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# DIVERSITY IN TEAMS

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What is the optimal diversity of expertise in a team? Prat (2002) shows that a supermodular production function (which describes strategic complementarity between individual outputs) implies a lower optimal diversity than a submodular function (which corresponds to strategic substitutability). We investigate how optimal diversity changes as the production function adjusts continuously from a supermodular to a submodular function. The Utilitarian objective gives an intuitive monotonic relationship. However, the Rawlsian objective, an objective that optimizes the worst scenario, generates a non-monotonic relationship: optimal diversity may fall with increasing weighting on the submodular component of the production function.

Keywords: cognitive diversity, supermodular and submodular functions, Rawlsian objective

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# 1 Introduction

Should an organization hire people with similar skills or with different skills? Prat (2002) argues that teams with similar skills perform better if jobs are strategic complements, while hiring people with different backgrounds are optimal if their jobs are strategic substitutes. For example, in synchronized swimming, teams with more similar skills would perform better. In contrast, in a math olympiad, teams with more diverse backgrounds will more likely solve all problems and win.

In reality, many jobs are not pure strategic complements or pure strategic substitutes. In light of Prat's results on the two extreme cases, as jobs become less complementary, does optimal diversity in workers' skills increase? We show that this might not be the case.

To illustrate our point, consider the following example. A hospital has two nurses who can perform CPR and its management has to decide where to locate each nurse along a lengthy corridor. A good CPR requires a quick initial response from a single nurse, followed by a quick response from a second nurse (who can take over to ensure that the first nurse is not fatigued and that high-quality chest compressions are delivered). If the hospital locates the nurses close to the ends of the corridor, then all patients have a similar chance of surviving: patients near the ends receive a very quick response from the first nurse but the second nurse will be very slow to come, while patients near the center receive relatively timely responses from both nurses. Now suppose that the nurses are equipped with a defibrillator. The addition of this technology means that the jobs become less complementary because a single nurse can have a greater impact when performing CPR. Then, to ensure that all patients have equal chances of surviving, the hospital locates the nurses at $1/4$ and $3/4$ of the corridor length. Indeed, with such disposition, a single nurse can timely reach any patient in their half of the corridor. This example shows that the optimal distance between the nurses can fall as their jobs become less complementary. The distance between the nurses reflects the difference, or the level of diversity, in nurses' ability to perform CPR for a given patient. Thus, the example demonstrates that, contrary to the initial intuition which is based on studying the extreme cases with pure complements and pure substitutes, optimal diversity may decrease as jobs become less complementary.

The example is based on the Rawlsian objective criterion, which maximizes the chances of surviving for the patient with the worst location. This objective is reasonable for a public service provider, such as a hospital, which aims to minimize the number of complaints from its customers. In contrast, the Utilitarian objective criterion, which maximizes the expected service quality for a representative customer and is more appropriate for a profit-maximizing firm, the results are reversed and more aligned with the initial intuition: optimal diversity always weakly increases as jobs become less complementary.

We contribute to the stream of literature which study how optimal team composition depends on the way the efforts of team members are aggregated. Franco et al. (2011); Kaya and Vereshchagina (2014); Bel et al. (2015); Glover and Kim (2021) focus on team incentive problem and thus assume that the choice of effort is endogenous. In contrast, our paper abstracts from incentive considerations and assume that the team structure and the production function do not affect the incentives of the team members. The closest paper to us is Prat (2002) who treat agents' efforts as exogenous and compares the optimal team composition for two production functions, submodular and supermodular. We build on Prat (2002)'s work in two ways. First, using a distant measure of diversity and allowing the production function vary in a continuous way from supermodular to submodular, we investigate how the optimal diversity changes as the production function becomes less complementary. Second, in addition to the Utilitarian objective, we look at the Rawlsian objective, which yields to a qualitatively different results.

## 2 Model

A principal must hire a team of two agents to perform a task.

Each agent $i = 1, 2$ has a particular cognitive type, or specialty, modelled as a point on a unit interval, $t_i \in [0, 1]$. Without loss of generality, assume that $t_2 \geq t_1$. The task is also modelled as a point on a unit interval, $t \in [0, 1]$. The closer the agent's cognitive type to the location of the task, the higher the agent's performance on that task. Formally, agent $t_i$'s output on task $t$ is given by a function $v(d_i)$ where $d_i = |t - t_i|$.

**Assumption 1.** *Function $v$ is strictly decreasing.*

The timeline is as follows. First, the principal chooses $t_1$ and $t_2$, with $t_2 \geq t_1$. Second, task $t$ materializes according to the uniform distribution on the support $[0, 1]$.

In the game, the agents act as dummies in the sense that they decide nothing. Thus, their payoffs are irrelevant for our analysis.

The principal's payoff is the joint output of the agents defined as $\omega\left(v(|t - t_1|), v(|t - t_2|)\right)$, where

$$\omega(x_1, x_2) = \beta \times \omega_s(x_1, x_2) + (1 - \beta) \times \omega_c(x_1, x_2), \quad \beta \in [0, 1]. \tag{1}$$

Parameter $\beta$ captures the degree of complementarity of jobs, which will become clear after we explain the functions $\omega_s$ and $\omega_c$.

It is natural to assume that higher performance of an agent benefits the principal. Formally:

**Assumption 2.** *Functions $\omega_s$ and $\omega_c$ are weakly increasing in each argument.*

For a given cognitive type, we assume that the identity of an agent does not affect the principal's payoff. Formally, functions $\omega_s$ and $\omega_c$ are symmetric:

**Assumption 3.** $\omega_s(x_1, x_2) = \omega_s(x_2, x_1)$ *and* $\omega_c(x_1, x_2) = \omega_c(x_2, x_1)$ *for any* $(x_1, x_2)$.

Function $\omega_s$ is submodular in the agents' individual outputs: for any $(\hat{x}_1, \hat{x}_2)$ and $(\check{x}_1, \check{x}_2)$,

$$\omega_s(\hat{x}_1, \hat{x}_2) + \omega_s(\check{x}_1, \check{x}_2) \geq \omega_s\left(\min\{\hat{x}_1, \check{x}_1\}, \min\{\hat{x}_2, \check{x}_2\}\right) + \omega_s\left(\max\{\hat{x}_1, \check{x}_1\}, \max\{\hat{x}_2, \check{x}_2\}\right), \tag{2}$$

while function $\omega_c$ is supermodular in the agents' individual outputs:

$$\omega_c(\hat{x}_1, \hat{x}_2) + \omega_c(\check{x}_1, \check{x}_2) \leq \omega_c\left(\min\{\hat{x}_1, \check{x}_1\}, \min\{\hat{x}_2, \check{x}_2\}\right) + \omega_c\left(\max\{\hat{x}_1, \check{x}_1\}, \max\{\hat{x}_2, \check{x}_2\}\right). \tag{3}$$

Submodularity and supermodularity is a generalization of the traditional notions of strategic substitutability and strategic complementarity, respectively (see Prat (2002)).[1,2] Thus, the lower the weight on the submodular component, $\beta$, the less complementary the agents' cognitive types are.

We may think of a high-$\beta$ task $t$ as a task that is best suited for type $t$ agent but also requires significant input from other agents. In contrast, a low-$\beta$ task $t$ does not require much input from agents other than type $t$ agent.

**Assumption 4.** $\omega_s(x_1, x_2) + \omega_c(x_1, x_2) = x_1 + x_2$.

Assumption 4 ensures that at $\beta = 0.5$, the principal's payoff $\omega(x_1, x_2) = 0.5(x_1 + x_2)$ satisfies (2) as equality (and, therefore, satisfies (3) as equality as well). In other words, at $\beta = 0.5$, submodular and supermodular components cancel each other, making the agents' contributions strategically independent. Assumption 4 implies that $\beta = 0.5$ corresponds to an additive task, for which the joint output of the agents is the sum of agents' contributions.

For example, $\omega_s(x_1, x_2) = \max\{x_1, x_2\}$ and $\omega_c(x_1, x_2) = \min\{x_1, x_2\}$. Then, $\beta = 0$ ($\beta = 1$) corresponds to a conjunctive (disjunctive) task, for which the joint output of the agents depends only on the output of the weakest (strongest) agent. This example corresponds to the extreme form of complementary and substitutability.[3]

Given assumption 4, we can rewrite (1) as

$$\omega(x_1, x_2) = (2\beta - 1) \times \omega_s(x_1, x_2) + (1 - \beta)(x_1 + x_2). \tag{4}$$

Thus, function $\omega(x_1, x_2)$ is supermodular for $\beta \leq 0.5$ and submodular for $\beta \geq 0.5$.

**Principal's objective.** The principal chooses $t_1$ and $t_2$ that maximize a certain objective. We consider two different objectives: the *Utilitarian objective*, defined

[1] Subscripts "s" and "c" in notations $\omega_s$ and $\omega_c$ stand for "substitutes" and "complements".

[2] For twice differentiable functions, condition (2) is equivalent to $\partial^2 \omega_s(x_1, x_2)/\partial x_1 \partial x_2 \leq 0$, while condition (3) is equivalent to $\partial^2 \omega_c(x_1, x_2)/\partial x_1 \partial x_2 \geq 0$.

[3] Categorization of tasks into additive, disjunctive and conjunctive is outlined in Steiner (1972).

as the expected value of the principal's payoff,

$$U = \int_0^1 \omega\left(v(|t - t_1|), v(|t - t_2|)\right) \mathrm{d}t; \tag{5}$$

and the *Rawlsian objective*, defined as the minimal value of the principal's payoff,

$$R = \min_{t \in [0,1]} \omega\left(v(|t - t_1|), v(|t - t_2|)\right). \tag{6}$$

We define *diversity* as the distance between the agents' types, $\Delta = t_2 - t_1$. Given the restriction $0 \leq t_1 \leq t_2 \leq 1$, diversity $\Delta$ can take any value from the interval $[0, 1]$.

Lemma 1 ensures that without loss of generality we can focus on the cognitive types which are symmetric around 0.5.

**Lemma 1.** *For any fixed $\Delta = t_2 - t_1$, $t_1 = (1-\Delta)/2$ and $t_2 = (1+\Delta)/2$ maximize both the Utilitarian and the Rawlsian objectives.*

Thus, we can rewrite the principal's objectives (5) and (6) as functions of $\Delta$, denoted as $U(\Delta)$ and $R(\Delta)$, respectively. We refer to the *optimal diversity* $\Delta^*$ as the value of $\Delta$, which maximizes one of the above objectives. Our goal is to investigate how the optimal diversity changes with $\beta$ and compare the results for different objectives.

**Leading example.** To illustrate our model, consider the following example. A team of students needs to complete an assignment. If any assignment is designed in such a way that it requires knowledge from different subjects, then the students' inputs are complementary because no single student can answer all questions by herself and team work is necessary for a successful performance — that is, $\beta$ is high. If any assignment is mostly focused on one subject (which maybe different for different assignments), then the students' inputs are less complementary because a single student with the appropriate background can do the majority of the assignment all be herself — that is, $\beta$ is low. The content of the assignment is unknown at the time of team formation. The Rawlsian objective is better suited for regular class assignments and describes the goal to complete any assignment with a positive (but not necessarily the highest) mark. The Utilitarian objective is better for competitions where the goal is to get a high mark with a high probability (potentially sacrificing the performance on some assignments).

