



NATIONAL RESEARCH
UNIVERSITY

National Research University Higher School of Economics
International Centre of Decision Choice and Analysis

NEW METHODS OF PATTERN ANALYSIS: THEORETICAL ASPECTS AND PRACTICAL APPLICATION

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Autumn School “Advances in Decision Analysis”



Internet Live Stats (2022):

5,2

billion Internet users worldwide

1,9

billion sites

176

billion e-mail sent (for one day)

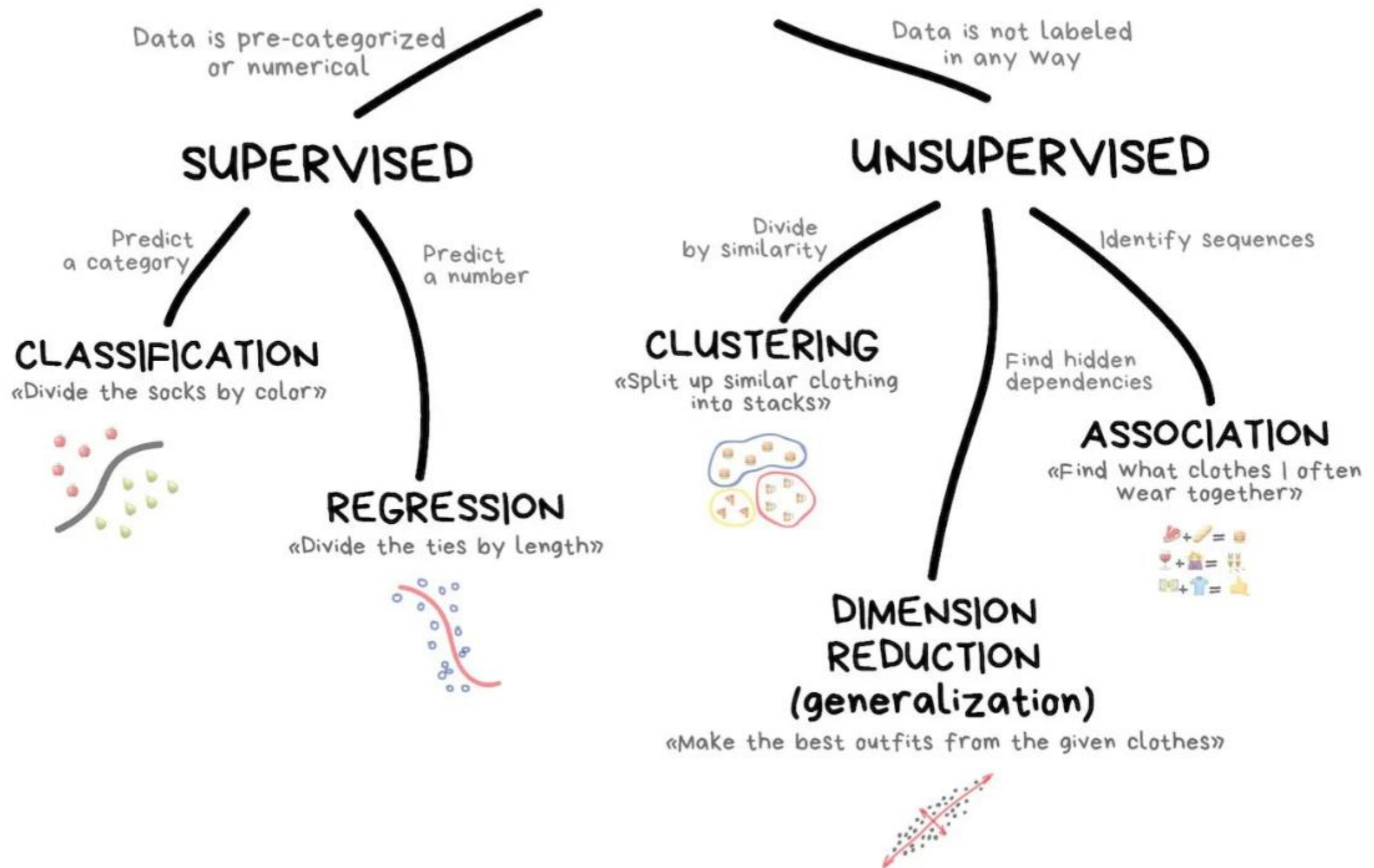
5,1

billion videos viewed on YouTube (for one day)

8

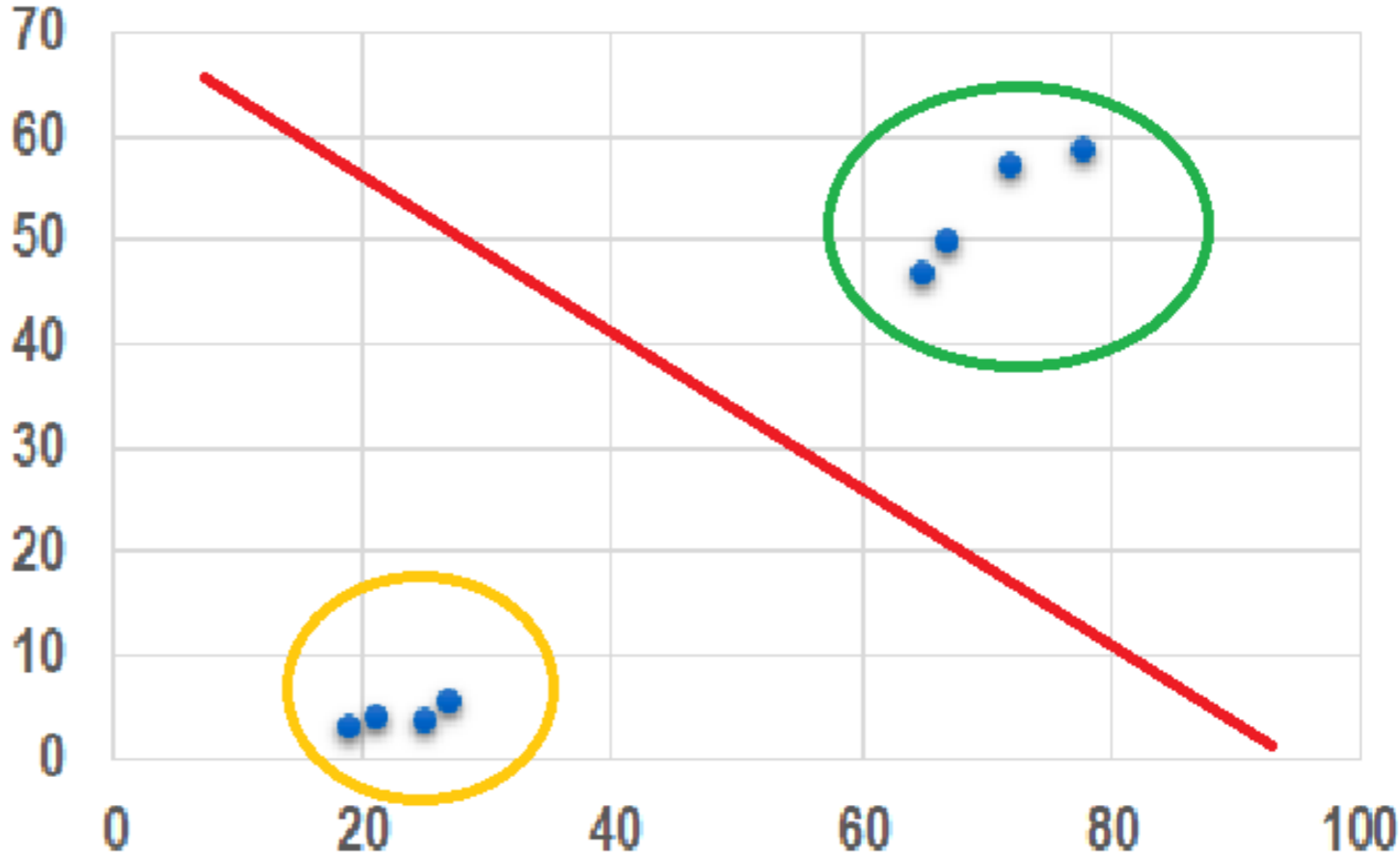
billion GB of Internet traffic (for one day)

CLASSICAL MACHINE LEARNING



CLASSIFICATION

	Height	Weight
Dog	65	46,8
Dog	67	49,9
???	72	57
Dog	78	58,6
Cat	25	3,5
Cat	27	5,5
???	21	4
Cat	19	3



Human vs Machine: Lung Tumor

Chest X-Rays image the lungs, heart, blood vessels, and bones. AI has been used to read and understand them.

Example:

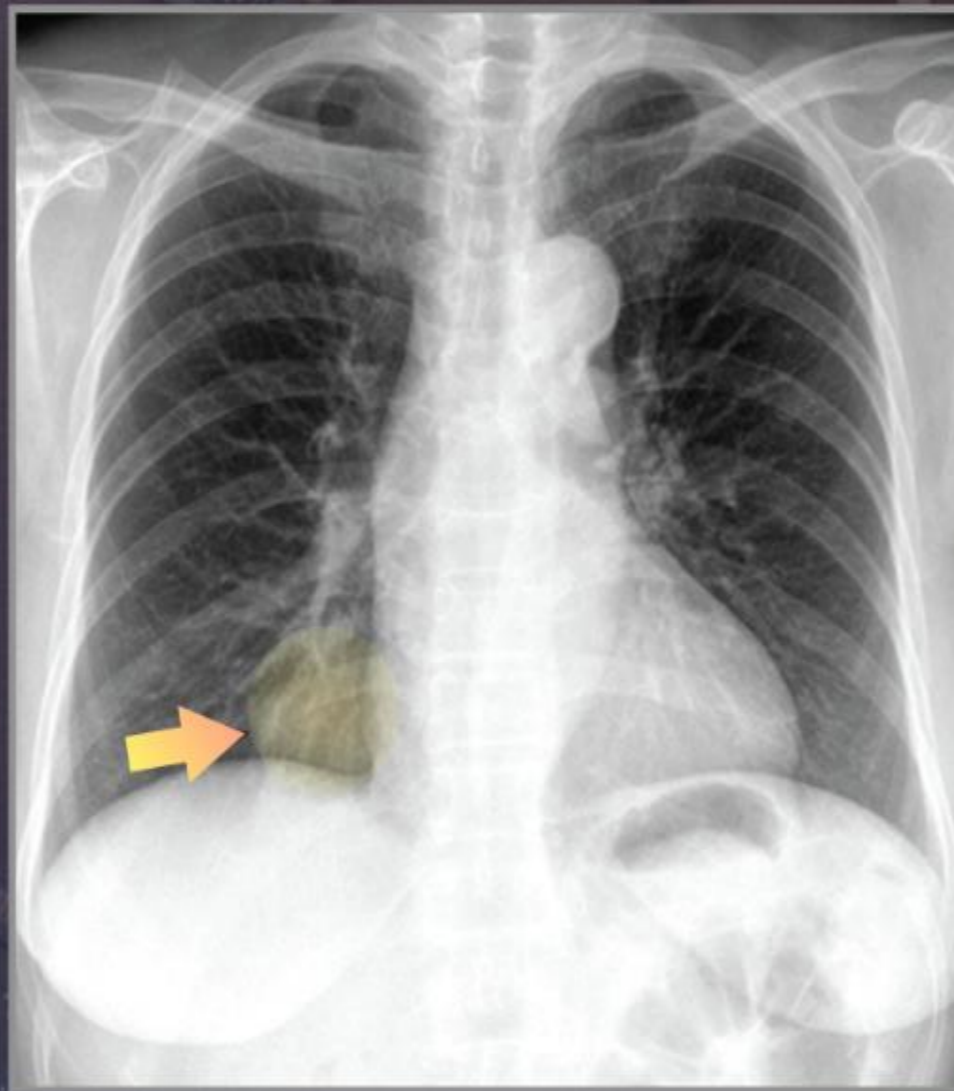
Lung Tumor

Computers:

Score: 0.291

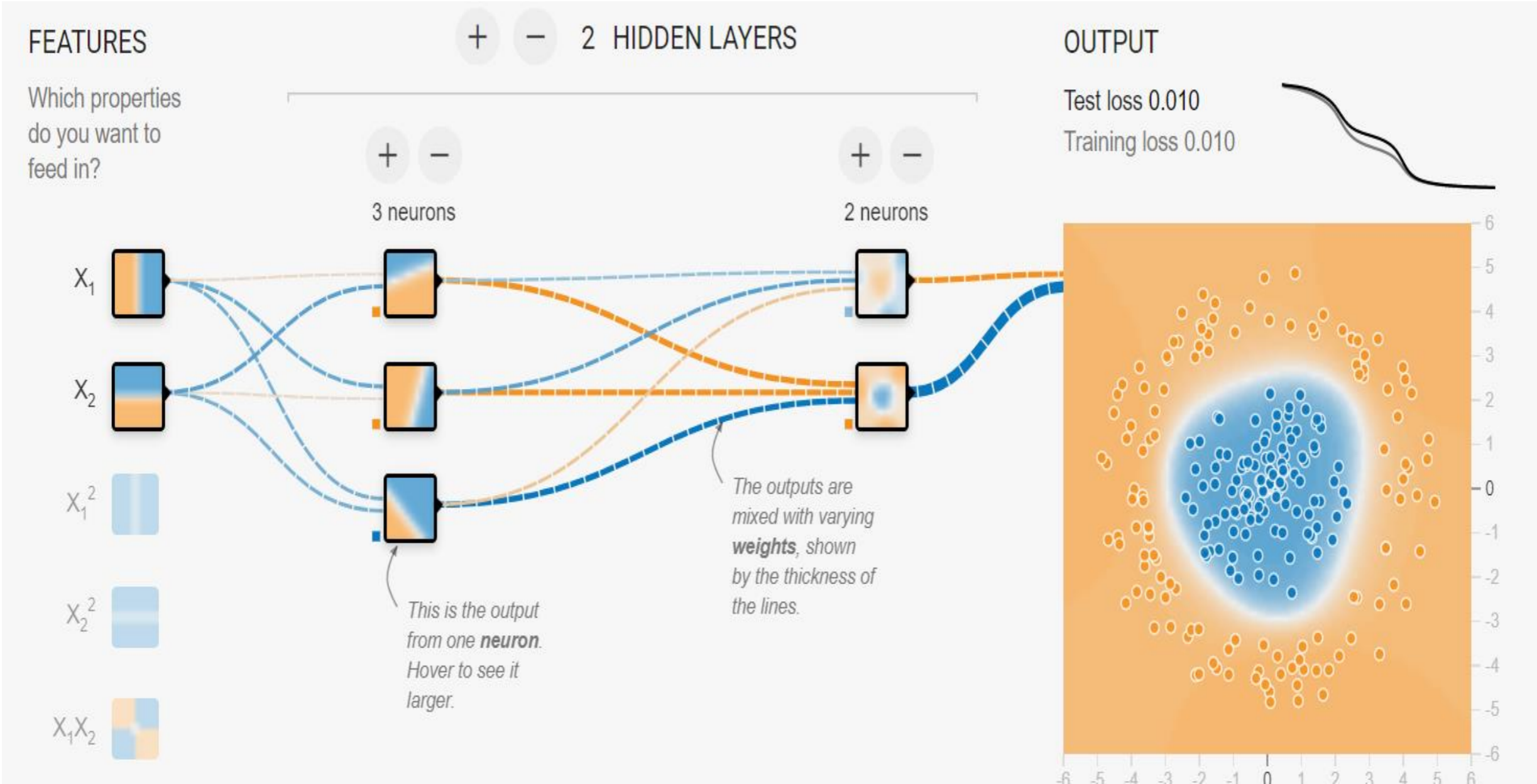
Doctors:

2/15 Detected



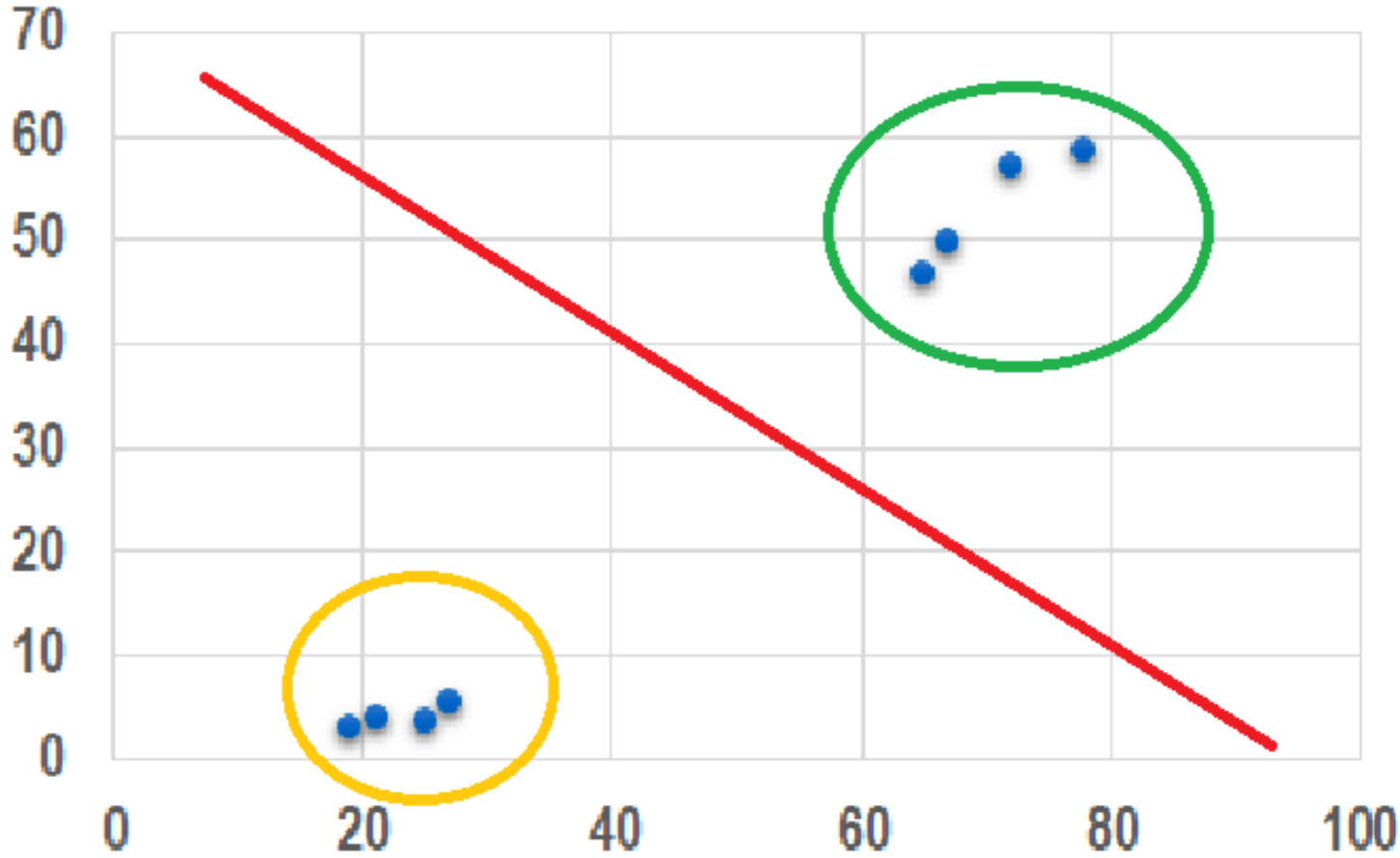
EXAMPLE

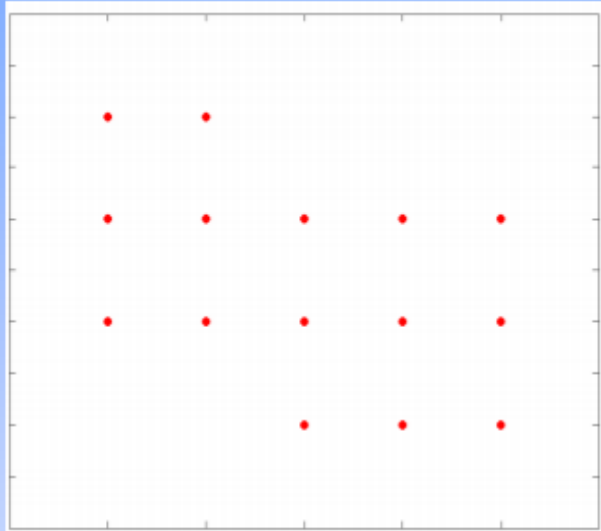
<http://playground.tensorflow.org>



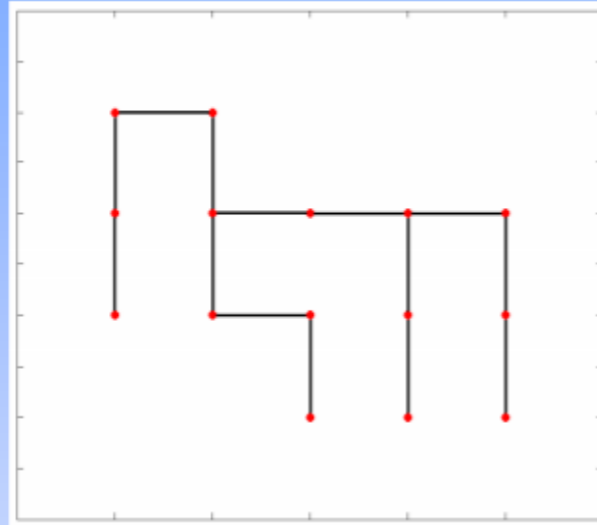
CLUSTERING

	Height	Weight
?	65	46,8
?	67	49,9
?	72	57
?	78	58,6
?	25	3,5
?	27	5,5
?	21	4
?	19	3

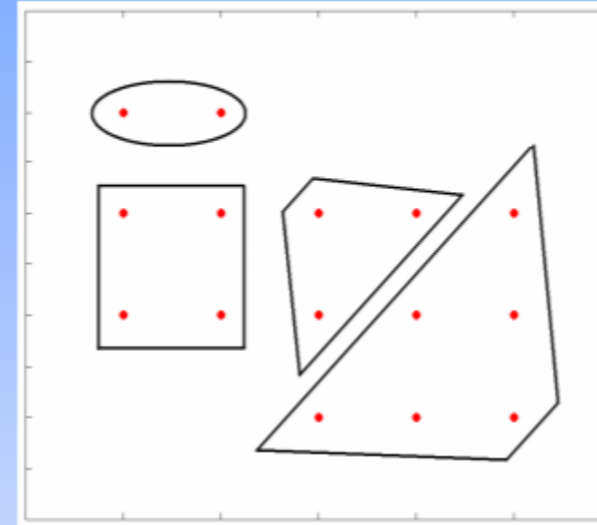




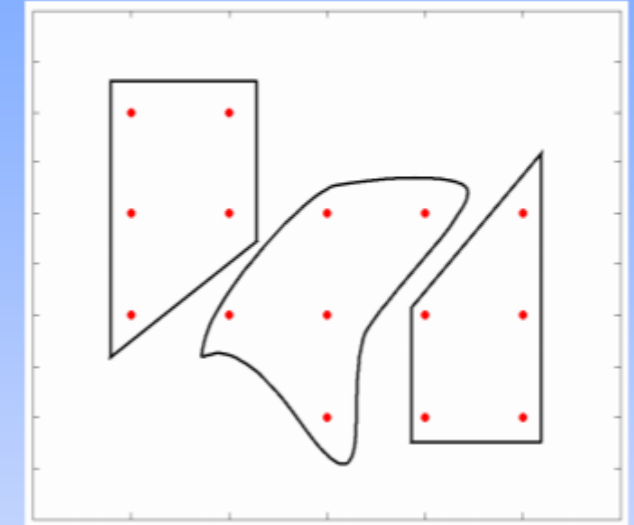
15 Data points



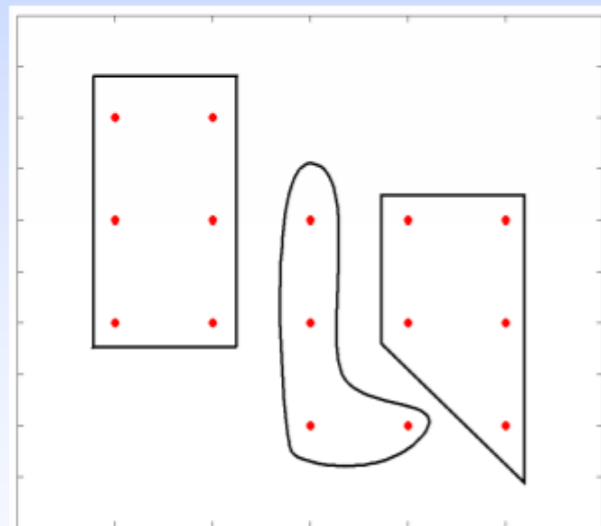
MST



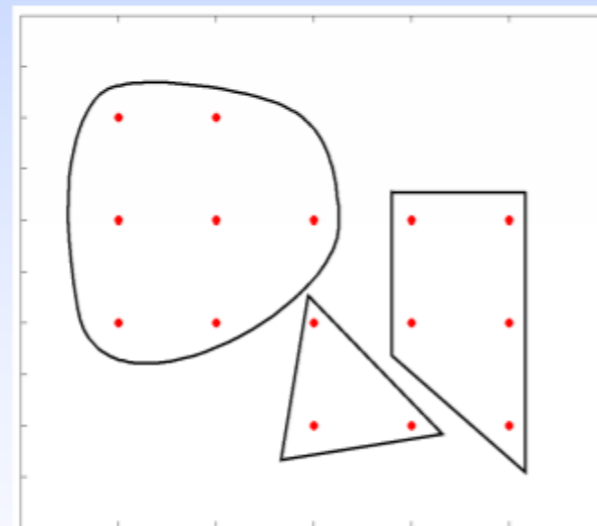
FORGY



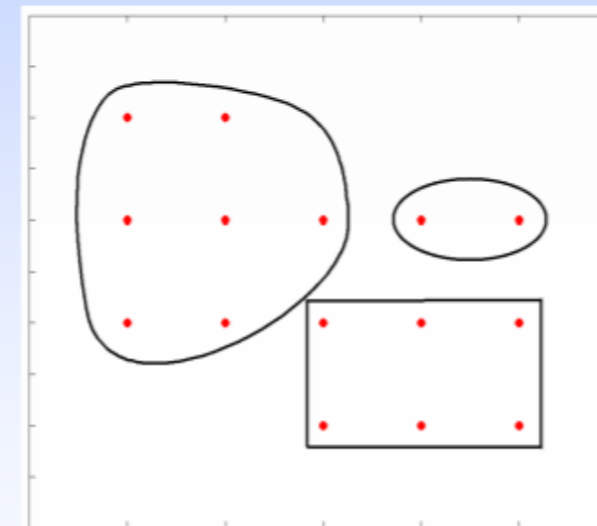
ISODATA



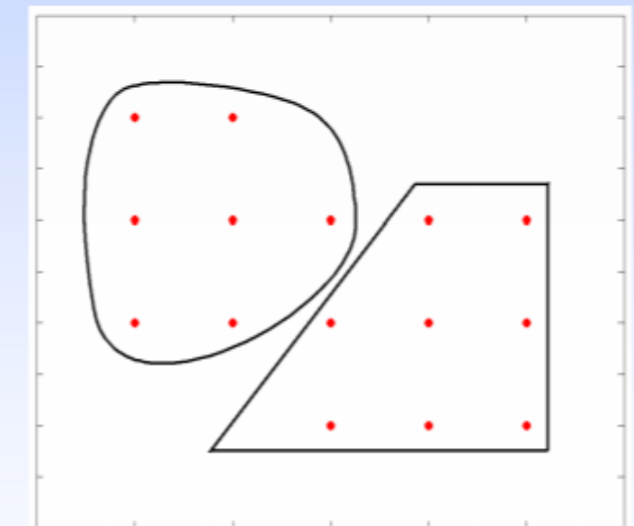
WISH



CLUSTER

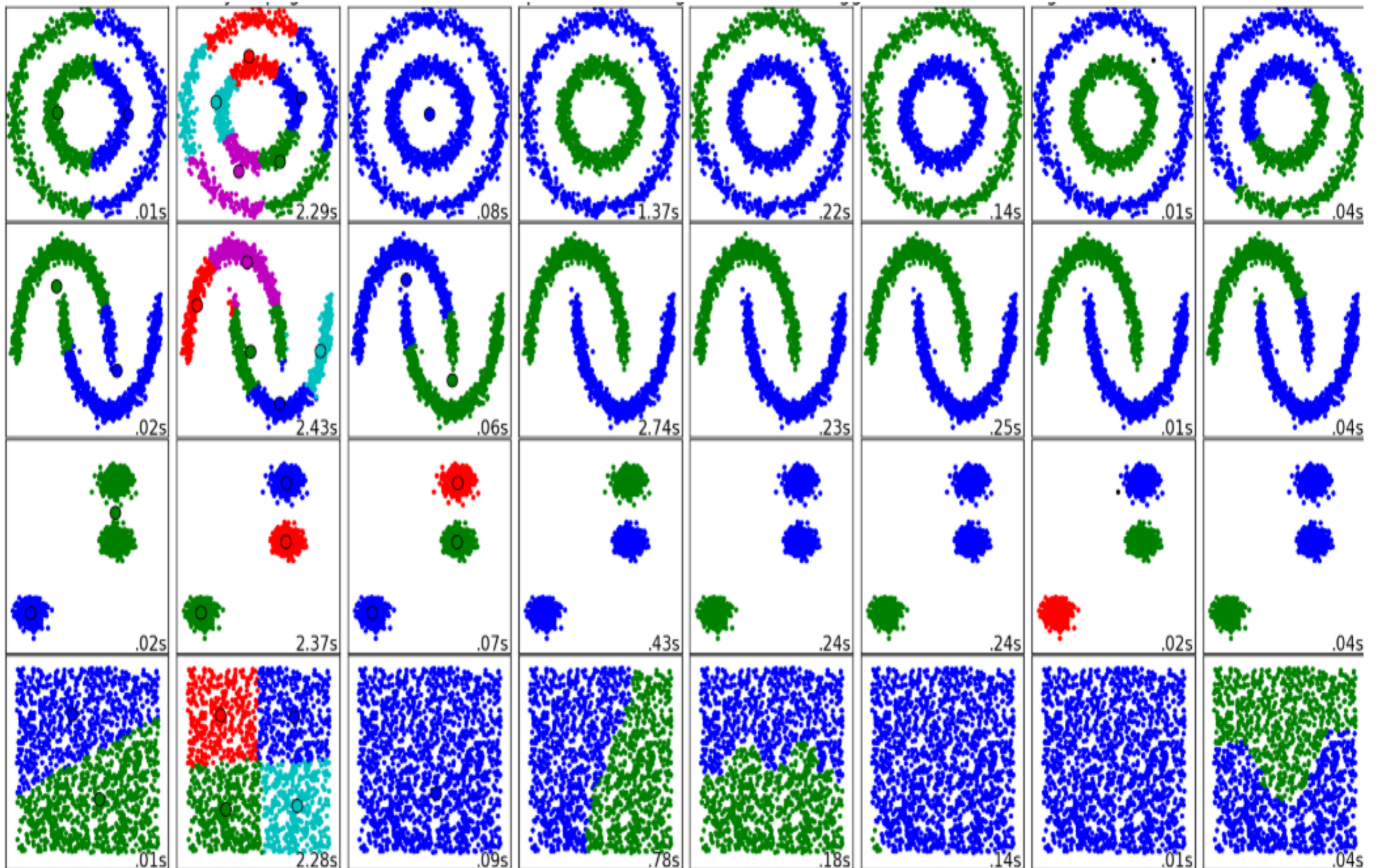


Complete Link



JP

Dubes and Jain, "Clustering Techniques: User's Dilemma", *Pattern Recognition*, 1976



