, dz\, d\bar{z} \tag{4}$$

where

$$\Delta\psi = q\, \Delta \log q + \frac{4}{q}\,, \qquad q = \frac{1}{\sqrt{g}}\,. \tag{5}$$

Now path integrals in the theory

$$\int \tilde{F}(g)\ \exp\{-\tilde{A}(g)\}\, dg \qquad (6)$$

are path integrals

$$\int F(\psi)\ \exp\{-A(\psi)\}\, d\psi \qquad (7)$$

over the Gaussian functional measure

$$\mu_\lambda(d\psi) = \frac{\exp\{-A(\psi)\}\, d\psi}{\int \exp\{-A(\psi)\}\, d\psi}\,. \qquad (8)$$

We consider a model of 2D gravity with the action quadratic in curvature and represent path integrals as integrals over the $SL(2,\,\mathbb{R})$ invariant Gaussian functional measure. We reduce these path integrals to the products of Wiener path integrals and calculate the correlation function of the metric in the first perturbative order.

Talk is based on V. V. Belokurov and E. T. Shavgulidze, *An approach to quantum 2D gravity, Physics Letters B Volume 836, 10 January 2023, 137633*

Classification of all constant solutions of SU(2) Yang-Mills equations

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We present a classification and an explicit form of all constant solutions of the Yang-Mills equations with SU(2) gauge symmetry for an arbitrary constant non-Abelian current in pseudo-Euclidean space $\mathbb{R}^{p,q}$ or Eulidean space $\mathbb{R}^n$ of arbitrary finite dimension $n = p + q$. Using singular value decomposition, hyperbolic singular value decomposition, and two-sheeted covering of orthogonal group by spin group, we solve the nontrivial system for constant solutions of the Yang-Mills equations of 3n cubic equations with 3n unknowns and 3n parameters in the general case. We present a new symmetry of this system of equations. All solutions in terms of the potential, strength, and invariant of the Yang-Mills field are presented. Nonconstant solutions of the Yang-Mills equations can be considered in the form of series of perturbation theory using all constant solutions as a zeroth approximation.

Some aspects of higher gauge theories

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It is well known that a 2-form field or a Kalb-Ramon field appears naturally in string theory.

Is there a non-Abelian generalization of the 2-form field? Also such field generalizes the Yang-Mills theory. A suitable generalization of the theory with 2-form is the higher Yang-Mills theory based on the Lie 2-group or equivalently a 2-crossed module [1; 2].

One can define higher analogues of curvatures for higher gauge theories with the trivial principal 2-bundle. It is also possible to write here the higher analogues of the Bianchi identities, the equations of motion and action [1].

In talk I will describe in more detail the higher Yang-Mills theory and its dimensional reduction on the circle.

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Supersingular and large solutions of semilinear parabolic and elliptic equations

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Exact conditions for the existence of non-negative super-singular (s.s.) solutions of these equations (solutions which a more singular at some points of the boundary of the domain, than any solution of the corresponding linear equation), as well as the structure of these solutions, were first obtained in the work of H. Brezis, L. Peletier, D. Terman (1986) in the Cauchy problem for a semilinear heat equation with a nonlinear absorption term $f(u)$.The study of another important class of strongly singular, so-called large (b.) solutions, that is, solutions taking an infinite value on the entire or on some part of the boundary of the domain, was initiated by the work of L. Bieberbach (1916), where the existence of these (b.)solutions was established for a semilinear elliptic equation with nonlinear absorption $f(u) = b^2 exp(u)$ in a bounded smooth 2-dimensional domain D. For the first time, the uniqueness of the (b.) solution was proved by C. Loevner, L. Nirenberg (1974) in case of bounded smooth n-dimensional domain D and nonlinear absorption $f(u) = u^{(n+2)(n-2)^{-1}}$.

We study existence and uniqueness conditions, as well as qualitative and asymptotic properties of (s.s.) and (b.) solutions of various classes of semilinear parabolic and elliptic equations with “nonhomogeneous” absorption structure $f(t, u)$ or $f(x, u)$ degenerating on the boundary of the region, or its part, or on some manifolds that lie in the region and whose boundary has a non-empty intersection with the boundary of the domain. Exact conditions are established for this degeneracy, which guarantee the existence or non-existence of (s.s.) or (b.) solutions. It is shown that for some problems these conditions are necessary and sufficient (criterion) for the existence of the solutions under discussion. In particular, it is shown that in the elliptic case with the absorption $f(x, u) = g(x)u^p, p > 1$, degenerating at the boundary of the domain, obtained condition on the degeneracy of $g(x)$ is only a new close to the exact sufficient condition for the uniqueness of the (b.)solution and, surprisingly, the sufficient and necessary condition for the existence of the (s.s.)solution. This connection justifies a parallel study of the discussed classes of solutions.

Some of these results were published in [1–7].

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A priori estimates of solutions to mixed problems for the Vlasov–Poisson system and kinetic of plasma in a mirror trap

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We consider the first mixed problem for the Vlasov–Poisson system describing a kinetics of high temperature plasma in a fusion reactor with external magnetic field. It was obtained a priory estimate of solutions of this problem with compact supports of distribution functions with respect to space variables.

This work ia supported by the Ministry of Science and Higher Education of the Russian Federation (megagrant agreement No. 075–15–2022–1115) .

Solution of a one-dimensional initial-boundary value problem for the Klein-Gordon equation on the semiaxis and its asymptotics

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In this work, the initial-boundary value problem for the Klein-Gordon equation on the semiaxises $z > 0, t > 0$ is formulated and solved. A one-dimensional system of equations of hydrothermodynamics, which describes the motion of atmospheric gas, in particular, the propagation of plane acoustic waves initiated by a source at the lower boundary of the region, is reduced to such a problem. Exact analytical solutions are obtained for a family of boundary functions and asymptotics are obtained.

The reported study was funded by Immanuel Kant Baltic Federal University, project No 122051300013-8.

Point to point controllability for systems with drift

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We discuss global controllability for autonomous systems of ODEs. In particular, we give a constructive proof of a global controllability result for such a system guided by bounded locally Lipschitz and divergence free (i.e. incompressible) vector field, when the phase space is the whole Euclidean space and the vector field satisfies so-called vanishing mean drift condition. For the case when the ODE is defined over some smooth compact connected Riemannian manifold, we significantly strengthen the assertion of the known controllability theorem in absence of nonholonomic constraints by proving that one can find a control steering the state vector from one given point to another by using the observations of only the state vector, i.e., in other words, by changing slightly the vector field, and such a change can be made small not only in uniform, but also in Lipschitz (i.e. C^1) topology. Joint work with Sergey Kryzhevicz.

Affine super Yangian and quantum Weyl groupoid

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The super Yangian $Y_\hbar(\hat{\mathfrak{g}}(\Pi)) = Y_\hbar(\hat{sl}(m|n))$ of the affine special linear Lie superalgebra $\hat{\mathfrak{g}}(\Pi) = \hat{sl}(m|n)$ is defined in the case of an arbitrary system of simple roots Π, first in terms of the so-called "minimalistic" system of generators and defining relations. We also introduce the super Yangian $\breve{Y}_\hbar(\hat{sl}(m|n))$ of the special linear Lie superalgebra in terms of the new system of Drinfeld generators, as well as in the case of an arbitrary system of simple roots of the Lie superalgebra $Y_\hbar(\hat{sl}(m|n))$. We prove that the super Yangians $Y_\hbar(\hat{sl}(m|n))$ and $\breve{Y}_\hbar(\hat{sl}(m|n))$ are isomorphic as associative superalgebras. We introduce the Weyl groupoid in the case of the super Yangian $Y_\hbar(\hat{sl}(m|n))$ and explicitly describe the action of the elements of the groupoid as isomorphisms in the category of super Yangians $Y_\hbar(\hat{\mathfrak{g}}(\Pi))$ of a special affine linear superalgebra defined by various systems of simple roots Π. We describe quasi Hopf structures on affine super Yangians defined by triangular decompositions.

A free boundary problem with temperature-dependent thermal conductivity with power-law nonlinearities

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The heat conduction equation is a non-linear equation when the temperature dependence of thermal parameters is taken into account. It is proved that the mathematical condition for reducing the one-dimensional nonlinear heat equation to a linear form is the constancy of the Storm condition for heat capacity and heat conduction.

The nonlinear heat equation for an isotropic solid has the form

$$\nabla(b(u)\nabla u) = a(u)u_t$$

Here u is the temperature of the solid, $b(u)$ is the thermal conductivity, $a(u) = \rho C_\rho$, where ρ is the density, C_ρ is the specific heat at constant pressure. The two quantities $b(u)$ and $a(u)$ are called “thermal parameters”. It is assumed that the metal has nonlinear thermal characteristics, so that the heat capacity $a(u)$ and thermal conductivity $b(u)$ satisfy the Storm condition (see, for example, [1–3]):

$$\frac{d}{du}\frac{\sqrt{\frac{a(u)}{b(u)}}}{a(u)} = \lambda = const > 0. \tag{1}$$

Condition (1) was originally obtained in [3] in the study of thermal conductivity in simple monatomic metals. In this work, it was shown that if this condition is met, the heat equation can be transformed to a linear form. There the condition (1) is checked for aluminum, silver, sodium, cadmium, zinc, copper and lead.

In this paper, we study a Florin-type problem with a free boundary without initial conditions in the following formulation.

Statement of the problem. Find a pair of functions $(s(t), u(t, x))$ such that the function $s(t)$ is continuously differentiable on the interval $0 < t \leq T$, $s(0) = 0$, $s(t) > 0$ and the function $u(t, x)$ in $D = \{(t; x) : 0 < t \leq T, 0 < x < s(t)\}$ satisfies

$$u_t = (u^{-2}u_x)x - au^{-3}, (t, x) \in D \tag{2}$$

is continuous in D with the derivative $u_x(t,x)$ and satisfies the conditions

$$u_x(t,0) = 0, 0 < t \leq T, \quad (3)$$

$$ux(t,s(t)) = 0, 0 < t \leq T, \quad (4)$$

$$u(t,s(t)) = g(s(t)), 0 < t \leq T. \quad (5)$$

Here $g(x) > 0$ is defined and continuous in the interval $0 \leq x \leq x_0$, $0 < s(t) < x_0, a > 0$. The study is carried out according to the following scheme. First, with the help of some transformations (hodographs), the problem is reduced to a problem with a free boundary for a new function $v(t,y)$ in some non-standard domain for a parabolic equation with the lowest term , without an initial condition with a homogeneous boundary condition of the second kind. Some initial a priori estimates for $v(t,y)$ are established and the uniqueness theorem for the solution is proved. It can be seen that problem (2)-(5) is a Florin-type problem, which is characterized by a number of features: the free boundary begins at the solid wall $x = 0$; a free boundary condition is implicitly specified for this boundary; the behavior of the free boundary is unknown. Therefore, below we consider a problem with an initial condition, which reduces to a Stefan-type problem. Their equivalence is proved. To solve the Stefan-type problem, a priori Schauder-type estimates are established and, on their basis, an existence theorem is proved. At the same time, for the unknown boundary, two-sided estimates are established from known curves that determine the behavior of the unknown boundary at $t \to 0$. At the end of the article, it was proved that with an unlimited increase in time, the free boundary tends to some constant x_0.