# 3 Utilitarian objective

In line with Proposition 1 in Prat (2002), null diversity is a solution whenever the principal's payoff $\omega(x_1, x_2)$ is supermodular, that is, whenever $\beta \leq 0.5$.

**Proposition 1.** *If $\beta \leq 0.5$, then $\Delta^* = 0$ maximizes the Utilitarian objective.*

According to Proposition 2 in Prat (2002), under additional assumptions, for submodular $\omega$ (that is, for $\beta > 0.5$), the set of optimal $\Delta$ contains $\Delta^* > 0$. In general, the uniqueness of the optimal diversity $\Delta^*$ is not guaranteed without additional assumptions (which should be even more restrictive than the one listed in Proposition 2 in Prat (2002)). These assumptions, however, are not required for our main result formulated in Theorem 1.[4] We take any local maximizer $\Delta^*$ of $U(\Delta)$ and prove that it must increase in $\beta$. More precisely, we require $\Delta^*$ to be a strict local maximizer — that is, $\Delta^*$ uniquely maximizes $U(\Delta)$ in some neighborhood — for $\Delta^*(\beta)$ to be a well-defined function.[5]

**Theorem 1.** *Consider $\beta > 0.5$ and let $\Delta^*(\beta) \in (0,1)$ be a strict local maximizer of the Utilitarian objective $U(\Delta)$. Then $\Delta^*(\beta)$ is strictly increasing in $\beta$.*

Theorem 1 confirms initial expectations that the optimal diversity always increases as jobs become less complementary. According to Theorem 1, once the

[4]Finding the least restrictive set of sufficient conditions which ensure the uniqueness of the optimal diversity $\Delta^*$ is a challenging task. To stay focused on our main goal — the comparative statics of $\Delta^*$ with respect to $\beta$ — we set this task aside.

[5]Alternatively, Theorem 1 can be formulated in terms of minimal and maximal optimal diversity: if $\Delta^*(\beta) \in (0,1)$ is a minimum (maximum) diversity that maximizes the Utilitarian objective $U(\Delta)$, then it is strictly increasing in $\beta$. IN general, as long as function $\Delta^*(\beta)$ is well-defined, the comparative statics result in Theorem 1 holds.

optimal diversity becomes positive at some $\beta > 0.5$, it strictly increases until either $\Delta^* = 1$ or $\beta = 1$, whichever happens first.

**Leading example.** If the organizers of a student competition commit to randomly choose a single subject for the competition assignment, then, to maximize the winning chances, the team should be homogeneous, i.e., with a strong focus on a single subject. Then, if the team is lucky and the chosen subject coincides with the team focus, they have very good chances winning the competition. However, if the organizers commit to include several subjects in the assignment, then the successful team must be heterogeneous.

Below we provide an example to illustrate Proposition 1 and Theorem 1. In this example, the uniqueness of the optimal diversity is easy to establish.

**Example 1.** Suppose $\omega_s(x_1, x_2) = \max\{x_1, x_2\}$ and $\omega_c(x_1, x_2) = \min\{x_1, x_2\}$. Then the optimal diversity is unique for all $\beta \in [0,1]$ and equal to 0 for $\beta \in [0, 0.5]$. For $\beta \in [0.5, 1]$, the optimal diversity increases from 0 to 0.5; at each $\beta \in [0.5, 1]$, the optimal diversity uniquely solves

$$\beta\left(v\left(\frac{\Delta^*}{2}\right) - v\left(\frac{1-\Delta^*}{2}\right)\right) = (1-\beta)\left(v\left(\frac{\Delta^*}{2}\right) - v\left(\frac{1+\Delta^*}{2}\right)\right). \tag{7}$$

The difference $v\left(\frac{\Delta^*}{2}\right) - v\left(\frac{1-\Delta^*}{2}\right)$ in left-hand side of (7) is the marginal benefit from increasing diversity when the task is disjunctive. It is equal to the difference in performance of the most productive agent when the task at at the center ($t = 0.5$) and when the task is at the corner ($t = 0$ or $t = 1$). Similarly, the difference $v\left(\frac{\Delta^*}{2}\right) - v\left(\frac{1+\Delta^*}{2}\right)$ in right-hand side of (7) is the marginal benefit from decreasing diversity when the task is conjunctive. Thus, equation (7) equates marginal benefits from increasing and decreasing diversity, with respective weights. It is intuitive from (7) and consistent with Theorem 1 that higher weight on the disjunctive task increases the optimal diversity. This monotonic relationship is illustrated in Figure 1 for $v(d) = -d^2$, in which case equation (7) gives $\Delta^* = \beta - 0.5$ for $\beta > 0.5$.

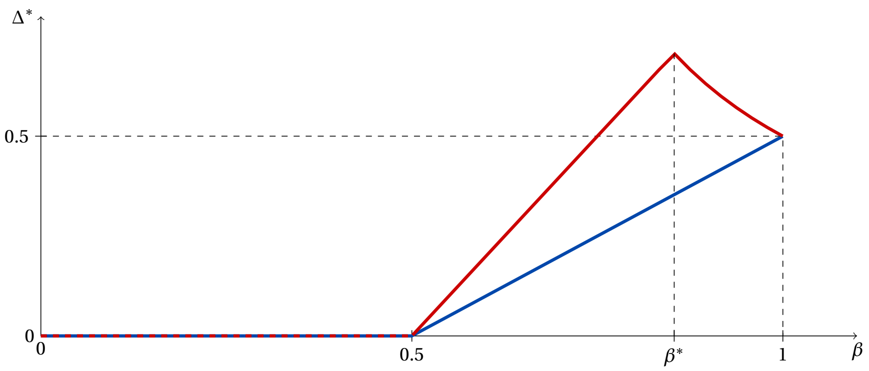



Figure 1: The optimal diversity for $\omega_s(x_1, x_2) = \max\{x_1, x_2\}$, $\omega_c(x_1, x_2) = \min\{x_1, x_2\}$, $v(d) = -d^2$. Blue graph corresponds to the Utilitarian objective; red graph corresponds to the Rawlsian objective.

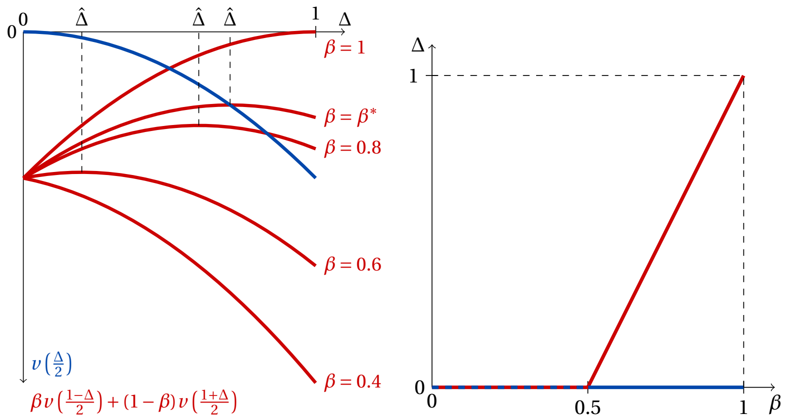
# 4 Rawlsian objective

The Rawlsian objective may deliver qualitatively different comparative statics. To illustrate this point, consider an example.

**Example 2.** Suppose $\omega_s(x_1, x_2) = \max\{x_1, x_2\}$ and $\omega_c(x_1, x_2) = \min\{x_1, x_2\}$. Suppose function $v$ is twice differentiable and strictly concave. Then the Rawlsian objective becomes

$$R = \min\left\{\beta v\left(\frac{1-\Delta}{2}\right) + (1-\beta) v\left(\frac{1+\Delta}{2}\right), v\left(\frac{\Delta}{2}\right)\right\}, \tag{8}$$

where the first term is the joint output of the agents for the corner tasks ($t = 0$ and $t = 1$) and the second term is the joint output for the task at the center ($t = 1/2$). The optimal diversity $\Delta^*$ that maximizes (8) is unique for all $\beta \in [0, 1]$, and it is equal to 0 for $\beta \in [0, 0.5]$ and equal to 0.5 for $\beta = 1$. Moreover, there exists $\beta^* \in (0.5, 1)$ such that the optimal diversity strictly increases for $\beta \in (0.5, \beta^*)$ and strictly decreases for $\beta \in (\beta^*, 1)$. Figure 1 illustrates the hump-shaped form of the optimal diversity function $\Delta^*(\beta)$ for $v(d) = -d^2$, in which case the maximization of (8) gives $\Delta^* = 2\beta - 1$ for $0.5 < \beta < \beta^* = (2 + \sqrt{2})/4 \approx 0.85$ and $\Delta^* = 1/(4\beta - 2)$ for $\beta > \beta^*$.




(a) Agents' joint output

(b) Diversity at the maximum

Figure 2: The diversity depicted on the right maximizes the joint output depicted on the left. Red graphs correspond to the corner tasks ($t = 0$ and $t = 1$). Blue graphs correspond to the center task ($t = 1/2$). Parameters: $\omega_s(x_1, x_2) = \max\{x_1, x_2\}$, $\omega_c(x_1, x_2) = \min\{x_1, x_2\}$, $v(d) = -d^2$.

The intuition is as follows. The joint output for the task at the center, $v\left(\frac{\Delta}{2}\right)$, is maximized at zero diversity (see the blue graphs in Figure 2). Thus, the only reason for the principal to choose a positive diversity can be to increase the joint output for the corner tasks (the first term in (8)).

The red graphs in Figure 2a illustrate the first term in (8), $\beta v\left(\frac{1-\Delta}{2}\right) + (1-\beta)v\left(\frac{1+\Delta}{2}\right)$, which describes the joint output of the agents for the corner tasks. The red graph in Figure 2b depicts the diversity that maximizes the joint output for the corner tasks illustrated in Figure 2a. Suppose that $\beta \leq 0.5$, so that the agents' types are complements. Then increasing diversity lowers the joint output for the corner tasks because it lowers the output for the weakest agent. Thus, the diversity that maximizes the joint output for the corner tasks is zero. Suppose that $\beta > 0.5$, so that the agents' types are substitutes. Then the joint output for the corner tasks is maximized at some positive diversity (for $v(d) = -d^2$, this diversity is equal to $2\beta - 1$). As the agents' types become less complementary (i.e., $\beta$ increases), the diversity that maximizes the joint output is increasing because the output of the weakest agent become less relevant for the joint output and, thus, the agents should be located closer to the corners to increase the output of the strongest agent. At the extreme $\beta = 1$, only the output of the strongest agent contributes to the joint output; thus, the joint output is maximized at the maximum diversity which locates the agents exactly at the corners.