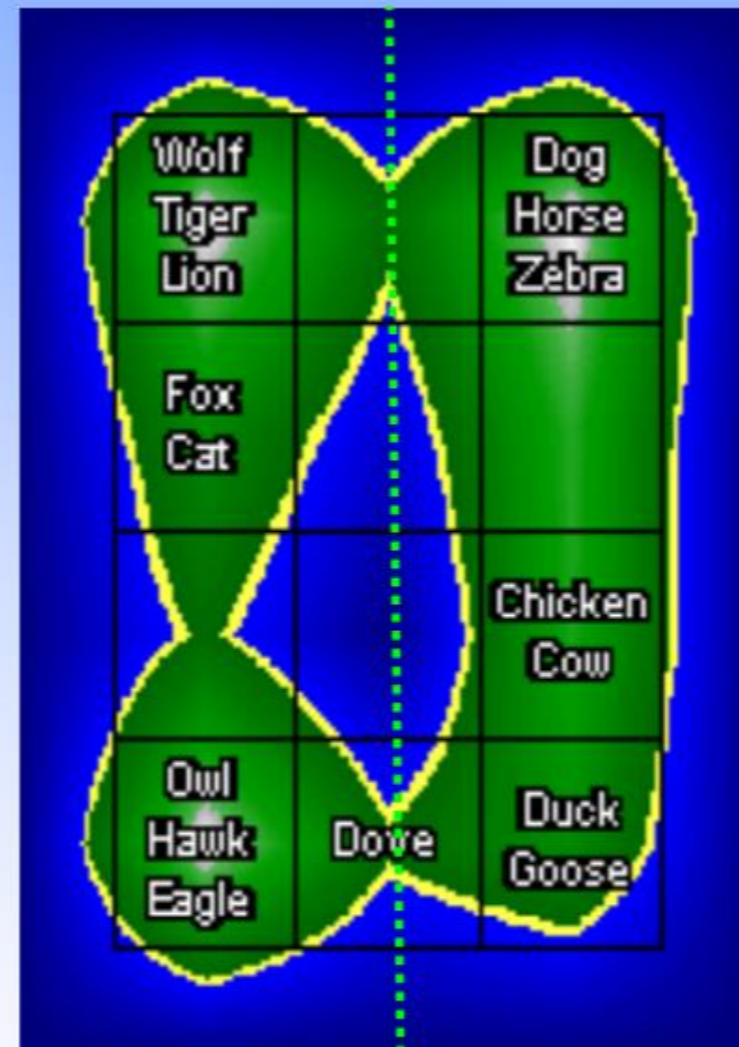
Mammals

Vs.

Birds



Large weight on appearance features



Large weight on activity features

Predators

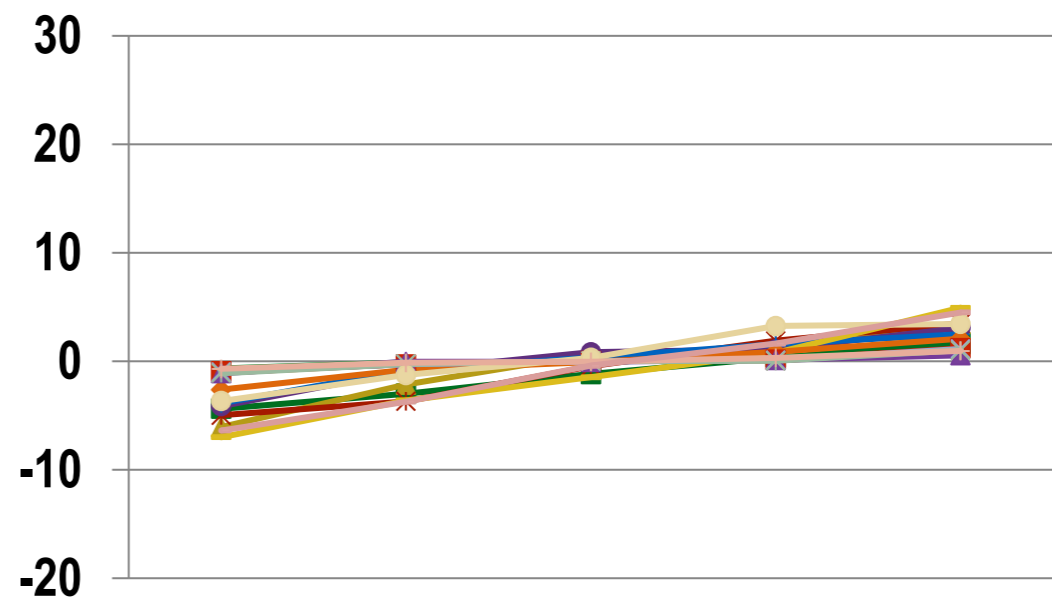
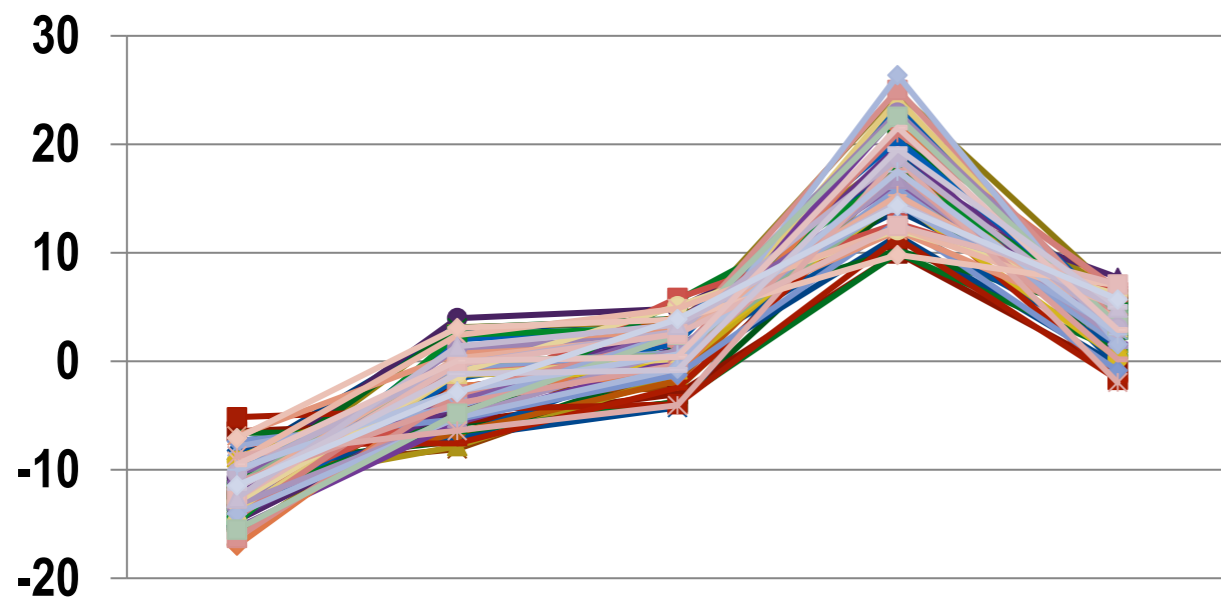
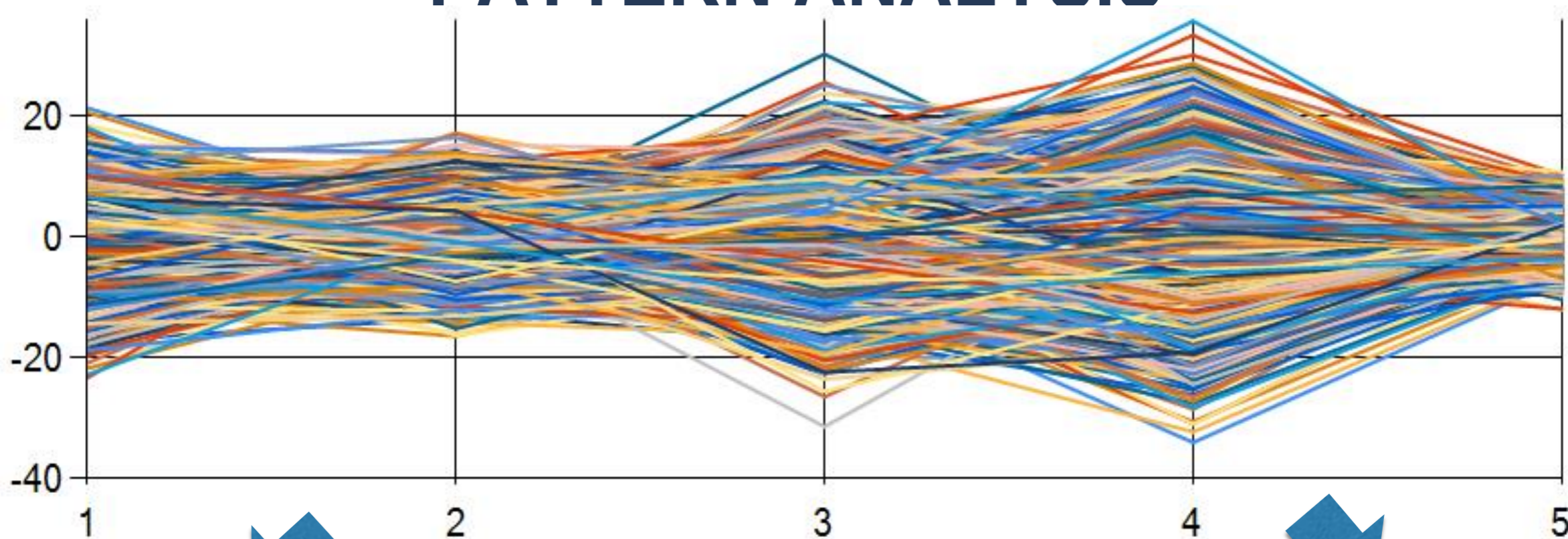
Vs.

Non-Predators

<https://pdfs.semanticscholar.org/1823/98ca4d85c25c64ba238dad10cafc92203660.pdf>

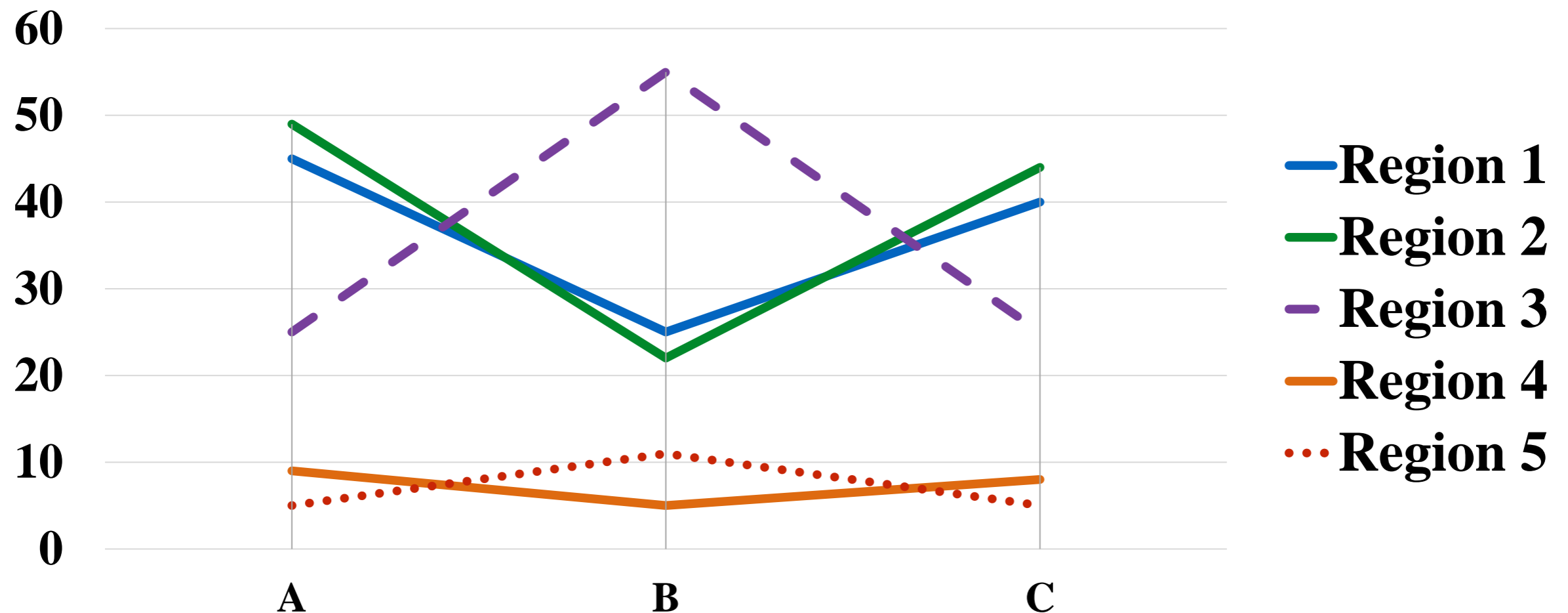
<http://www.ofai.at/~elias.pampalk/kdd03/animals/>

PATTERN ANALYSIS



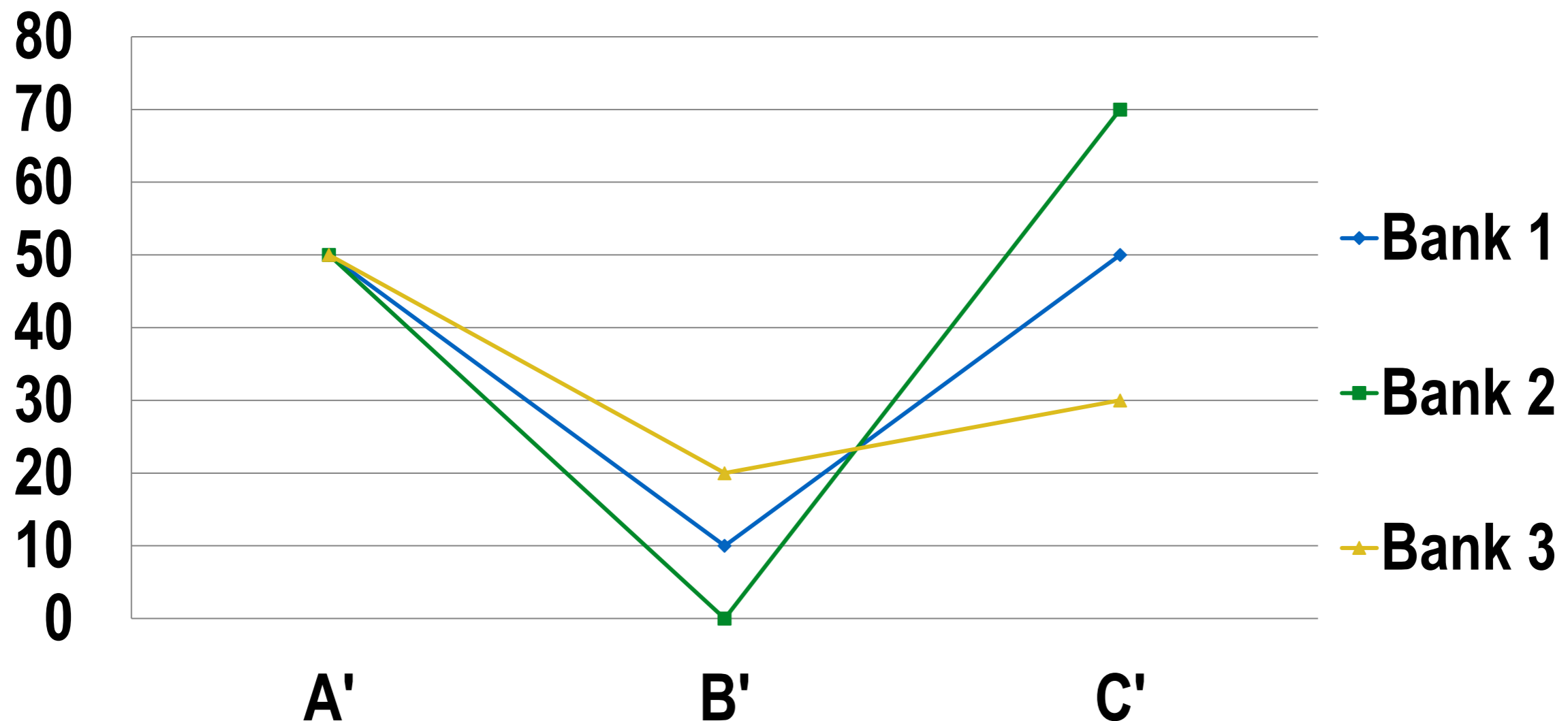


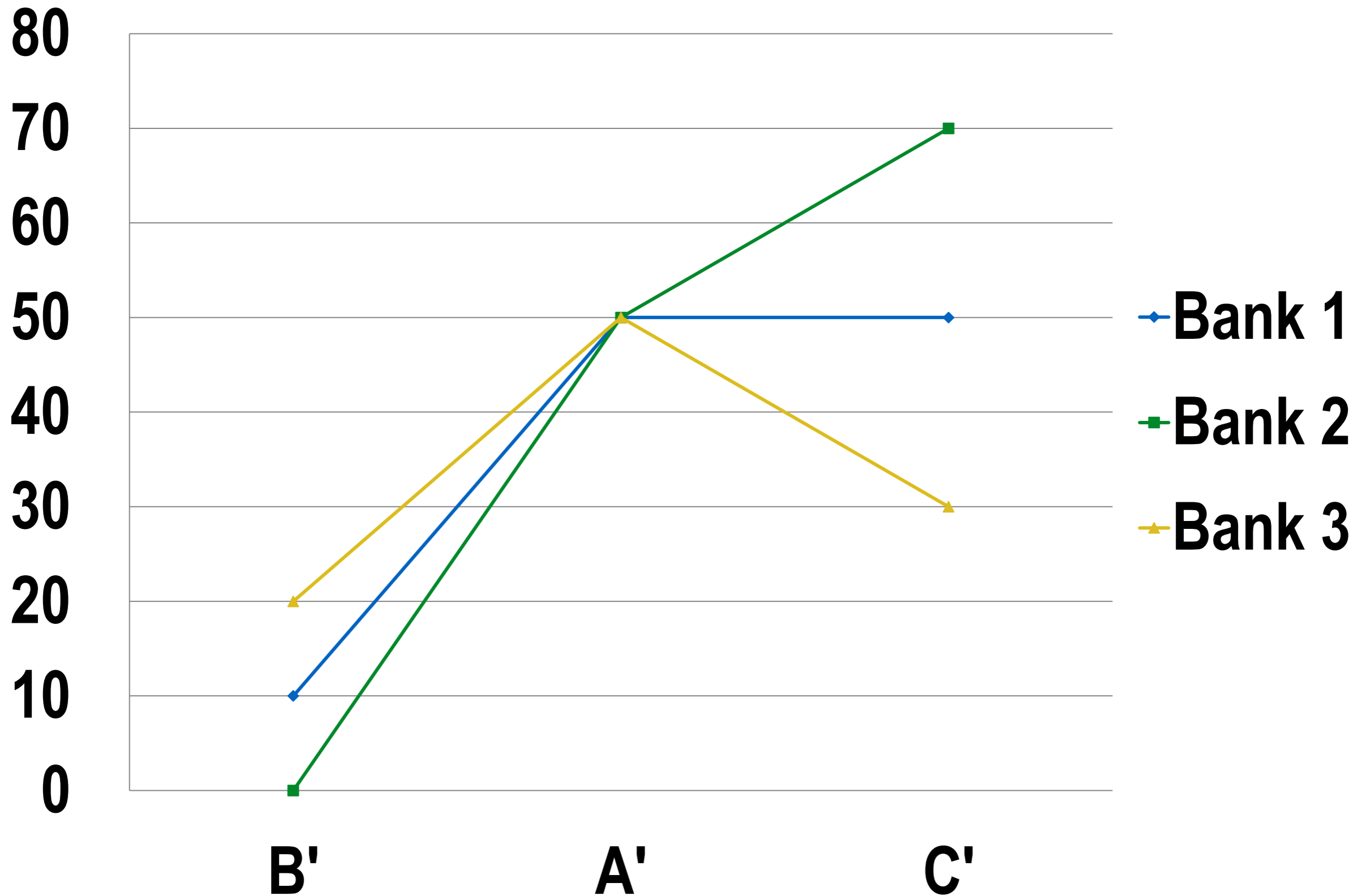
Regions	A	B	C
Region 1	45	25	40
Region 2	49	22	44
Region 3	25	55	25
Region 4	9	5	8
Region 5	5	11	5