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Solvability problem of elliptic functional differential equations with orthotropic contractions in weighted spaces

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We study a solvability problem of elliptic functional-differential equations with contractions and expansions of arguments in the principal part.

$$A_R u \equiv -\sum_{i,j=1}^{2} (R_{ij} u_{x_i})_{x_j} = f(x_1, x_2),$$

$$R_{ij} v(x) = a_{ij0} v(x_1, x_2) + a_{ij1} v(q^{-1} x_1, p x_2) + a_{ij,-1} v(q x_1, p^{-1} x_2), \quad p, q > 1.$$

Functional-differential equations could have power singularities of solutions on a boundary or inside a bounded domain and it turned out to be natural to consider them in weighted spaces. Some solvability results were obtained in weighted spaces introduced by V.A. Kondratiev to study elliptic problems in domains with angular or conical points. A new type of weighted spaces with the weight associated with the orthotropic contractions is constructed.

This work is supported by the Ministry of Science and Higher Education of the Russian Federation (megagrant agreement No. 075-15-2022-1115).

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Structure Of Essential Spectrum And Discrete Spectra Of The Energy Operator Of Six Electron Systems In The Hubbard Model. Third Singlet State

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The Hubbard model first appeared in 1963 in the works [1]. We consider of the energy operator of six-electron systems in the Hubbard model and investigated the structure of essential spectra and discrete spectrum of the system for third singlet state. Hamiltonian of the considering system has the form

$$H = A\sum_{m,\gamma} a^{+}_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^{+}_{m,\gamma}a_{m+\tau,\gamma} + U\sum_{m} a^{+}_{m,\uparrow}a_{m,\uparrow}a^{+}_{m,\downarrow}a_{m,\downarrow}.$$

Here A is the electron energy at a lattice site, B is the transfer integral between neighboring sites, $\tau = \pm e_j,\ j = 1, 2, ..., \nu$, where e_j are unit mutually orthogonal vectors, which means that summation is taken over the nearest neighbors, U is the parameter of the on-site Coulomb interaction of two electrons, γ is the spin index, $\gamma =\uparrow$ or $\gamma =\downarrow$, $\uparrow$ and $\downarrow$ denote the spin values $\frac{1}{2}$ and $-\frac{1}{2}$, and $a^{+}_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m \in Z^{\nu}$.

In the six electron systems has a octet state, and quintet states, and triplet states, and singlet states. The energy of the system depends on its total spin S. Along with the Hamiltonian, the N_e electron system is characterized by the total spin $S,\ S = S_{max}, S_{max} - 1, ..., S_{min}, S_{max} = \frac{N_e}{2}, S_{min} = 0, \frac{1}{2}$.

Hamiltonian H commutes with all components of the total spin operator $S = (S^{+}, S^{-}, S^{z})$, and the structure of eigenfunctions and eigenvalues of the system therefore depends on S. The Hamiltonian H acts in the antisymmetric Fock space $\mathsf{H}_{as} = l_2^{as}((Z^{\nu})^6)$, where $l_2^{as}((Z^{\nu})^6)$ is the subspace of antisymmetric functions of $l_2((Z^{\nu})^6)$. Let φ_0 be the vacuum vector in the space H_{as}. The third singlet state corresponds to the free motion of six electrons over the lattice and their interactions with the basic functions ${}^3s^0_{p,q,r,t,k,n\in Z^{\nu}} = a^{+}_{p,\uparrow}a^{+}_{q,\uparrow}a^{+}_{r,\downarrow}a^{+}_{t,\downarrow}a^{+}_{k,\downarrow}a^{+}_{n,\uparrow}\varphi_0$. The subspace ${}^3\mathsf{H}^0_s$, corresponding to the second singlet state is the set of all vectors of the form ${}^3\psi^0_s = \sum_{p,q,r,t,k,n\in Z^{\nu}} f(p,q,r,t,k,n)^3s^0_{p,q,r,t,k,n\in Z^{\nu}}$,

$f \in l_2^{as}$, where l_2^{as} is the subspace of antisymmetric functions in the space $l_2((Z^\nu)^6)$. We denote by ${}^3H_s^0$ the restriction of operator H to the subspace ${}^3\mathsf{H}_s^0$.

Theorem 1. *If $\nu = 1$ and $U < 0$, then the essential spectrum of the operator ${}^3H_s^0$ consists of the union of seven segments: $\sigma_{ess}({}^3H_s^0) = [a+c+e, b+d+f] \cup [a+c+z_3, b+d+z_3] \cup [a+e+z_2, b+f+z_2] \cup [a+z_2+z_3, b+z_2+z_3] \cup [c+e+z_1, d+f+z_1] \cup [c+z_1+z_3, d+z_1+z_3] \cup [e+z_1+z_2, f+z_1+z_2]$, and discrete spectrum of the operator ${}^3H_s^0$ consists of no more one eigenvalue: $\sigma_{disc}({}^3H_s^0) = \{z_1+z_2+z_3\}$, or $\sigma_{disc}({}^3H_s^0) = \emptyset$, here and hereafter $a = -2A - 4B\cos\frac{\Lambda_1}{2}$, $b = -2A + 4B\cos\frac{\Lambda_1}{2}$, $c = 2A - 4B\cos\frac{\Lambda_2}{2}$, $d = 2A + 4B\cos\frac{\Lambda_2}{2}$, $e = 2A - 4B\cos\frac{\Lambda_3}{2}$, $f = 2A + 4B\cos\frac{\Lambda_3}{2}$, and $z_1 = -2A + \sqrt{9U^2 + 16B^2\cos^2\frac{\Lambda_1}{2}}$, $z_2 = 2A - \sqrt{9U^2 + 16B^2\cos^2\frac{\Lambda_2}{2}}$, and $z_3 = 2A - \sqrt{U^2 + 16B^2\cos^2\frac{\Lambda_3}{2}}$, where $\Lambda_1 = \lambda+\gamma$, $\Lambda_2 = \mu+\theta$, and $\Lambda_3 = \eta+\xi$, and $\lambda, \mu, \gamma, \theta, \eta, \xi$ are the quasimomentum of the electrons.*

Theorem 2. *If $\nu = 1$ and $U > 0$, then the essential spectrum of the operator ${}^3H_s^0$ consists of the union of seven segments: $\sigma_{ess}({}^3H_s^0) = [a+c+e, b+d+f] \cup [a+c+\widetilde{z}_3, b+d+\widetilde{z}_3] \cup [a+e+\widetilde{z}_2, b+f+\widetilde{z}_2] \cup [a+\widetilde{z}_2+\widetilde{z}_3, b+\widetilde{z}_2+\widetilde{z}_3] \cup [c+e+\widetilde{z}_1, d+f+\widetilde{z}_1] \cup [c+\widetilde{z}_1+\widetilde{z}_3, d+\widetilde{z}_1+\widetilde{z}_3] \cup [e+\widetilde{z}_1+\widetilde{z}_2, f+\widetilde{z}_1+\widetilde{z}_2]$, and discrete spectrum of the operator ${}^3H_s^0$ consists of no more one eigenvalue: $\sigma_{disc}({}^3H_s^0) = \{\widetilde{z}_1+\widetilde{z}_2+\widetilde{z}_3\}$, or $\sigma_{disc}({}^3H_s^0) = \emptyset$, here $\widetilde{z}_1 = -2A - \sqrt{9U^2 + 16B^2\cos^2\frac{\Lambda_1}{2}}$, $\widetilde{z}_2 = 2A + \sqrt{9U^2 + 16B^2\cos^2\frac{\Lambda_2}{2}}$, and $\widetilde{z}_3 = 2A + \sqrt{U^2 + 16B^2\cos^2\frac{\Lambda_3}{2}}$.*

Let $\nu = 3$, and $U < 0$, $\Lambda_1 = (\Lambda_1^0, \Lambda_1^0, \Lambda_1^0)$, $\Lambda_2 = (\Lambda_2^0, \Lambda_2^0, \Lambda_2^0)$, and $\Lambda_3 = (\Lambda_3^0, \Lambda_3^0, \Lambda_3^0)$, and $W-$ Watson integral.

Theorem 3. *If $U < 0$, and $U < -\frac{12B\cos\frac{\Lambda_3^0}{2}}{W}$, and $\cos\frac{\Lambda_3^0}{2} > \frac{1}{3}\cos\frac{\Lambda_2^0}{2}$, and $\cos\frac{\Lambda_2^0}{2} > \cos\frac{\Lambda_1^0}{2}$, then the essential spectrum of the operator ${}^3H_s^0$ consists of the union of seven segments: $\sigma_{ess}({}^3H_s^0) = [a_1+c_1+e_1, b_1+d_1+f_1] \cup [a_1+c_1+z_3', b_1+d_1+z_3'] \cup [a_1+e_1+z_2', b_1+f_1+z_2'] \cup [a_1+z_2'+z_3', b_1+z_2'+z_3'] \cup [c_1+e_1+z_1', d_1+f_1+z_1'] \cup [c_1+z_1'+z_3', d_1+z_1'+z_3'] \cup [e_1+z_1'+z_2', f_1+z_1'+z_2']$, and discrete spectrum of the operator ${}^3H_s^0$ consists of no more one eigenvalue: $\sigma_{disc}({}^3H_s^0) = \{z_1'+z_2'+z_3'\}$, or $\sigma_{disc}({}^3H_s^0) = \emptyset$, here $a_1 = -2A - 12B\cos\frac{\Lambda_1^0}{2}$, $b_1 = -2A + 12B\cos\frac{\Lambda_1^0}{2}$, $c_1 = 2A - 12B\cos\frac{\Lambda_2^0}{2}$, $d_1 = 2A + 12B\cos\frac{\Lambda_2^0}{2}$, $e_1 = 2A - 12B\cos\frac{\Lambda_3^0}{2}$, $f_1 = 2A - 12B\cos\frac{\Lambda_3^0}{2}$, and z_1', z_2', and z_3', are the same concrete numbers.*

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Topological conjugacy of chaotic homeomorphism groups of surfaces with boundary

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Chaotic homeomorphism groups of topological surfaces with boundary are investigated. Follow to [1], we call a homeomorphism group G of a topological manifold X chaotic if there exists a dense orbit of G and the union of finite orbits is dense in X.

Recall that a homeomorphism group G of a metric space (X, d) is called sensitive to initial conditions if there exists a number $\delta > 0$ such that for every open subset $U \subset X$ there exists an element $g \in G$ such that $diam(g(U)) \geq \delta$.

We consider homeomorphism groups as continuous topological groups endowed with the discrete topology. The well known result of E. Kontorovich and M. Megrelishvili [4] on sensitivity of continuous actions of semigroups implies that that chaotic homeomorphism groups are sensitive to initial conditions. Therefore, definition of a chaotic homeomorphism group mentioned above can be considered as an analogue of the notion of a chaotic dynamical system in the sense of R. L. Devaney [2].

We use toral linked twist mappings and develop the method suggested in [1] to construct chaotic homeomorphism groups G_i, $i \in \mathbb{N}$, of surfaces with boundary. The case of empty boundary is not not excluded. We also investigate chaotic homeomorphism groups on non-compact topological surfaces.

A distinctive feature of this paper is the investigation of topological conjugacy of homeomorphism groups generated by toral linked twist mappings, based on the use of the properties of the topological space of fixed points of these groups.