Under the Rawlsian objective (8), the principal wants to increase the joint output both for the tasks at the corners and for the task at the center. When the agents' types are complements ($\beta \leq 0.5$), there is no conflict between the corners and the center task (both the blue and the red graphs in Figure 2b are zero) and the optimal diversity is zero. However, when the agents' types are substitutes ($\beta > 0.5$), the joint output for the corner tasks is maximized at some positive diversity, while the joint output for the center task is still maximized at zero diversity.

For $0.5 < \beta < \beta^*$, the minimum joint output of the agents is achieved only at the corner tasks. Indeed, according to Figure 2a, for $\beta < \beta^*$, the point $\hat{\Delta}$ at which the red graph achieves its maximum is located to the left of the point at which the red graph crosses the blue graph. Hence, the minimum of the two graphs,

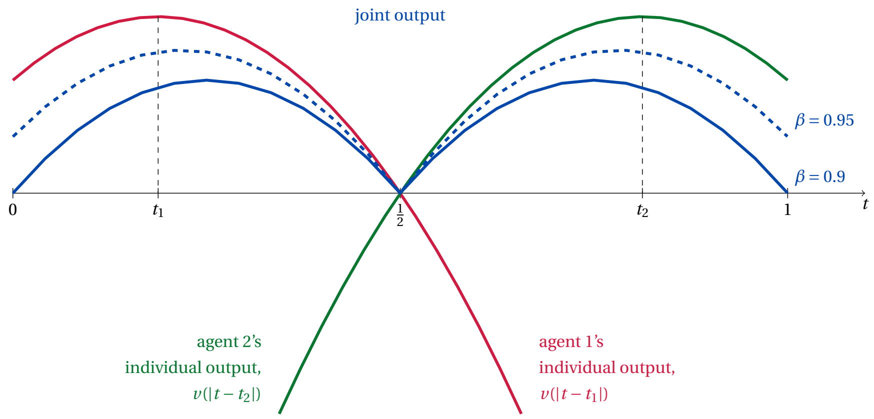



Figure 3: Agents' joint and individual outputs for $\omega_s(x_1, x_2) = \max\{x_1, x_2\}$, $\omega_c(x_1, x_2) = \min\{x_1, x_2\}$, $v(d) = -d^2$. The agents' types are chosen optimally for $\beta = 0.9 > \beta^*$.

which corresponds to the Rawlsian objective (8), is maximized at $\hat{\Delta}$, which implies that the minimum joint output of the agents is achieved only at the corner tasks. Thus, for $\beta < \beta^*$, as the agents' types become less complementary (i.e., as $\beta$ increases), the principal increases diversity to increase the performance of the best agent for the corner tasks.

For $\beta > \beta^*$, the minimum joint output of the agents is achieved both at the corner tasks and at the center task. Indeed, according to Figure 2a, for $\beta > \beta^*$, $\hat{\Delta}$ is located to the right of the point at which the red graph crosses the blue graph. Hence, the minimum of the two graphs, which corresponds to the Rawlsian objective (8), is maximized at the crossing point of the red graph and the blue graph, which implies that the minimum joint output of the agents is achieved both at the corner tasks and at the center task. Thus, for $\beta > \beta^*$, the principal faces the corners/center trade-off: the optimal diversity equalizes the joint output of the agents at the corner tasks and at the center task. Blue solid graph in Figure 3 illustrates the joint output of the agents for all tasks $t \in [0, 1]$ when the agents' types are choices optimally. In agreement with our theoretical result, at the optimal diversity, the joint output of the agents achieves the minimum at the corner tasks and at the center task. Blue dashed graph in Figure 3 illustrates how the joint output changes as the agents' types become less complementary (i.e., as $\beta$ increases) for a fixed diversity. As $\beta$ increases, the joint output moves closer to the function $\omega_s(v(|t-t_1|), v(|t-t_2|)) = \max\{v(|t-t_1|), v(|t-t_2|)\}$, which is the maximum of the red graph and the green graph in Figure 3. According to Figure 3, the joint output of the agents at the corner tasks increases, while their output at the center task does not change. Thus, for $\beta > \beta^*$, as the agents' types become less complementary (i.e., as $\beta$ increases), the principal optimally lowers the diversity to reestablish the balance between the corner tasks and the center task.

The intuition behind non-monotonicity of the optimal diversity under the Rawlsian objective is robust to other functions $\omega_s$ and $\omega_c$. The optimal diversity decreases whenever the minimum joint output is achieved at both the corners and the center. Proposition 2 proves that the corners are always in the set of tasks that minimize the joint output. Proposition 3 states that whenever some task in the center, i.e., in-between the two types ($t \in (t_1, t_2)$), is also a minimizer, the corners/center trade-off emerges and reverses the comparative statics. Proposition 4 confirms that in the absence of the corners/center trade-off, the comparative statics is the same as for the Utilitarian objective. Theorem 2 establishes that the corners/center trade-off appears only for sufficiently high $\beta$, that is, for $\beta > \beta^*$, so that the optimal diversity is either monotone (if $\beta^* = 1$) or has a hump-shaped form (if $\beta^* < 1$).

**Proposition 2.** *Let $\Delta^*$ be a strict local maximizer of $R(\Delta)$. Then, the corner tasks ($t = 0$ and $t = 1$) deliver the minimum in $R(\Delta^*)$.*

**Proposition 3.** *Suppose that $\beta \in (0.5, 1)$ and that function $v$ is differentiable and strictly concave. Let $\Delta^*(\beta)$ be a strict local maximizer of $R(\Delta)$. Suppose further that at $\Delta = \Delta^*(\beta)$, there is a task $t \in (t_1, t_2)$ which delivers the minimum in $R(\Delta)$. Then $\Delta^*(\beta)$ belongs to the interval $(0.5, 1]$ and it is strictly decreasing in $\beta$.*

**Proposition 4.** *Suppose that $\beta \in (0.5, 1)$ and that function $v$ is differentiable and strictly concave. Suppose that $\Delta^*(\beta) \in (0, 1)$ is a strict local maximizer of $R(\Delta)$ and there is no task $t \in (t_1, t_2)$ which delivers the minimum in $R(\Delta)$. Then $\Delta^*(\beta)$ is strictly increasing in $\beta$.*

**Theorem 2.** *Suppose that function $v$ is differentiable and strictly concave. Suppose $\Delta^*(\beta) \in (0,1)$ is a strict local maximizer of $R(\Delta)$. Then, there exists $\beta^* \in [0.5,1]$ such that $\Delta^*(\beta)$ strictly increases for $\beta \in (0.5, \beta^*)$ and strictly decreases for $\beta \in (\beta^*, 1)$.*

Theorem 2 established the existence of threshold $\beta^* \in [0.5,1]$ beyond Example 2: once the optimal diversity becomes positive at some $\beta > 0.5$, it is strictly increasing until either $\Delta^* = 1$ or $\beta = \beta^*$; after $\beta = \beta^*$, the optimal diversity is strictly decreasing. Thus, the optimal diversity decreases as jobs become less complementary ($\beta$ increases) if and only if $\beta^* < 1$. How common is it to have $\beta^* < 1$? Example 2 shows that it is possible. Proposition 5 provides sufficient conditions for $\beta^* < 1$, thus confirming that Example 2 is not an exception.

**Proposition 5.** *Suppose that*

*1) function $v$ is twice differentiable and strictly concave;*

*2) functions $\omega_s$ and $\omega_c$ are twice differentiable;*

*3) the following condition holds for all admissible $x_1$ and $x_2$:*

$$\frac{\partial^2 \omega_s(x_1, x_2)}{\partial x_1 \partial x_2} \max_{d \in [0,1]} \left\{ \frac{v'(d)}{v'(1-d)} \right\} < \frac{\partial^2 \omega_s(x_1, x_2)}{\partial x_1^2} - \frac{\partial \omega_s(x_1, x_2)}{\partial x_1} \max_{d \in [0,1]} \left\{ -\frac{v''(d)}{v'(d)^2} \right\}. \tag{9}$$

*Then there exists $\beta^* < 1$ such that for all $\beta \in (\beta^*, 1)$, $R(\Delta, \beta)$ is uniquely maximized at some $\Delta^*(\beta) \in (0,1)$, and $\Delta^*(\beta)$ strictly decreases for $\beta \in (\beta^*, 1)$.*

In words, condition (9) requires that the submodular component of the joint output function is sufficiently submodular. Indeed, by assumption 1, the maximum on the left-hand side is positive. The submodularity property (2) regulates the sign of the mixed derivative $\frac{\partial^2 \omega_s(x_1, x_2)}{\partial x_1 \partial x_2}$ — see footnote 2. Thus, condition (9) holds whenever the mixed derivative $\frac{\partial^2 \omega_s(x_1, x_2)}{\partial x_1 \partial x_2}$ is sufficiently negative, that is, $\omega_s$ is sufficiently submodular.

Although the conditions 1)-3) are more restrictive than necessary (for example, Example 2 does not satisfy all of them), they still cover a large class of functions, such as the one described in Example 3.

**Example 3.** Suppose that

$$v(d) = \frac{2^{a+1} - (d+1)^{a+1}}{2^{a+1} - 1}, \quad \omega_s(x_1, x_2) = \frac{x_1 + x_2}{2} - k\left(x_1 x_2 + w(x_1) + w(x_2)\right). \quad (10)$$

Then conditions 1)-3) from Proposition 5, in addition to all assumptions listed in Section 2, are satisfied if

$$a > 0, \quad k > 0, \quad w'(x) \geq 0, \quad w''(x) \leq 0, \quad w'''(x) \geq 0, \quad (11)$$

$$1 + w'(0) < \frac{1}{2k} < w'(1) + \frac{a+1}{\left(2^{a+1} - 1\right) a}\left(2^a - w''(1)\right). \quad (12)$$

For example, if $w(x) = 0$, then condition (12) becomes $k \in \left(\frac{(1-2^{-a-1})a}{a+1}, \frac{1}{2}\right)$, which is nonempty for any $a \in (0, 1.5]$. Another example is $w(x) = \ln(x+1)$, $a = 0.01$, $k \in (0.004, 0.25)$.

**Leading example.** If a class assignment is known to cover a single but randomly chosen subject, then, to ensure that any assignment is at least partially completed, each student in the team should have a wide area of expertise. Then, the team members can successfully communicate with each other and together succeed even in a narrowly focused assignment. In contrast, if the assignment is known to potentially cover several subjects, then the optimal team consists of students with very different backgrounds. Indeed, if an assignment is mostly focused on one subject, it will be partially completed by the student with major in this subject but all other students will neither understand nor contribute to the task. At the same time, if an assignment equally covers all subjects and formulated in such a way that all students can participate in the discussion, then, despite that no single member of the team has enough training to complete the assignment, the students' joint efforts allow the team to get a positive (though not highest) mark on this assignment.