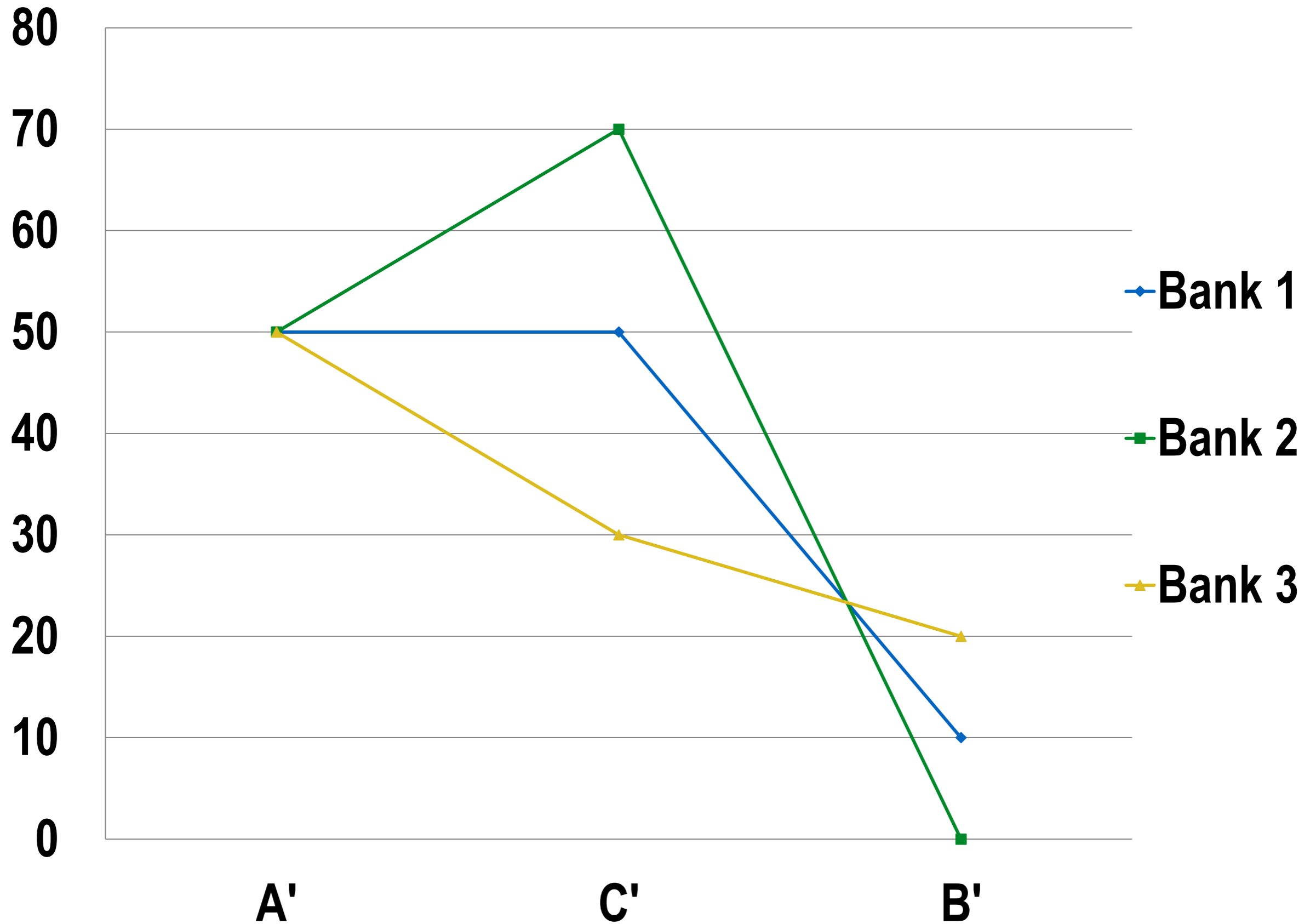


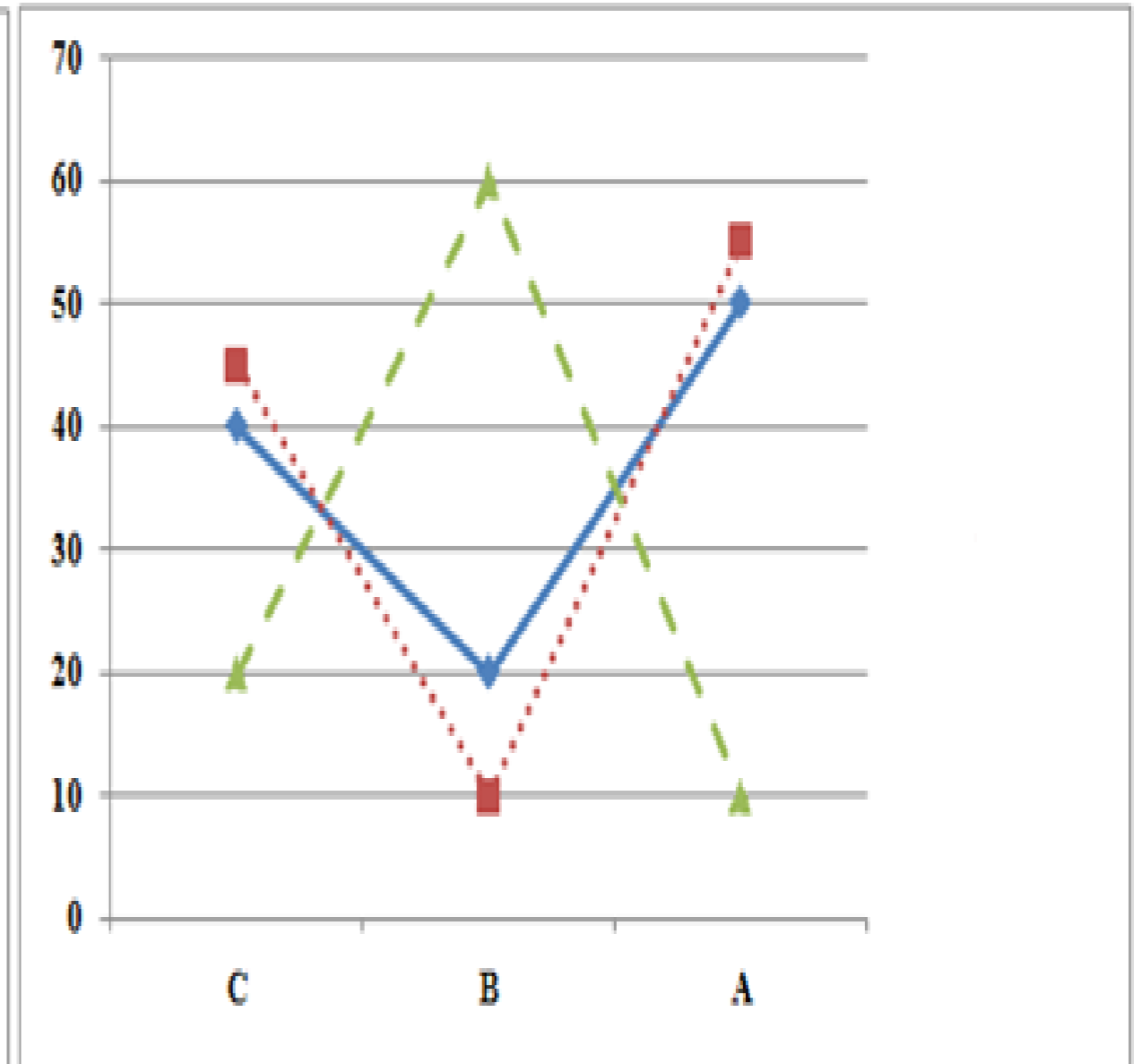
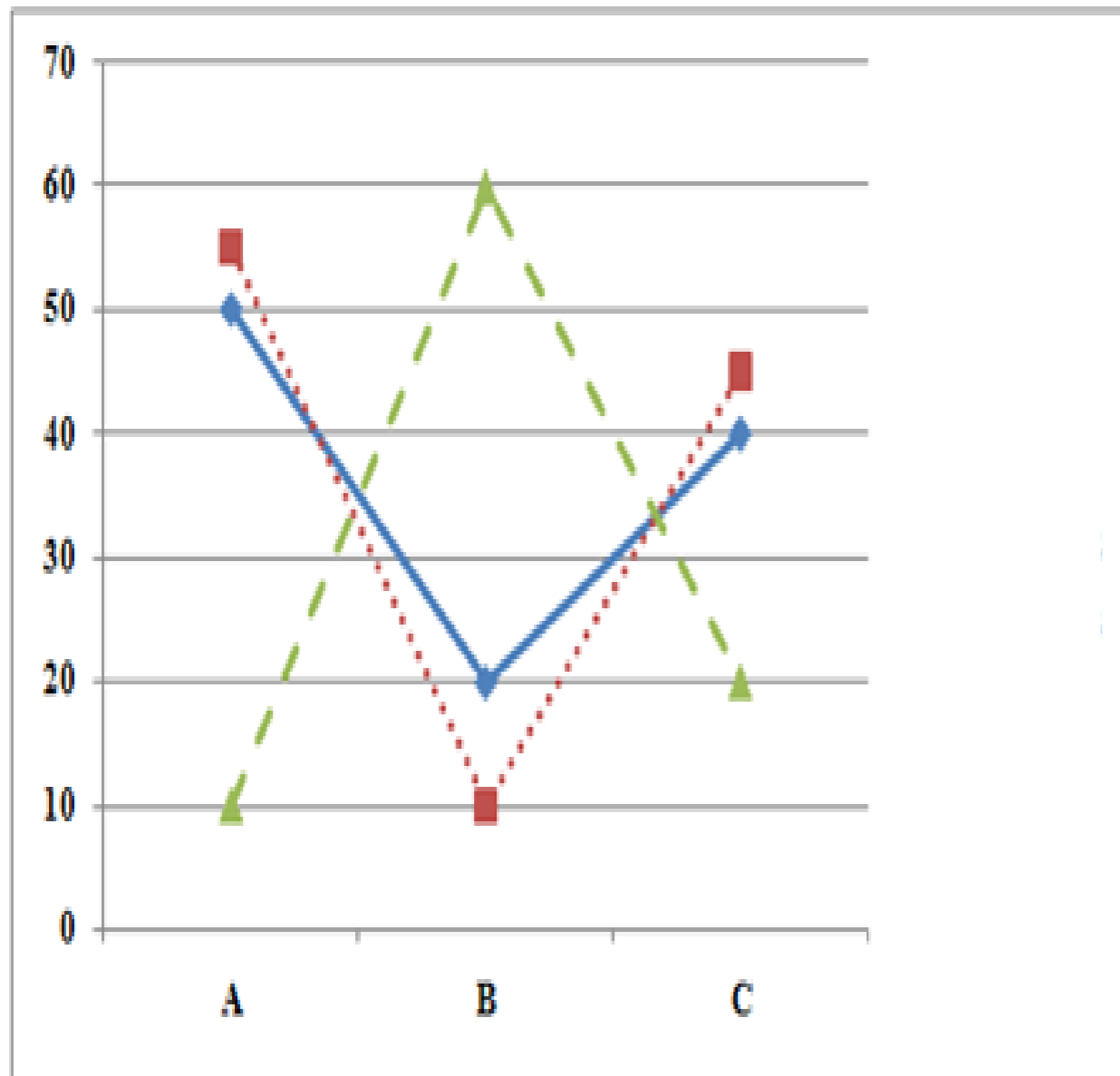


Bank	A'	B'	C'
Bank 1	50	10	50
Bank 2	50	0	70
Bank 3	50	20	30











METHODS

1. Ordinal-fixed pattern clustering;
2. Ordinal-invariant pattern clustering;
3. Diffusion-invariant pattern clustering;
4. Ordinal-interval pattern clustering;
5. Bayesian pattern analysis



ORDINAL-INVARIANT PATTERN CLUSTERING

$$q_i = \sum_{s=1}^{n-1} 10^{s-1} z_i^{n-s}$$

$$q_i^{dop} = \sum_{s=1}^{n-2} \sum_{j=s+2}^n 10^{j-(s+2)} e_i^{sj}$$

$$z_i^s = \begin{cases} 1, & \text{if } x_{is} < x_{is+1} \\ 0, & \text{if } x_{is} = x_{is+1} \\ 2, & \text{if } x_{is} > x_{is+1} \end{cases}$$

$$e_i^{sj} = \begin{cases} 1, & \text{if } x_{is} < x_{is+1} \\ 0, & \text{if } x_{is} = x_{is+1} \\ 2, & \text{if } x_{is} > x_{is+1} \end{cases}$$

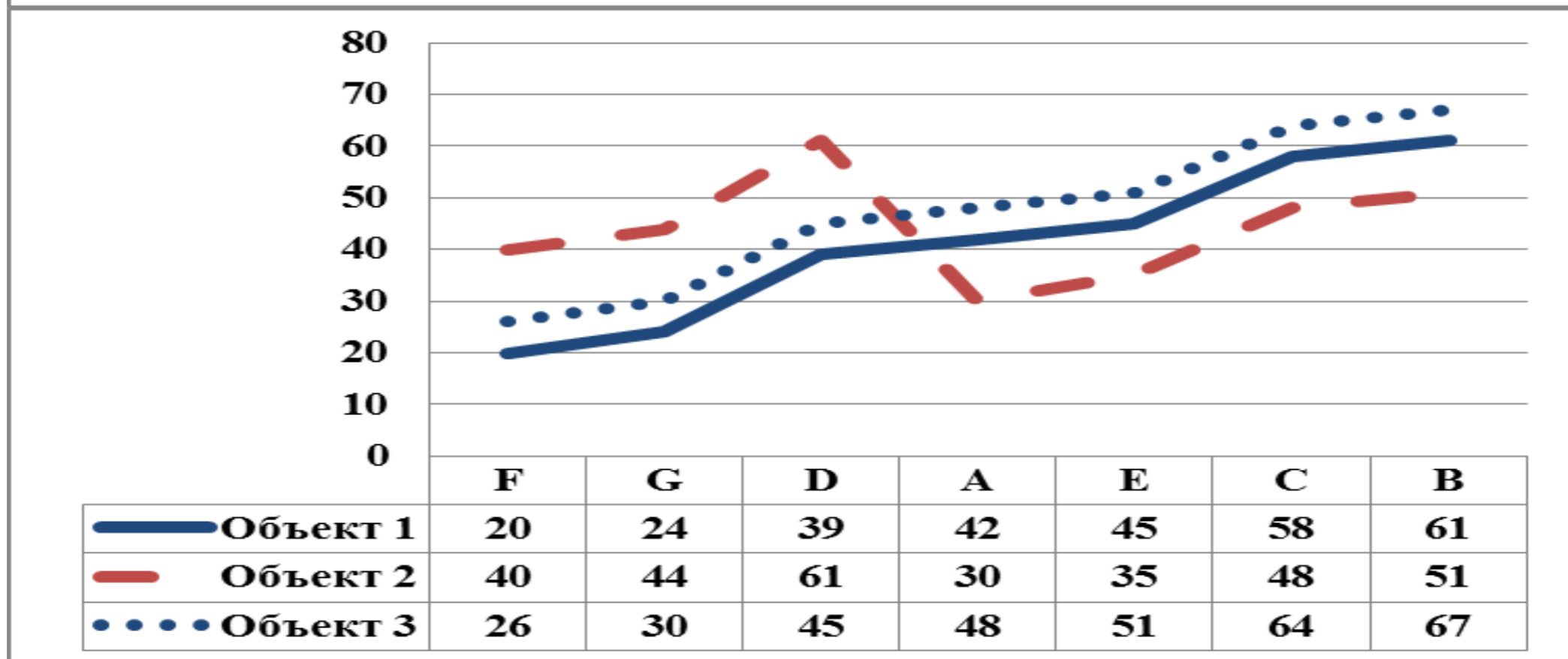
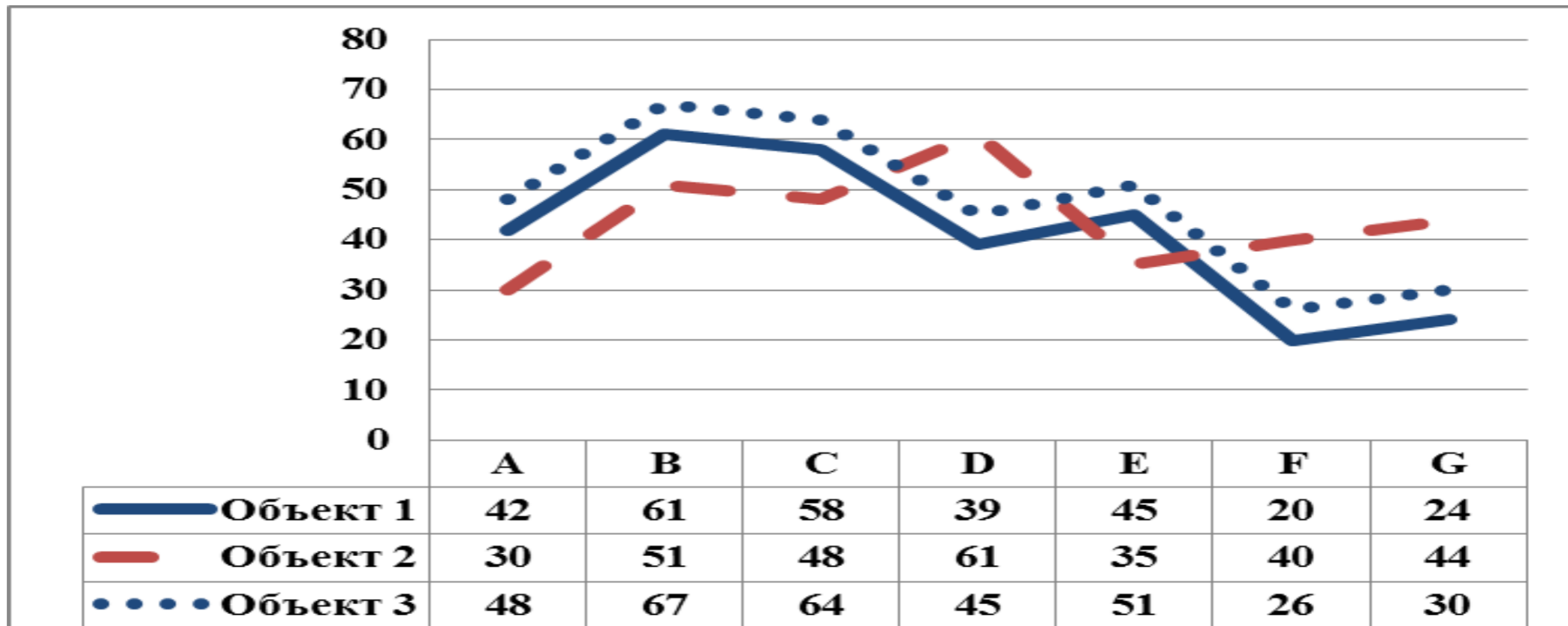
$$Z_{fix} = \frac{m^2(m-1)(n-1)}{2}$$

$$Z_{inv} = \frac{m^2 n (m-1)(n-1)}{4}$$

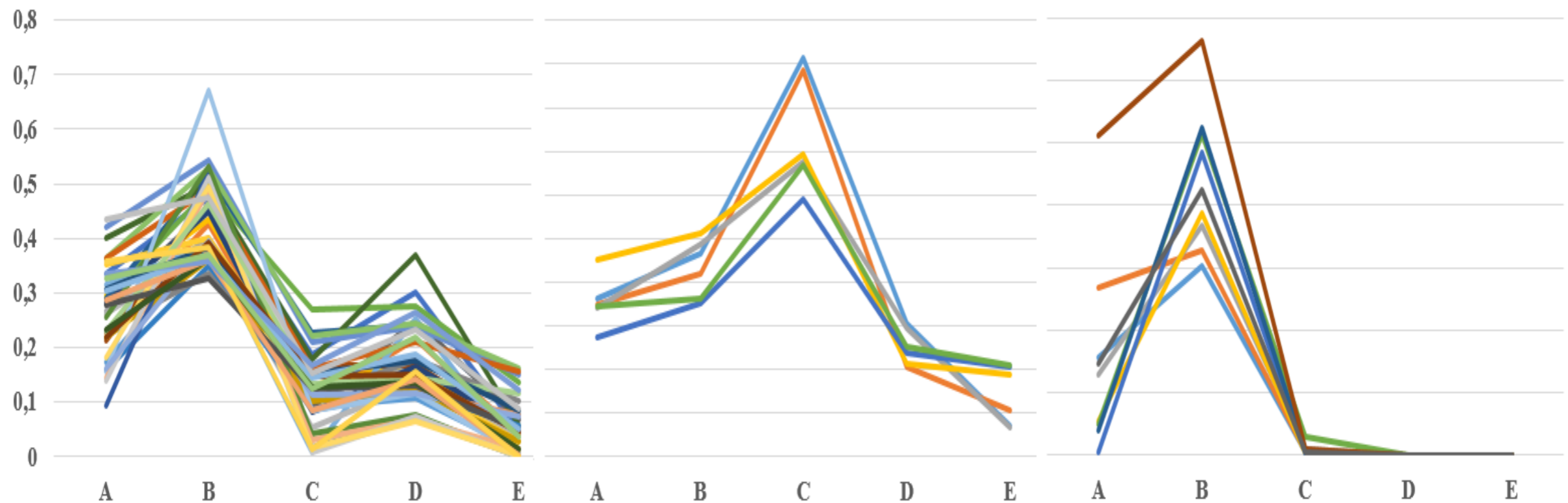


BASIC PROPERTIES

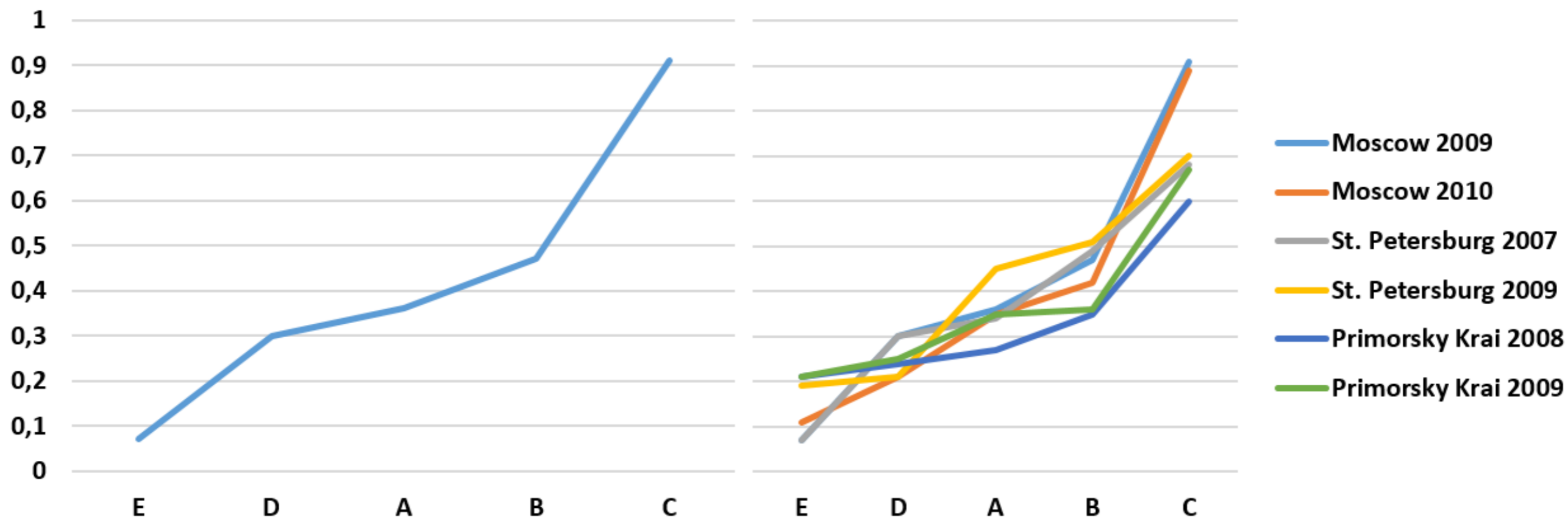
1. Clusters obtained using ordinal-invariant pattern clustering do not intersect.
2. If there exists a sequence of indicators in which their values form a strictly monotonically increasing / decreasing sequence for each object of the original set X , then this set is an ordinal-invariant pattern cluster.



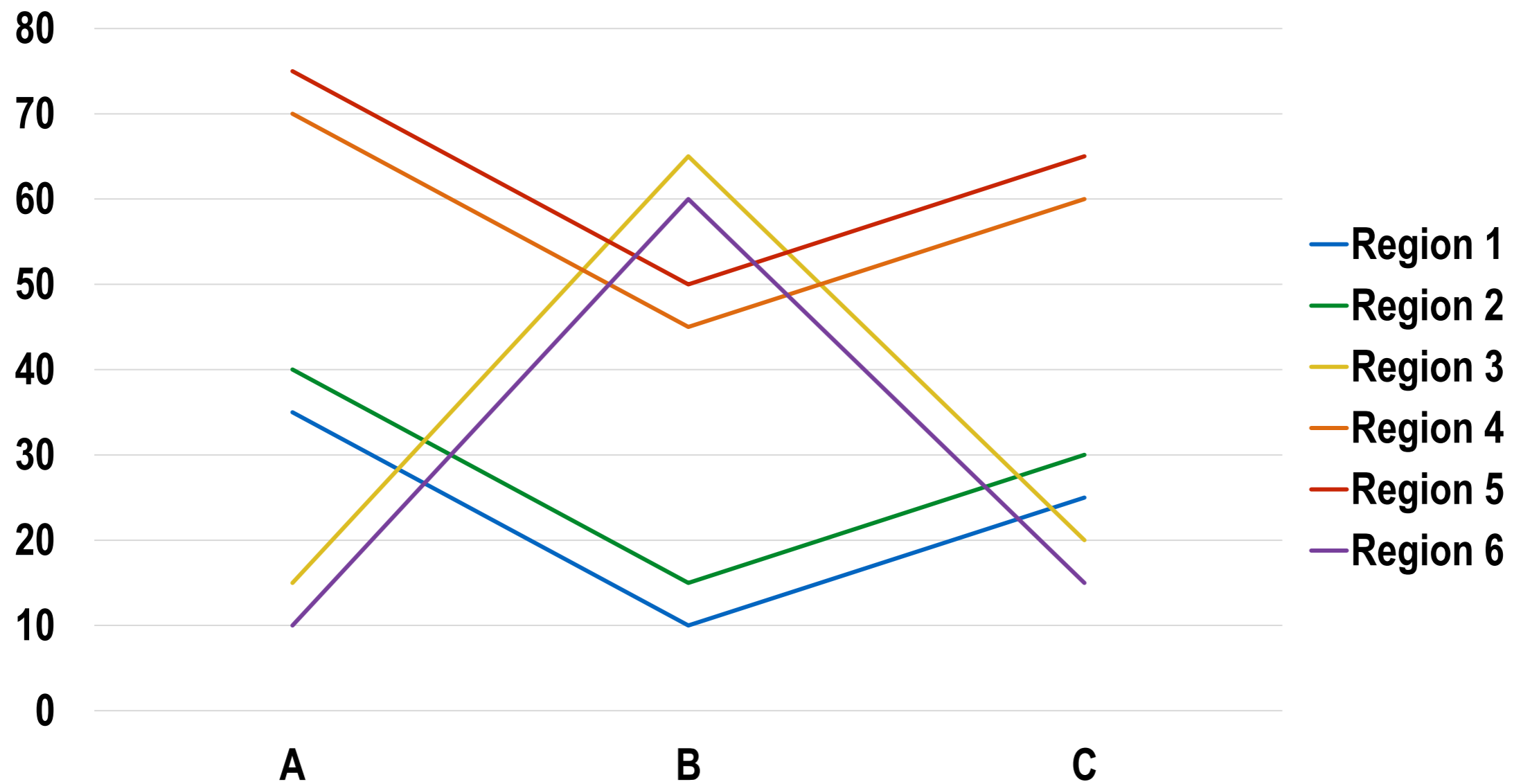
3. There is an order of the indicators for objects of ordinal-invariant pattern-cluster, in which their values are monotonous non decreasing/non increasing sequence for each object.