In particular, for every compact surface M with a boundary, an infinite countable family $\{G_i \,|\, i \in \mathbb{N}\}$ of pairwise topologically non conjugated chaotic homeomorphism groups isomorphic to $\mathbb{Z}$ is constructed.

The investigation of chaotic homeomorphism groups of compact surfaces were supported by the Russian Science Foundation (grant no. 22-21-000304).

The results on homeomorphisms groups of non-compact surfaces were obtained with the support from the Laboratory of Dynamical Systems and Applications NRU HSE, grant of the Ministry of science and higher education of the RF (ag. no. 075-15-2022-1101).

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Linear Interpolation of Program Contrl with Respect to a Multidimensional Parameter in the Convergence Problem

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We consider a control system containing a constant n-dimensional vector parameter, the approximate value of which is reported to the control person only at the movement start moment.

Only the set of possible values of this indeterminate vector parameter is known in advance.

For this control system, the problem of approaching the target set at a given time is posed.

At the same time, it is considered that the control person is not able to carry out in real time the cumbersome calculations associated with the construction of such resolvability structures as reachable sets and integral funnels.

Therefore, to solve this problem, it is proposed to calculate in advance several "nodal" resolvability controls for the parameter values, which are the nodes of the grid covering the set of possible parameter values.

In the event that at the movement start moment it turns out that the value of the parameter does not coincide with any of the grid nodes, it is supposed to quickly calculate the program control using linear interpolation formulas. However, this procedure can be effective only if a linear combination of controls corresponding to the same "guide" in the terminology of N. N. Krasovsky's extreme aiming method is used.

For effective application of linear interpolation, it is proposed to construct 2^n "nodal" resolvability controls for each node of the grid and, in addition, use the method of dividing the control into main and compensating. Due to the application of the latter method, the calculated resolvability set turns out to be somewhat smaller than the actual one, but the accuracy of transferring the system state to the target set increases.

This research was supported by the Russian Science Foundation (grant no. 19-11-00105, `https://rscf.ru/en/project/19-11-00105`*).*

Pseudo-differential equations in non-smooth domains

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1. A general concept. This talk is devoted to describing the structure of a special class of linear bounded operators on a manifold with non-smooth boundary. Our description is based on local principle and theory of envelopes [1; 3]. This approach leads to studying invertibility conditions for model pseudo-differential operators in canonical domains.

2. Local situation. Starting point for the study is the model pseudo-differential equation [6] in the cone $C \subset \mathbf{R}^m$

$$(Au)(x) = v(x), \quad x \in C, \quad x \in C, \tag{1}$$

where $A : H^s(C) \to H^{s-\alpha}(C)$ is a pseudo-differential operator with the symbol $A(\xi)$ satisfying the condition

$$c_1(1+|\xi|)^\alpha \le |A(\xi)| \le c_2(1+|\xi|)^\alpha.$$

Constructions of solutions for the equation (1) was obtained for certain cones C [5].

3. Asymptotic analysis. Each cone C has certain parameters as a rule, for example $C_+^a = \{x \in \mathbf{R}^2 : x = (x_1, x_2), x_2 > a|x_1|, a > 0\}$ with the parameter a and $C_+^{a,b} = \{x \in \mathbf{R}^3 : x = (x_1, x_2, x_3), x_3 > a|x_1| + |x_2|, a, b > 0\}$ with two parameters a, b. It is very natural question that if we have a solution of the equation (1) then what is its limit value if the parameters tend to their endpoint values 0 or ∞. Some cases were discussed in [2].

4. Discrete analysis. One can consider a discrete variant of the equation (1) using the following constructions for functions of a discrete variable $u_d(\tilde{x}), \tilde{x} \in h\mathbf{Z}^m, h > 0$. Let $C_d = h\mathbf{Z}^m \cap C, \hbar = h^{-1}, \mathbf{T} = [-\pi, \pi]$ and $\tilde{A}_d(\xi)$ be a measurable periodic function in $\mathbf{R}^m$ with basic square of periods $\hbar\mathbf{T}^m$. A digital pseudo-differential operator A_d with the symbol $\tilde{A}_d(\xi)$ in the discrete cone C_d is called an operator of the following type

$$(A_d u_d)(\tilde{x}) = \sum_{\tilde{y} \in h\mathbf{Z}^2} h^2 \int_{\hbar\mathbf{T}^2} \tilde{A}_d(\xi) e^{i(\tilde{x}-\tilde{y})\cdot\xi} \tilde{u}_d(\xi) d\xi, \quad \tilde{x} \in C_d,$$

where $\tilde{u}_d(\xi)$ denotes the discrete Fourier transform of u_d [4].

We can introduce discrete analogues of spaces $H^s(C_d)$ and for the special case $C = \mathbf{R}^m_+$ it is possible to obtain solvability conditions for the discrete analogue of the equation (1). It was shown that discrete solutions have approximation properties for small h. The similar results were obtained for a discrete quadrant in a plane. Moreover, a comparison between discrete and continuous solutions are given also for some canonical domains.

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Maximally symmetric bifurcations of Liouville tori of billiard books that contain foci.

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Consider a two-dimensional CW-complex whose two-dimensional cells are flat domains bounded by arcs of confocal quadrics. The one-dimensional cells of the complex are the segments of the boundaries of elementary billiards — the segments between the angles of the boundary curves. We enumerate all two-dimensional cells and assign to each one-dimensional edge of the complex — "spine" of the book — a cyclic permutation of the numbers of sheets adjacent to this edge. Project all billiard sheets isometrically onto a plane. If the image of several edges of a CW-complex under this projection is the same arc of the plane, then we combine the cycles assigned to them into one permutation (these cycles are obviously independent). For the continuity of particle motion through the book, we require commutation of permutations in zero-dimensional cells. In projection terms, this means that the permutations assigned to the arcs of two quadrics in a neighborhood of the intersection point of the latter commute. We call this two-dimensional complex with assigned permutations a *billiard book* [3].

The billiard movement according to the book is defined as follows. Inside two-dimensional cells, the movement does not change. Let, while moving along the sheet with number i, the material point hits the spine of the book, then after the impact it will continue its movement along the sheet $\sigma(i)$.

Since billiards bounded by arcs of confocal quadrics are integrable [2], the corresponding billiard books are also integrable. Our goal is to describe the resulting Lagrangian fibration. It always has one special fiber whose trajectories lie on straight lines passing through the foci of a family of quadrics that form the boundaries of the sheets of the billiard book.

Proposition *Consider the billiard book which is glued from convex domains A_1, each of which is bounded by one arc of the ellipse and one arc of the hyperbola (which are confocal). Commuting permutations ρ (elliptic boundary) and σ (hyperbolic) are assigned to the spines of the book. Consider the permutation*

$\omega = \sigma \circ \rho$ and expand it into a product of independent cycles. Renumber the sheets of the billiard book so that

$$\omega = (1 \,..\, k)(k+1 \,..\, 2k) \,..\, (n-k+1 \,..\, n).$$

Then the number $m = \frac{n}{k}$ (the number of independent cycles in the decomposition of the permutation ω) is equal to the number of critical circles of the 3-atom (bifurcation of Liouville tori) describing the bifurcation at the focal level. The atom itself has the following form depending on the permutations of σ and ρ.

1. *If k is odd, then the atom on the focal level is the atom A^{*m}.*
2. *If the numbers k and l are even (where $l = \rho^m(1)$), then the atom belongs to the series of maximally symmetric atoms Y_m (see [1]).*
3. *If the number k is even and the number l is odd, then the atom belongs to the series of maximally symmetric atoms X_m (see [1]).*

The work was supported by Russian Science Foundation, project 22-71-10106.

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Higher order traps in quantum control landscapes

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An important problem in quantum control theory is the study of the existence of traps in quantum control landscapes. Traps are controls from which it is difficult to escape by local search optimization methods. The notion of the higher order trap was introduced in [1], where quantum control landscapes with 3-rd order traps were discovered. In the talk we show that traps of the order $(2N-3)$ exist for N-level controllable quantum systems with the "chained" interaction Hamiltonian. We show that the quantum control landscape for the problem of controlled generation of single-qubit phase shift quantum gates for small times is free of traps [4]. We also investigate the detailed structure of the quantum control landscape for this problem [3].

The work is partially supported by the project of the Russian Science Foundation grant No. 22-11-00330, "Priority 2030" federal leadership program and Ministry of Science and Higher Education (grant No. 075-15-2020-788).

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Numerical analysis of 3rd and 7th order traps in quantum control landscapes for some three-level and four-level quantum systems

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Quantum control is an important direction in modern science with various existing and prospective applications in quantum technologies [1; 2; 5]. An important problem in quantum control is the analysis of quantum control landscapes, including estabolishing either presence or absence of traps, n-order traps, since is helps to determine a correct algorithm for finding optimal control fields [3; 7; 8]. In this talk, we discuss our results with numerical analysis of control landscapes for various quantum systems, including three-level systems with traps of orders 3 and 7 and four-level systems [9].

This talk presents our recent results on 3rd and 7th-order traps for some three-level and four-level closed quantum systems with free and iinteraction Hamiltonians of the form

$$H_0 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}, \qquad V = \begin{pmatrix} 0 & v_1 & 0 \\ v_1^* & 0 & v_2 \\ 0 & v_2^* & 0 \end{pmatrix}; \tag{1}$$

$$H_0 = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}, \qquad V = \begin{pmatrix} 0 & v_{12} & v_{13} & 0 \\ v_{12}^* & 0 & v_{23} & v_{24} \\ v_{13}^* & v_{23}^* & 0 & v_{34} \\ 0 & v_{24}^* & v_{34}^* & 0 \end{pmatrix}. \tag{2}$$

The dynamics of such systems are described by the Schrodinger equation

$$i\frac{dU_t^f}{dt} = (H_0 + f(t)V)U_t^f, \qquad U_t^f = \mathbb{I} \tag{3}$$

We study critical points for the target functional $J_O[f] = \mathrm{Tr}(OU_T^f \rho_0 U_T^{f\dagger}) \to$ max. Here $\rho_0 = |i\rangle\langle i|$ is the initial state of the system, assuming it is pure, O is the target observable. Such a target functional describes a wide range of quantum phenomena, e.g., breaking desired chemical bonding, creation of selective atomic and molecular excitations, etc. A control $f_0 \in L_2([0,T],\mathbb{R})$ is a trap if f_0 is a point of local extremum of the target control functional, but not global [6]. We numerically investigate the control landscapes in a vicinity of some 3rd and 7th order traps, finding significant differences between them. This talk is a part of a larger research on the study of higher-order traps in quantum control landscapes.

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Asymptotics of the eigenvalues of seven-diagonal Toeplitz matrices of a special form

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This work is devoted to the construction of a uniform asymptotics in the dimension of the matrix n tending to infinity of all eigenvalues in the case of a seven-diagonal Toeplitz matrix with a symbol having a zero of the sixth order, while the cases of symbols with zeros of the second and fourth orders were considered earlier. On the other hand, the results obtained refine the results of the classical work of Parter and Widom on the asymptotics of the extreme eigenvalues. We also note that the obtained formulas showed high computational efficiency both in sense of accuracy (already for relatively small values of n) and in sense of speed.