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# A Proofs

## A.1 Proof of Lemma 1

We first split each objective function into four parts:

$$U = \underbrace{\int_0^{t_1} \omega\left(v(t_1 - t), v(t_2 - t)\right) \mathrm{d}t}_{U_{01}} + \underbrace{\int_{t_1}^{\frac{t_1+t_2}{2}} \omega\left(v(t - t_1), v(t_2 - t)\right) \mathrm{d}t}_{U_{1m}}$$
$$+ \underbrace{\int_{\frac{t_1+t_2}{2}}^{t_2} \omega\left(v(t - t_1), v(t_2 - t)\right) \mathrm{d}t}_{U_{m2}} + \underbrace{\int_{t_2}^{1} \omega\left(v(t - t_1), v(t - t_2)\right) \mathrm{d}t}_{U_{21}}, \quad \text{(A.1)}$$

$$R = \min\left\{ \underbrace{\min_{t\in[0,t_1]} \omega\left(v(t_1 - t), v(t_2 - t)\right)}_{R_{01}}, \underbrace{\min_{t\in\left[t_1, \frac{t_1+t_2}{2}\right]} \omega\left(v(t - t_1), v(t_2 - t)\right)}_{R_{1m}}, \right.$$
$$\left. \underbrace{\min_{t\in\left[\frac{t_1+t_2}{2}, t_2\right]} \omega\left(v(t - t_1), v(t_2 - t)\right)}_{R_{m2}}, \underbrace{\min_{t\in[t_2,1]} \omega\left(v(t - t_1), v(t - t_2)\right)}_{R_{21}} \right\}. \quad \text{(A.2)}$$

Applying the change of variable $\tau = t_1 + t_2 - t$ and using $\omega(x_1, x_2) = \omega(x_2, x_1)$, which is followed from assumption 3, we get that the third part of each objective is equal to the second part:

$$U_{m2} = \int_{t_1}^{\frac{t_1+t_2}{2}} \omega\left(v(t_2 - \tau), v(\tau - t_1)\right) \mathrm{d}\tau = U_{1m}, \quad \text{(A.3)}$$

$$R_{m2} = \min_{\tau\in\left[t_1, \frac{t_1+t_2}{2}\right]} \omega\left(v(t_2 - \tau), v(\tau - t_1)\right) = R_{1m}. \quad \text{(A.4)}$$

We then rewrite each of the remaining three parts so that each depends on $t_1$ and $t_2$ through the sum $t_1+t_2 \equiv s$ and the difference $t_2-t_1 \equiv \Delta$. Since $0 \le t_1 \le t_2 \le 1$, the difference $t_2-t_1=\Delta$ belongs to $[0,1]$ and, for a fixed $\Delta$, the sum $t_1+t_2=s$ belongs to $[\Delta, 2-\Delta]$.

Applying the change of variable $\tau = \frac{t_1+t_2}{2} - t$, we get that the first part becomes

$$U_{01} = \int_{\Delta/2}^{s/2} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right) \mathrm{d}\tau, \tag{A.5}$$
$$R_{01} = \min_{\tau \in [\Delta/2, s/2]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right).$$

Applying the change of variable $\tau = \frac{t_2-t_1}{2} - t_1 + t$, we get that the second part of each objective depends only on $t_1$ and $t_2$ only through $t_2 - t_1 \equiv \Delta$:

$$U_{1m} = \int_{\Delta/2}^{\Delta} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right) \mathrm{d}\tau, \tag{A.6}$$
$$R_{1m} = \min_{\tau \in [\Delta/2, \Delta]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right).$$

Applying the change of variable $\tau = t - \frac{t_1+t_2}{2}$ and using $\omega(x_1, x_2) = \omega(x_2, x_1)$, we rewrite the forth part of each objective as

$$U_{21} = \int_{\Delta/2}^{1-s/2} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right) \mathrm{d}\tau, \tag{A.7}$$
$$R_{21} = \min_{\tau \in [\Delta/2, 1-s/2]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right).$$

**Utilitarian objective**

To prove that $s=1$ is optimal for any fixed $\Delta \in [0,1]$, we differentiate (A.1) with respect to $s$ and use (A.3), (A.5), (A.6) and (A.7) to get

$$\frac{\partial U(\Delta, s)}{\partial s} = \frac{1}{2}\left\{ \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right)\Big|_{\tau = s/2} - \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right)\Big|_{\tau = 1-s/2} \right\}. \tag{A.8}$$

Since $\omega(x_1, x_2)$ is weakly increasing in each argument by assumption 2 and $\nu$ is strictly decreasing by assumption 1, function $\omega\left(\nu\left(\tau - \frac{\Delta}{2}\right), \nu\left(\tau + \frac{\Delta}{2}\right)\right)$ is weakly decreasing in $\tau$. Thus,

$$\frac{\partial U(\Delta, s)}{\partial s} \geq 0 \text{ for } s < 1, \quad \frac{\partial U(\Delta, s)}{\partial s} \leq 0 \text{ for } s > 1, \tag{A.9}$$

which implies that $s = 1$ is optimal.

**Rawlsian objective**

Since

$$R(\Delta, s) = \min\{R_{01}, R_{1m}, R_{21}\} \tag{A.10}$$

by (A.2) and (A.4) and since $R_{1m}$ in (A.6) does not depend on $s$, to prove that $R(\Delta, s)$ is maximized at $s = 1$, it is sufficient to prove that $\min\{R_{01}, R_{21}\}$ is maximized at $s = 1$. By (A.5) and (A.7),

$$\min\{R_{01}, R_{21}\} = \min_{\tau \in \left[\frac{\Delta}{2}, \max\left\{\frac{s}{2}, 1 - \frac{s}{2}\right\}\right]} \omega\left(\nu\left(\tau - \frac{\Delta}{2}\right), \nu\left(\tau + \frac{\Delta}{2}\right)\right), \tag{A.11}$$

which is weakly decreasing in $\max\left\{\frac{s}{2}, 1 - \frac{s}{2}\right\}$. The minimum of $\max\left\{\frac{s}{2}, 1 - \frac{s}{2}\right\}$ is achieved at $s = 1$.

## A.2 Proof of Proposition 1

For $\beta \leq 0.5$, function $\omega(x_1, x_2)$ is supermodular. Hence, for any $t_1$, $t_2$ and $t$,

$$\begin{aligned} &\omega(\nu(|t - t_1|), \nu(|t - t_2|)) + \omega(\nu(|t - t_2|), \nu(|t - t_1|)) \\ \leq\; &\omega\left(\nu(|t - t_1|), \nu(|t - t_1|)\right) + \omega\left(\nu(|t - t_2|), \nu(|t - t_2|)\right). \end{aligned} \tag{A.12}$$

By assumption 3, function $\omega(x_1, x_2)$ is symmetric, which implies

$$\omega(\nu(|t - t_2|), \nu(|t - t_1|)) = \omega(\nu(|t - t_1|), \nu(|t - t_2|)). \tag{A.13}$$

Substituting $\omega(\nu(|t - t_2|), \nu(|t - t_1|))$ from (A.13) into (A.12), taking the expectation of (A.12) over $t$ and using the definition (5) of the Utilitarian objective, we get

$$2U(t_1, t_2) \leq U(t_1, t_1) + U(t_2, t_2), \tag{A.14}$$

which implies

$$U(t_1,t_2) \le \max\{U(t_1,t_1), U(t_2,t_2)\} \le \max_{t\in[0,1]} U(t,t). \tag{A.15}$$

Since (A.15) holds for any $0 \le t_1 \le t_2 \le 1$ and the right-hand side of (A.15) does not depend on $t_1$ and $t_2$, we can take the maximimum over $t_1$ and $t_2$ of the left-hand side of (A.15) and get

$$\max_{0\le t_1\le t_2\le 1} U(t_1,t_2) \le \max_{t\in[0,1]} U(t,t). \tag{A.16}$$

The right-hand side of (A.16) is obviously less or equal the left-hand side of (A.16). Hence, inequality (A.16) must always hold as equality. Result (A.16) implies that there always exist $t_1^*$ and $t_2^*$ such that $t_1^* = t_2^*$ and they maximize $U(t_1,t_2)$ over $0 \le t_1 \le t_2 \le 1$. By Lemma 1, for any given $t_1 = t_2 = t$, $t = 0.5$ maximizes $U(t,t)$. Hence, $t_1^* = t_2^* = 0.5$ maximize $U(t_1,t_2)$ over $0 \le t_1 \le t_2 \le 1$.

### A.3 Proof of Theorem 1

By the first and the second order conditions, interior $\Delta^*(\beta)$ is a strict local maximizer of the Utilitarian objective $U(\Delta,\beta)$ if and only if

$$\frac{\partial U(\Delta^*(\beta),\beta)}{\partial\Delta} = 0, \quad \frac{\partial^2 U(\Delta^*(\beta),\beta)}{\partial\Delta^2} < 0. \tag{A.17}$$

By the implicit function theorem applied to the equality in (A.17),

$$\frac{\mathrm{d}\Delta^*(\beta)}{\mathrm{d}\beta} = -\left.\frac{\partial^2 U(\Delta^*(\beta),\beta)}{\partial\Delta\partial\beta}\right/\frac{\partial^2 U(\Delta^*(\beta),\beta)}{\partial\Delta^2}. \tag{A.18}$$

The denominator in (A.18) is negative by the inequality in (A.17). Hence, $\Delta^*(\beta)$ is strictly increasing if

$$\frac{\partial^2 U(\Delta^*(\beta),\beta)}{\partial\Delta\partial\beta} > 0. \tag{A.19}$$

Denote by $U_s(\Delta) = U(\Delta,1)$ the Utilitarian objective when the weight on the submodular component is one, and by $U_a(\Delta) = U(\Delta,0.5)$ the Utilitarian objective for an additive task. Then, decomposition (4) implies that for any $\beta \in [0,1]$, $U(\Delta,\beta)$ is a weighted sum of $U_s(\Delta)$ and $U_a(\Delta)$:

$$U(\Delta,\beta) = (2\beta-1)U_s(\Delta) + 2(1-\beta)U_a(\Delta). \tag{A.20}$$

Using (A.20), we can rewrite the equality in (A.17) as

$$(2\beta-1)U_s'(\Delta^*(\beta))+2(1-\beta)U_a'(\Delta^*(\beta))=0 \tag{A.21}$$

and inequality (A.19) as

$$U_s'(\Delta^*(\beta))>U_a'(\Delta^*(\beta)). \tag{A.22}$$

Thus, it is sufficient to show that (A.21) implies (A.22). Substituting $U_s'(\Delta^*(\beta))$ from (A.21) into (A.22), we get

$$-\frac{2(1-\beta)}{2\beta-1}U_a'(\Delta^*(\beta))>U_a'(\Delta^*(\beta)). \tag{A.23}$$

Since $\beta>0.5$ by assumption, (A.23) is equivalent to $U_a'(\Delta^*(\beta))<0$. Thus, to prove the theorem, it is sufficient to prove that

$$U_a'(\Delta)<0, \quad \text{for all } \Delta\in(0,1). \tag{A.24}$$

We calculate the Utilitarian objective for an additive task by substituting $s=1$ and $\omega(x_1,x_2)=0.5(x_1+x_2)$ into the expression (A.1), which is simplified using (A.3), (A.5), (A.6) and (A.7):

$$U_a(\Delta)=\int_{\Delta/2}^{1/2}\left(v\left(\tau-\frac{\Delta}{2}\right)+v\left(\tau+\frac{\Delta}{2}\right)\right)\mathrm{d}\tau+\int_{\Delta/2}^{\Delta}\left(v\left(\tau-\frac{\Delta}{2}\right)+v\left(\frac{3\Delta}{2}-\tau\right)\right)\mathrm{d}\tau. \tag{A.25}$$

Differentiating (A.25) yields

$$U_a'(\Delta)=\frac{1}{2}\left(v\left(\frac{1+\Delta}{2}\right)-v\left(\frac{1-\Delta}{2}\right)\right), \tag{A.26}$$

which is negative for all $\Delta>0$ by assumption 1.