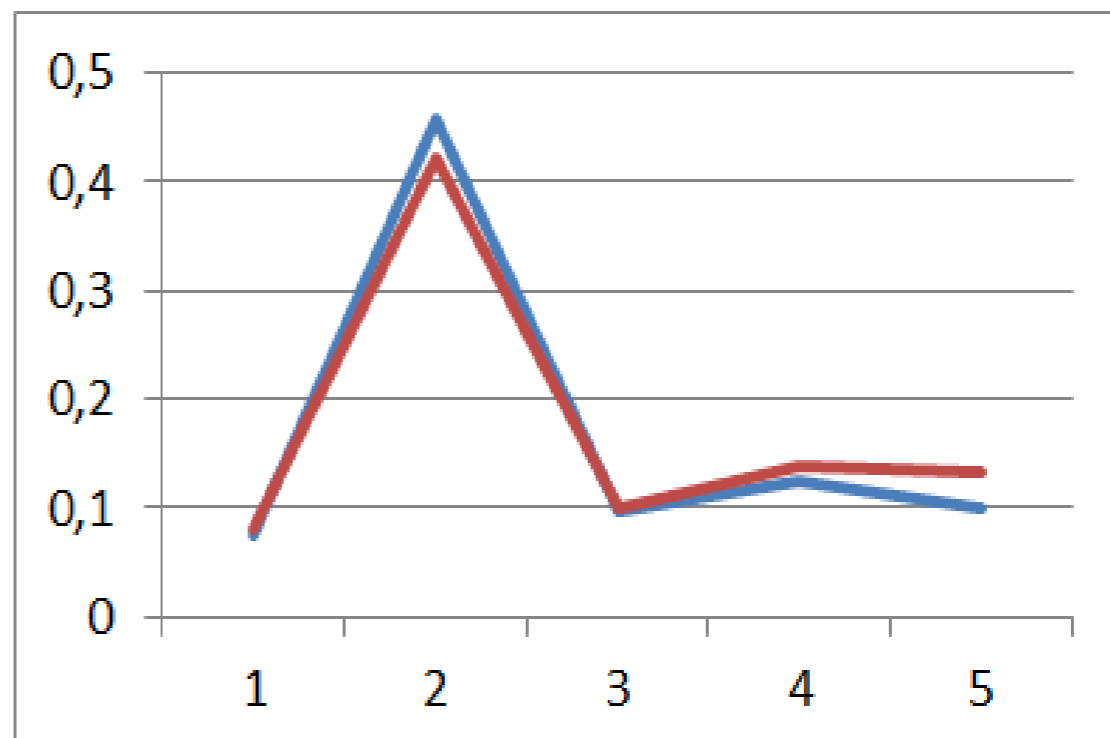
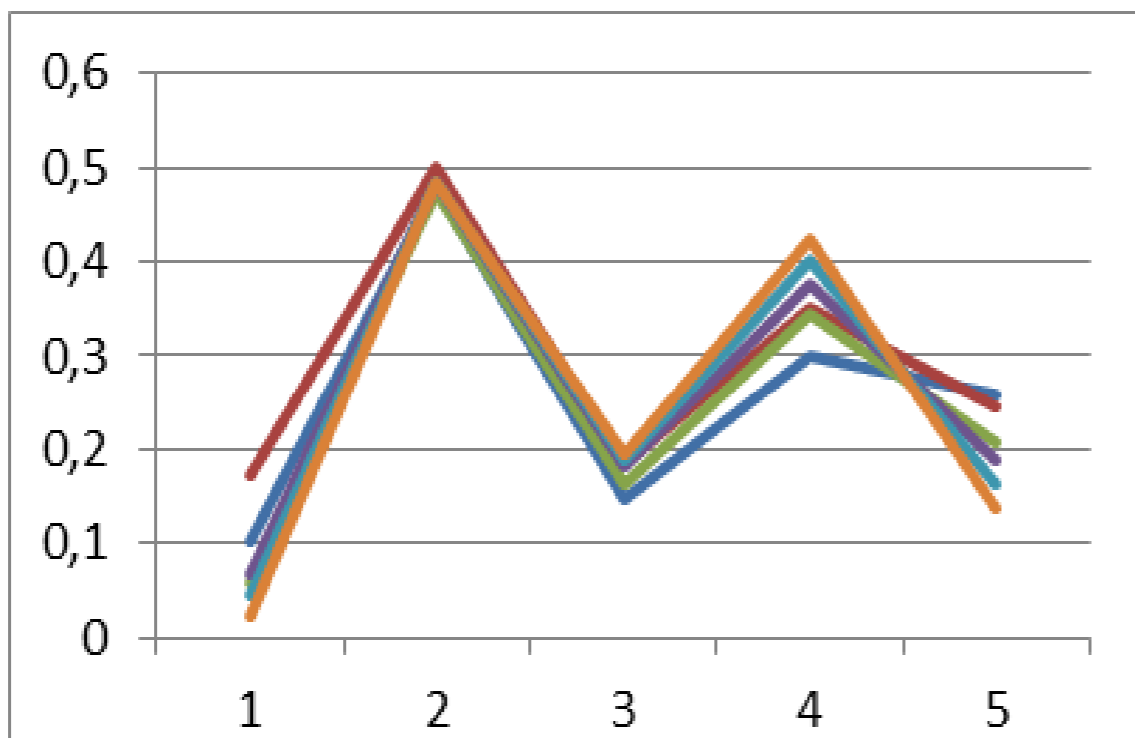
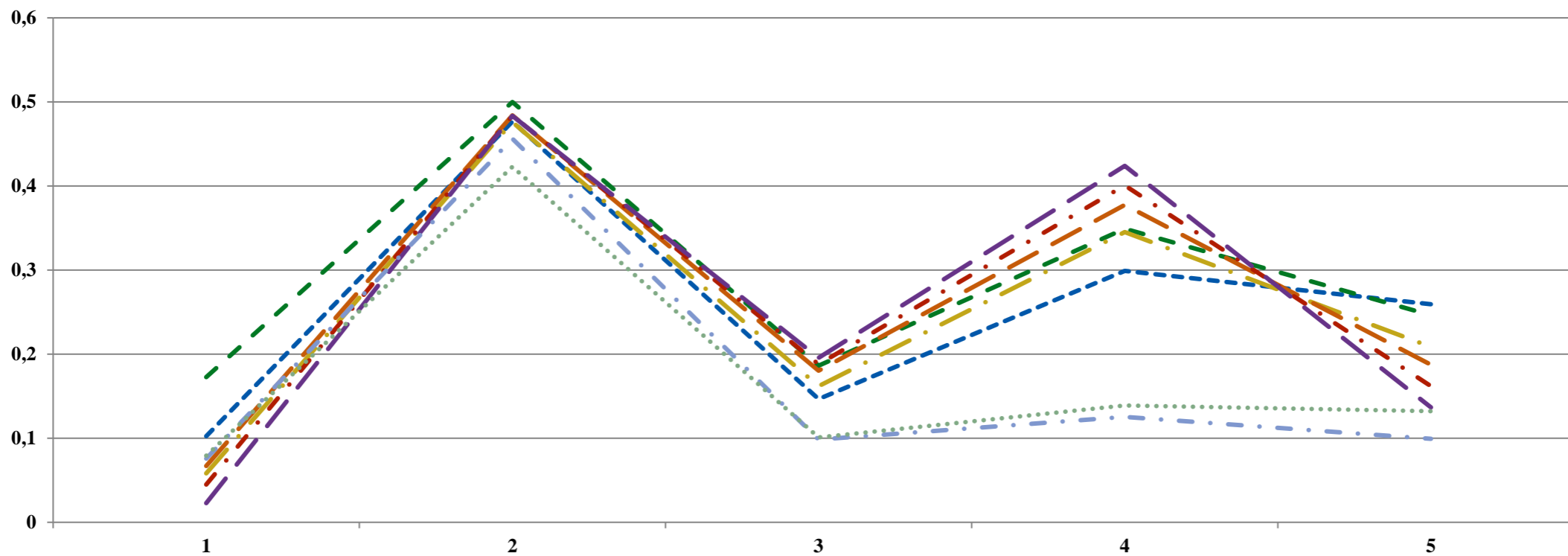


Region/year	A	B	C	D	E
Moscow 2009	0,36	0,47	0,91	0,3	0,07
Moscow 2010	0,35	0,42	0,89	0,21	0,11
St. Petersburg 2007	0,34	0,49	0,68	0,3	0,07
St. Petersburg 2009	0,45	0,51	0,7	0,21	0,19
Primorsky Krai 2008	0,27	0,35	0,6	0,24	0,21
Primorsky Krai 2009	0,35	0,36	0,67	0,25	0,21



DIFFUSION-INVARIANT PATTERN CLUSTERING





Theorem. Let two objects x_1 и x_2 , described by the vectors $x_1 = (x_{11}, x_{12}, \dots, x_{1m})$ and $x_2 = (x_{21}, x_{22}, \dots, x_{2m})$ respectively, belong to the same Euclidean cluster distances – $d(x_i, z) < R_{v_{dif}}$ between the object and the center of the cluster z , described by the vector $z = (z_1, z_2, \dots, z_m)$, where $R_{v_{dif}}$ is the cluster radius. We form new objects x_1^* and x_2^* by exchanging the corresponding values of one of the coordinates (for definiteness, the first) of the original objects x_1 and x_2 : $x_1^* = (x_{21}, x_{12}, \dots, x_{1m})$; $x_2^* = (x_{11}, x_{22}, \dots, x_{2m})$. Then:

- 1) The Euclidean distances between the original objects $d(x_1, x_2)$ and between the newly formed objects $d(x_1^*, x_2^*)$ are equal.
- 2) At least one of the newly formed objects belongs to this cluster v_{dif} .

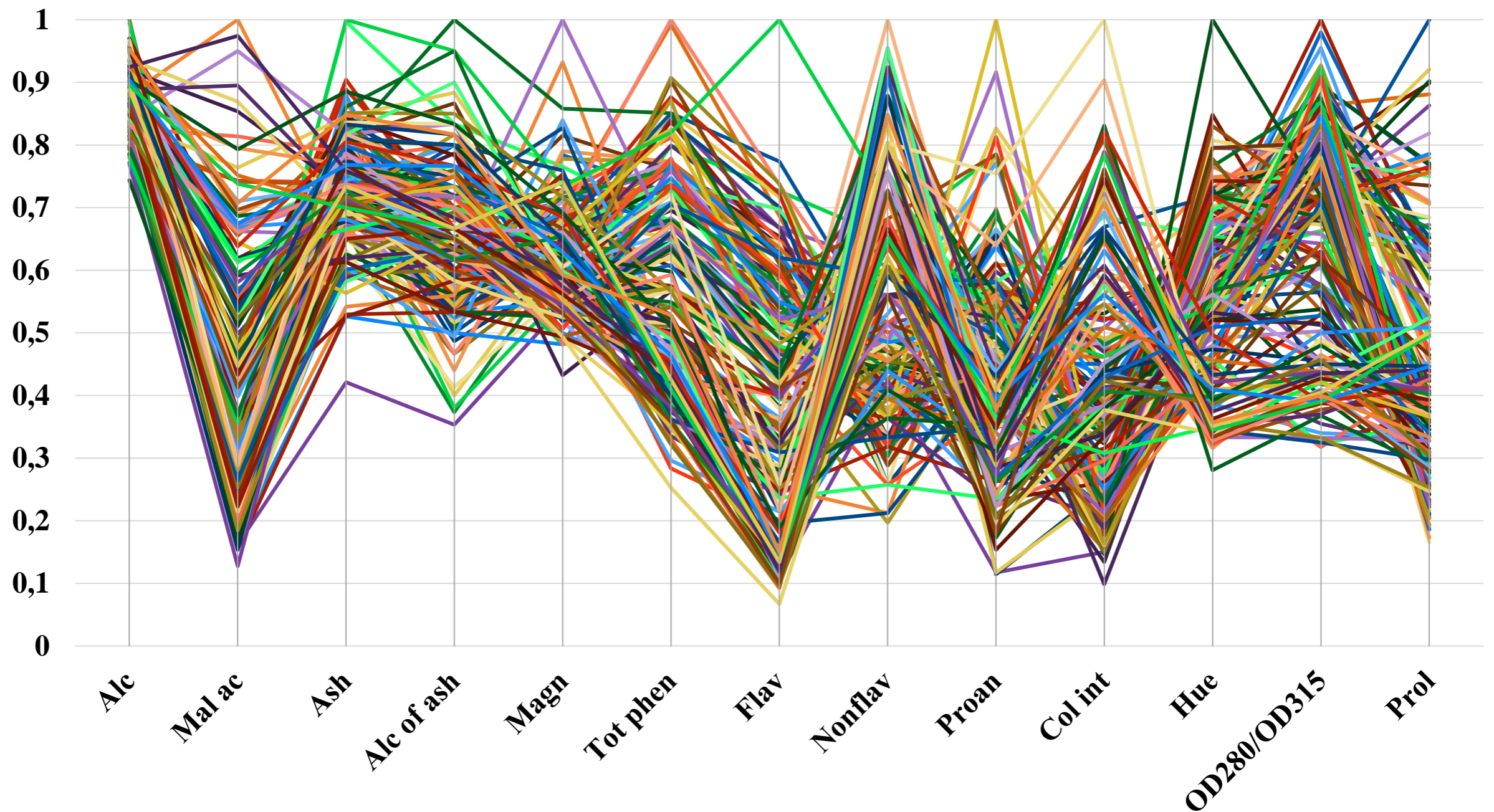


“In most cases, calculations are made with approximate numbers and, moreover, approximately. Therefore, even for the exact method of solving the problem, at each stage of the calculation, an error of actions and a rounding error arise... Under adverse circumstances, the total error can be so great that the result obtained will have only an illusory value” *

*DEMIDOVICH B.P., MARON I.A. Fundamentals of Computational Mathematics (in Russian)



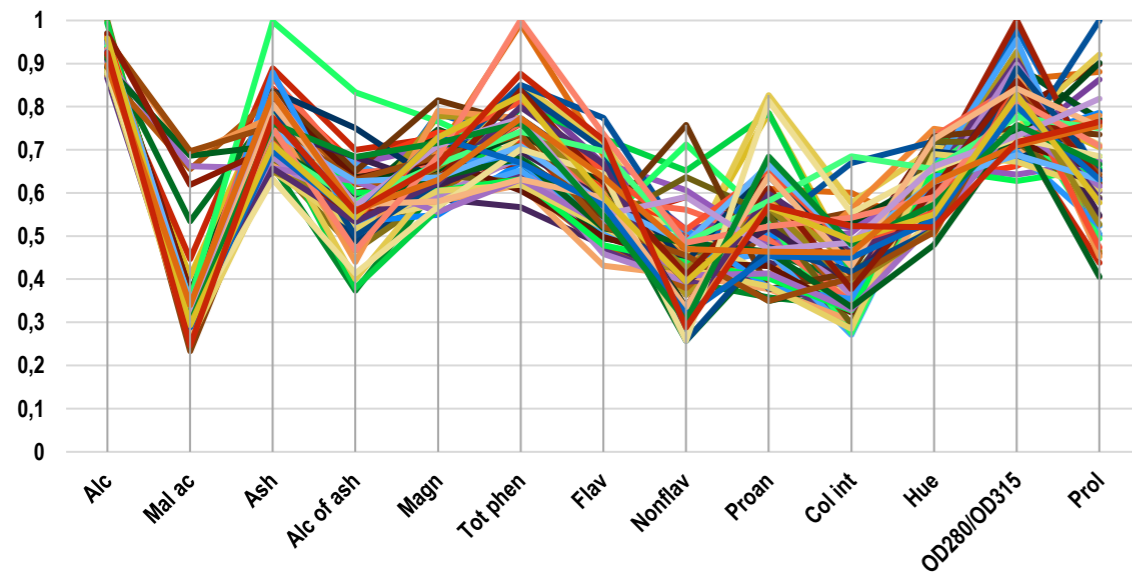
WINE DATA SET



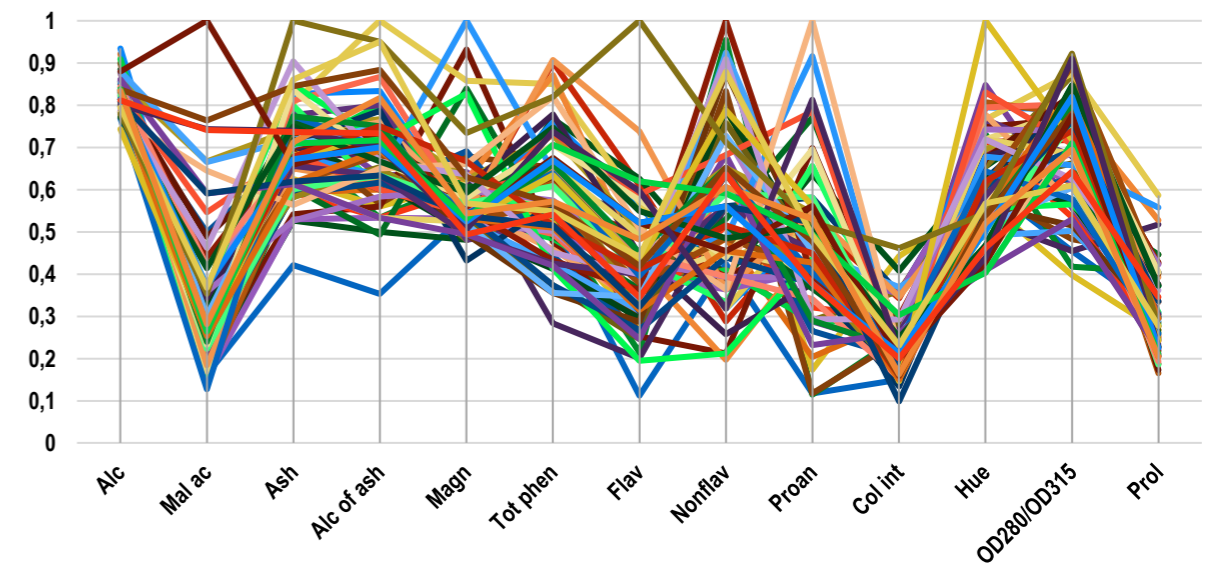


VINE DATA SET

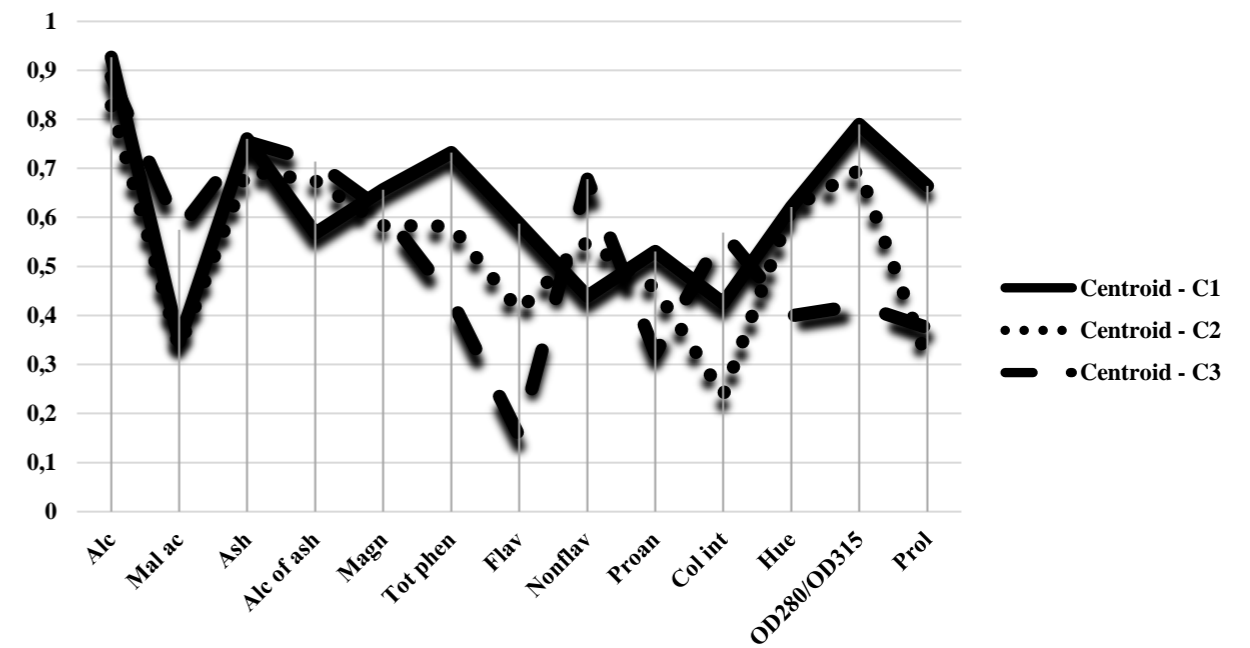
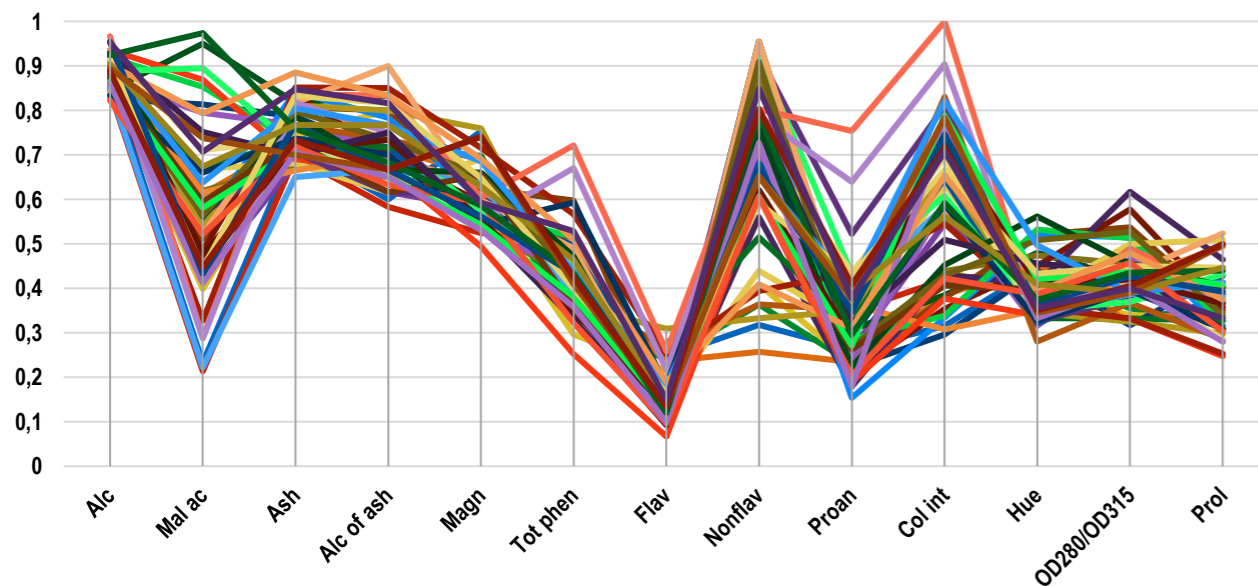
1