Isomonodromic deformations on an elliptic curve

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We consider the problem of isomonodromic deformation of a 2×2-Fuchsian system on a compact Riemann surface of genus g as an isomonodromic deformation of a logarithmic connection in a rank two semistable bundle of degree zero. Such a problem with fixed singular points was first considered by H. Esnault and E. Viehweg. In the case when the curve is elliptic, the isomonodromic deformation of the twisted system is obtained in the paper of D. A. Korotkin, J. A. H. Samtleben "On the quantization of isomonodromic formations on the torus". This deformation is given by a system which is equivalent to the Knizhnik-Zamolodchikov-Bernard system.

Using an interpretation based on semistable bundles of degree zero, we describe the local structure of the theta-divisor of an isomonodromy deformation on an elliptic curve, that is, the set of points where the deformation coefficients tend to infinity.

Generalized measures and symmetries in infinite-dimensional spaces

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According to Noether's theorem, each continuous symmetry of a physical system corresponds to a certain conservation law. Continuous symmetry is the invariance with respect to a family of continuous transformations. We will start with a proof of Noether's theorem in a book published by Kleinert. In this book, all formulas related to fields do not specify the domains in infinite-dimensional spaces, so they need to be improved. Since invariances and conservation laws in ordinary Euclidean spaces have been systematically studied, we study the translational invariance of generalized measures in Hilbert space from a new paper.

Nonlocal Problems with an Integral Condition for Mixed-Type Equations

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We study how a solution of the boundary value problem with a nonlocal integral condition for a mixed type equation with singular coefficients in a rectangular domain depends on numerical parameters occurring in the equation.

Let $D = \{(x,y) : 0 < x < l, -\alpha < y < \beta\}$, where l, α, and β are given positive real numbers, is a rectangular domain. Set $D_+ = D \cap \{y > 0\}$ and $D_- = D \cap \{y < 0\}$.

Statement of the problem. Find a function $u(x,y)$ that satisfies the following conditions:

$$u(x,y) \in C(\overline{D}) \cap C^2(D_+ \cup D_-);$$

$$u_{xx} + (\operatorname{sgn} y) u_{yy} + \frac{p}{x} u_x + \frac{q}{|y|} u_y = 0, \quad (x,y) \in D_+ \cup D_-; \tag{1}$$

$$\lim_{y\to 0+} y^q u_y(x,y) = \lim_{y \to 0-} (-y)^q u_y(x,y), \quad 0 < x < l;$$

$$u(x,\beta) = \varphi(x), \quad u(x,-\alpha) = \psi(x), \quad 0 \le x \le l;$$

$$\lim_{x\to 0+} x^p u_x(x,y) = 0, \quad 0 < |p| < 1, \quad -\alpha \le y \le \beta;$$

$$\int\limits_0^l x^p u(x,y)\, dx = A, \quad -\alpha \le y \le \beta,$$

where p ($p > -1, p \neq 0$), q ($0 < |q| < 1$), and A are given real numbers; $\varphi(x)$ and $\psi(x)$ are given sufficiently smooth functions satisfying the conditions

$$\int\limits_0^l x^p \varphi(x)\, dx = \int\limits_0^l x^p \psi(x)\, dx = A.$$

A nonlocal boundary-value problem for Eq. (1) with $q = 0$ was investigated in [1] and [3]; with $q \neq 0$ was investigated in [4] (see also the monograph [2]).

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Navier–Stokes equations, the algebraic aspect

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We show that the Navier-Stokes equations are subject to meaningful analysis within the framework of the algebraic approach to differential equations. The resulting equations for finding algebraic characteristics of Navier-Stokes equations, such as symmetries and cohomologies, are essentially complicated. One may hope to find their partial solutions at least, especially using analytical computational packets (Mathematica, for example).

О численно-аналитическом методе для уравнения Бюргерса

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Рассматривается пространственно одномерная периодическая начально-краевая задача для уравнения Бюргерса:

$$\frac{\partial u}{\partial t} - \lambda \frac{\partial^2 u}{\partial x^2} + u(t,x)\frac{\partial u}{\partial x} = f(t,x), \qquad x \in [-1,1], \qquad t \in [0,T],$$

$$u(0,x) = u_0(x), \qquad u(t,-1) = u(t,1), \qquad \frac{\partial u}{\partial x}(t,-1) = \frac{\partial u}{\partial x}(t,1).$$

В докладе представлен новый численно-аналитический метод, основанный на использовании явно-неявной схемы дискретизации по времени и решении на каждом временно́м слое линейной задачи для обыкновенного дифференциального уравнения с постоянными коэффициентами.

Новизна подхода заключается в аналитическом методе решения вспомогательной линейной задачи в классе непрерывных периодических кусочно-аналитических функций на отрезке $[-1,1]$ с применением явного вида соответствующей функции Грина. Эффективность такого аналитического метода обусловлена тем, что он имеет всего лишь линейную алгоритмическую сложность по количеству N узлов пространственной аппроксимации. Использование интегрального представления решения позволяет избежать потери точности, которая возникает вследствие применения каких-либо разностных аппроксимаций производных искомого решения по x. Отсутствие такого рода аппроксимаций в вычислительной схеме представляется особенно актуальным в задачах, где коэффициент при старшей производной по пространственной переменной является малым.

Выполнена численная реализация построенного метода [2] и проведено его сопоставление с известным точным решением [3].

Отметим, что аналогичный подход при решении сингулярно возмущенной краевой задачи был применен в работе [1], где в интегральном представлении использовался главный член асимптотики функции Грина, построенный методом ВКБ.

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О существовании энтропийного решения для уравнения с мерозначным потенциалом

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В области $\Omega \subset \mathbb{R}^m$ рассматривается задача Неймана для уравнения

$$-\mathrm{div}(a(x,u,\nabla u)) + b_0(x,u,\nabla u) + b_1(x,u)\mu = f, \quad f \in L_1(\Omega), \tag{1}$$

где μ - неотрицательная мера Радона. На границе $\partial\Omega$ ставится условие Неймана: $a(x,u,\nabla u)\cdot n = 0$.

Векторное поле $a(x,u,\nabla v)$ в (1) удовлетворяет при $x \in \Omega$ условиям:

$$\overline{M}(x,|a(x,r,y)|) \le g(r)\left(G(x) + M(x,y)\right), \quad r \in \mathbb{R}, \quad y \in \mathbb{R}^m, \quad G(x) \in L_1(\Omega)$$

$$a(x,r,y)\cdot y \ge c_0 M(x,y) - G(x), \quad r \in \mathbb{R}, \quad c_0 > 0,$$

$$(a(x,r,y) - a(x,r,z))\cdot(y-z) > 0,\ y \ne z, \quad y,z \in \mathbb{R}^m,\ r \in \mathbb{R}, \quad x \in \Omega.$$

Кроме того, пусть каратеодориевые функции b_i удовлетворяют неравенствам:

$$|b_0(x,s,y)| \le g(r)(\widetilde{G}_0(x) + M(x,y)), |s| \le r, |x| \le r;$$

$$|b_1(x,s)| \le g(r)\widetilde{G}_1(x), |s| \le r, |x| \le r,$$

где $\widetilde{G}_0 \in L_{1,\mathrm{loc}}(\Omega)$, $\widetilde{G}_1 \in L_{1,\mu,\mathrm{loc}}(\Omega)$;

$$b_0(x,r,y)r \ge 0. \tag{1}$$

Пусть существует возрастающая функция $\widetilde{g}(r)$, $r > 0$, $\lim\limits_{r\to\infty} \widetilde{g}(r) = \infty$, такая, что

$$|b_1(x,s)| > \widetilde{g}(r),\ s \ge r,\ x \in \Omega. \tag{2}$$

Пространство $\mathcal{H}_M^1(\Omega)$ определим как замыкание множества $C_0^\infty(\overline{\Omega})$ в *-слабой топологии пространства $L_M(\Omega)^m$. Требуем, чтобы пространство $\mathcal{H}_M^1(\Omega)$ не содержало констант.

Определение. Энтропийным решением задачи Неймана для уравнения (1) называется функция u такая, что $T_k(u) \in \mathcal{H}_M^1(\Omega)$ при всех $k > 0$ и при всех $\xi \in C_0^\infty(\overline{\Omega})$ корректно неравенство

$$\int\limits_\Omega (a(x,u,\nabla u)\cdot\nabla T_k(u-\xi) + (b_0(x,u,\nabla u) - f)T_k(u-\xi))dx + \\ + \int\limits_\Omega b_1(x,u)T_k(u-\xi)d\mu \leq 0.$$

Теорема. *Пусть выполнены условия на a, b_i, тогда существует энтропийное решение задачи Неймана для уравнения (1).*

Конформное отображение L-образной области в аналитическом виде

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Проблема параметров интеграла Кристоффеля–Шварца для конформного отображения f канонической области на L-образную решена в аналитическом виде при произвольных геометрических параметрах области. Неизвестный прообраз представлен в виде ряда по степеням малого параметра с явно выписанными коэффициентами, для которых получена оценка их модуля. Найдены асимптотики для эффекта кроудинга (скучивания прообразов), ярко выраженного для удлиненной области. Для вычисления отображения f и обратного к нему f^{-1} даны ряды с явными коэффициентами, области сходимости которых в совокупности покрывают всю (замкнутую) отображаемую область. Сочетание с дробно-линейными отображениями и эллиптическим синусом позволило получить отображение полуплоскости, круга и прямоугольника на L-образную область. Численная реализация построенных отображений показала высокую эффективность применяемых методов.

Задачи определения квазистационарных электромагнитных полей в слабо неоднородных средах

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Для описания электромагнитных явлений при моделировании значительной части современных технологических процессов используются квазистационарные приближения для системы уравнений Максвелла [4]. Нерелятивистское магнитное приближение заключается в пренебрежении током смещения и характерно для медленно протекающих процессов в средах с достаточно высокой проводимостью [1; 5]. Нерелятивистское электрическое приближение, формально заключающееся в условии потенциальности электрического поля, используется для описания достаточно медленных процессов в средах с низкой проводимостью, в частности, при моделировании электромагнитных процессов в нижних слоях атмосферы [3; 7]. Квазистационарное приближение, обобщающее классические нерелятивистские квазистационарные приближения, получило название приближение Дарвина [6; 10]. В этом приближении выделяются потенциальная и соленоидальная компоненты электрического поля, в системе уравнений Максвелла сохраняется потенциальная часть тока смещения.

Вопросы иерархии квазистационарных приближений рассматриваются, в частности, в работах [1; 5; 6; 8].

В настоящей работе для сравнения решений задач определения квазистационарных электромагнитных полей в неоднородных средах используется параметр $\|\mathrm{grad}\sigma\|$, характеризующий степень неоднородности среды. В случае слабо неоднородных сред анализируется асимптотическое разложение электромагнитных полей в различных приближениях по этому параметру.

Предлагаются и обосновываются итерационные алгоритмы построения промежуточных квазистационарных приближений, связывающих квазистационарное приближение Дарвина и классические нерелятивистские квазистационарные приближения.

Исследование выполнено за счет гранта Российского научного фонда № 23-21-00440, `https://rscf.ru/project/23-21-00440`

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Популяционная генетика и теория обучения

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Рассмотрена модель популяционной генетики типа Лотки-Вольтерры с мутациями на статистическом многообразии. Мутации в модели описываются диффузией на статистическом многообразии с генератором в виде оператора Лапласа-Бельтрами с метрикой Фишера–Рао, то есть модель объединяет популяционную генетику и информационную геометрию. Такая модель описывает обобщение модели теории машинного обучения, модели порождающих соревновательных сетей (GAN), на случай популяций порождающих соревновательных сетей. Введённая модель описывает контроль переобучения для порождающих соревновательных сетей.