## A.4 Example 1

Rewriting the expression (A.1) using (A.3), (A.5), (A.6) and (A.7), and then substituting $s=1$ and $\omega(x_1,x_2)=\beta\max\{x_1,x_2\}+(1-\beta)\min\{x_1,x_2\}$, we get the following expression for the Utilitarian objective

$$\begin{aligned}U(\Delta,\beta)=2&\int_{\Delta/2}^{1/2}\left(\beta v\left(\tau-\frac{\Delta}{2}\right)+(1-\beta)v\left(\tau+\frac{\Delta}{2}\right)\right)\mathrm{d}\tau \\ &+2\int_{\Delta/2}^{\Delta}\left(\beta v\left(\tau-\frac{\Delta}{2}\right)+(1-\beta)v\left(\frac{3\Delta}{2}-\tau\right)\right)\mathrm{d}\tau.\end{aligned} \tag{A.27}$$

Differentiating (A.27) yields

$$\frac{\partial U(\Delta,\beta)}{\partial \Delta} = \beta\left(v\left(\frac{1+\Delta}{2}\right) - v\left(\frac{1-\Delta}{2}\right)\right) + (1-2\beta)\left(v\left(\frac{1+\Delta}{2}\right) - v\left(\frac{\Delta}{2}\right)\right). \tag{A.28}$$

Suppose $\beta \in [0, 0.5]$. Since function $v$ is strictly decreasing by assumption 1, the derivative (A.29) is negative for all $\Delta > 0$. Thus, the optimal diversity is unique and equal to 0.

Suppose $\beta \in [0.5, 1]$. Rewriting the derivative (A.29) as

$$\frac{\partial U(\Delta,\beta)}{\partial \Delta} = (1-\beta)v\left(\frac{1+\Delta}{2}\right) - \beta v\left(\frac{1-\Delta}{2}\right) + (2\beta-1)v\left(\frac{\Delta}{2}\right), \tag{A.29}$$

we can see that it is strictly decreasing in $\Delta$ because function $v$ is strictly decreasing by assumption 1. Thus, $U(\Delta,\beta)$ is concave in $\Delta$ and, therefore, the optimal diversity is unique. Moreover, the optimal diversity is equal to 0.5 for $\beta = 1$ and belongs to the interval $(0, 0.5)$ for $\beta \in (0.5, 1)$ because

$$\frac{\partial U(0,\beta)}{\partial \Delta} = (2\beta-1)\underbrace{\left(v\left(0\right) - v\left(\frac{1}{2}\right)\right)}_{>0}, \quad \frac{\partial U(0.5,\beta)}{\partial \Delta} = (1-\beta)\underbrace{\left(v\left(\frac{3}{4}\right) - v\left(\frac{1}{4}\right)\right)}_{<0}, \tag{A.30}$$

where the signs follow from assumption 1. Thus, the optimal diversity solves $\frac{\partial U(\Delta^*,\beta)}{\partial \Delta} = 0$, which is equivalent to (7). The optimal diversity increases in $\beta$ because from (A.29),

$$\frac{\partial^2 U(\Delta,\beta)}{\partial \Delta \partial \beta} = \frac{1}{\beta}\left(\frac{\partial U(\Delta,\beta)}{\partial \Delta} + \underbrace{v\left(\frac{\Delta}{2}\right) - v\left(\frac{1+\Delta}{2}\right)}_{>0}\right), \tag{A.31}$$

which implies that the mixed derivative is positive whenever $\frac{\partial U(\Delta,\beta)}{\partial \Delta} = 0$.

## A.5 Example 2

Rewriting the expression (A.2) using (A.4), (A.5), (A.6) and (A.7), and then substituting $s = 1$ and $\omega(x_1, x_2) = \beta \max\{x_1, x_2\} + (1-\beta)\min\{x_1, x_2\}$, we get the following expression for the Rawlsian objective

$$R(\Delta,\beta) = \min\left\{R_{01}(\Delta,\beta), R_{1m}(\Delta,\beta)\right\}, \tag{A.32}$$

$$R_{01}(\Delta,\beta) = \min_{\tau\in[\frac{\Delta}{2},\frac{1}{2}]} \left\{\beta \max\left\{v\left(\tau-\frac{\Delta}{2}\right), v\left(\tau+\frac{\Delta}{2}\right)\right\}\right.$$
$$\left. + (1-\beta)\min\left\{v\left(\tau-\frac{\Delta}{2}\right), v\left(\tau+\frac{\Delta}{2}\right)\right\}\right\}, \tag{A.33}$$

$$R_{1m}(\Delta,\beta) = \min_{\tau\in[\frac{\Delta}{2},\Delta]} \left\{\beta \max\left\{v\left(\tau-\frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2}-\tau\right)\right\}\right.$$
$$\left. + (1-\beta)\min\left\{v\left(\tau-\frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2}-\tau\right)\right\}\right\}. \tag{A.34}$$

By assumption 1, function $v$ is strictly decreasing. Hence,

$$v\left(\tau-\frac{\Delta}{2}\right) \geq v\left(\tau+\frac{\Delta}{2}\right) \quad \text{for all } \tau, \quad v\left(\tau-\frac{\Delta}{2}\right) \geq v\left(\frac{3\Delta}{2}-\tau\right) \quad \text{for } \tau\leq\Delta, \tag{A.35}$$

which allows to rewrite (A.33) and (A.34) as

$$R_{01}(\Delta,\beta) = \min_{\tau\in[\frac{\Delta}{2},\frac{1}{2}]} \left\{\beta v\left(\tau-\frac{\Delta}{2}\right) + (1-\beta)\, v\left(\tau+\frac{\Delta}{2}\right)\right\}, \tag{A.36}$$

$$R_{1m}(\Delta,\beta) = \min_{\tau\in[\frac{\Delta}{2},\Delta]} \left\{\beta v\left(\tau-\frac{\Delta}{2}\right) + (1-\beta)\, v\left(\frac{3\Delta}{2}-\tau\right)\right\}. \tag{A.37}$$

Since $v$ is decreasing, the objective function in (A.36) is decreasing in $\tau$ and the minimum in (A.36) is achieved at $\tau = 1/2$:

$$R_{01}(\Delta,\beta) = \beta v\left(\frac{1-\Delta}{2}\right) + (1-\beta)\, v\left(\frac{1+\Delta}{2}\right). \tag{A.38}$$

The second derivative w.r.t. $\tau$ of the objective function in (A.37) is negative because $v$ is strictly concave by assumption. Hence, the minimum in (A.37) is achieved either at $\tau = \Delta/2$ or at $\tau = \Delta$:

$$R_{1m}(\Delta,\beta) = \min\left\{\beta v(0) + (1-\beta)\, v(\Delta),\, v\left(\frac{\Delta}{2}\right)\right\}. \tag{A.39}$$

Substituting (A.38) and (A.39) into (A.32) and using assumption 1 to claim that $v(0) \geq v\left(\frac{1-\Delta}{2}\right)$ and $v(\Delta) \geq v\left(\frac{1+\Delta}{2}\right)$, we get

$$R(\Delta,\beta) = \min\left\{\beta v\left(\frac{1-\Delta}{2}\right) + (1-\beta)\, v\left(\frac{1+\Delta}{2}\right),\, v\left(\frac{\Delta}{2}\right)\right\}. \tag{A.40}$$

Since $v$ is strictly concave by assumption, the first term in (A.40), $\beta v\left(\frac{1-\Delta}{2}\right) + (1-\beta) v\left(\frac{1+\Delta}{2}\right)$, is strictly concave in $\Delta$. In other words, there exists $\hat{\Delta}(\beta) \in [0,1]$ such that the first term in (A.40) is strictly increasing in $\Delta \in [0, \hat{\Delta}(\beta))$ and strictly decreasing in $\Delta \in (\hat{\Delta}(\beta), 1]$.

Since $v$ is strictly decreasing by assumption 1, the second term in (A.40), $v\left(\frac{\Delta}{2}\right)$, is strictly decreasing in $\Delta \in [0,1]$. Thus, both terms in (A.40) are strictly decreasing in $\Delta \in (\hat{\Delta}(\beta), 1]$, which implies that

$$R(\hat{\Delta}(\beta), \beta) > R(\Delta, \beta) \quad \text{for all } \Delta \in (\hat{\Delta}(\beta), 1]. \tag{A.41}$$

Suppose $\beta \leq 0.5$. Then, at $\Delta = 0$, since $v$ is strictly decreasing, the first term in (A.40) is weakly decreasing in $\Delta$ because its derivative $(0.5-\beta)v'(0.5)$ is non-positive. Hence, $\hat{\Delta}(\beta) = 0$ and both terms in (A.40) are strictly decreasing in $\Delta \in [0,1]$, which implies that $\Delta^* = 0$ uniquely maximizes $R(\Delta, \beta)$.

Suppose $\beta > 0.5$. Then, at $\Delta = 0$, the first derivative of the first term in (A.40), $(0.5-\beta)v'(0.5)$, is positive. Hence, $\hat{\Delta}(\beta) > 0$. For $\Delta \in [0, \hat{\Delta}(\beta))$, the first term in (A.40) is strictly increasing, while the the second term in (A.40) is strictly decreasing. At $\Delta = 0$, since $v$ is strictly decreasing, the first term in (A.40), $v(0.5)$, is strictly less than the second term in (A.40), $v(0)$. Thus, there exists $\Delta^*(\beta) \in (0, \hat{\Delta}(\beta)]$ such that $R(\Delta, \beta)$ is equal to the first term and strictly increasing for $\Delta \in [0, \Delta^*(\beta))$, and equal to the second term and strictly decreasing for $\Delta \in (\Delta^*(\beta), \hat{\Delta}(\beta)]$. This $\Delta^*(\beta)$ uniquely maximizes $R(\Delta, \beta)$ on $\Delta \in [0, \hat{\Delta}(\beta)]$ and, therefore, by (A.41), on $\Delta \in [0,1]$.

Suppose $\beta = 1$. Then, $\hat{\Delta}(\beta) = 1$ because the first term in (A.40) is strictly increasing in $\Delta \in [0,1]$ since $v$ is strictly decreasing. At $\Delta = 0.5$, the first and the second terms in (A.40) are equal. Hence, $\Delta^*(\beta) = 0.5$.