2

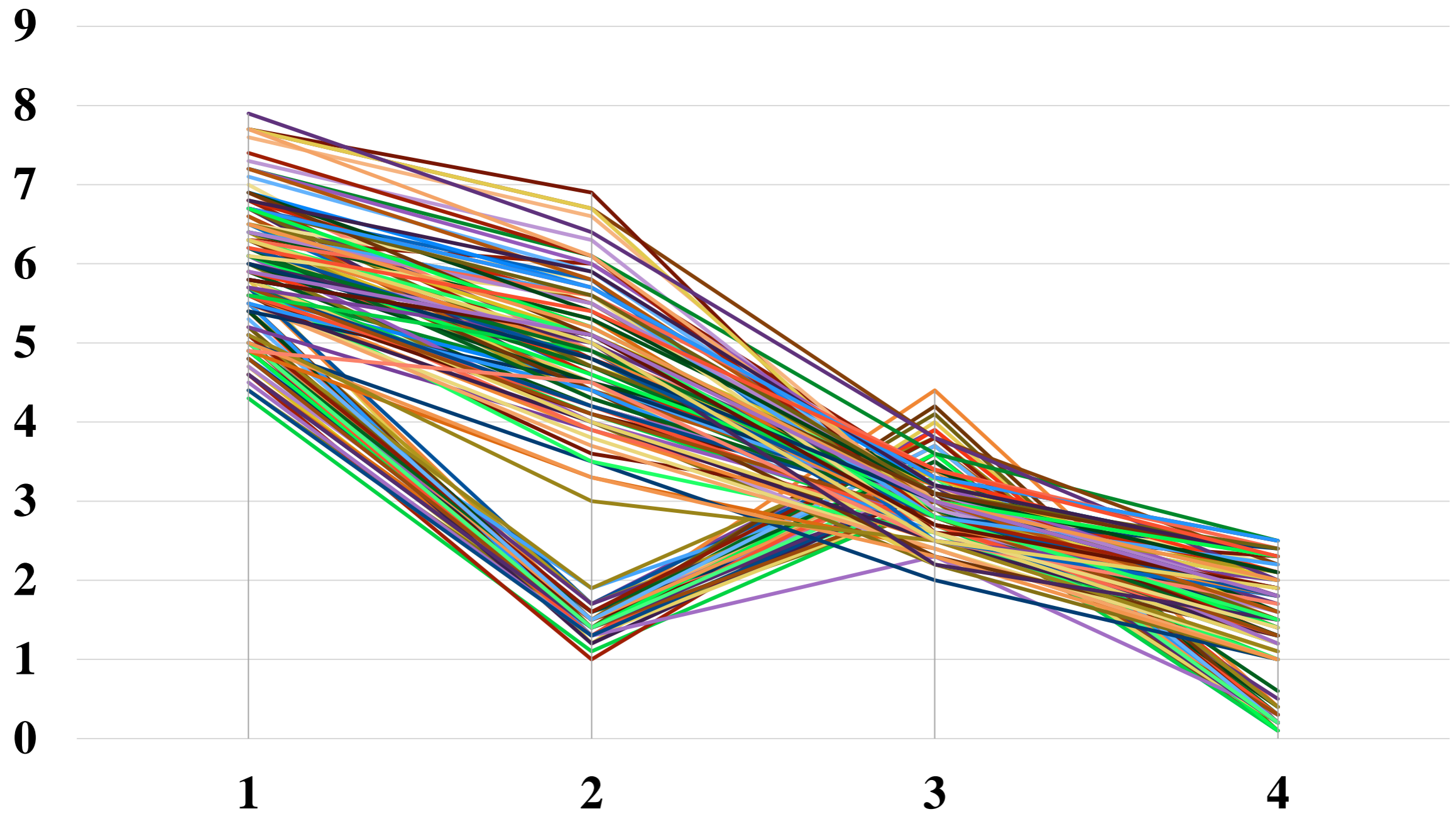


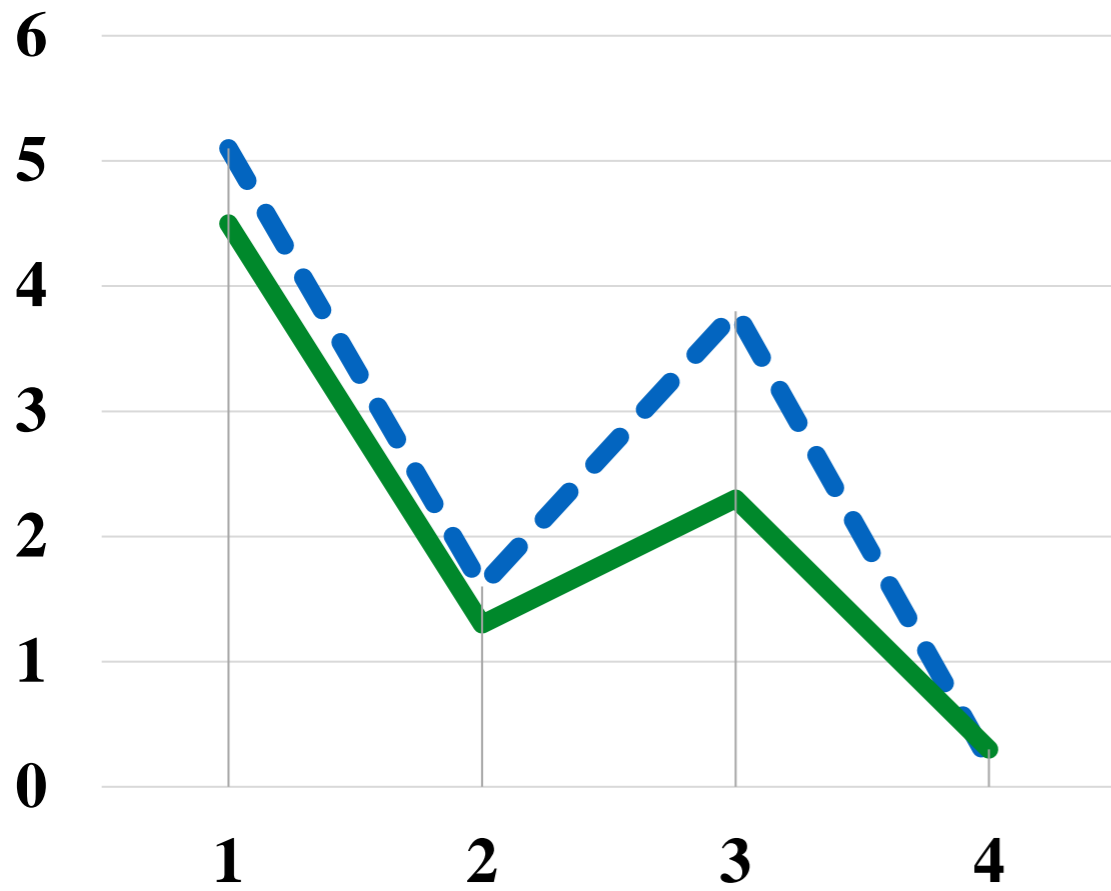
3



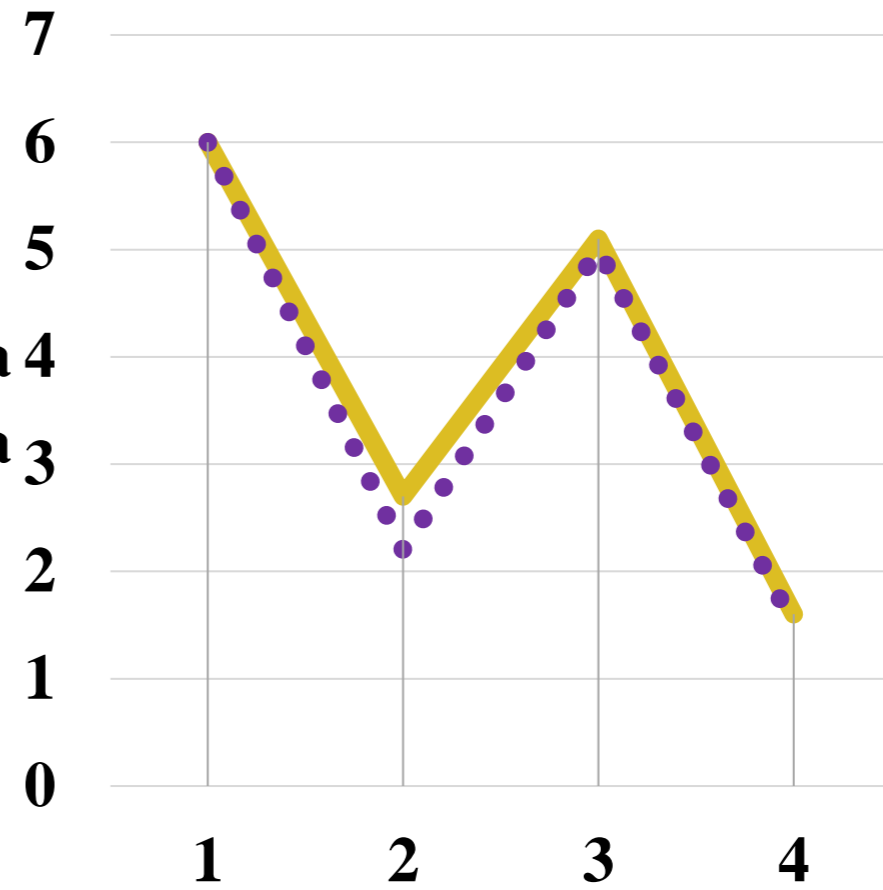


IRIS DATA





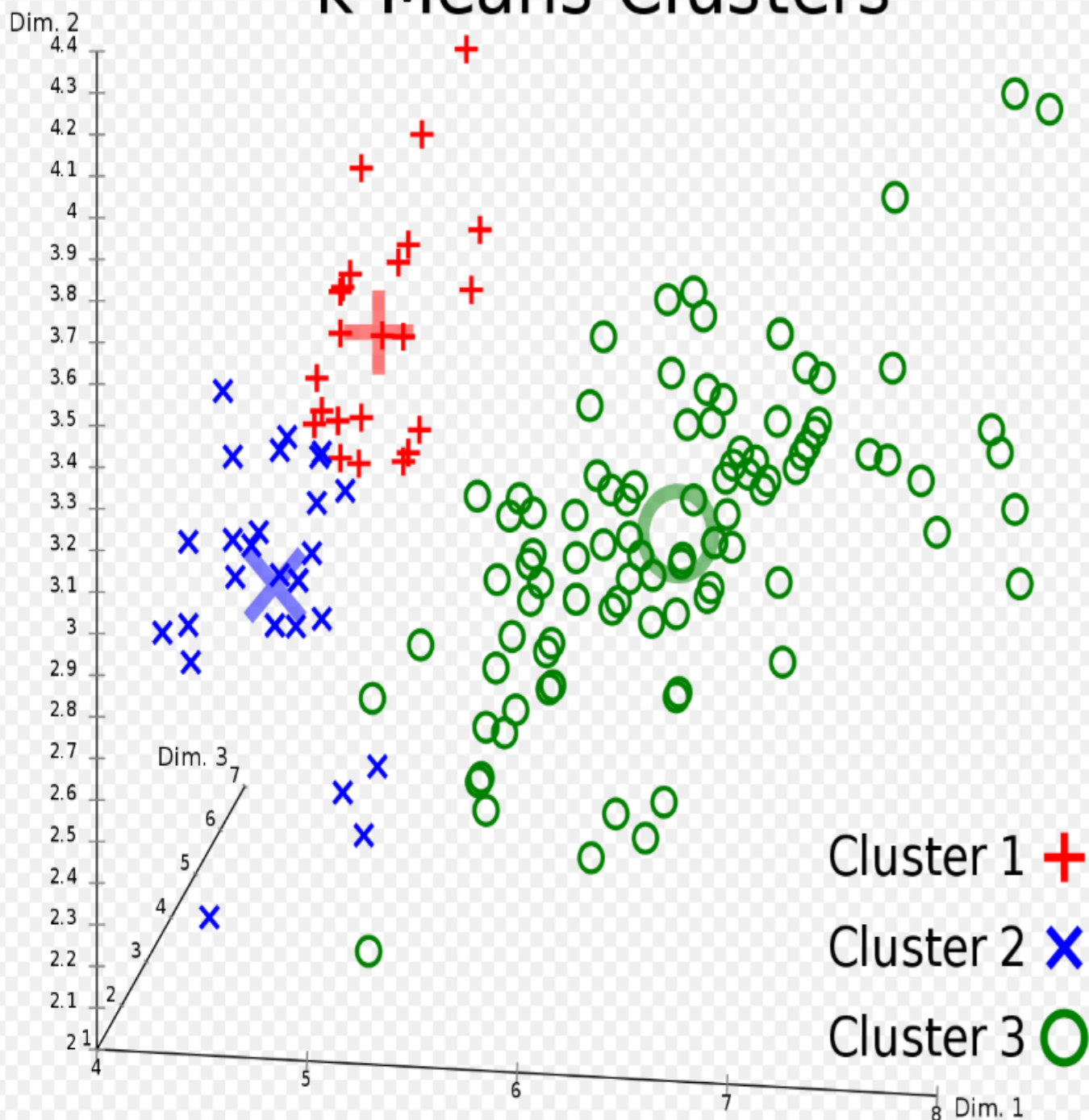
Setosa 4
Setosa 3



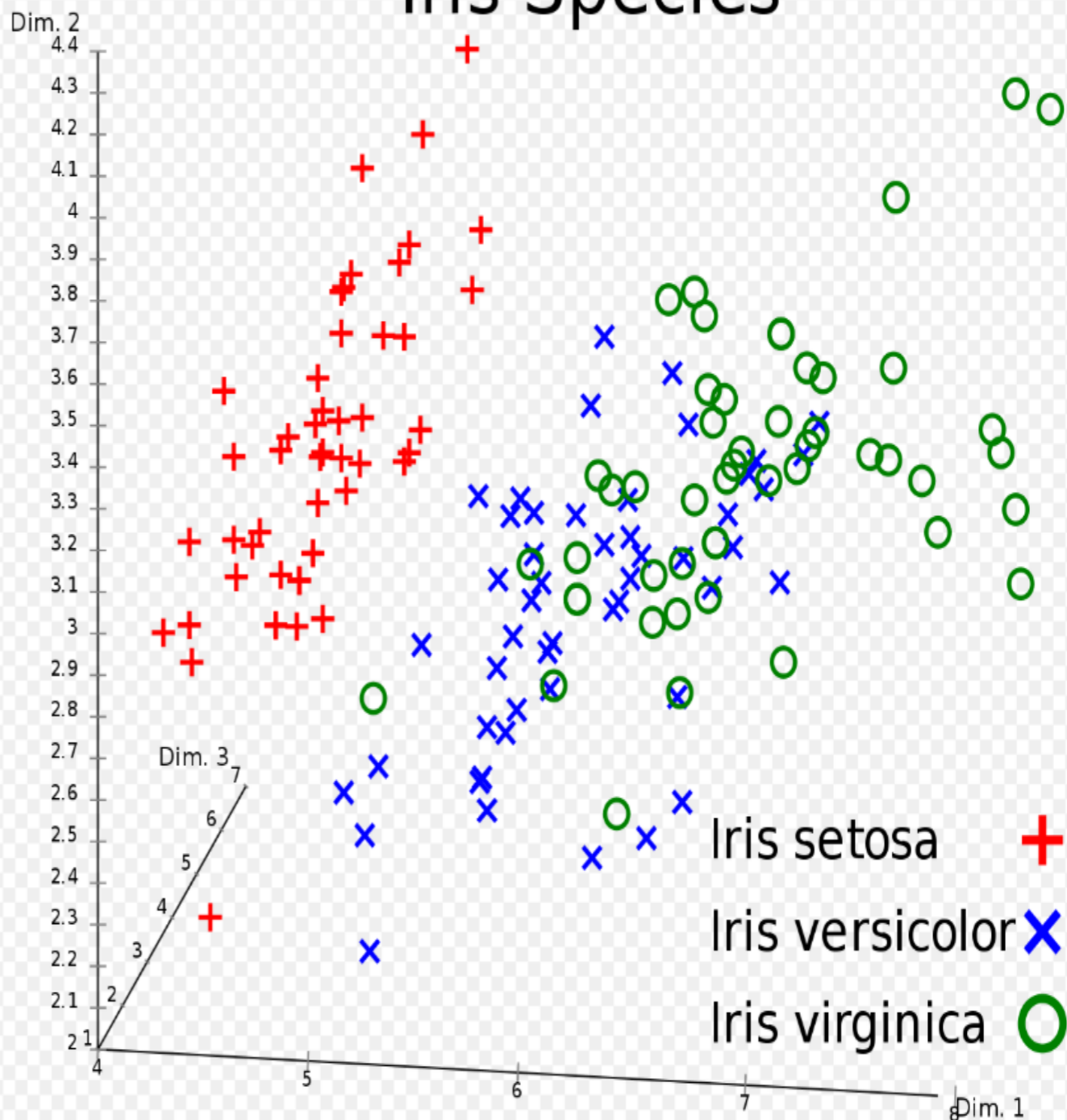
Versicolor
Virginica

K-means

k-Means Clusters

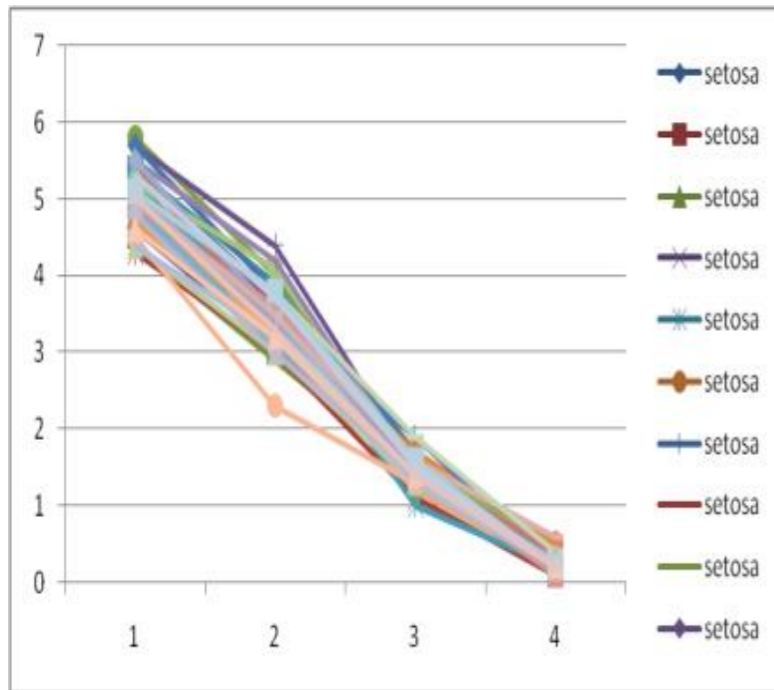


Iris Species

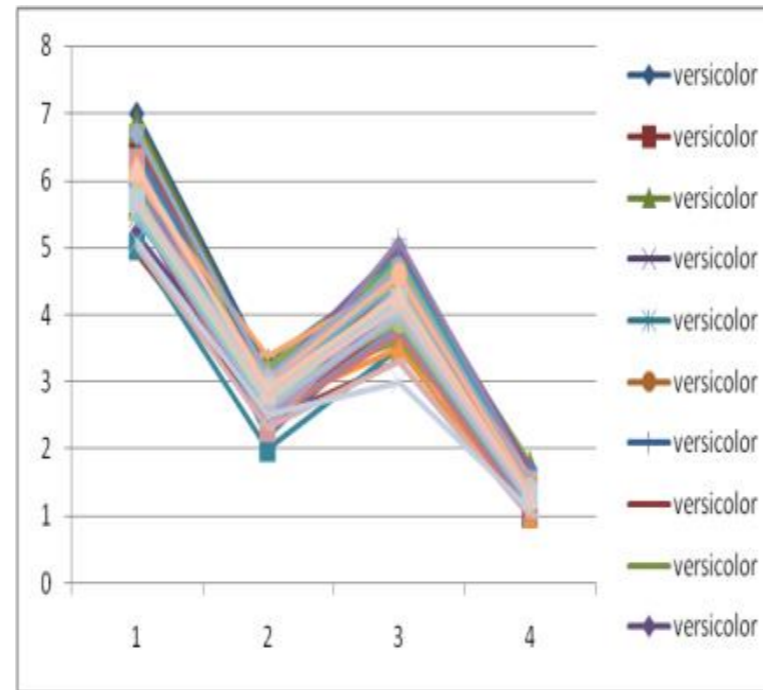




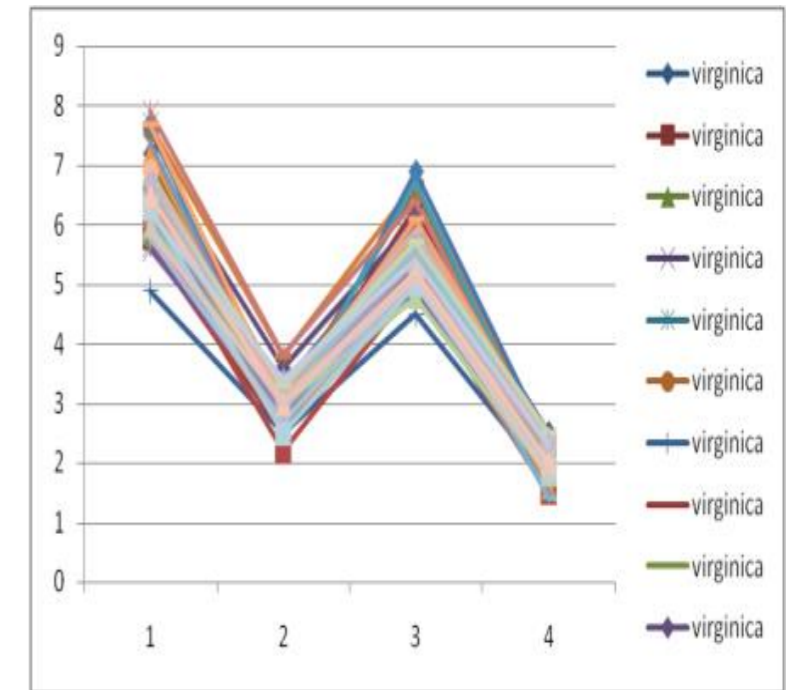
Setosa



Versicolor



Virginica



IRIS	NUMBER OF IRIS	MISTAKES
<i>Iris Setosa</i>	50	0
<i>Iris Versicolor</i>	50	1
<i>Iris Virginica</i>	50	4

Balance Scale Data Set Siegler (1976)

The “Balance Scale Data set” data set is modeled for psychological experiments and contains 625 lines. This kit is also often used to test cluster analysis methods. Each line reflects the values of five indicators ($p_0; p_1; p_2; p_3; p_4$), which characterize the indicators and the state of the lever weights. The indicator $p_0 = \{ "B", "L", "R" \}$ characterizes the state of the weights: "B" - balance; the deviation of the scale arrow to the left ("L" - left) or to the right ("R" - right).

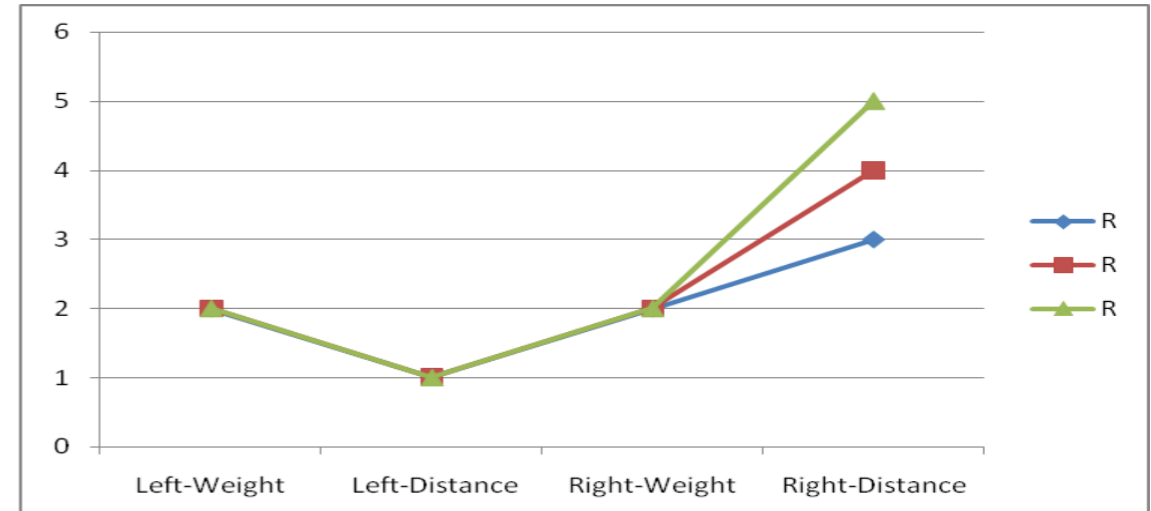
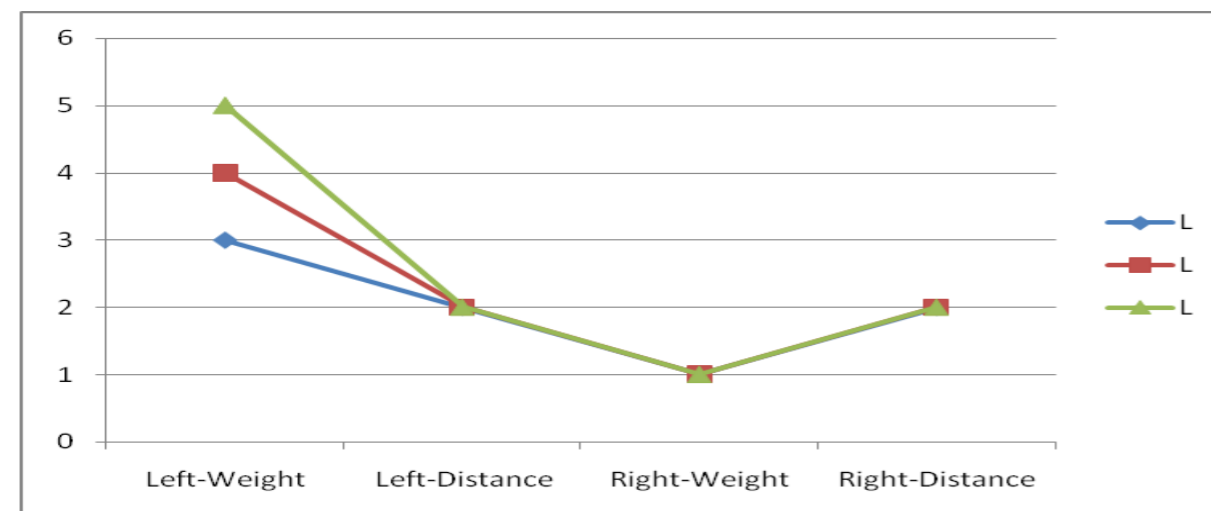
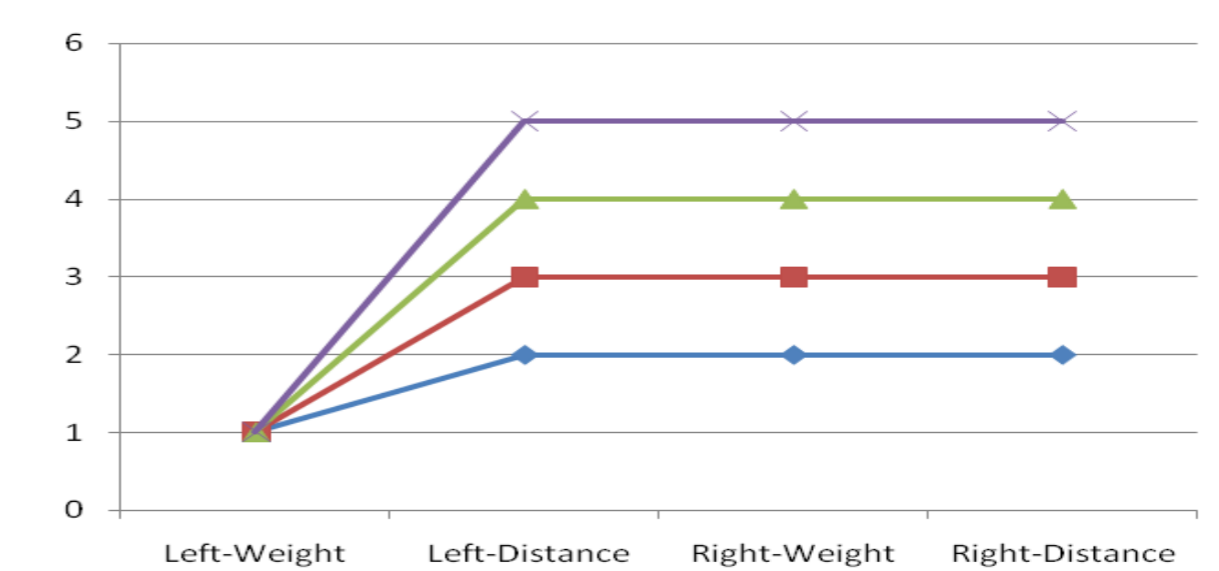
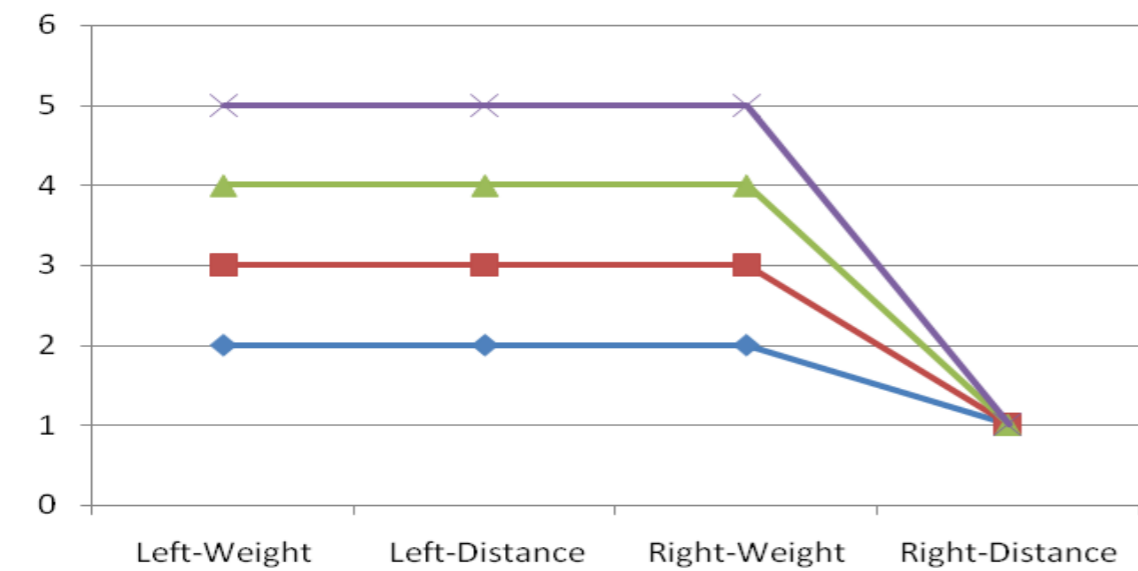
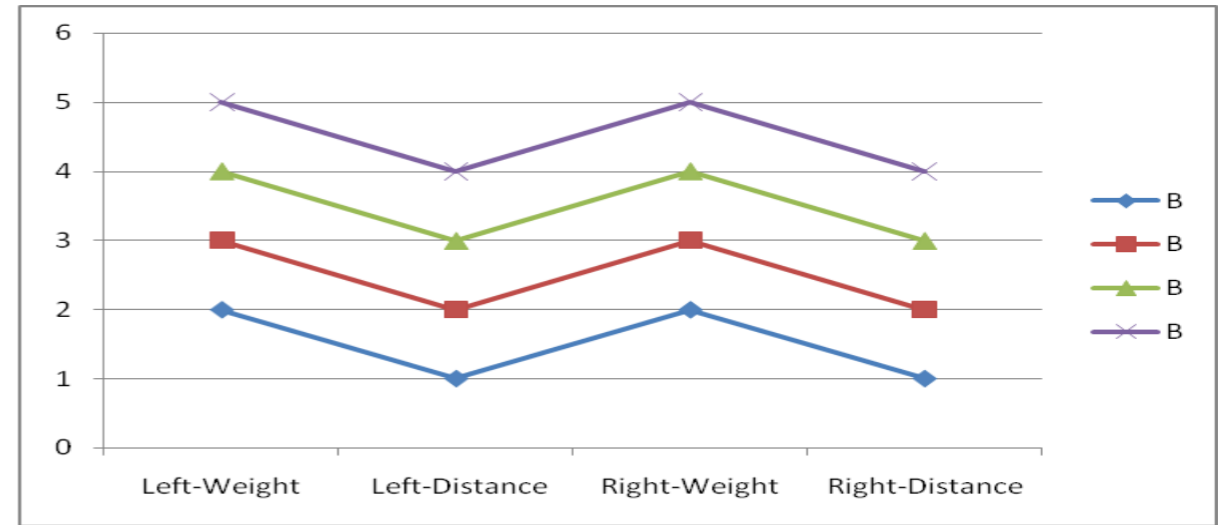
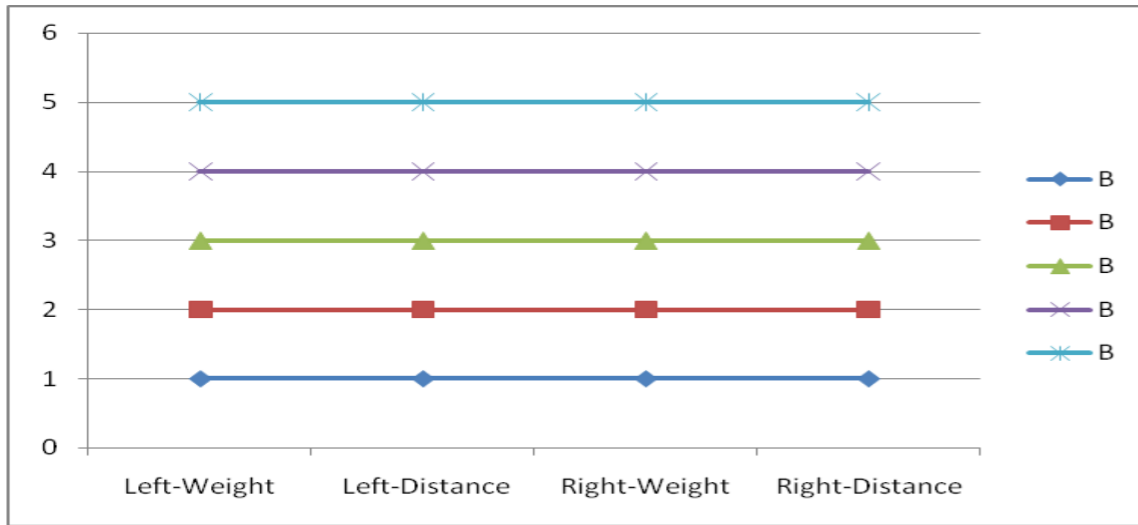
The remaining parameters p_1, p_2, p_3 , and p_4 take integer values from one to five and characterize:

$p_1 = \{ 1, 2, 3, 4, 5 \}$ - weight of the left bowl of scales;

$p_2 = \{ 1, 2, 3, 4, 5 \}$ - the length of the lever on which the left bowl is suspended;

$p_3 = \{ 1, 2, 3, 4, 5 \}$ - weight of the right bowl of scales; and

$p_4 = \{ 1, 2, 3, 4, 5 \}$ - the length of the lever on which the right bowl is suspended.





STATE CAPACITY

National Research University Higher School of Economics

State Capacity - the ability of the state to perform basic functions (external security, internal order, legitimacy, good governance, ensuring conditions for development).