Асимптотики решений дифференциальных уравнений в окрестности иррегулярных особых точек в пространстве функций k-экспоненциального роста. Проблема Пуанкаре

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Проблема построения равномерных асимптотик решений дифференциальных уравнений с голоморфными коэффициентами в окрестности иррегулярных особых точек, в том числе бесконечности является классической задачей аналитической теории и в общем виде была сформулирована Пуанкаре в работах [6; 7]. В этих работах Пуанкаре сформулировал вопрос о общем виде асимптотических разложений в окрестности иррегулярной особой точки. Ответ на этот вопрос дается в настоящей работе. В данной работе мы построим общий вид этих асимптотик в пространстве функций k-экспоненциального роста. Без ограничения общности будем считать, что особой точкой уравнения является ноль.

Рассмотрим уравнение

$$\begin{gathered} a_n\left(x\right)\left(\tfrac{d}{dx}\right)^n u\left(x\right)+a_{n-1}\left(x\right)\left(\tfrac{d}{dx}\right)^{n-1} u\left(x\right)+ldots \\ +a_i\left(x\right)\left(\tfrac{d}{dx}\right)^i u\left(x\right)+\ldots+a_0\left(x\right)u\left(x\right)=0 \end{gathered} \tag{1}$$

Здесь $a_n\left(x\right)$ — функции голоморфные в некоторой окрестности нуля.

Целью нашего исследования является построение асимптотик решений уравнения (1) при $x\to 0$, в предположении, что $x=0$ является иррегулярной особой точкой. Общий вид асимптотик в окрестности регулярных особых точек хорошо известен, это конормальные асимптотики.

Как показано в работе [3] уравнение (1) с иррегулярной особенностью в нуле может быть записано в виде

$$\hat{H}u(x)=\left(-\frac{1}{k}x^{k+1}\frac{d}{dx}\right)^n u\left(x\right)+\sum_{i=0}^{n-1}a_i^0\left(x\right)\left(-\frac{1}{k}x^{k+1}\frac{d}{dx}\right)^i u\left(x\right)=0 \tag{2}$$

Где $k\in N$, $a_i^0\left(x\right)$ — функции голоморфные в окрестности нуля. В работе [1] найдено минимальное натуральное k. Заметим, что тот же результат будет и в случае, когда коэффициенты $a_{n-1}(x)$ будут иметь мероморфную особенность в нуле.

Определение. *Символом дифференциального оператора $\hat{H}$ называется функция*

$H(r,p) = p^n + \sum_{i=0}^{n-1} a_i^0(r)p^i$. *Основным символом Оператора $\hat{H}$ называется полином*

$$H_0(p) = H(0,p) = p^n + \sum_{i=0}^{n-1} a_i^0(0)p^i.$$

Вопрос о виде равномерной асимптотики в окрестности иррегулярной особой точки проще всего решается в случае, когда корни основного символа $H_0(p)$ являются простыми. В работах [3; 5] доказано, что асимптотики в этом случае имеют вид

$$\sum_{i=1}^{n} e^{P_i(\frac{1}{x})} x^{\sigma_i} \sum_{k=0}^{\infty} A_i^k x^k,$$

Где $P_i(y) = \lambda_i y^k + \alpha_i^{k-1} y^{k-1} + \ldots + \alpha_i^1 y$, σ_i — комплексное число, $\sum_{k=0}^{\infty} A_i^k x^k$ — асимптотический ряд. Простому j-му корню полинома $H_0(p)$ будет соответствовать асимптотический член вида $e^{P_j(\frac{1}{x})} x^{\sigma_{ji}} \sum_{k=0}^{\infty} A_j^k x^k$, $j = 1, \ldots, n$. В случае кратных корней задача построения асимптотик значительно сложнее. В работах [2; 4] построены асимптотики решений в окрестности бесконечности в пространствах функций экспоненциального роста для уравнения (1) в случае, когда $a_n(x) = 1$. Заметим, что бесконечность вообще говоря является иррегулярной особой точкой. В общем случае на вопрос о виде асимптотик в окрестности произвольной иррегулярной особой точки отвечает

Теорема. *Любая асимптотика соответствующая нулевому корню основного символа уравнения (2) в пространстве функций k-экспоненциального роста представима в виде суммы асимптотических членов вида*

$u_i(x) \approx \exp\left(P_i\left(x^{-\frac{1}{l_i}}\right)\right) x^{\sigma_i} \sum_{k=0}^{\infty} a_k^i x^{\frac{k}{l_i}}$*,i=1,...,n* $l_i \in N$, σ_i *— комплексные числа,* $P_i(x)$ *является полиномом степень которого не превыщает* $(k-1)l_i$, $\sum_{k=0}^{\infty} a_k^i x^i$ *— асимптотический ряд.*

Заметим, что корень основного символа $p_i \neq 0$ сдвигается в ноль с помощью экспоненциальной подстановки $u(x) = \exp \frac{p_i}{x^{k-1}} u_i(x)$.

Теорема доказана с помощью применения методов ресургентного анализа и метода повторного квантования, основой которого является интегральное представление Лапласа–Бореля [8].

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О цилиндрических отображениях с убегающими орбитами и углах поворота.

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Как известно, для цилиндрического отображения над иррациональным поворотом окружности с гладкой функцией справедлива теорема о возвращении траектории. Однако, над любым иррациональным поворотом существуют непрерывные цилиндри-ческие отображения с убегающими в бесконечность орбитами. Первый такой пример для конкретного угла, построенный несколько в других терминах, восходит к А. Пуанкаре. Позже были построены примеры для любого иррационального угла; при этом изучались дополнительные свойства этих отображений, такие как, например, скорость убегания орбит или массивность множества таких орбит.

Существует ряд примеров цилиндрических отображений, имеющих убегающие орбиты, с функциями, удовлетворяющими условию Гёльдера.

Рассматривается зависимость существования и свойств таких отображений от показателя Гёльдера и свойств угла поворота.

Об условиях существования граничного значения у полигармонической функции

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Доказывается представление m-гармонической в области Q функции в виде суммы ортогональных полигармонических слагаемых, устанавливающее взаимно-однозначное соответствие с m-гармоническими в Q функциями. Устанавливается связь с представлением Альманси. Показана справедливость теоремы В. П. Михайлова о существовании граничного значения у полигармонической функции, при достаточно общих предположениях.

1. Пусть $Q \subset R^n$ – ограниченная область с достаточно гладкой границей $S = \partial Q$. Обозначим

$$e_m(Q) = \left\{ E_{m,n}(x-y) \mid x \in Q, y \in R^n \backslash \overline{Q} \right\},\ m \geqslant 1$$

множество сдвигов фундаментального решения m-гармонического уравнения [1] и $G_m(Q)$ – замыкание линейной оболочки span $\{e_m(Q)\}$ в норме $L_2(Q)$. Полученное множество $G_m(Q)$ будем называть полигармоническим подпространством.

2. Рассмотрим разложение $L_2(Q) = G_1(Q) \oplus N_1(Q)$. Обозначим $\Delta : L_2(Q) \to N_1(Q)$ – соответствующее расширение оператора Лапласа, а $\Delta^{-1} : N_1(Q) \to L_2(Q)$ – обратный оператор. Естественным образом для $k \geqslant 1$ определяется степень Δ^{-k}.

Теорема 1. *Если $f \in G_m(Q)$, $(m \geqslant 1)$, то существуют гармонические из $G_1(Q)$ функции g_0, g_1, ..., g_{m-1}, такие что*

$$f = g_0 + \Delta^{-1} g_1 + \ldots + \Delta^{-(m-1)} g_{m-1} \ (1)$$

и такое представление единственно.

В работе [4] получен частный случай разложения (1) для полианалитических функций в единичном круге. Представление (1) является глобальным представлением полигармонической функции в области Q в отличие от разложения Альманси [5], которое, как известно, является локальным представлением. Более того, в отличие от разложения Альманси, разложение (1) является ортогональным. При помощи разложения Альманси в [1] изучается граничное поведение полигармонических функций в зависимости от граничного поведения ее компонент Альманси.

3. Справедливо следующее утверждение.

Теорема 2. *Пусть* $k \geqslant 1$*, тогда*

$$\Delta^{-k} g_k = g_k * E_{k,n} - P_k(g_k * E_{k,n}),$$

где P_k *– проектор на подпространство* $G_k(Q)$.

На основе свойств оператора Δ^{-k}, $k \geqslant 1$ исследуется граничное поведение полигармонических функций из пространства $G_m(Q)$. Полученные свойства позволяют установить справедливость теоремы В. П. Михайлова [2; 3] – эквивалентность существования L_2-предела полигармонической функции на границе области Q и ее L_2-компактности вблизи границы.

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Динамические свойства ренормализационной группы в обобщенной фермионной иерархической модели

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Пусть $T = \{0, 1, 2, \ldots\}$ и $V_k^s = \{j \in T : k \cdot 2^s \leq j < (k+1) \cdot 2^s\}$, где $k \in T$, $s \in N = \{1, 2, 3, \ldots\}$. Иерархическое расстояние $d_2(i,j)$, $i, j \in T$, $i \neq j$ определено, как $d_2(i,j) = 2^{s(i,j)}$, где $s(i,j) = \min\{s :$ есть $k \in T$ такое, что $i, j \in V_k^s\}$ Пусть $T^2 = T \times T$, $k = (k_1, k_2) \in T^2$, $V_k^s = \{(j_1, j_2) \in T^2 : k_1 \cdot 2^s \leq j_1 < (k_1+1) \cdot 2^s,\ k_2 \cdot 2^s \leq j_2 < (k_2+1) \cdot 2^s\}$. Для любого $k = (k_1, k_2) \in T^2$, $l = (l_1, l_2) \in T^2$, $k \neq l$ определено $s(k,l) = \max(s(k_1, l_1), s(k_2, l_2))$. Иерархическое расстояние на T^2 определено, как $d_2(k,l) = 2^{s(k,l)}$. Рассмотрим 4-компонентное фермионное поле $\psi^*(i) = (\bar{\psi}_1(i), \psi_1(i), \bar{\psi}_2(i), \psi_2(i)), \quad i \in T^2$, где все компоненты которого, являются образующими алгебры Грассмана.

Действие ренормализационной группы Каданова-Вильсона $r(\alpha)$ на ψ^* определеяется формулой $\psi^{*\prime}(i) \equiv (r(\alpha)\psi^*)(i) = 2^{-\alpha/2} \sum\limits_{j \in V_i^1} \psi^*(j)$, где

$\alpha \in R$ — параметр ренормгруппы.

Определим следующие функции на T^2: $d(k, l; \lambda) = d_2(k, l)$, если $s(k_1, l_1) \neq s(k_2, l_2)$; $d(k, l; \lambda) = \lambda d_2(k, l)$, если $s(k_1, l_1) = s(k_2, l_2)$, $f(k, l; \lambda; \alpha) = d^\alpha(k, l; \lambda)$, если $k \neq l$, $\quad f(k, k; \lambda; \alpha) = \frac{2+\lambda^\alpha}{4(1-2^{-(2+\alpha)})}$, λ — действительный параметр, $\lambda > 0$. В работе [1] показано, что гауссовское фермионное поле с нулевым средним и бинарной корреляционной функцией $\langle \psi_n(k) \bar{\psi}_m(l) \rangle = \delta_{n,m} f(k, l; \lambda; \alpha - 4), \quad n, m = 1, 2, \quad k, l \in T^2$, где $\delta_{n,m}$ — символ Кронекера, инвариантно относительно преобразования ренормгруппы с параметром α.