Consider the optimal diversity function $\Delta^*(\beta)$ on $\beta \in (0.5, 1)$.

At $\Delta = 1$, the first derivative of the first term in (A.40) is equal to $-0.5\beta v'(0) + 0.5(1-\beta)v'(1)$. Since $v$ is strictly decreasing, this derivative is positive for $\beta > \bar{\beta}$ and negative for $\beta < \bar{\beta}$, where

$$\bar{\beta} = \frac{v'(1)}{v'(0) + v'(1)}. \tag{A.42}$$

Note that $\bar{\beta} \in (0.5, 1)$ because $v'(1) < v'(0) < 0$ since $v$ is strictly decreasing and strictly concave.

Suppose $\beta \in [\bar{\beta}, 1)$. Then, the first term in (A.40) is strictly increasing for all $\Delta \in [0,1)$, which means that $\hat{\Delta}(\beta) = 1$. At $\Delta = 1$, the first term in (A.40), $\beta v(0) + (1-$

$\beta)v(1)$, is strictly greater than the second term in (A.40), $v(0.5)$, because

$$\begin{aligned}
&\beta v(0)+(1-\beta)v(1)-v(0.5) && \text{(A.43)}\\
&\overset{v(0)>v(1),\,\beta\geq\bar{\beta}}{\geq} \bar{\beta}\,(v(0)-v(0.5))+(1-\bar{\beta})\,(v(1)-v(0.5))\\
&\overset{\bar{d}\in(0.5,1),\,\underline{d}\in(0,0.5)}{=} -\bar{\beta}v'(\underline{d})0.5+(1-\bar{\beta})v'(\bar{d})0.5\\
&\overset{v''<0}{>} -\bar{\beta}v'(0)0.5+(1-\bar{\beta})v'(1)0.5 \overset{\text{(A.42)}}{=} 0.
\end{aligned}$$

Suppose $\beta\in(0.5,\bar{\beta})$. Then, the maximum of the first term in (A.40) is interior, which means that $\hat{\Delta}(\beta)\in(0,1)$ satisfies

$$(1-\beta)v'\left(\frac{1+\Delta}{2}\right)=\beta v'\left(\frac{1-\Delta}{2}\right)\quad \text{at } \Delta=\hat{\Delta}(\beta). \tag{A.44}$$

By the implicit function theorem applied to (A.44),

$$\frac{\mathrm{d}\hat{\Delta}(\beta)}{\mathrm{d}\beta}=\frac{v'\left(\frac{1-\Delta}{2}\right)+v'\left(\frac{1+\Delta}{2}\right)}{\frac{\beta}{2}v''\left(\frac{1-\Delta}{2}\right)+\frac{1-\beta}{2}v''\left(\frac{1+\Delta}{2}\right)}\overset{v'<0,\,v''<0}{>}0\quad \text{at } \Delta=\hat{\Delta}(\beta). \tag{A.45}$$

Consider the difference between the first and the second terms in (A.40) at $\Delta=\hat{\Delta}(\beta)$. This difference is increasing in $\beta$ because

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\beta}\left\{\beta v\left(\frac{1-\hat{\Delta}(\beta)}{2}\right)+(1-\beta)v\left(\frac{1+\hat{\Delta}(\beta)}{2}\right)-v\left(\frac{\hat{\Delta}(\beta)}{2}\right)\right\} && \text{(A.46)}\\
&\overset{\text{(A.44)}}{=}\underbrace{v\left(\frac{1-\hat{\Delta}(\beta)}{2}\right)-v\left(\frac{1+\hat{\Delta}(\beta)}{2}\right)}_{>0 \text{ since } v'<0}-\frac{1}{2}v'\left(\frac{\hat{\Delta}(\beta)}{2}\right)\underbrace{\frac{\mathrm{d}\hat{\Delta}(\beta)}{\mathrm{d}\beta}}_{>0}>0,
\end{aligned}$$

positive at $\beta=\bar{\beta}$ by (A.43) and negative at $\beta=0.5$ because $\hat{\Delta}(0.5)=0$ and $v(0.5)<v(0)$.

Thus, there exists a unique $\beta^*\in(0.5,\bar{\beta})$ such that at $\Delta=\hat{\Delta}(\beta)$, the first term in (A.40) is strictly greater than the second term in (A.40) for all $\beta\in(\beta^*,1)$, and the first term is strictly lower than the second term for all $\beta\in(0.5,\beta^*)$.

Suppose $\beta\in(0.5,\beta^*)$. Then, the optimal diversity $\Delta^*(\beta)$ is equal to $\hat{\Delta}(\beta)$ and, thus, strictly increasing by (A.45).

Suppose $\beta\in(\beta^*,1)$. Then, the optimal diversity $\Delta^*(\beta)$ is less than $\hat{\Delta}(\beta)$ and equalizes the first and the second terms in (A.40):

$$\beta v\left(\frac{1-\Delta^*(\beta)}{2}\right)+(1-\beta)v\left(\frac{1+\Delta^*(\beta)}{2}\right)=v\left(\frac{\Delta^*(\beta)}{2}\right). \tag{A.47}$$

By the implicit function theorem applied to (A.47),

$$\frac{\mathrm{d}\Delta^*(\beta)}{\mathrm{d}\beta} = \frac{v\left(\frac{1-\Delta^*(\beta)}{2}\right) - v\left(\frac{1+\Delta^*(\beta)}{2}\right)}{\frac{\beta}{2}v'\left(\frac{1-\Delta^*(\beta)}{2}\right) - \frac{1-\beta}{2}v'\left(\frac{1+\Delta^*(\beta)}{2}\right) + \frac{1}{2}v'\left(\frac{\Delta^*(\beta)}{2}\right)}. \tag{A.48}$$

The numerator in (A.48) is positive because $v$ is strictly decreasing and $\Delta^*(\beta) > 0$. The denominator in (A.48) is negative because $v'\left(\frac{\Delta^*(\beta)}{2}\right) < 0$ and the first term in (A.40) is increasing at $\Delta = \Delta^*(\beta) < \hat{\Delta}(\beta)$. Thus, (A.48) is negative.

## A.6 Proof of Proposition 2

Due to the symmetry ($t_1 = (1-\Delta)/2$ and $t_2 = (1+\Delta)/2$, $R_{01} = R_{21}$ when $s = 1$ by (A.5) and (A.7), $R_{m2} = R_{1m}$ by (A.4)), we can focus on $t \in [0, 0.5]$:

$$R(\Delta) = \min\{R_{01}(\Delta), R_{1m}(\Delta)\}, \tag{A.49}$$

$$R_{01}(\Delta) = \min_{t\in\left[0,\frac{1-\Delta}{2}\right]} \omega\left(v\left(\frac{1-\Delta}{2} - t\right), v\left(\frac{1+\Delta}{2} - t\right)\right), \tag{A.50}$$

$$R_{1m}(\Delta) = \min_{t\in\left[\frac{1-\Delta}{2},\frac{1}{2}\right]} \omega\left(v\left(t - \frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2} - t\right)\right). \tag{A.51}$$

Function $R_{1m}(\Delta)$ is weakly decreasing in $\Delta$ because the objective function in (A.51) is weakly decreasing in $\Delta$ by assumptions 1 and 2 and the interval $\left[\frac{1-\Delta}{2}, \frac{1}{2}\right]$ over which the objective function is minimized expands as $\Delta$ increases.

Suppose $\Delta = \Delta^* > 0$ is a strict local maximizer of $R(\Delta)$. Then, choosing $\Delta$ slightly lower than $\Delta^*$ must hurt the principal (i.e., must lower $R(\Delta)$). Since $R_{1m}(\Delta)$ is weakly decreasing in $\Delta$, choosing $\Delta$ slightly lower than $\Delta^*$ strictly lowers $R(\Delta)$ only if $R_{1m}(\Delta^*) \geq R_{01}(\Delta^*)$.

If $\Delta^* = 0$, then the interval $\left[\frac{1-\Delta^*}{2}, \frac{1}{2}\right]$ collapses to a single point $t = \frac{1-\Delta^*}{2}$, which also belongs to the interval $\left[0, \frac{1-\Delta^*}{2}\right]$ in (A.50). Thus, $R_{1m}(\Delta^*) \geq R_{01}(\Delta^*)$.

Thus, if $\Delta^*$ is a strict local maximizer of $R(\Delta)$, then $R_{1m}(\Delta^*) \geq R_{01}(\Delta^*)$, which implies that the minimum under the Rawlsian objective is achieved at $t \in \left[0, \frac{1-\Delta}{2}\right]$.

For the interval $t \in \left[0, \frac{1-\Delta}{2}\right]$, the minimum is achieved at $t = 0$:

$$R_{01}(\Delta) = \omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right), \tag{A.52}$$

because the objective function in (A.50) is increasing in $t$ by assumptions 1 and 2. Thus, the minimum under the Rawlsian objective is achieved at $t = 0$.

### A.7 Proof of Proposition 3

Before we proceed to the main argument, we prove the following claim:

**Claim A.1.** For $\beta \in (0.5, 1)$, function $\omega$ is strictly increasing in each argument.

*Proof.* By decomposition (4), for $\beta \in (0.5, 1)$, $\omega$ is strictly increasing in each argument by assumption 2. □

By assumption, there exists a task $t \in [t_1, t_2]$ which delivers the minimum in $R(\Delta)$ at $\Delta = \Delta^*(\beta)$. Thus, since $t = 0$ also delivers the minimum in $R(\Delta)$ by Proposition 2, (A.51) is equal to (A.50):

$$F(\Delta, \beta) \equiv R_{1m}(\Delta, \beta) - R_{01}(\Delta, \beta) = 0 \quad \text{at } \Delta = \Delta^*(\beta). \tag{A.53}$$

Function $F(\Delta, \beta)$ can be equal to 0 only for $\Delta > 0.5$. To see it, we use expressions (A.5) (with $s = 1$) and (A.6) for $R_{01}$ and $R_{1m}$, respectively:

$$\begin{aligned} R_{01} &= \min_{\tau \in [\Delta/2, 1/2]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right), \\ R_{1m} &= \min_{\tau \in [\Delta/2, \Delta]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right). \end{aligned} \tag{A.54}$$

Since function $v$ is strictly decreasing by assumption 1,
$v\left(\tau + \frac{\Delta}{2}\right) < v\left(\frac{3\Delta}{2} - \tau\right)$ for all $\tau > \Delta/2$. Then, since function $\omega$ is strictly increasing in the second argument by Claim A.1,

$$\omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right) < \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right), \quad \text{for all } \tau > \frac{\Delta}{2}. \tag{A.55}$$

Consider any $\Delta \leq 0.5$. Then, (A.54) and (A.55) imply

$$R_{1m} \overset{(A.54),\, \Delta \leq 0.5}{\geq} \min_{\tau \in [\Delta/2, 1/2]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right) \overset{(A.54),(A.55)}{\geq} R_{01}. \tag{A.56}$$

Moreover, the second inequality in (A.56) becomes equality only if $\tau = \Delta/2$ delivers the minimum in $R_{01}$. However, $\tau = \Delta/2$ never delivers the minimum in $R_{01}$ because $\omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\tau + \frac{\Delta}{2}\right)\right)$ is strictly decreasing in $\tau$ by assumption 1 and

Claim A.1. Thus, $R_{1m} > R_{01}$ for all $\Delta \le 0.5$, which implies that $\Delta^*(\beta)$ belongs to the interval $(0.5, 1]$.