BASIC SYSTEM OF INDICATORS (1995-2005-2015)

UNSC_NY	Membership in the Security Council
IMF_vote	Share of votes in the IMF
Military_exp	Military expenditures
army	Number of military
Nuclear_Weapon	Availability of nuclear weapons
export_share	Share in world exports
GDP_share	Share in world GDP
R&D_worldshare	Share in world Research & Development
Territory	Share in world territory
POPUL_share	Share in world population
Oil_Gas	Share in world oil and gas reserves
University	Number of universities in the top-500 (Shanghai ranking)
Nobel	Number of Nobel Prizes
Non-resident patents	Number of applications for patents from non-residents

CORRELATION ANALYSIS (2005)

	UNSC_NY2005	IMF_vote2005	Military_exp2005	army2005	Nuclear_Weapons2005	export_share2005	GDP_share2005	R&D_worldshare2005	Territory 2005	POPUL_share2005	Oil_Gas2005	University2005	Nobel2005	Non-resident patents 2005
UNSC_NY2005	1													
IMF_vote2005	0,73	1												
Military_exp2005	0,59	0,9	1											
army2005	0,57	0,52	0,5	1										
Nuclear_Weapons2005	0,86	0,66	0,64	0,68	1									
export_share2005	0,71	0,9	0,7	0,57	0,59	1								
GDP_share2005	0,66	0,97	0,95	0,52	0,63	0,84	1							
R&D_worldshare2005	0,58	0,93	0,93	0,45	0,56	0,79	0,98	1						
Territory 2005	0,62	0,48	0,41	0,6	0,63	0,44	0,43	0,36	1					
POPUL_share2005	0,39	0,32	0,25	0,81	0,5	0,41	0,31	0,23	0,46	1				
Oil_Gas2005	0,33	0,25	0,14	0,3	0,37	0,19	0,13	0,09	0,59	0,11	1			
University2005	0,63	0,95	0,96	0,47	0,62	0,83	0,98	0,96	0,4	0,25	0,1	1		
Nobel2005	0,66	0,91	0,94	0,4	0,65	0,76	0,92	0,89	0,34	0,16	0,1	0,96	1	
Non-resident patents 2005	0,57	0,85	0,9	0,65	0,61	0,75	0,92	0,91	0,54	0,48	0,14	0,9	0,79	1

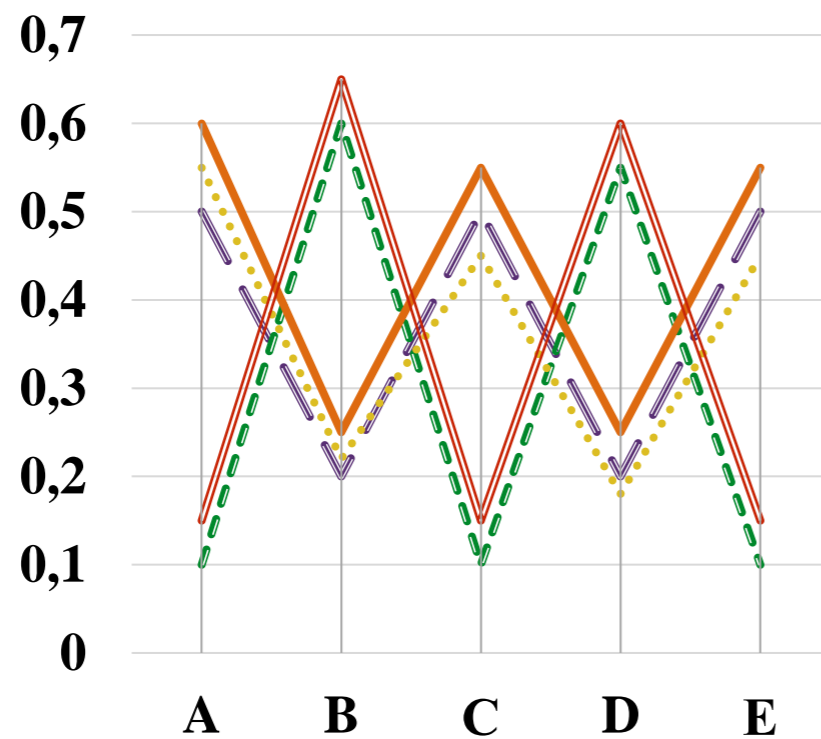


BASIC SYSTEM OF INDICATORS

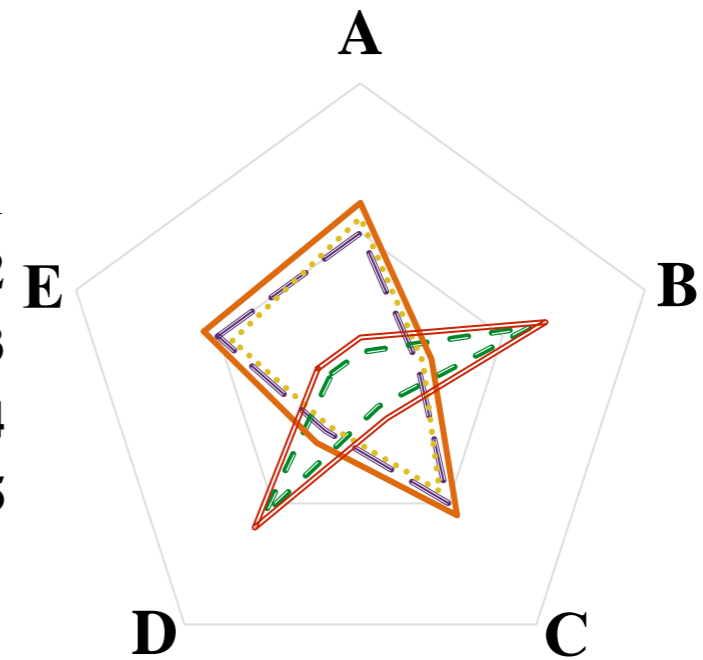
- *Mil_exp* – share of military spending in a country's GDP;
- *Mil_pers* – share of military personnel in the total population;
- *Taxes* – share of tax revenue in a country's GDP;
- *Safety* – parameter, calculated as the inverse of the total level of homicides and victims of conflicts;
- *WGI* – indicator characterizing the quality of state institutions.

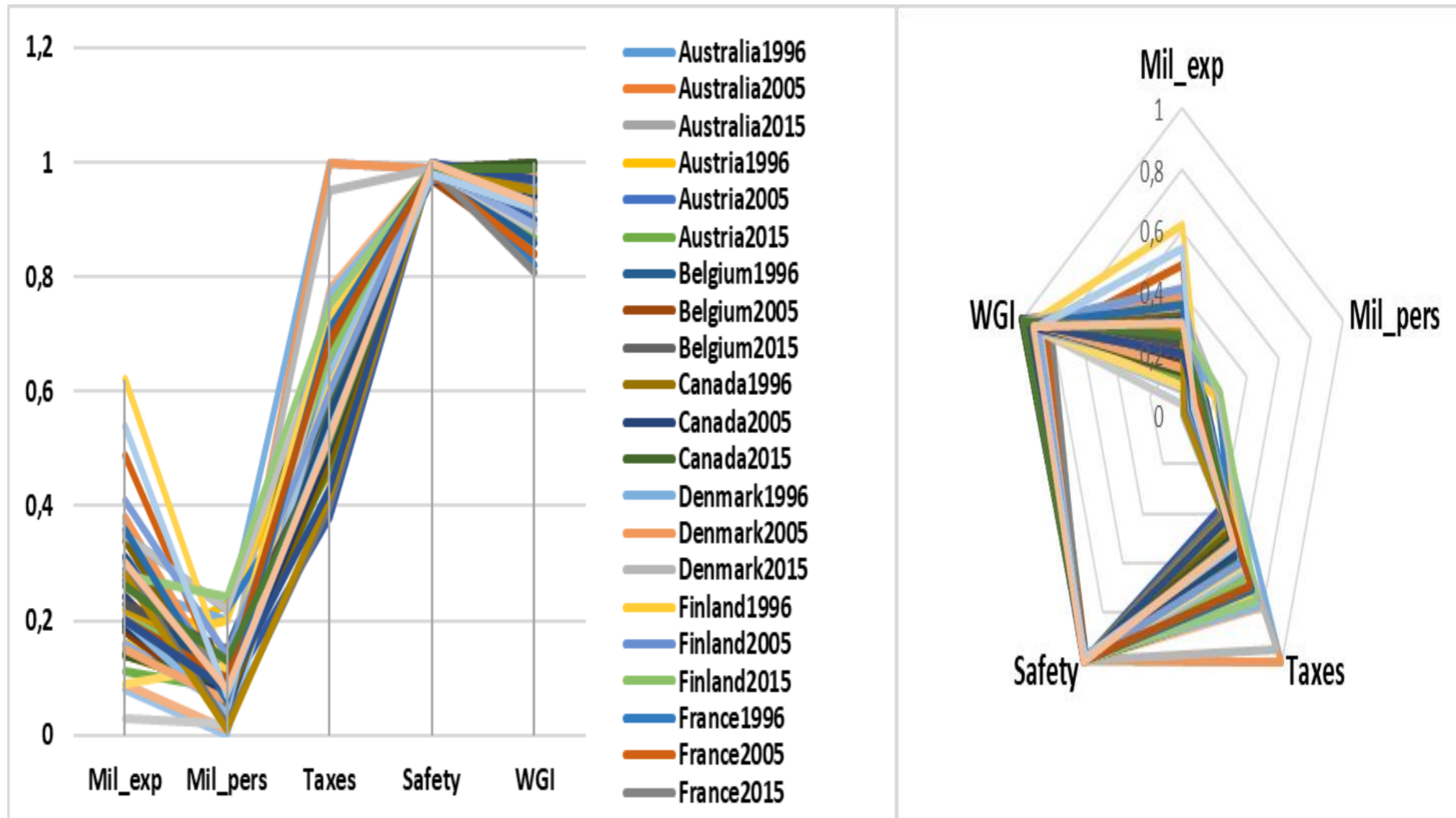


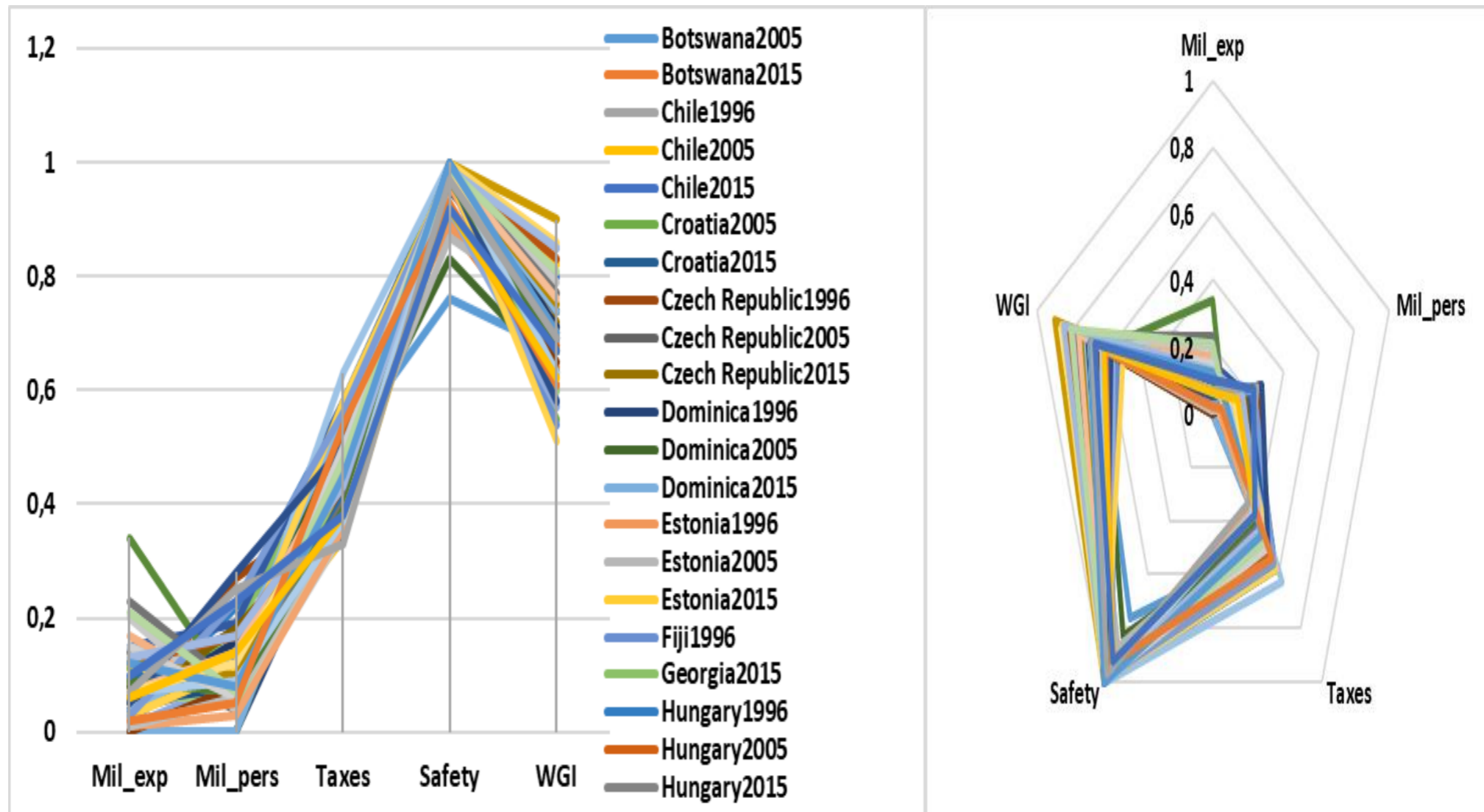
<i>Country</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
Country 1	0,5	0,2	0,5	0,2	0,5
Country 2	0,1	0,6	0,1	0,55	0,1
Country 3	0,55	0,22	0,45	0,18	0,45
Country 4	0,6	0,25	0,55	0,25	0,55
Country 5	0,15	0,65	0,15	0,6	0,15

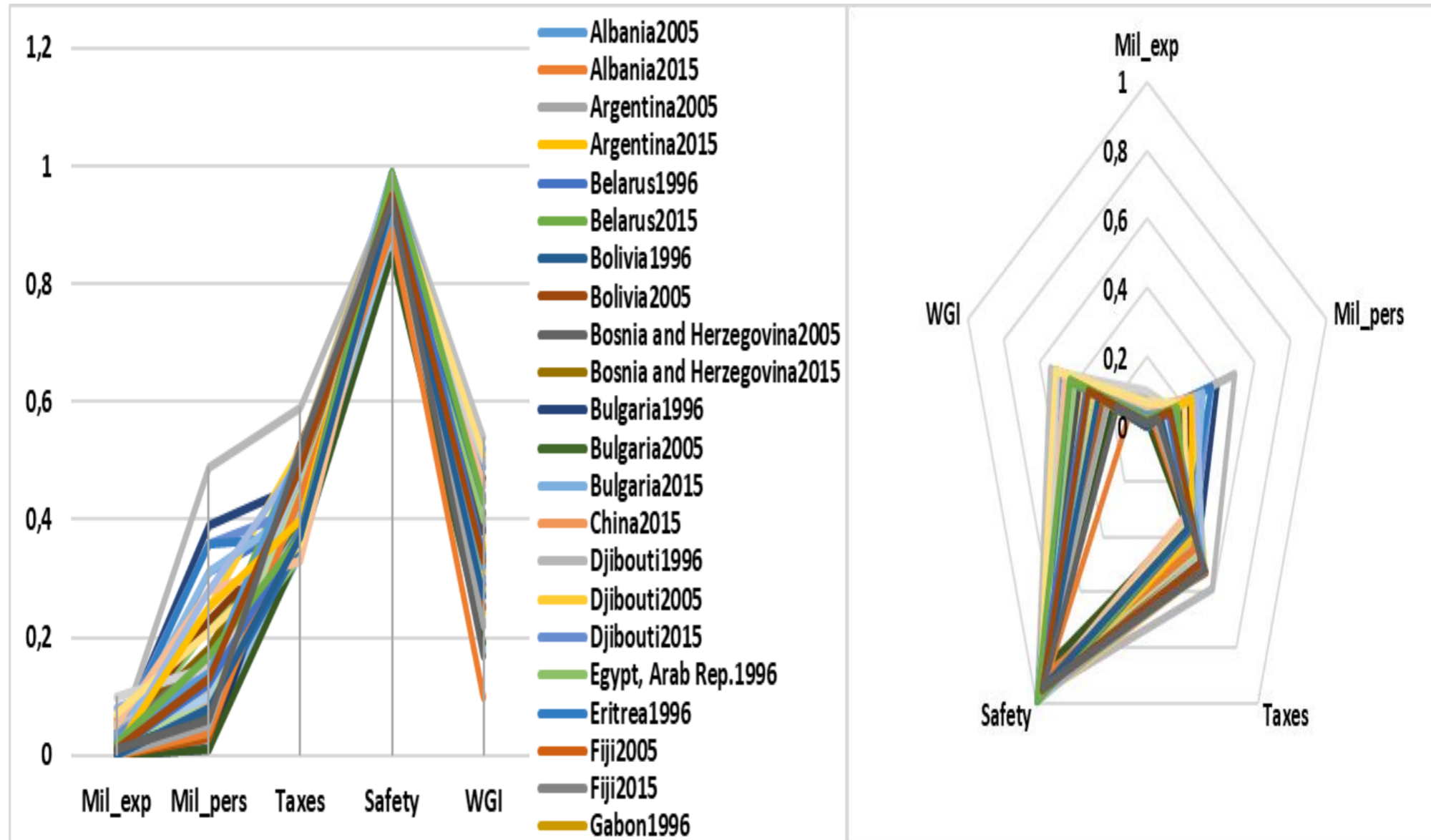


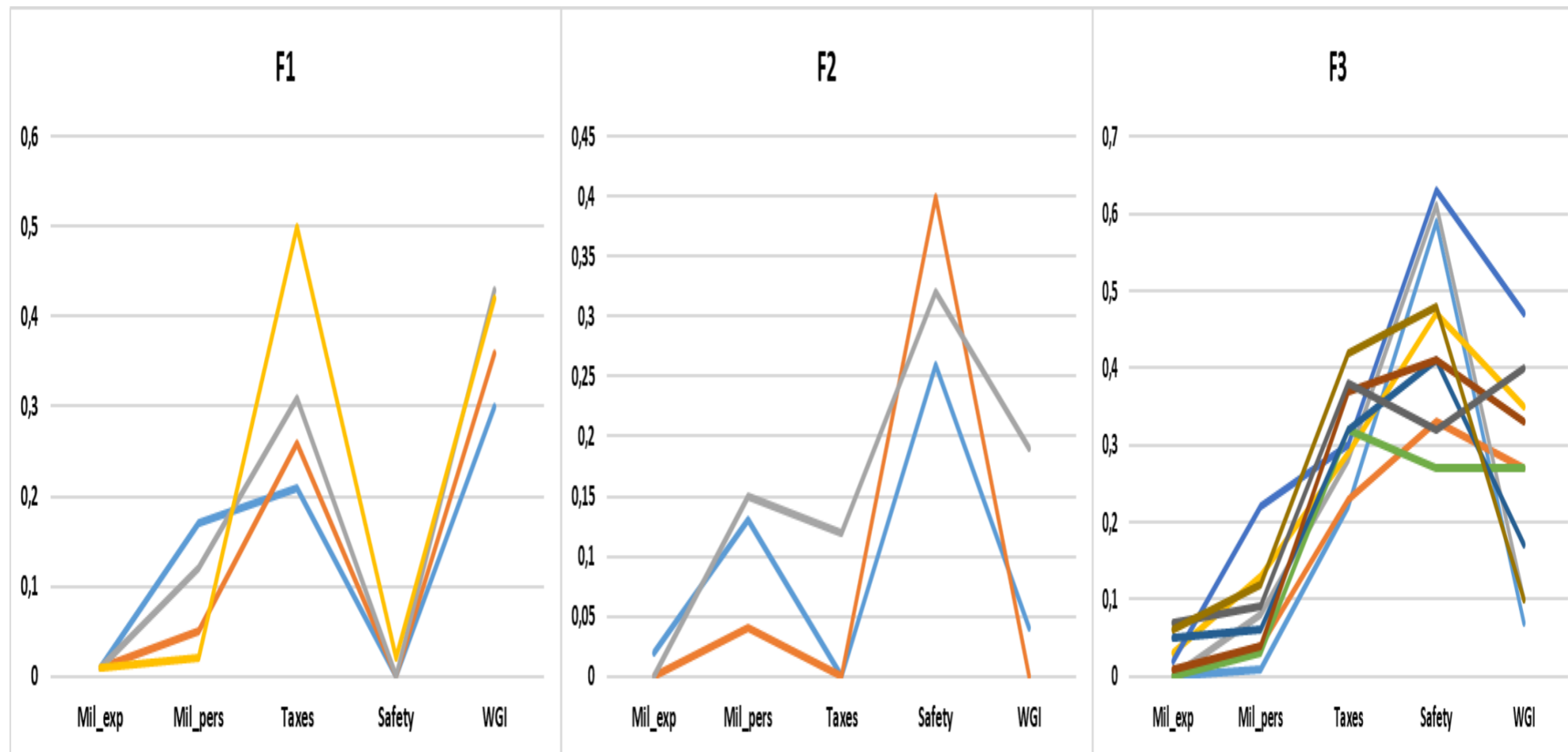
- Страна 1
- - Страна 2
- ... Страна 3
- Страна 4
- Страна 5

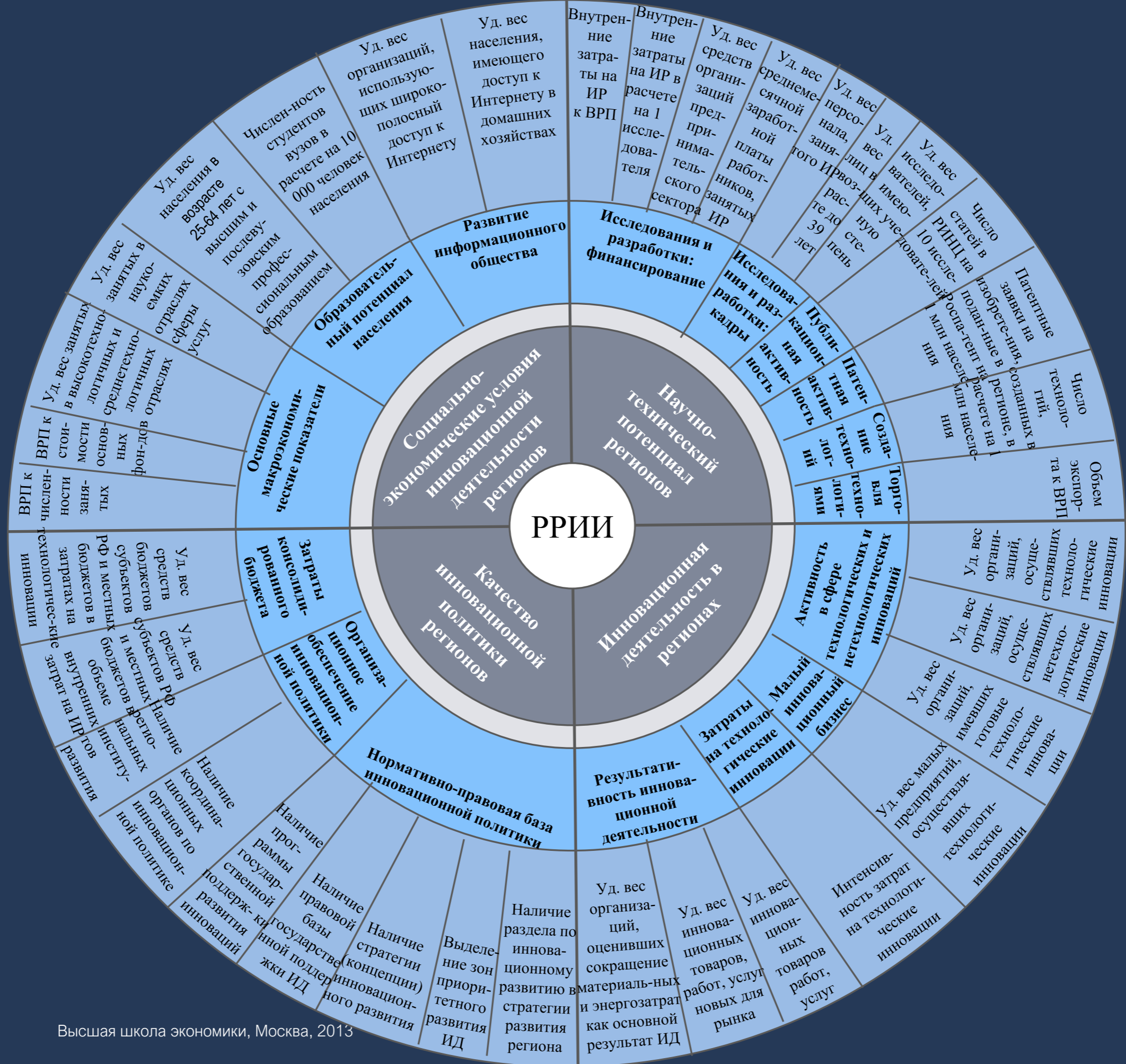












Basic system of indicators

Block 1. Socio-economic conditions

Block 2. Educational potential

Block 3. The effectiveness of research and development

Block 4. The potential of innovation

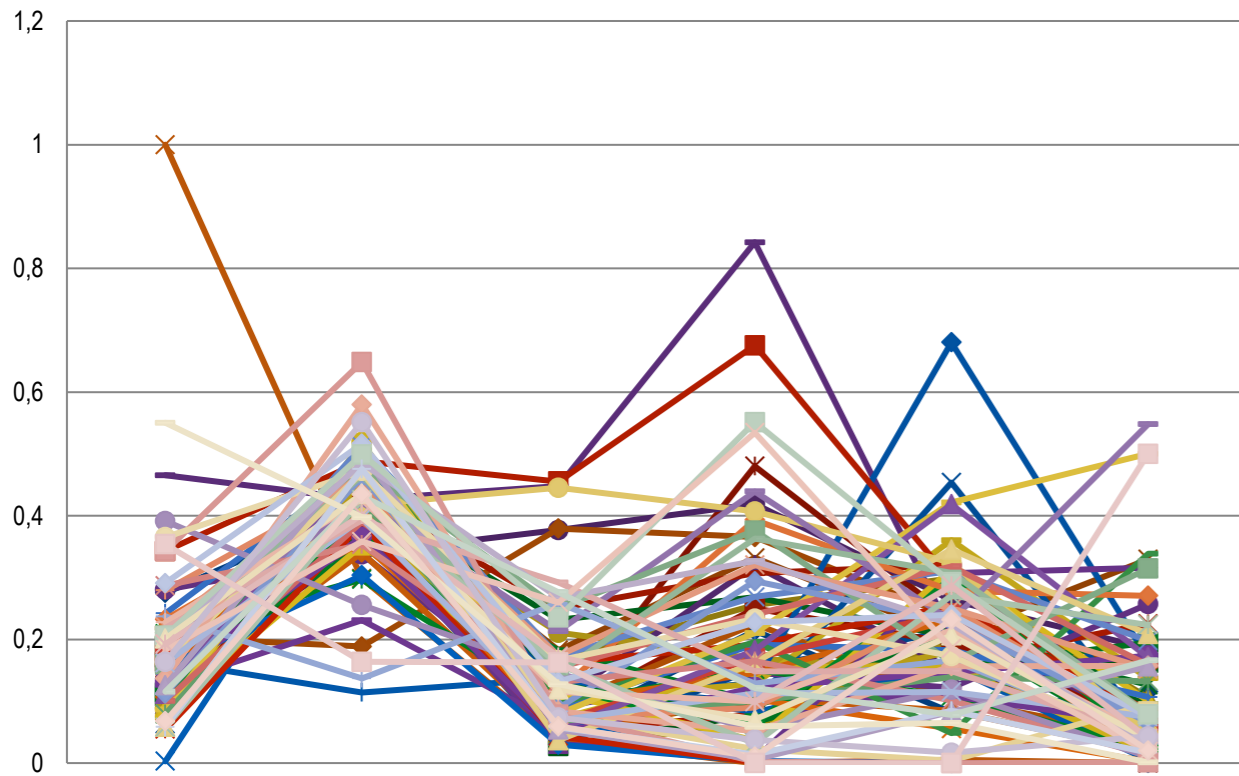
Block 5. The effectiveness of innovation