Негауссовские поля будем описывать с помощью добавления к вышеописанному гауссовскому полю докального потенциала, представленного в виде грассмановозначной плотности «свободной меры» общего вида $u(\psi^*) = c_0 + c_1(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2) + c_2\bar{\psi}_1\psi_1\bar{\psi}_2\psi_2$.

Вычисления показывают, что действие преобразования ренормгруппы на негауссовское поле сводится к отображению в пространстве коэффициентов $R(\alpha;\delta)$: $R(\alpha;\delta)(c_0,c_1,c_2)=(c_0^{'},c_1^{'},c_2^{'})$, где

$$c_0^{'}=c_0^4(\delta-1)^2+c_0^3c_1(-4\delta^2+6\delta-2)+c_0^3c_2(\delta^2-\delta+\frac{1}{4})+c_0^2c_1^2(5\delta^2-3\delta-\frac{5}{4})+$$
$$+c_0^2c_1c_2(-2\delta^2-\delta+1)+c_0^2c_2^2(\frac{\delta^2}{4}+\frac{1}{2})+c_0c_1^3(-2\delta^2-3\delta+3)+$$
$$+c_0c_1^2c_2(\frac{\delta^2}{2}+5\delta-\frac{3}{2})+c_0c_1c_2^2(-\delta-1)+\frac{c_0c_2^3}{4}+c_1^4(\frac{\delta^2}{4}+\delta)+c_1^3c_2(-\delta-1)+\frac{3c_1^2c_2^2}{4},$$
$$c_1^{'}=\gamma(c_0^3c_1(\delta-1)^2+c_0^3c_2(-\frac{\delta^2}{2}\frac{3\delta}{4}-\frac{1}{4})+c_0^2c_1^2(-\frac{7\delta^2}{2}+\frac{21\delta}{4}-\frac{7}{4})+c_0^2c_1c_2(\frac{5\delta^2}{2}-2\delta)+$$
$$+c_0^2c_2^2(-\frac{\delta^2}{4}-\frac{\delta}{2}+\frac{1}{2})+c_0c_1^3(\frac{7\delta^2}{2}-2\delta-1)+c_0c_1^2c_2(-3\delta^2-\frac{3\delta}{2}+\frac{3}{2})+c_0c_1c_2^2(\frac{\delta^2}{2}+2\delta)+$$
$$+c_0c_2^3(-\frac{\delta}{4}-\frac{1}{4})+c_1^4(-\frac{3\delta^2}{4}-2\delta+2)+c_1^3c_2(\frac{\delta^2}{2}+4\delta-1)+c_1^2c_2^2(-\frac{7\delta}{4}-\frac{7}{4})+c_1c_2^3),$$
$$c_2^{'}=\gamma^2(c_0^3c_2(\frac{\delta^2}{4}-\frac{\delta}{2}+\frac{1}{4})+c_0^2c_1^2(\frac{3\delta^2}{4}-\frac{3\delta}{2}+\frac{3}{4})+c_0^2c_1c_2(-2\delta^2+3\delta-1)+$$
$$+c_0^2c_2^2\left(\frac{3\delta^2}{4}-\delta+\frac{1}{2}\right)+c_0c_1^3(-2\delta^2+3\delta-1)+c_0c_1^2c_2(4\delta^2-2\delta-\frac{3}{2})+$$
$$+c_0c_1c_2^2(-2\delta^2-\delta+1)+c_0c_2^3(\frac{\delta^2}{4}+\frac{\delta}{2}+\frac{1}{4})+c_1^4(\frac{5\delta^2}{4}-\delta)+$$
$$+c_1^3c_2(-2\delta^2-3\delta+3)+c_1^2c_2^2(\frac{3\delta^2}{4}+\frac{11\delta}{2}-\frac{5}{4})++c_1c_2^3(-2\delta-2)+c_2^4)$$

Здесь δ — параметр, зависящий от α и λ, $\gamma=2^{\alpha-2}$. В докладе мы описываем инвариантные множества отображения $R(\alpha;\delta)$. Легко видеть, что точки $A_0=(1,0,0)$ и $A_1=(0,0,1)$ являются неподвижными точками отображения $R(\alpha;\delta)$ при всех α и δ. Точка A_0 соответствует гауссовской неподвижной точке. Точке A_1 соответствует плотность $u(\psi^*)=\bar{\psi}_1\psi\bar{\psi}_2\psi_2$ (грассманова дельта-функция). Мы описываем динамику отображения ренормгруппы $R(\alpha;\delta)$ в окрестности этих неподвижных точек.

Работа выполнена за счет средств Программы стратегического академического лидерства Казанского (Приволжского) федерального университета (ПРИОРИТЕТ-2030).

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К вопросу о вложении гильбертовых пространств с воспроизводящим ядром

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Во многих задачах комплексного анализа часто возникает вопрос, будет ли данное гильбертово пространство с воспроизводящим ядром (RKHS) содержаться в более широком RKHS. Мы исследуем следующую задачу. Пусть дано некоторое RKHS H_1 состоящее из функций, заданных на некотором множестве точек $\Omega_1 \subset \mathbf{C}^n$. Также имеется некоторое RKHS H_2 также состоящее из функций, заданных на множестве точек $\Omega_1 \subset \mathbf{C}^n$. Вопрос: какие условия гарантируют вложение пространств $H_1 \subset H_2$? Мы рассмотрим эту задачу в несколько другой постановке. Пусть H – некоторое гильбертово пространство с воспроизводящим ядром, состоящее из функций, заданных на множестве $\Omega \subset \mathbf{C}^m$, $m \geq 1$, т.е. для произвольной точки $\xi \in \Omega$ функционал δ_ξ, ставящий в соответствие любой функции $f \in H$ значение функции f в точке $\xi \in \Omega$, является линейным и непрерывным функционалом над H. Предположим, что $\{e_1(\cdot, \xi)\}_{\xi \in \Omega_1}$, $\{e_2(\cdot, \xi)\}_{\xi \in \Omega_1}$ – некоторые полные системы функций в H, $\Omega_1 \subset \mathbf{C}^n$, $n \geq 1$. Обозначим

$$\widetilde{f}(z) \overset{def}{=} (e_1(\cdot, z), f)_H \, \forall z \in \Omega_1, \quad \widetilde{H} = \{\widetilde{f}, \, f \in H\},$$
$$(\widetilde{f}_1, \widetilde{f}_2)_{\widetilde{H}} \overset{def}{=} (f_2, f_1)_H, \, \|\widetilde{f}_1\|_{\widetilde{H}} = \|f_1\|_H \quad \forall \widetilde{f}_1, \widetilde{f}_2 \in \widetilde{H},$$
$$\widehat{f}(z) \overset{def}{=} (e_2(\cdot, z), f)_H \, \forall z \in \Omega_1, \quad \widehat{H} = \{\widehat{f}, \, f \in H\},$$
$$(\widehat{f}_1, \widehat{f}_2)_{\widehat{H}} \overset{def}{=} (f_2, f_1)_H, \, \|\widehat{f}_1\|_{\widehat{H}} = \|f_1\|_H \quad \forall \widehat{f}_1, \widehat{f}_2 \in \widehat{H}.$$

Необходимо найти условие, при выполнении которого, пространства $\widehat{H}$ и $\widetilde{H}$ обладают свойством $\widehat{H} \subset \widetilde{H}$, т.е. $\widehat{H}$ как множество функций содержится в $\widetilde{H}$ и найдется постоянная $C > 0$ такая, что

$$\|f\|_{\widetilde{H}} \leq C \|f\|_{\widehat{H}} \quad \forall f \in \widehat{H}.$$

Утверждение 1 *Для того чтобы* $\widehat{H} \subset \widetilde{H}$ *необходимо и достаточно, чтобы нашелся линейный непрерывный оператор* $A\colon H \to H$ *такой, что*

$$A\colon e_2(\cdot, z) \mapsto e_1(\cdot, z) \quad \forall z \in \Omega_1.$$

Утверждение 2 *Пусть* $\{e_j(\cdot, z)\}_{z\in\Omega_1}$, $j = 1, 2$ *– две ортоподобные системы разложения в пространстве* H *с одной и той же мерой* μ. *Предположим, что найдется линейный непрерывный оператор* $\mathcal{T}\colon H \to H$ *такой, что системы функций* $\{e_j(\cdot, z)\}_{z\in\Omega_1}$, $j = 1, 2$ *согласованы с оператором* $\mathcal{T}$*, т.е.*

$$(e_1(\cdot, z_1), e_2(\cdot, z_2))_H = \overline{(e_1(\cdot, z_2), \mathcal{T}e_2(\cdot, z_1))_H} \quad \forall z_1, z_2 \in \Omega_1.$$

Тогда $\widehat{H} \subset \widetilde{H}$.

Следует отметить, что вопрос совпадения RKHS ранее изучался авторами в работах [1; 3]. В отличии от результатов этих работ, операторы A и $\mathcal{T}$ в утверждении 1, утверждении 2 не предполагаются обратимыми. В докладе обсуждаются возможные применения полученных результатов, в частности, будет рассмотрен вопрос о следах функций из пространства Баргмана – Фока на решетках фон Неймана (см. [2]).

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Задачи теории чисел в КТП на решётке

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При проведении исследований в области квантовой теории поля (КТП) на решётке была обнаружена и доказана математическая теорема.

Пусть $\mathbb{Z}(N)$ — кольцо остатков деления на N. Обычно мы используем представление вида $\mathbb{Z}(N) = \{0, 1, 2, \ldots, N-1\}$.

Рассмотрим d-мерную решётку $\mathbb{Z}^d(N) = \underbrace{\mathbb{Z}(N) \times \mathbb{Z}(N) \times \ldots \times \mathbb{Z}(N)}_{d}$.

Пусть $c_{Ndm}(k)$, где $m \in \mathbb{N}$, — число узлов $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_d)$, где $\xi_i \in \mathbb{Z}(N)$, решётки $\mathbb{Z}^d(N)$ таких, что $\sum\limits_{i=1}^{d} \xi_i^m \equiv k \pmod N$.

Теорема 1 *Для произвольного N при $d \geqslant 3$, и для $N = 2^n$ при $d \geqslant 2$, где $n \in \mathbb{N}$*

$$\forall k \in \mathbb{Z}(N) \quad c_{Nd2}(k) \equiv 0 \pmod N.$$

В доказательстве этой теоремы рассмотрены четыре случая:

1. N — простое число. Для этого случая теорема доказана с помощью производящих функций и символа Лежандра.
2. $N = p^n$, где p — простое число, $n \in \mathbb{N}$, $n > 1$. Для доказательства использован метод индукции, где предполагается, что теорема выполнена для $N = p^{n-2}, p^{n-1}$, а в качестве базы взято выполнение теоремы для $n = 0, 1$. Использовано разбиение решётки на две части, обнуление количества узлов доказано для каждой из них, из чего затем следует обнуление количества узлов для всей решётки.
3. $N = 2^n$, где $n \in \mathbb{N}$, $n > 1$. Доказательство аналогично предыдущему случаю, но в качестве предположения индукции взято выполнение теоремы для $N = 2^{n-3}, 2^{n-2}, 2^{n-1}$, а в качестве базы используется выполнение теоремы для $n = 0, 1, 2$. База индукции проверяется численно.
4. N — составное число, не являющееся степенью простого числа. Каждое такое N можно представить в виде произведения n степеней неодинаковых простых чисел. Для доказательства применён метод индукции,

где предполагается, что теорема выполнена для N, являющегося произведением $n-1$ степеней неодинаковых простых чисел, а в качестве базы взято выполнение теоремы для $n=1$.