Following the above logic, we also argue that function $F(\Delta,\beta)$ can be equal to 0 only if

$$R_{1m} = \min_{\tau\in[1/2,\Delta]} \omega\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right); \tag{A.57}$$

that is, given that $\tau = \Delta - 0.5 + t$ by the change of variables described before (A.6), the task at which the minimum in $R_{1m}$ is achieved must belong to the interval $[2t_1, 0.5]$. Since by (A.53), $F(\Delta,\beta) = 0$ at $\Delta = \Delta^*(\beta)$, equality (A.57) holds whenever $\Delta = \Delta^*(\beta)$.

Function $F(\Delta,\beta)$ is strictly decreasing in $\Delta$ at point $\Delta = \Delta^*(\beta)$. Indeed, as we argue after (A.51), $R_{1m}(\Delta,\beta)$ is weakly decreasing in $\Delta$. Thus, to prove that $F(\Delta,\beta)$ is strictly decreasing in $\Delta$ at point $\Delta = \Delta^*(\beta)$, it is sufficient to argue that $R_{01}(\Delta,\beta)$ is strictly increasing in $\Delta$ at point $\Delta = \Delta^*(\beta)$. Towards contradiction, suppose $R_{01}(\Delta,\beta)$ is weakly decreasing in $\Delta$ at point $\Delta = \Delta^*(\beta)$. Then, since $R_{1m}(\Delta,\beta)$ is weakly decreasing in $\Delta$ and since $\Delta^*(\beta)$ is positive, choosing $\Delta$ slightly lower $\Delta^*(\beta)$ is feasible and must weakly increase both $R_{01}(\Delta,\beta)$ and $R_{1m}(\Delta,\beta)$ — hence, weakly increase $R(\Delta,\beta)$. Thus, choosing $\Delta$ slightly lower $\Delta^*(\beta)$ weakly benefits the principal, which contradicts the assumption that $\Delta^*(\beta)$ is a strict local maximizer.

Since function $F(\Delta,\beta)$ is strictly decreasing in $\Delta$ at point $\Delta = \Delta^*(\beta)$, to prove that $\Delta^*(\beta)$ is strictly decreasing, by the implicit function theorem applied to (A.53), it is sufficient to show that $F(\Delta,\beta)$ is strictly decreasing in $\beta$ at point $\Delta = \Delta^*(\beta)$.

Applying decomposition (4) to (A.52) and to (A.57), we get

$$R_{01}(\Delta,\beta) = (2\beta-1)\omega_s\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right) + (1-\beta)\left(v\left(\frac{1-\Delta}{2}\right) + v\left(\frac{1+\Delta}{2}\right)\right), \tag{A.58}$$

$$\begin{aligned} R_{1m}(\Delta,\beta) = &\min_{\tau\in[1/2,\Delta]} (2\beta-1)\omega_s\left(v\left(\tau - \frac{\Delta}{2}\right), v\left(\frac{3\Delta}{2} - \tau\right)\right) \\ &+ (1-\beta)\left(v\left(\tau - \frac{\Delta}{2}\right) + v\left(\frac{3\Delta}{2} - \tau\right)\right) \quad \text{at } \Delta = \Delta^*(\beta). \end{aligned} \tag{A.59}$$

Expressions (A.58) and (A.59), together with the definition (A.53) of $F(\Delta,\beta)$, imply

$$\frac{\partial F(\Delta,\beta)}{\partial\beta} = \frac{2}{2\beta-1}F(\Delta,\beta) + \frac{1}{2\beta-1}\left(v\left(\frac{1-\Delta}{2}\right) + v\left(\frac{1+\Delta}{2}\right)\right.$$
$$\left. - v\left(\tau(\Delta)-\frac{\Delta}{2}\right) - v\left(\frac{3\Delta}{2}-\tau(\Delta)\right)\right) \quad \text{at } \Delta = \Delta^*(\beta), \tag{A.60}$$

where $\tau(\Delta) \in [1/2, \Delta]$ delivers the minimum in (A.59). Since $F(\Delta,\beta) = 0$ by (A.53), the derivative (A.60) becomes

$$\frac{\partial F(\Delta,\beta)}{\partial\beta} = \frac{f(\tau(\Delta),\Delta)}{2\beta-1} \quad \text{at } \Delta = \Delta^*(\beta), \tag{A.61}$$

where

$$f(\tau,\Delta) \equiv v\left(\frac{1-\Delta}{2}\right) + v\left(\frac{1+\Delta}{2}\right) - v\left(\tau-\frac{\Delta}{2}\right) - v\left(\frac{3\Delta}{2}-\tau\right). \tag{A.62}$$

Function $f(\tau,\Delta)$ is strictly convex in $\tau$ since $v$ is strictly concave. Then, since the derivative $\frac{\partial f(\tau,\Delta)}{\partial\Delta}$ is equal to 0 at $\tau = \Delta$, function $f(\tau,\Delta)$ is strictly decreasing in $\tau \in [1/2, \Delta]$. At $\tau = 1/2$, $f\left(\frac{1}{2},\Delta\right) = v\left(\frac{1+\Delta}{2}\right) - v\left(\frac{3\Delta-1}{2}\right)$ is negative for $\Delta < 1$ by assumption 1 and equal to 0 at $\Delta = 1$. Thus, $f(\tau,\Delta) < 0$ for all $\tau \in [1/2, \Delta]$ if $\Delta < 1$, and $f(\tau,\Delta) < 0$ for all $\tau \in (1/2, \Delta]$ if $\Delta = 1$.

Suppose $\Delta = \Delta^*(\beta) < 1$. Then, the derivative (A.61) is negative because $\frac{1}{2\beta-1} > 0$ since $\beta > 0.5$ by assumption and because $f(\tau,\Delta) < 0$ for all $\tau \in [1/2, \Delta]$.

Suppose $\Delta = \Delta^*(\beta) = 1$. Then there exists $\tau > 1/2$ which delivers the minimum in (A.59). This follows from the assumption that there is a task $t \in (t_1, t_2)$ which delivers the minimum in $R(\Delta)$. Indeed, the interval $(t_1, t_2)$ becomes $(0, 1)$ at $\Delta = 1$; then, given that $\tau = \Delta - 0.5 + t = 0.5 + t$ by the change of variables described before (A.6), a minimizer $t \in (0, 0.5]$ corresponds to a minimizer $\tau \in (0.5, 1]$. Then, the derivative (A.61) is negative because $\frac{1}{2\beta-1} > 0$ since $\beta > 0.5$ by assumption and because $f(\tau,\Delta) < 0$ for all $\tau \in (1/2, \Delta]$.

## A.8 Proof of Proposition 4

Since there is no task $t \in (t_1, t_2)$ which delivers the minimum in $R(\Delta)$, $R_{01} < R_{1m}$. Thus, $\Delta^*(\beta)$ is a strict local maximizer of $R_{01}(\Delta,\beta)$. Hence, to show that $\Delta^*(\beta)$ is strictly increasing, it is sufficient to show that the mixed derivative $\frac{\partial R_{01}(\Delta,\beta)}{\partial\beta}$ is positive at $\Delta = \Delta^*(\beta)$.

By (A.58),

$$\frac{\partial R_{01}(\Delta,\beta)}{\partial\beta} = \frac{2}{2\beta-1}R_{01}(\Delta,\beta) - \frac{1}{2\beta-1}\left(v\left(\frac{1-\Delta}{2}\right)+v\left(\frac{1+\Delta}{2}\right)\right). \tag{A.63}$$

Hence,

$$\frac{\partial^2 R_{01}(\Delta,\beta)}{\partial\beta\partial\Delta} = \frac{2}{2\beta-1}\frac{\partial R_{01}(\Delta,\beta)}{\partial\Delta} + \frac{1}{2(2\beta-1)}\left(v'\left(\frac{1-\Delta}{2}\right)-v'\left(\frac{1+\Delta}{2}\right)\right). \tag{A.64}$$

At $\Delta = \Delta^*(\beta)$, the mixed derivative (A.64) is positive because $\frac{\partial R_{01}(\Delta,\beta)}{\partial\Delta} = 0$ by optimality of $\Delta = \Delta^*(\beta)$, $\beta > 0.5$ by assumption, and $v'\left(\frac{1-\Delta}{2}\right) > v'\left(\frac{1+\Delta}{2}\right)$ because $v$ is strictly concave and $\Delta = \Delta^*(\beta) > 0$.

## A.9 Proof of Theorem 2

Suppose there exist $0.5 < \underline{\beta} < \bar{\beta} < 1$ such that $\Delta^*(\underline{\beta}) = \Delta^*(\bar{\beta}) \equiv \check{\Delta} \in (0,1)$ and $\Delta^*(\beta)$ is weakly decreasing at $\beta = \underline{\beta}$. Then, by Proposition 4, there is a task $t \in (t_1, t_2)$ which delivers the minimum in $R(\check{\Delta}, \underline{\beta})$. Hence, $F(\check{\Delta}, \underline{\beta}) = 0$, where $F$ is defined in (A.53). Following the argument in the proof of Proposition 3, we conclude that $F(\check{\Delta}, \beta)$ is decreasing in $\beta$ whenever
$F(\check{\Delta}, \beta) = 0$ and $\beta \in (0.5, 1)$. Then, since $F(\check{\Delta}, \underline{\beta}) = 0$, $F(\check{\Delta}, \bar{\beta})$ must be negative. Hence, there is a task $t \in (t_1, t_2)$ which delivers the minimum in $R(\check{\Delta}, \bar{\beta})$. Then, by Proposition 3, $\Delta^*(\beta)$ is strictly decreasing at $\beta = \bar{\beta}$.

## A.10 Proof of Proposition 5

In light of Proposition 3 and Theorem 2, to prove Proposition 5, it is sufficient to show that there exists $\beta^* \in (0.5, 1)$ such that for all $\beta \in (\beta^*, 1)$, conditions in Proposition 3 hold, that is, $R(\Delta)$ is uniquely maximized at some $\Delta^* \in (0,1)$ and there is a task $t \in (t_1, t_2)$ which delivers the minimum in $R(\Delta^*)$.