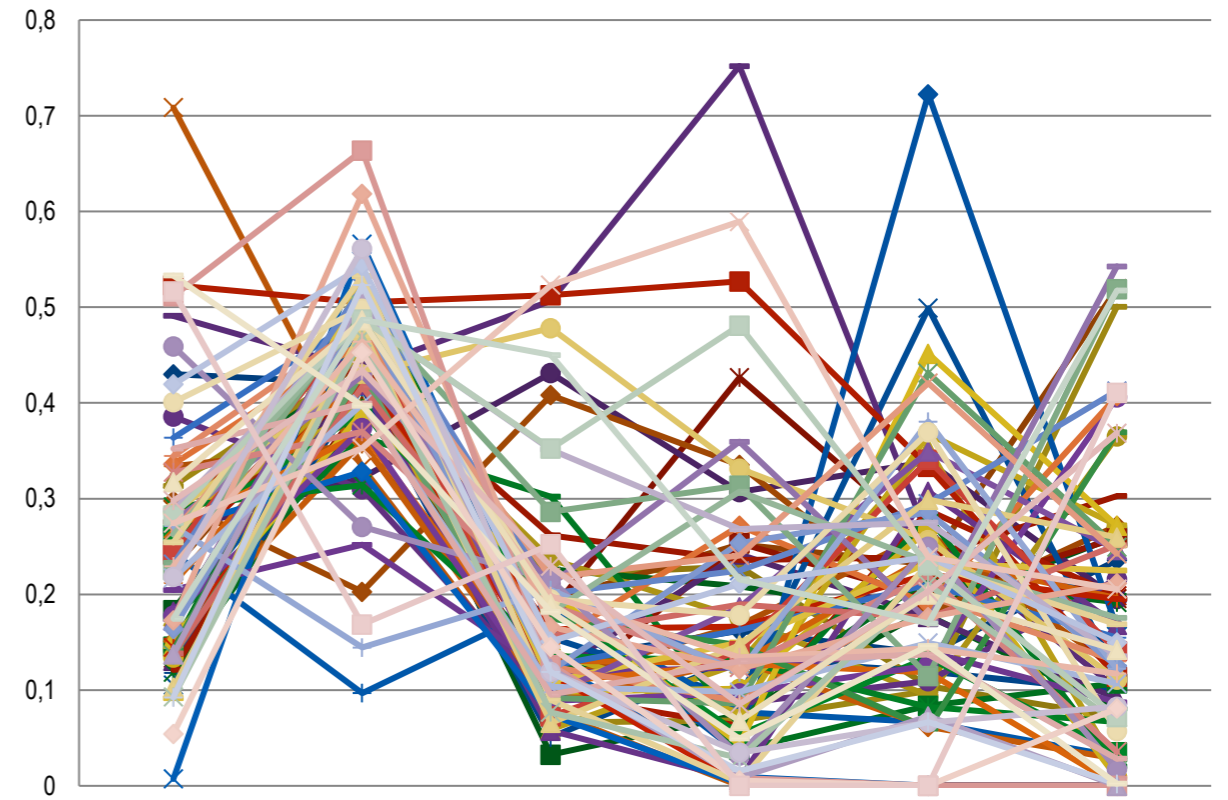
<i>2007</i>	<i>Block 1</i>	<i>Block 2</i>	<i>Block X</i>	<i>Block 3</i>	<i>Block 4</i>	<i>Block 5</i>
Block 1	1					
Block 2	-0,2	1				
Block X	0,16	-0,1	1			
Block 4	0,11	0,11	0,7	1		
Block 5	-0,2	0,14	0,22	0,33	1	
Block 6	-0,1	0,05	0,05	0,13	0,21	1



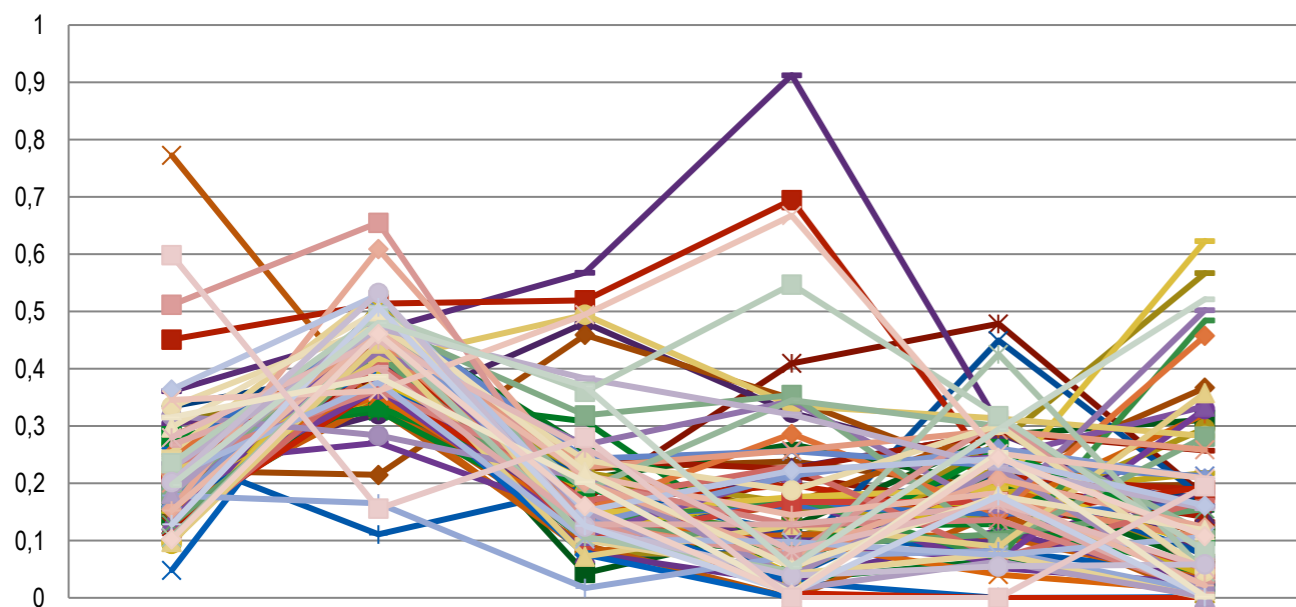
2007



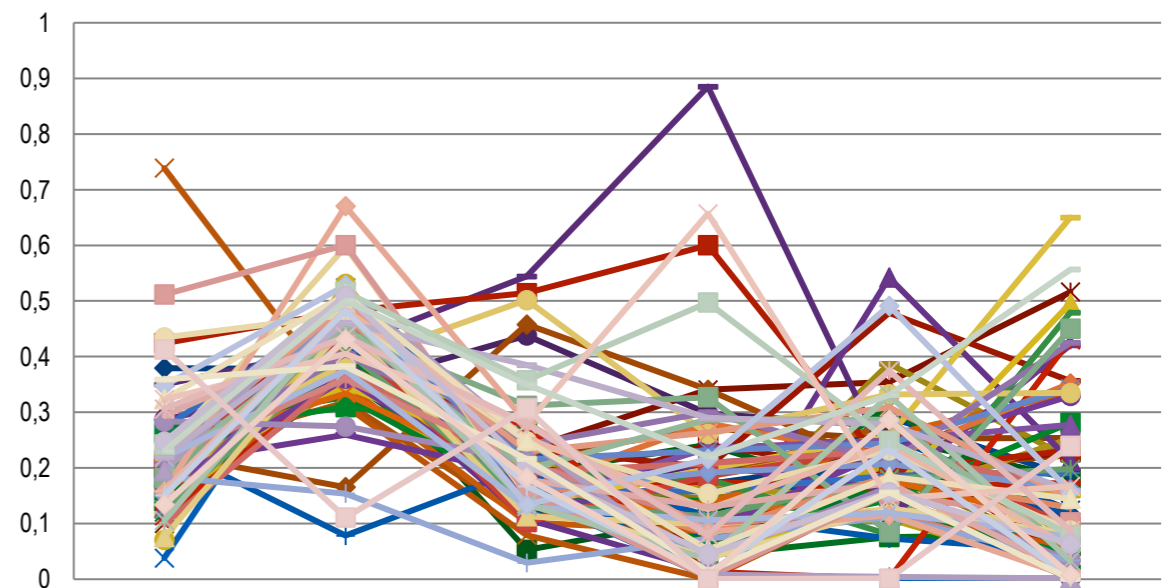
2008



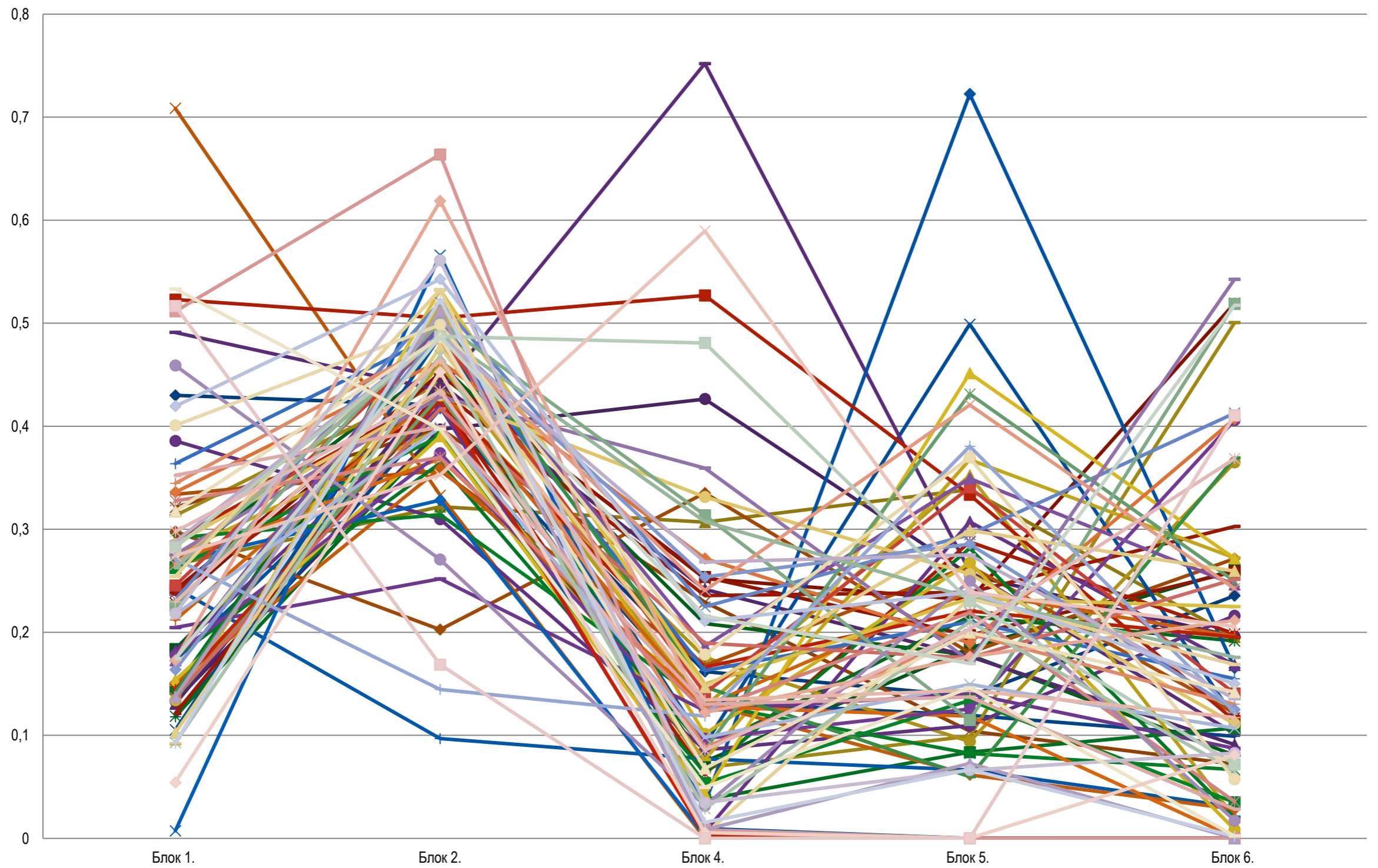
2009



2010

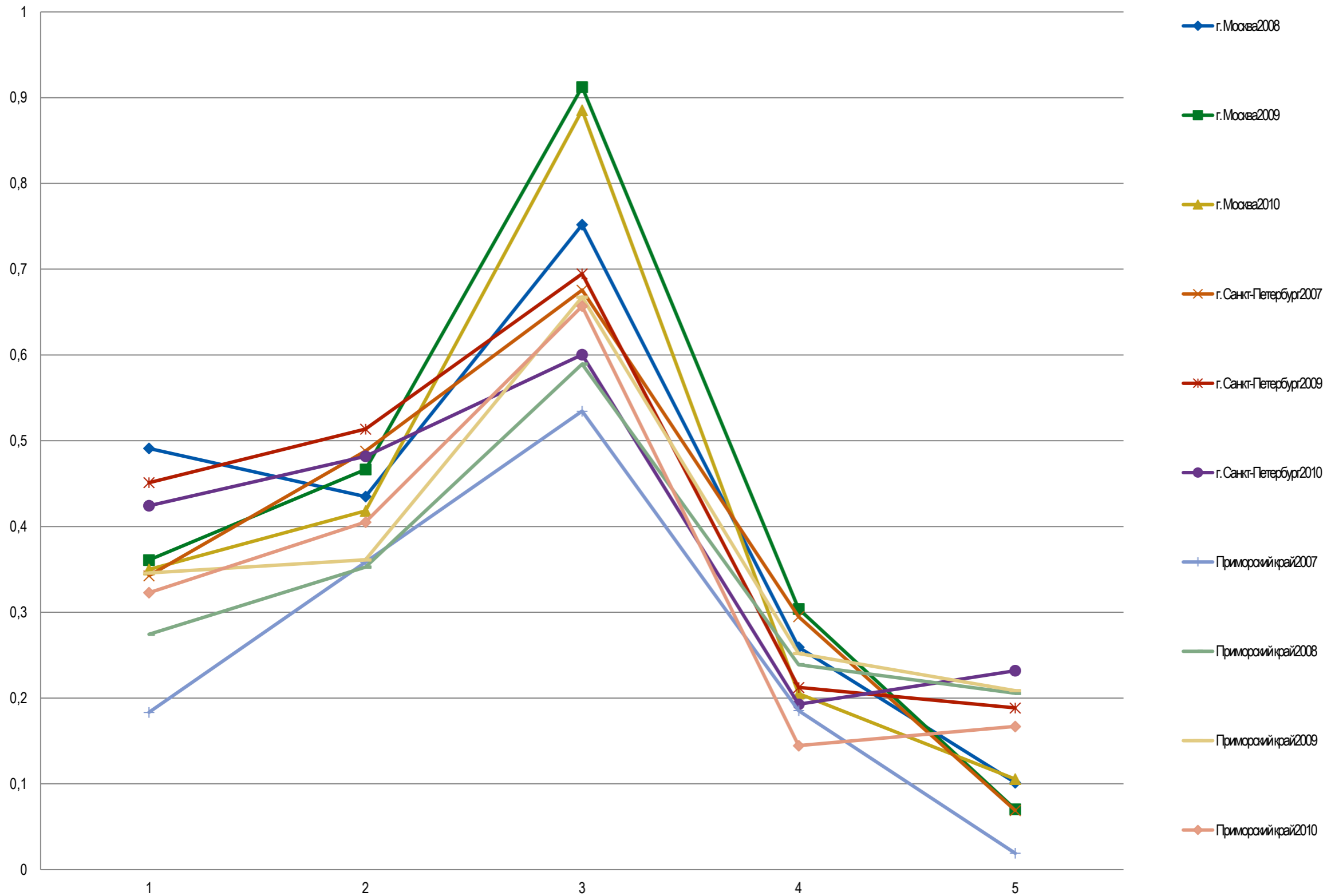


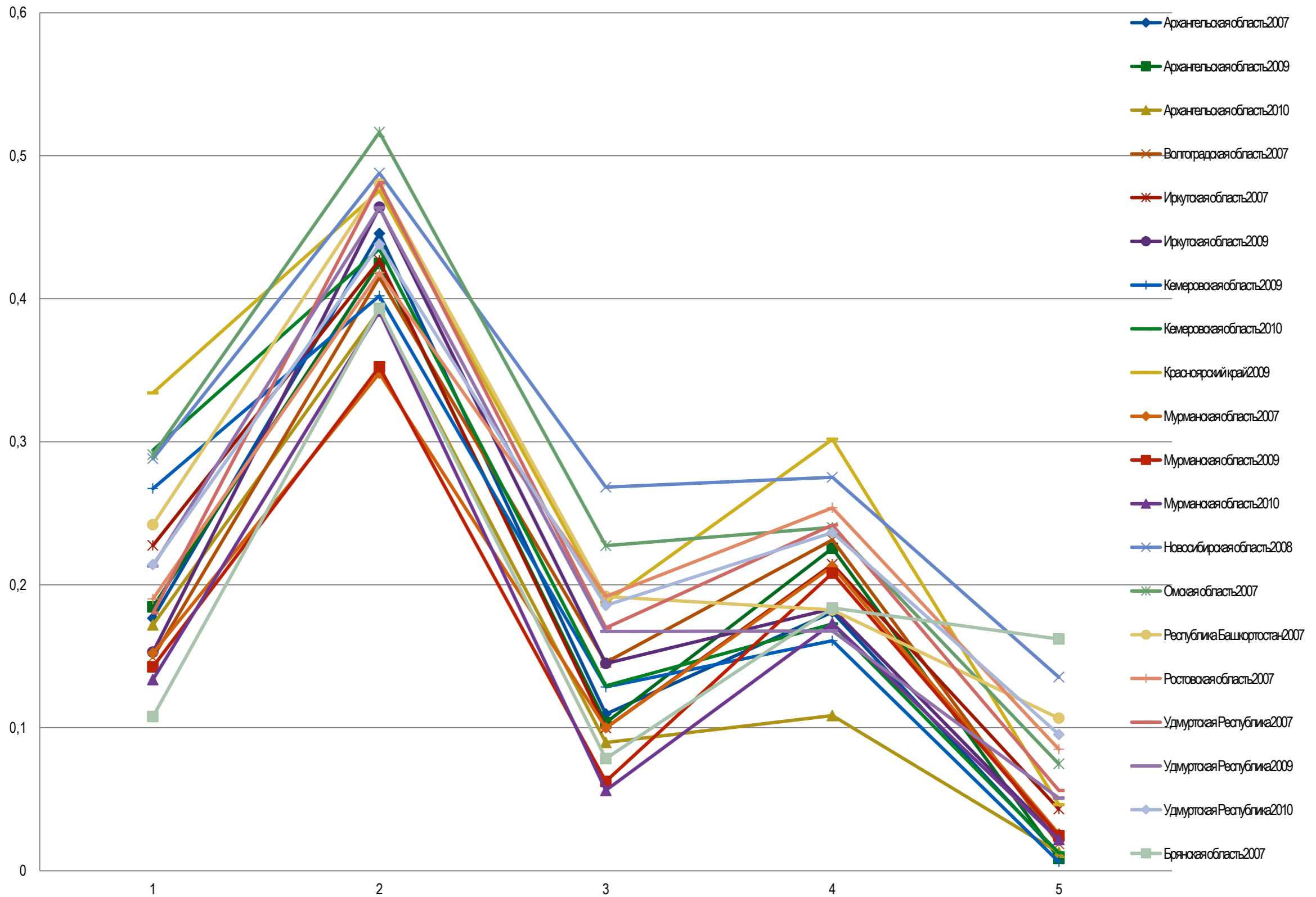
2008



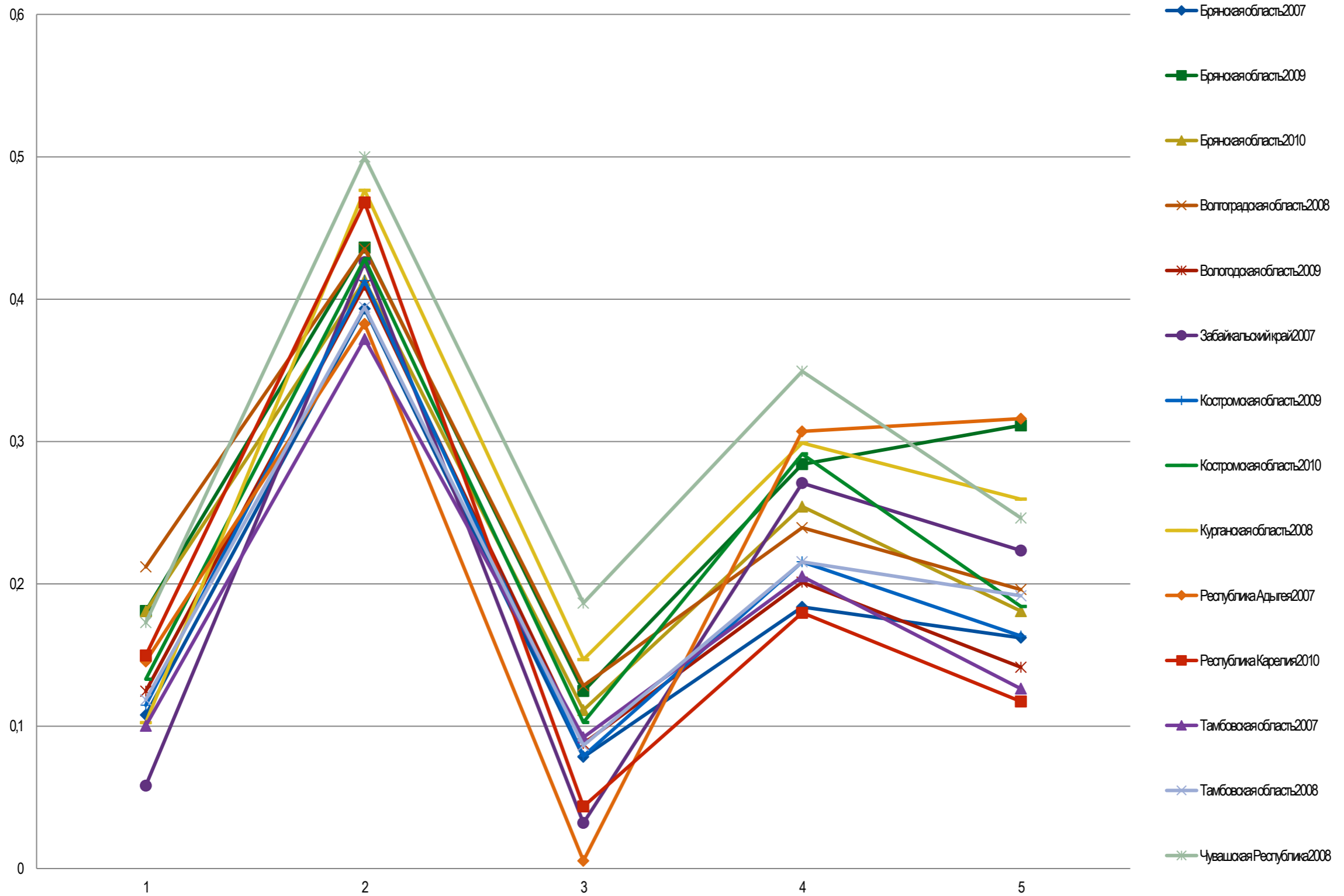


1



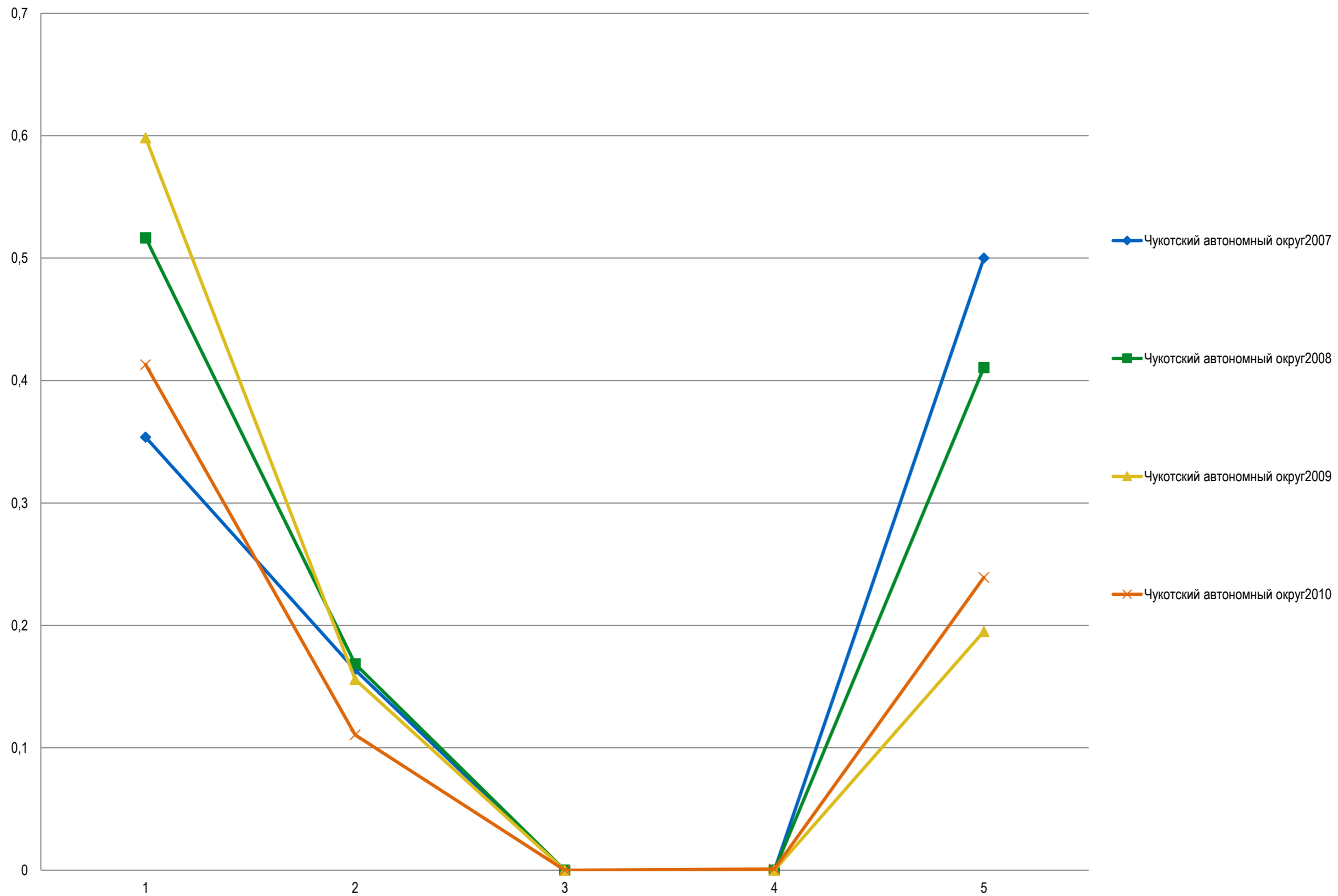


3





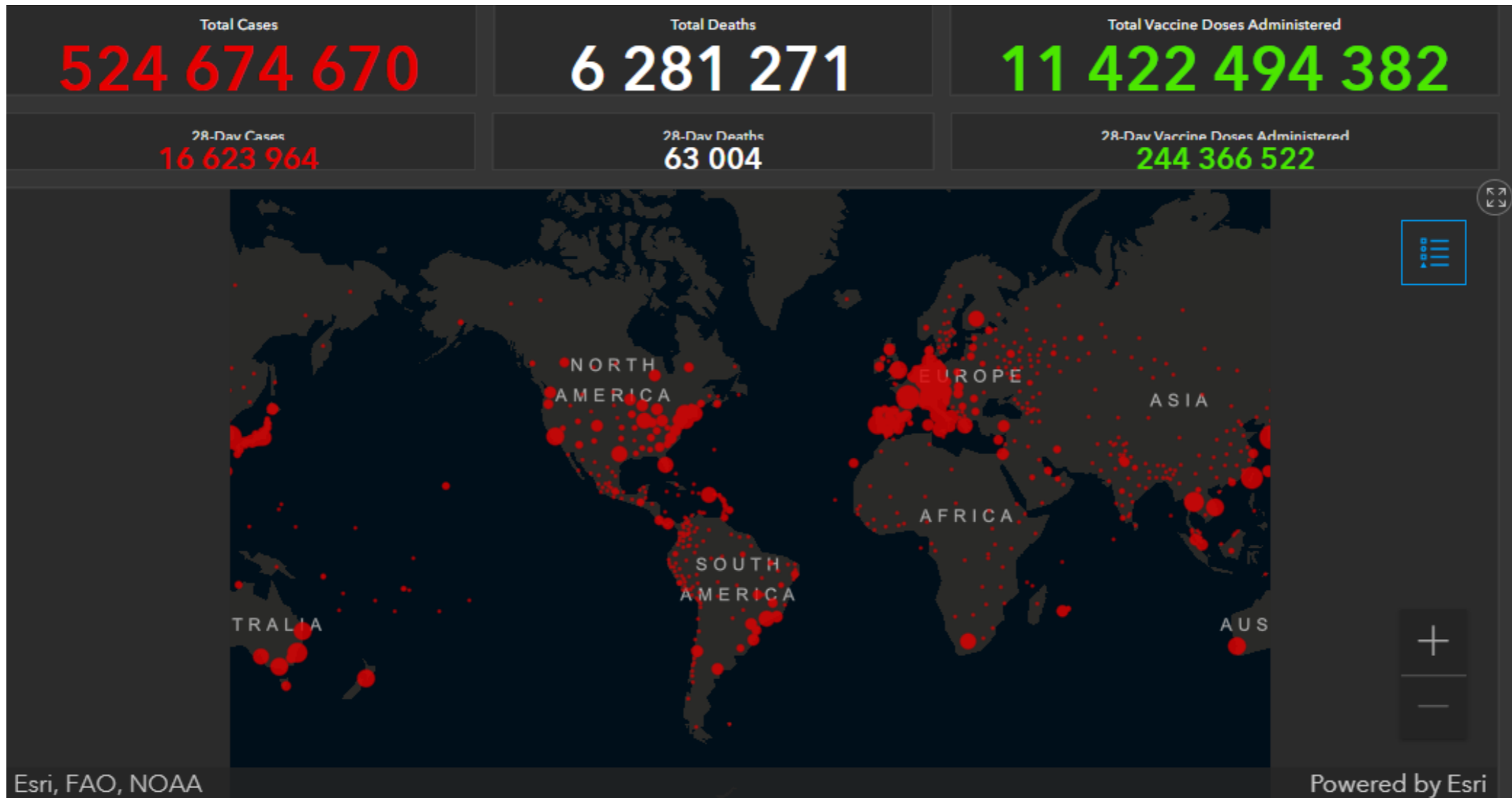
20





COVID-19 CASES

National Research University
Higher School of Economics



COVID-19 Dashboard by the Center for Systems Science and Engineering (CSSE)
at Johns Hopkins University (JHU): <https://coronavirus.jhu.edu/map.html>
Accessed 18 May 2022



DATA USED

National Research University
Higher School of Economics

- The intensity of separate quarantine measures
- Number of infected people
- Education Level
- Law-abidingness

The source: The Oxford COVID-19 Government Response Tracker

On everyday basis from January 1, 2020
Covers more than 180 countries

<https://www.bsg.ox.ac.uk/research/research-projects/coronavirus-government-response-tracker>



UNIVERSITY OF
OXFORD



DATA USED

National Research University
Higher School of Economics

Quarantine measures, Criteria, Grades

Criterion	Grades
C1. School closing	4
C2. Workplace closing	4
C3. Cancel public events	3
C4. Restrictions on gathering size	5
C5. Close public transport	3
C6. Stay at home requirements	4
C7. Restrictions on internal movement	3
C8. Restrictions on international travel	5

Law-abidingness – to which extent people follow the rules stated by government (expert estimations)



AGGREGATION MODEL OF QUARANTINE MEASURES

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Higher School of Economics

The threshold aggregation of quarantine measure is a lexicographic ordering of criterial values. Countries are compared in a pairwise way beginning from criteria with minimal values being equal 0.