Доказательство теоремы для случаев 2, 3, 4, то есть когда N не является простым, составляет основное содержание доклада.

Была обнаружена, но не доказана, более общая закономерность, частным случаем которой является утверждение этой теоремы для произвольного N при $d \geqslant 3$:

Гипотеза 1 *Для произвольных* N, m *при* $d \geqslant p+1$

$$\forall k \in \mathbb{Z}(N) \quad c_{Ndm}(k) \equiv 0 \pmod{N}.$$

Таким образом, устанавливается гипотетическая связь между выделенным положением трёхмерной решётки и тем фактом, что при подсчёте узлов решётки суммируются именно вторые степени их координат.

Обобщённая гипотеза была проверена численно для случаев $m \leqslant 8, N \leqslant 1000$; $m \leqslant 35, N \leqslant 300$; $m \leqslant 100, N \leqslant 37$. Результат: контрпримеров не обнаружено.

Если же $d \leqslant m$, то существует N, для которого утверждение теоремы не выполняется. Это утверждение было численно проверено для $m \leqslant 100$. Контрпримеров также не обнаружено.

В КТП на решётке этой теореме соответствует перенормировка в нуль энергии вакуума, пробегающей дискретные значения, на трёхмерной импульсной решётке. Поскольку перенормировка происходит для произвольного размера решётки, есть основания полагать возможность переноса построенной теоретико-числовой перенормировки на непрерывный случай с помощью предельного перехода $N \to \infty$.

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Вещественные полюсы третьего трансцендента Пенлеве

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Рассматривается специальное третье уравнение Пенлеве

$$\frac{d^2u}{dx^2} = \frac{1}{u}\left(\frac{du}{dx}\right)^2 - \frac{1}{x}\frac{du}{dx} + 4\frac{(N-1)\,u^2 - N}{x} + 4u^3 - \frac{4}{u}, \quad N = \text{const}. \quad (1)$$

Вещественные решения уравнения (1) применяются во многих моделях математической физики, таких как модель Гросса-Виттена-Вадьи, модель Полмайера-Редже и модели распределения осциллирующих мод в сильно демпфированном контакте Джозефсона [2]. Все такие решения на положительной полуоси имеют гладкую осциллирующую асимптотику в нуле и последовательность полюсов в точках $x = a_k$, начиная с некоторого $x = a_0 > 0$.

$$u(x) = \pm\frac{1}{2(x-a_k)} - \frac{2(N-1)\pm 1}{4a_k} + b_k(x-a) + O(x-a_k)^2, \;\; x \to a_k. \quad (2)$$

Асимптотика полюсов при больших x при фиксированном N вычисляется методом изомонодромных деформаций [1]

$$a_k = \pi\left(k + \frac{N+1}{2}\right) - \frac{1}{2}\ln 8\pi k + O(1), \quad k \to +\infty$$

В работе рассматривается другой асимптотический предел, $N \to \infty$, $k = O(1)$. Для этой цели применяется следующее преобразование Беклунда

$$u(x) = \frac{v'(x)}{4v(x)\,(v(x)-1)} - \frac{N}{2x\,v(x)},$$

где $y(x)$ является решением уравнения Пенлеве V

$$v'' = \frac{1}{2}\left(\frac{1}{v-1} + \frac{1}{v}\right)(v')^2 - \frac{v'}{x} - 8v(v-1) + 2\frac{N^2}{x^2}\frac{v-1}{v}. \quad (3)$$

При больших N уравнение (2) имеет гладкое решение вида

$$v(x) = \left(1 + [\alpha J_N(2x) + \beta Y_N(2x)]^2\right)\left(1 + O(N^{-1})\right),$$

где J_N, Y_N - функции Бесселя со значком N. Тогда согласно преобразованию Беклунда в окрестности полюса $x = a$ имеем

$$u(x) = \frac{1}{2}\frac{\alpha J_N'(2x) + \beta Y_N'(2x)}{\alpha J_N(2x) + \beta Y_N(2x)} + O(N^{-1}) = \frac{1}{2(x-a)} - \frac{2N-1}{4a} + b(x-a) + \dots$$

Произвольные постоянные α и β определяются из алгебраических уравнений

$$\begin{aligned} &\alpha J_N(2a) + \beta Y_N(2a) = 0, \\ &\alpha J'_N(2a) + \beta Y'_N(2a) = \tfrac{N}{a\sqrt{3}}\sqrt{1 - \tfrac{3}{N} + \tfrac{a^2(24b-16)+5}{4N^2}} \ . \end{aligned} \qquad (4)$$

Теорема *Пусть вещественное решение уравнения (1) имеет полюс (2), тогда оно имеет счетный набор вещественных полюсов, которые при $N \to \infty$ совпадают с нулями функции Бесселя $\alpha J_N(2x) + \beta Y_N(2x)$, где коэффициенты α, β определяются из уравнений (4).*

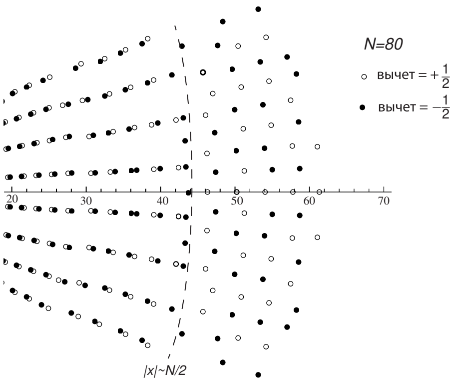

Рис. 1: Распределение полюсов в комплексной плоскости. Все вещественные полюсы находятся справа от точки $x = N/2$.

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О свойстве монотонности решений систем относительно начальных условий

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Рассмотрим автономную систему дифференциальных уравнений

$$\dot{x} = f(x), \quad x \in R^n, \tag{1}$$

в предположении, что вектор-функция $f(x)$ и ее производные $\partial f_i/\partial x_j$ $(i, j = 1, \ldots, n)$ непрерывны. Обозначим через $\varphi(t, x)$ решение данной системы, удовлетворяющее начальному условию $\varphi(0, x) = x$. Для решения многих прикладных задач желательно, чтобы решения системы (1) обладали следующим свойством монотонности относительно начальных условий:

Свойство 1. *Пусть* $x(0) \in R^n$, $y(0) \in R^n$ *такие, что* $x(0) \leqslant y(0)$. *Тогда*

$$\varphi(t, x(0)) \leqslant \varphi(t, y(0)), \;\; t \geqslant 0.$$

Здесь и далее неравенство $x \leqslant y$, записанное для векторов $x \in R^n$, $y \in R^n$, будем понимать, как неравенства $x_i \leqslant y_i$, $i = 1, \ldots, n$.

Рассмотрим сначала линейную систему дифференциальных уравнений

$$\dot{x} = Ax,$$

где A — постоянная матрица размеров $n \times n$. Известно, что решение данной системы можно записать в виде $\varphi(t, x) = e^{At}x$, где e^{At} — матричная экспонента. Матрица A называется *экспоненциально неотрицательной*, если $e^{At} \geqslant 0$ для всех $t \geqslant 0$. Матрица A называется *матрицей Метцлера*, если ее элементы удовлетворяют неравенствам $a_{ij} \geqslant 0$ при $i \neq j$, $i = 1, \ldots, n$, см. [1].

Лемма 1 (см. [1; 2]). *Матрица A является экспоненциально неотрицательной тогда и только тогда, когда она является матрицей Метцлера.*

Из леммы 1 очевидно, следует, что если A — матрица Метцлера и $x \leqslant y$, то $\varphi(t,x) = e^{At}x \leqslant e^{At}y = \varphi(t,y)$ для любого $t \geqslant 0$.

Вернемся к рассмотрению нелинейной системы (1). Функции f_i в правой части этой системы могут зависеть не от всех переменных, в частности, они могут быть постоянными.

Условие 1. *Пусть множество* $D \subseteq R^n$ *положительно инвариантно относительно системы* (1). *Каждая из функций* f_i *является возрастающей на множестве* D *по всем переменным, от которых она явным образом зависит, за исключением переменной* x_i, $i = 1, \ldots, n$.

Доказано, что свойство 1 выполнено для любого дифференциального уравнения $\dot{x} = f(x)$. Имеет место следующее утверждение.

Теорема 1. *Пусть выполнено условие* 1. *Тогда, если для* $x(0) \in D$, $y(0) \in D$ *имеет место неравенство* $x(0) \leqslant y(0)$, *то* $\varphi(t, x(0)) \leqslant \varphi(t, y(0))$ *для всех* $t \geqslant 0$.

Отметим, что если система (1) линейная и выполнено условие 1, то матрица A данной системы является матрицей Метцлера.

Рассматривается также одна из задач, для исследования которой применяется теорема 1 — это задача оценки средней временной выгоды для систем со случайными параметрами, которая выполнена с вероятностью единица.

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Об интегральных инвариантах уравнений Биркгофа для бесконечномерных систем

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1. Постановка задач. Пусть состояние бесконечномерной потенциальной системы определяется вектор-функцией $u(x,t) = (u^1(x,t), u^2(x,t), \cdots, u^{2n}(x,t))$, $(x,t) \in Q_T = \Omega \times (0,T)$, Ω — ограниченная область из $\mathbb{R}^m$ с кусочно гладкой границей $\partial\Omega$.

Предположим, что при этом действие по Гамильтону имеет вид

$$F[u] = \int\limits_0^T \int\limits_\Omega \left[\sum_{i=1}^{2n} R_i(x,t,u_\alpha)\, u_t^i - B(u_\alpha)\right] dxdt, \qquad (1)$$
$$\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_m),\, |\alpha| = \sum_{i=1}^{m} \alpha_i,\, |\alpha| = \overline{0,s},$$

где $R_i = R_i(x,t,u_\alpha)$, $B = B(u_\alpha)$ — заданные достаточно гладкие функции, $u_t^i = \dfrac{\partial u^i}{\partial t}$, $i = \overline{1,2n}$, $u_\alpha = D_\alpha u = \dfrac{\partial^{|\alpha|} u}{(\partial x_1)^{\alpha_1}(\partial x_2)^{\alpha_2} \cdots (\partial x_m)^{\alpha_m}}$.