By decomposition (4), there exists $\beta^* \in (0.5, 1)$ such that for all $\beta \in (\beta^*, 1)$, inequality (9) translates to

$$\frac{\partial^2\omega(x_1,x_2)}{\partial x_1\partial x_2}\max_{d\in[0,1]}\left\{\frac{v'(d)}{v'(1-d)}\right\} < \frac{\partial^2\omega(x_1,x_2)}{\partial x_1^2} - \frac{\partial\omega(x_1,x_2)}{\partial x_1}\max_{d\in[0,1]}\left\{-\frac{v''(d)}{v'(d)^2}\right\}. \tag{A.65}$$

By (A.49), sufficient conditions for $R(\Delta)$ to be uniquely maximized at some $\Delta^* \in (0,1)$ and for the existence of a task $t \in (t_1, t_2)$ which delivers the minimum in $R(\Delta^*)$ are the following:

a) $R_{01}(\Delta)$ is strictly increasing in $\Delta \in (0,1)$;

b) $R_{1m}(\Delta)$ is strictly decreasing in $\Delta \in (0,1)$;

c) $R_{01}(0) < R_{1m}(0)$;

d) $R_{01}(1) > R_{1m}(1)$.

Condition a) holds for all $\beta \in (\beta^*, 1)$ because

$$R'_{01}(0) \overset{(A.52)}{=} \frac{1}{2} v'\left(\frac{1}{2}\right)\left(\frac{\partial\omega\left(v\left(\frac{1}{2}\right), v\left(\frac{1}{2}\right)\right)}{\partial x_1} - \frac{\partial\omega\left(v\left(\frac{1}{2}\right), v\left(\frac{1}{2}\right)\right)}{\partial x_2}\right) \tag{A.66}$$

is equal to 0 by the symmetry assumption 3, and

$$\begin{aligned} R''_{01}(\Delta) \overset{(A.52)}{=} \frac{v'\left(\frac{1-\Delta}{2}\right)^2}{4} &\left(\frac{v''\left(\frac{1-\Delta}{2}\right)}{v'\left(\frac{1-\Delta}{2}\right)^2} \frac{\partial\omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right)}{\partial x_1}\right. \\ &\left.+ \frac{\partial^2\omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right)}{\partial x_1^2} - \frac{v'\left(\frac{1+\Delta}{2}\right)}{v'\left(\frac{1-\Delta}{2}\right)} \frac{\partial^2\omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right)}{\partial x_1 \partial x_2}\right) \\ + \frac{v'\left(\frac{1+\Delta}{2}\right)^2}{4} &\left(\frac{v''\left(\frac{1+\Delta}{2}\right)}{v'\left(\frac{1+\Delta}{2}\right)^2} \frac{\partial\omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right)}{\partial x_2}\right. \\ &\left.+ \frac{\partial^2\omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right)}{\partial x_2^2} - \frac{v'\left(\frac{1-\Delta}{2}\right)}{v'\left(\frac{1+\Delta}{2}\right)} \frac{\partial^2\omega\left(v\left(\frac{1-\Delta}{2}\right), v\left(\frac{1+\Delta}{2}\right)\right)}{\partial x_1 \partial x_2}\right) \end{aligned} \tag{A.67}$$

is positive: each line in (A.67) is positive by assumptions 1 and 3 and condition (A.65); differentiation in (A.66) and (A.67) is a valid operation because functions $v$ and $\omega$ are twice differentiable by assumptions 1) and 2) of Proposition 5.

Condition b) holds for all $\beta \in (0.5, 1)$ because the objective function in (A.51) is strictly decreasing in $\Delta$ by assumption 1 and Claim A.1 and the interval $\left[\frac{1-\Delta}{2}, \frac{1}{2}\right]$ over which the objective function is minimized expands as $\Delta$ increases.

Condition c) holds for all $\beta \in (0.5, 1)$ because

$$R_{01}(0) - R_{1m}(0) \overset{(A.52),(A.51)}{=} \omega\left(v\left(\frac{1}{2}\right), v\left(\frac{1}{2}\right)\right) - \omega\left(v(0), v(0)\right) \tag{A.68}$$

is negative by assumption 1 and Claim A.1.

Condition d) holds for all $\beta \in (\beta^*, 1)$. Indeed,

$$R_{01}(1) - R_{1m}(1) \overset{(A.52),(A.51)}{=} \omega\left(v(0), v(1)\right) - \min_{t \in [0, \frac{1}{2}]} \omega\left(v(t), v(1-t)\right), \tag{A.69}$$

is positive because function $\omega\left(v(t), v(1-t)\right)$ is strictly decreasing in $t \in \left[0, \frac{1}{2}\right]$:

$$\left.\frac{\mathrm{d}\omega\left(v(t), v(1-t)\right)}{\mathrm{d}t}\right|_{t=1/2} = v'\left(\frac{1}{2}\right)\left(\frac{\partial\omega\left(v\left(\frac{1}{2}\right), v\left(\frac{1}{2}\right)\right)}{\partial x_1} - \frac{\partial\omega\left(v\left(\frac{1}{2}\right), v\left(\frac{1}{2}\right)\right)}{\partial x_2}\right) \tag{A.70}$$

is equal to 0 by the symmetry assumption 3, and

$$\begin{aligned}\frac{\mathrm{d}^2\omega\left(v(t), v(1-t)\right)}{\mathrm{d}t^2} &= v'(t)^2\left(\frac{v''(t)}{v'(t)^2}\frac{\partial\omega\left(v(t), v(1-t)\right)}{\partial x_1}\right. \\ &\quad \left. + \frac{\partial^2\omega\left(v(t), v(1-t)\right)}{\partial x_1^2} - \frac{v'(1-t)}{v'(t)^2}\frac{\partial^2\omega\left(v(t), v(1-t)\right)}{\partial x_1 \partial x_2}\right) \\ &\quad + v'(1-t)^2\left(\frac{v''(1-t)}{v'(1-t)^2}\frac{\partial\omega\left(v(t), v(1-t)\right)}{\partial x_2}\right. \\ &\quad \left. + \frac{\partial^2\omega\left(v(t), v(1-t)\right)}{\partial x_2^2} - \frac{v'(t)}{v'(1-t)}\frac{\partial^2\omega\left(v(t), v(1-t)\right)}{\partial x_1 \partial x_2}\right)\end{aligned} \tag{A.71}$$

is positive by the argument similar to the one we use to show that (A.67) is positive.

## A.11 Example 3

Assumption 1 holds because

$$v'(d) \overset{(10)}{=} -\frac{(a+1)(d+1)^a}{2^{a+1}-1} \tag{A.72}$$

is negative for all $a > 0$, $d \in [0, 1]$.

Given that function $v(d)$ defined in (10) is decreasing in $d \in [0, 1]$ from 1 to 0, the range on which the joint output function $\omega(x_1, x_2)$ must satisfy the desired properties is

$$0 \le x_1 \le 1, \quad 0 \le x_2 \le 1. \tag{A.73}$$

Assumption 3 holds because function $\omega_s(x_1, x_2)$ defined in (10) is symmetric in its arguments.

By symmetry, to verify assumption 2, it is sufficient to show the monotonicity of functions $\omega_s$ and $\omega_c$ with respect to the first argument:

$$\frac{\partial\omega_s(x_1,x_2)}{\partial x_1} \overset{(10)}{=} \frac{1}{2} - k\left(x_2 + w'(x_1)\right) \overset{(A.73),\,(11)}{\geq} \frac{1}{2} - k\left(1 + w'(0)\right) \overset{(11),\,(12)}{>} 0, \tag{A.74}$$

$$\frac{\partial\omega_c(x_1,x_2)}{\partial x_1} \overset{\text{assumption 4}}{=} 1 - \frac{\partial\omega_s(x_1,x_2)}{\partial x_1} \overset{(A.74)}{=} \frac{1}{2} + k\left(x_2 + w'(x_1)\right) \overset{(A.73),\,(11)}{>} 0. \tag{A.75}$$

Function $\omega_s(x_1,x_2)$ defined in (10) is submodular because

$$\frac{\partial^2\omega_s(x_1,x_2)}{\partial x_1\partial x_2} = -k \overset{(11)}{<} 0. \tag{A.76}$$

Condition 1) from Proposition 5 holds because

$$v''(d) \overset{(10)}{=} -\frac{a(a+1)(d+1)^{a-1}}{2^{a+1}-1} \tag{A.77}$$

is negative for all $a>0$, $d\in[0,1]$.

Condition 2) from Proposition 5 holds if function $w$ is twice differentiable.

Condition 3) from Proposition 5 holds because

$$\max_{d\in[0,1]}\left\{-\frac{v''(d)}{v'(d)^2}\right\} \overset{(A.72),\,(A.77)}{=} \max_{d\in[0,1]} \frac{\left(2^{a+1}-1\right)a}{(a+1)(d+1)^{a+1}} = \frac{\left(2^{a+1}-1\right)a}{a+1}, \tag{A.78}$$

$$\max_{d\in[0,1]}\left\{\frac{v'(d)}{v'(1-d)}\right\} \overset{(A.72)}{=} \max_{d\in[0,1]}\left(\frac{1+d}{2-d}\right)^a = 2^a, \tag{A.79}$$

$$\begin{aligned}
&\frac{\partial^2\omega_s(x_1,x_2)}{\partial x_1^2} - \frac{\partial\omega_s(x_1,x_2)}{\partial x_1}\frac{\left(2^{a+1}-1\right)a}{a+1} - \frac{\partial^2\omega_s(x_1,x_2)}{\partial x_1\partial x_2}2^a \\
&\quad\overset{(10)}{=} -kw''(x_1) - \left(\frac{1}{2} - k\left(x_2 + w'(x_1)\right)\right)\frac{\left(2^{a+1}-1\right)a}{a+1} + 2^a k \\
&\quad\overset{(A.73),\,(11)}{\geq} -kw''(1) - \left(\frac{1}{2} - kw'(1)\right)\frac{\left(2^{a+1}-1\right)a}{a+1} + 2^a k \overset{(11),\,(12)}{>} 0.
\end{aligned} \tag{A.80}$$

**Разнообразие в командах** [Электронный ресурс] : препринт WP9/2023/03 / М. Донг, Т. Майская, В. Смирнов, О. Тейлор, Э. Уэйт ; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (200 Кб). – М. : Изд. дом Высшей школы экономики, 2023. – (Серия WP9 «Исследования по экономике и финансам»). – 38 с. (На англ. яз.)

Каково оптимальное разнообразие экспертизы в команде? Прат (2002) показывает, что супермодульная производственная функция (описывающая стратегическую взаимодополняемость между отдельными продуктами) предполагает меньшее оптимальное разнообразие, чем субмодульная функция (которая соответствует стратегической взаимозаменяемости). Мы исследуем, как меняется оптимальное разнообразие по мере непрерывного изменения производственной функции от супермодулярной к субмодулярной. Утилитарная цель дает интуитивную монотонную связь. Однако цель Ролза, оптимизирующая наихудший сценарий, порождает немонотонную зависимость: оптимальное разнообразие может падать с увеличением веса субмодульного компонента производственной функции.

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*Препринт WP9/2023/03*
*Серия WP9*
*Исследования по экономике и финансам*

Донг Миаомиао, Майская Татьяна,
Смирнов Владимир, Тейлор Оливия, Уэйт Эндрю

**Разнообразие в командах**

(*на английском языке*)

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