Будем рассматривать функционал (1) на множестве

$$D(N) = \Big\{ u \in U = \left(U^1, \cdots, U^{2n}\right) : u^i \in U^i = C_{x,t}^{2s,1}\left(\overline{\Omega} \times [0,T]\right) : u^i\big|_{t=0} =$$
$$= \varphi_0^i(x),$$
$$u^i\big|_{t=T} = \varphi_1^i(x),\, \frac{\partial^\nu u^i}{\partial n_x^\nu}\bigg|_{\Gamma_T} = \psi_\nu^i(x,t),\, i = \overline{1,2n},\, |\nu| = \overline{0,s-1} \Big\},$$

где $\overline{\Omega} = \partial\Omega \cup \Omega$, $\Gamma_T = \partial\Omega \times (0,T)$, n_x — внешняя нормаль к $\partial\Omega$; φ_0^i, φ_1^i, $\psi_\nu^i(x,t)$ — заданные достаточно гладкие функции.

2. Система уравнений движения.

Теорема 1. *Экстремали функционала* (1) *являются решениями система уравнений*

$$N_i \equiv \sum_{k=1}^{2n} \sum_{|\beta|=0}^{s} \left[\sum_{|\alpha|=0}^{s} (-1)^{|\alpha|} \binom{\alpha}{\beta} D_{\alpha-\beta} \left(\frac{\partial R_k}{\partial u_\alpha^i} \right) - \frac{\partial R_i}{\partial u_\beta^k} \right] D_\beta u_t^k - \frac{\partial R_i}{\partial t} - \\ - \sum_{|\alpha|=0}^{s} (-1)^{|\alpha|} D_\alpha \frac{\partial B}{\partial u_\alpha^i} = 0, i = \overline{1, 2n}, \tag{2}$$

где

$$\binom{\alpha}{\beta} = \begin{cases} \binom{\alpha_1}{\beta_1} \binom{\alpha_1}{\beta_1} \cdots \binom{\alpha_m}{\beta_m}, \text{если } \forall i \in \{1, 2, \cdots, m\} : \alpha_i \geq \beta_i, \\ 0, \text{если } \exists i \in \{1, 2, \cdots, m\} : \alpha_i < \beta_i, \end{cases}$$

$$\binom{\alpha_i}{\beta_i} = \frac{\alpha_i!}{\beta_i! \, (\alpha_i - \beta_i)!}.$$

Отметим, что из (2) как частный случай следуют уравнения Биркгофа [1; 2].

3. Интегральные инварианты. Пусть $u = u\,(\lambda; x, t)$, $\lambda \in \Lambda = [0, 1]$ — произвольное однопараметрическое множество элементов из U непрерывно дифференцируемых по λ. Его можно рассматривать как кривую C в U. Будем считать, что $u\,(0; x, t) = u\,(1; x, t) \;\forall\, (x, t) \in Q_T$, т. е. кривая замкнута.

Введем обозначение $\delta u = \dfrac{\partial u\,(\lambda; x, t)}{\partial \lambda} d\lambda$.

Теорема 2. *Система уравнений* (2) *имеет интегральный инвариант первого*
порядка вида

$$\int_\Lambda \int_\Omega \sum_{i=1}^{2n} R_i \delta u^i dx = \oint_C \int_\Omega \sum_{i=1}^{2n} R_i \delta u^i dx.$$

4. Заключение. Из вариационного принципа с использованием заданного действия по Гамильтону получены весьма общие уравнения движения бесконечномерных систем. Как частный случай из них следуют известные уравнения Биркгофа. Найден линейный интегральный инвариант первого порядка.

Публикация выполнена в рамках проекта № 002092-0-000 Российского университета дружбы народов им. Патриса Лумумбы.

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Обратные задачи атмосферного электричества в квазистационарных приближениях

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Исследование широкого класса электромагнитных процессов в атмосфере Земли возможно в рамках квазистационарных приближений для системы уравнений Максвелла [5]. В частности, нерелятивистское электрическое приближение приводит к уравнению глобальной электрической цепи [4; 8]. Для изучения электромагнитных полей в верхних слоях атмосферы может использоваться нерелятивистское магнитное приближение [1; 6]. Более полно и точно описать рассматриваемые явления в атмосфере в целом позволяет квазистационарное приближение, основанное на сохранении в системе уравнений Максвелла потенциальной части тока смещения [2; 3; 7; 9].

В работе обсуждаются постановки обратных задач финального и граничного наблюдения для системы уравнений Максвелла в различных квазистационарных приближениях в неоднородных средах.

Исследование выполнено за счет гранта Российского научного фонда № 23-21-00440, `https://rscf.ru/project/23-21-00440`

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Исследование асимптотик решений дифференциальных уравнений 2-го порядка с мероморфными коэффициентами

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В работе рассмотрим дифференциальное уравнение 2-го порядка с иррегулярной особенностью ($n \geq 2$)

$$u''(r) + b^0(r)u'(r) + b^1(r)u(r) = 0, \tag{1}$$

здесь $b^0(r)$, $b^1(r)$-мероморфные функции. Без ограничения общности будем считать, что они имеют особенность в нуле. В работе [1] показано, что уравнение (1) может быть приведено к виду

$$\left(-r^n \frac{d}{dr}\right)^2 u + a_1(r) \left(-r^n \frac{d}{dr}\right) u + a_0(r)u = 0, \tag{2}$$

где $a_1(r)$,$a_0(r)$ – мероморфные функции ($n \geq 2$).

В работах [2—5] было доказано, что если полином $H_0(p)$ имеет простые корни в точках p_1,…,p_m, тогда асимптотика решения в пространстве функций $(n-1)$ экспоненциального роста уравнения $H\left(-r^n \frac{d}{dr}, r\right) u = 0$ имеет вид:

$$u(r) \approx \sum_{j=1}^{m} e^{\left(\frac{p_j}{r^{n-1}} + \sum_{i=1}^{n-2} \frac{\lambda_i}{r^{n-1-i}}\right)} r^{\sigma_j} \sum_{i=0}^{\infty} b_i^j r^i, \tag{3}$$

здесь числа σ_j и λ_i, а b_i^j – некоторые числовые коэффициенты.

Цель данной работы – найти коэффициенты в асимптотических разложениях решений для уравнения (2).

Запишем уравнение (2) в виде:

$$\left(-r^n\frac{d}{dr}\right)^2 u + a_1^0\left(-r^n\frac{d}{dr}\right)u + a_0^1 ru + a_0^2 r^2 u + \dots + a_0^{n-1}r^{n-1}u + \\ \left[ra_1(r)\left(-r^n\frac{d}{dr}\right) + r^n a_0(r)\right]u = 0. \quad (4)$$

основной символ имеет вид $H_0(p) = p^2 + a_1^0 p = p\left(p + a_1^0\right)$.

Предположим, что уравнений имеет простые корни. Это значит, что $a_1^0 \neq 0$.

Теорема 2 *Пусть n – четное число. Асимптотический член решения уравнения (2), соответствующий нулевому корню основного символа при условии $a_1^0 \neq 0$ имеет вид*

$$r^\delta e^{\left(\sum_{i=1}^{n-2}\frac{\lambda_i}{r^{n-i-1}}\right)}\sum_{i=0}^{\infty} b_i r^i.$$

Числа $\lambda_i, i = 1, 2, \dots, n-2$ однозначно определяются из системы уравнений

$$\begin{aligned}
&a_1^0\lambda_1(n-2) = -a_0^1,\\
&{\lambda_1}^2(n-2)^2 + a_1^0\lambda_2(n-3) = -a_0^2,\\
&\dots\\
&{\lambda_k}^2(n-k-1)^2 + a_1^0\lambda_{2k}(n-2k-1) = -a_0^{2k},\\
&a_1^0\lambda_{2k+1}(n-2k-2) = -a_0^{2k+1},\\
&\dots\\
&{\lambda_{\frac{n-2}{2}}}^2\left(n - \frac{n-2}{2} - 1\right)^2 + a_1^0\lambda_{n-2} = -a_0^{n-2}.
\end{aligned}$$

и

$$\delta = \frac{a_0^{n-1}}{a_1^0}.$$

Пусть n – нечетное числа. Коэффициенты $\lambda_1, \dots, \lambda_{n-2}$, определяются *из системы уравнений*

$$\begin{aligned}
&a_1^0\lambda_1(n-2) = -a_0^1,\\
&{\lambda_1}^2(n-2)^2 + a_1^0\lambda_2(n-3) = -a_0^2,\\
&\dots\\
&{\lambda_k}^2(n-k-1)^2 + a_1^0\lambda_{2k}(n-2k-1) = -a_0^{2k},\\
&a_1^0\lambda_{2k+1}(n-2k-2) = -a_0^{2k+1},\\
&\dots\\
&a_1^0\lambda_{n-2} = -a_0^{n-2}.
\end{aligned}$$

и

$$\delta = \frac{{\lambda_{\frac{n-1}{2}}}^2\left(n-\frac{n-1}{2}-1\right)^2 + a_0^{n-1}}{a_1^0}.$$

Где $\sum_{i=0}^{\infty} b_i r^i$ *– соответствующий асимптотический ряд.*

Чтобы построить асимптотику, соответствующую корню основного символа $p = -a_1^0 \neq 0$ сделаем замену: $u(r) = e^{\frac{\lambda_0}{r^{n-1}}}u_0(r), \lambda_0 = -a_1^0$, мы получим:

$$\begin{aligned}
\left(-r^n\frac{d}{dr}\right)^2 u_0(r) + \left(3a_1^0 - 2na_1^0\right)\left(-r^n\frac{d}{dr}\right)u_0(r) + a_0^1 r u_0(r) + a_0^2 r^2 u_0(r)\\
+ \dots + a_0^{n-1}r^{n-1}u_0(r)\\
+ \left[r a_1^{(0)}(r)\left(-r^n\frac{d}{dr}\right) + r^n a_0^{(0)}(r)\right]u_0(r) = 0.
\end{aligned} \tag{5}$$

Новый основной символ $H_0(p) = p^2 + \left(3a_1^0 - 2na_1^0\right)p$. То есть корень основного символа с помощью экспоненциальной замены сдвинут в ноль. Остальные расчеты аналогичны предыдущему случаю, где $p = 0$.

Пусть теперь $a_1^0 = 0$. В этом случае корень уравнения кратный и уравнение (2) приводится к виду

$$\left(-r^n\frac{d}{dr}\right)^2 u + r^{k_1+1}a_1^1(r)\left(-r^n\frac{d}{dr}\right)u + r^{k_0+1}a_0^0(r)u = 0, \tag{6}$$

здесь $a_1^1(r) = \sum_{i=k_1}^{\infty} a_i^1 r^{i-k_1}, a_0^0(r) = \sum_{i=k_0}^{\infty} a_i^0 r^{i-k_0}, H_0(p) = p^2$. Корень

$p = 0$ имеет кратность 2.

Теорема 3 *Пусть $k_1 + 1 \neq \frac{k_0+1}{2}$, тогда существует такое число $x > 0$ такое, что уравнение с кратным корнем в нуле при делении на r^{2x} сводится либо к уравнению Фуксова типа, либо к уравнению с простыми корнями основного символа.*

Пусть $k_1+1 = \frac{k_0+1}{2} < n-1$, тогда положим $x = k_1+1 = \frac{k_0+1}{2}$. Основной символ равен $H_0(p) = p^2 + a^1_{k_1} p + a^0_{k_0}$. Если этот многочлен имеет кратный корень, то сдвинув его в ноль сведем последнее уравнение к уравнению вида (6) с вырождением порядка $n - x$, где $x = k_1 + 1 = \frac{k_0+1}{2}$.

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ISBN 978-5-6048945-7-6