A country for which the number of criteria with 0 values is lower dominates a country with more 0 values. If for a pair of countries the number of criteria with 0 values is equal, then the criteria with value 1 are compared. The process proceeds until maximal values of criteria.

Gathering restrictions

- C2. Workplace closing
- C3. Cancel public events
- C4. Restrictions on gathering size
- C6. Stay at home requirements

Government prohibitions

- C1. School closing
- C5. Close public transport
- C7. Restrictions on internal movement
- C8. Restrictions on international travel



AGGREGATION MODEL OF QUARANTINE MEASURES

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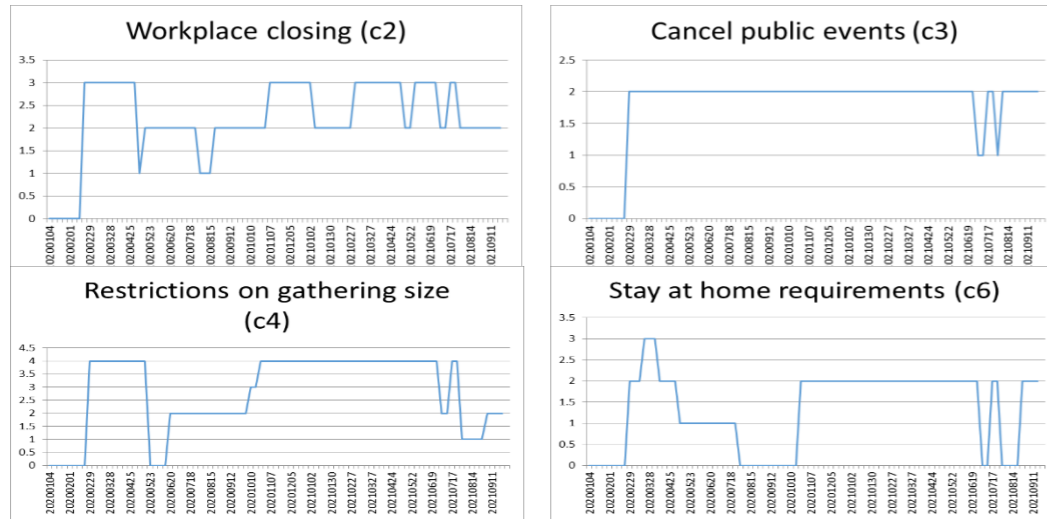
Threshold aggregation of quarantine measures
Example for 3 parameters

	Measure X	Measure Y	Measure Z
Country A	average	average	average
Country B	tough	tough	mild

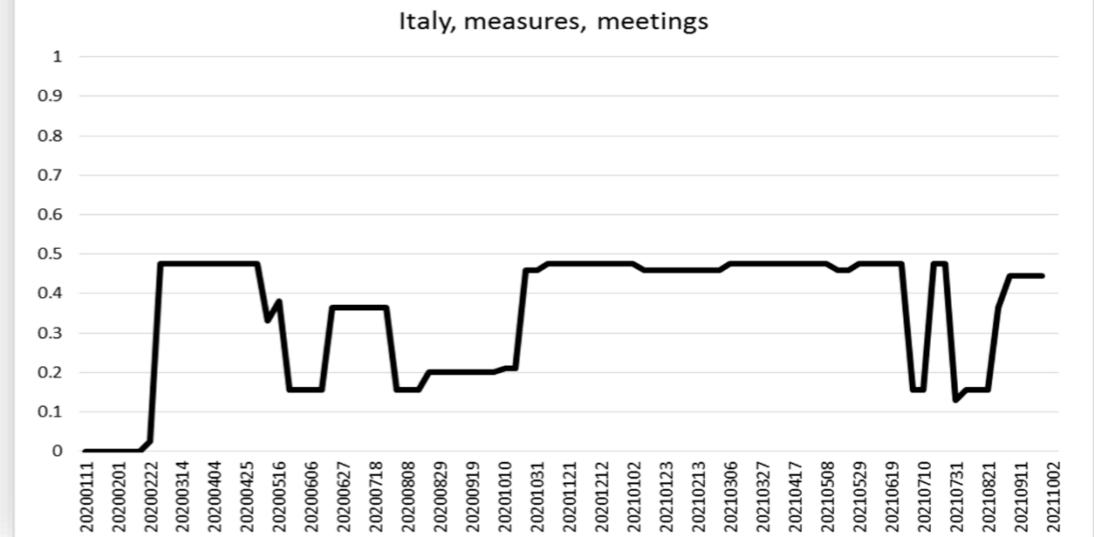
Aggregation – country A has stronger measures than country B



Measures (gathering)



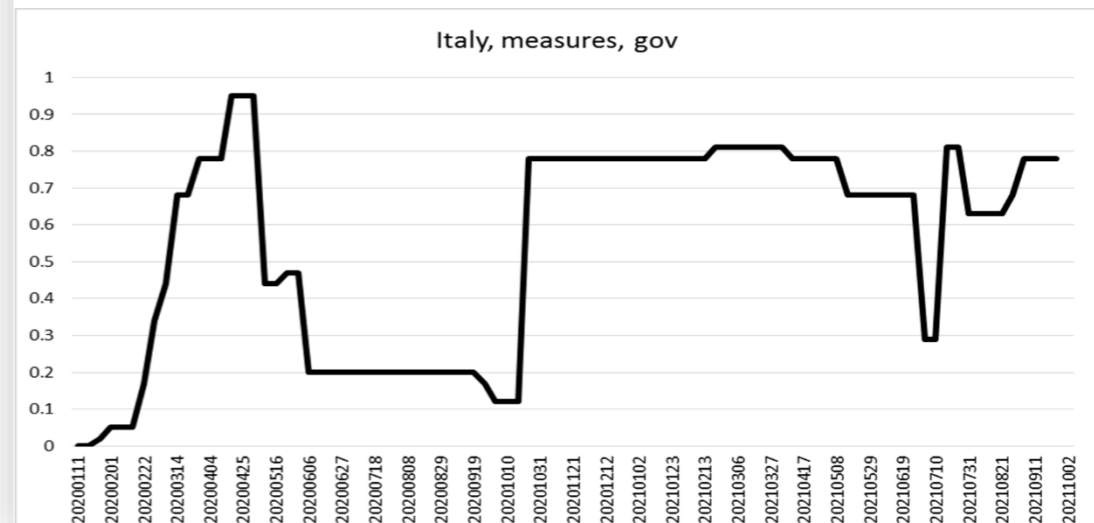
Aggregated measures (gathering)



Measures (government)



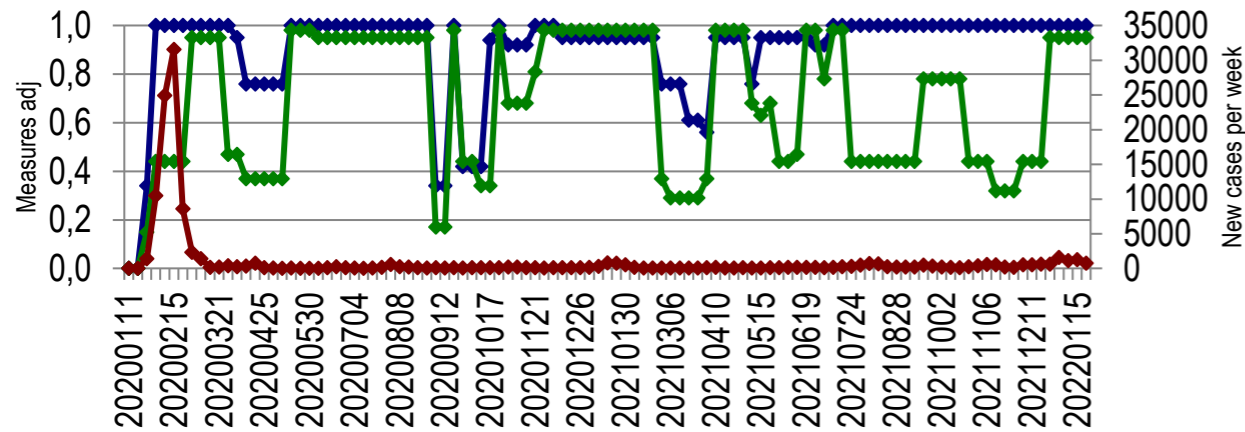
Aggregated measures (government)



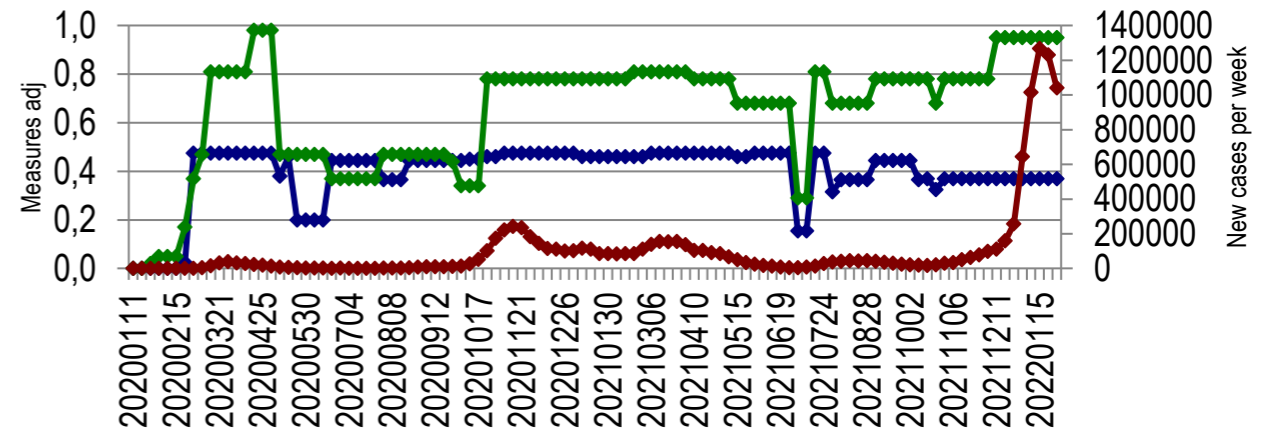


AGGREGATED MEASURES: GATHERING AND ENTRANCE RESTRICTION AND NEW CASES PER WEEK

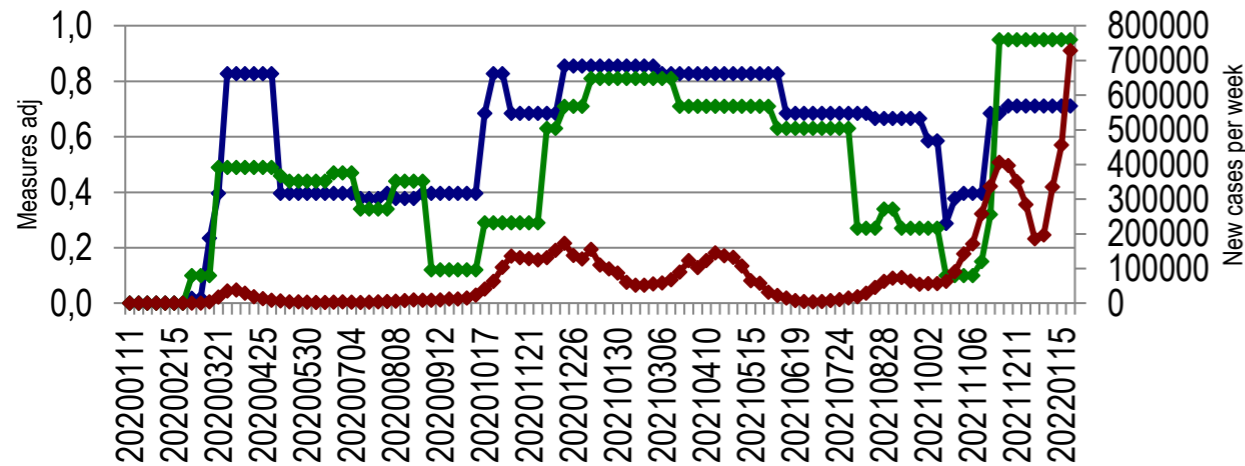
China



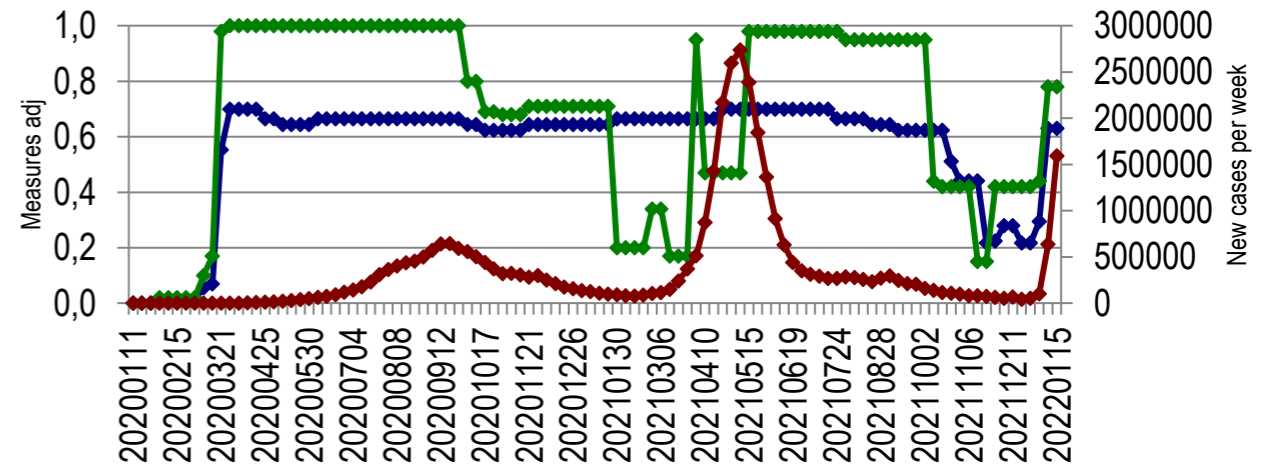
Italy



Germany



India



New cases

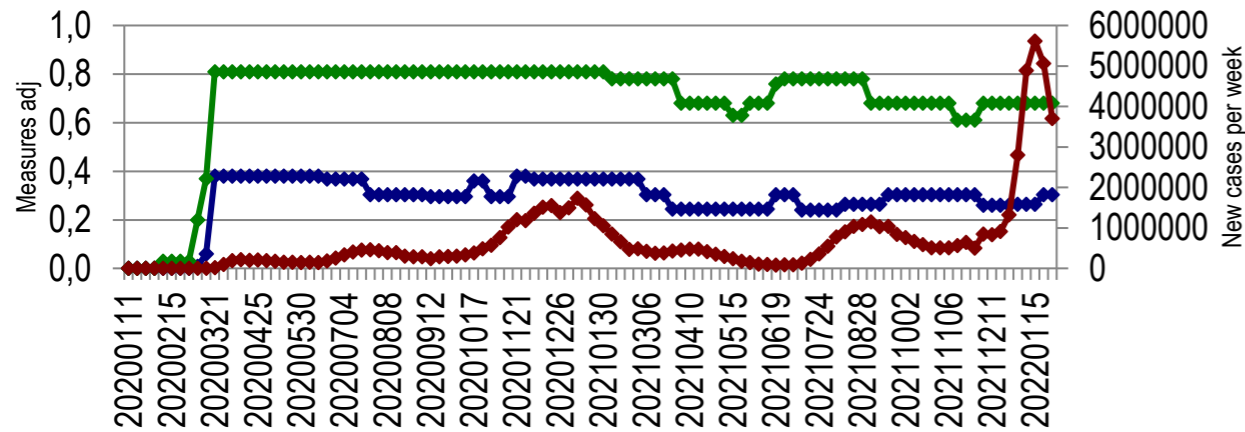
Measures: Meet.

Measures: Gov.

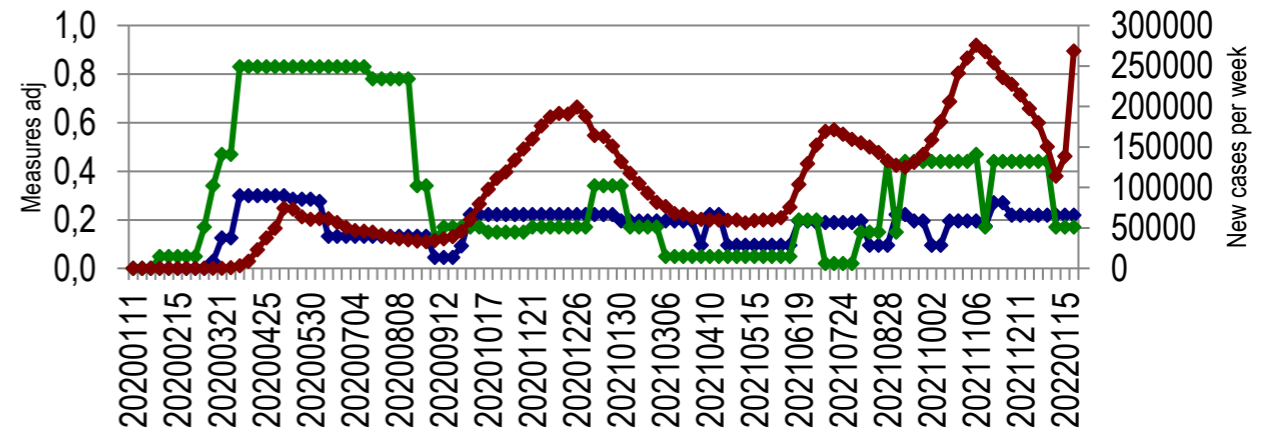


AGGREGATED MEASURES: GATHERING AND ENTRANCE RESTRICTION AND NEW CASES PER WEEK

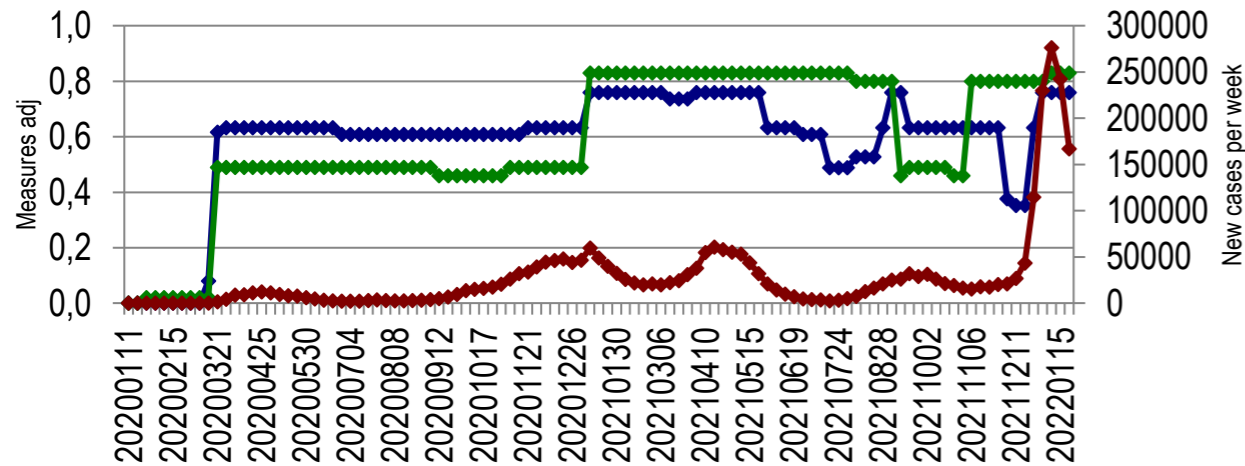
USA



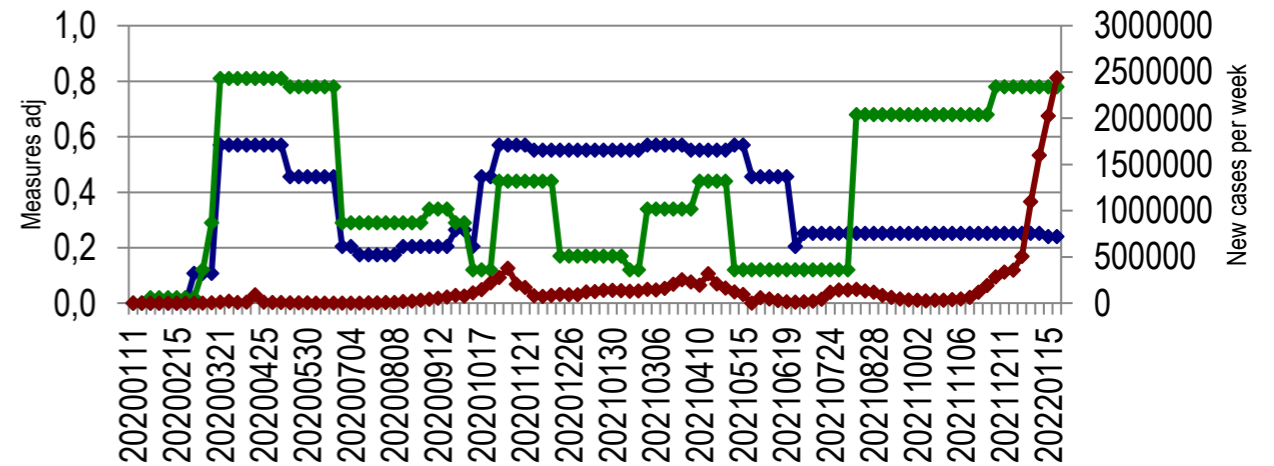
Russia



Canada



France



New cases

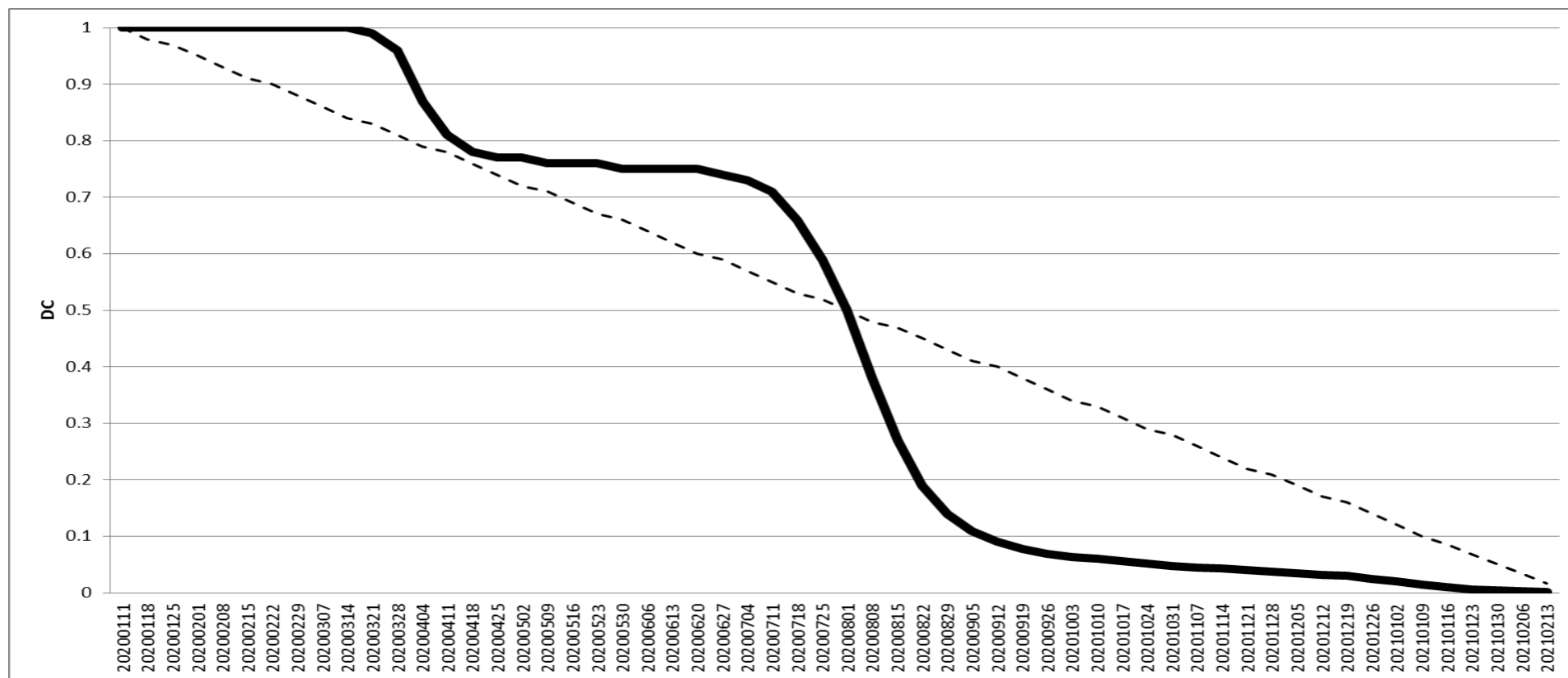
Measures: Meet.

Measures: Gov.



THE MODEL OF TIME-SERIES ANALYSIS OF THE NUMBER OF INFECTED PEOPLE

For the selected time interval we evaluate the sum of the values of a series for all period (of the length n), and for a current k -th moment we evaluate the share of values from this moment up to the end of the period.





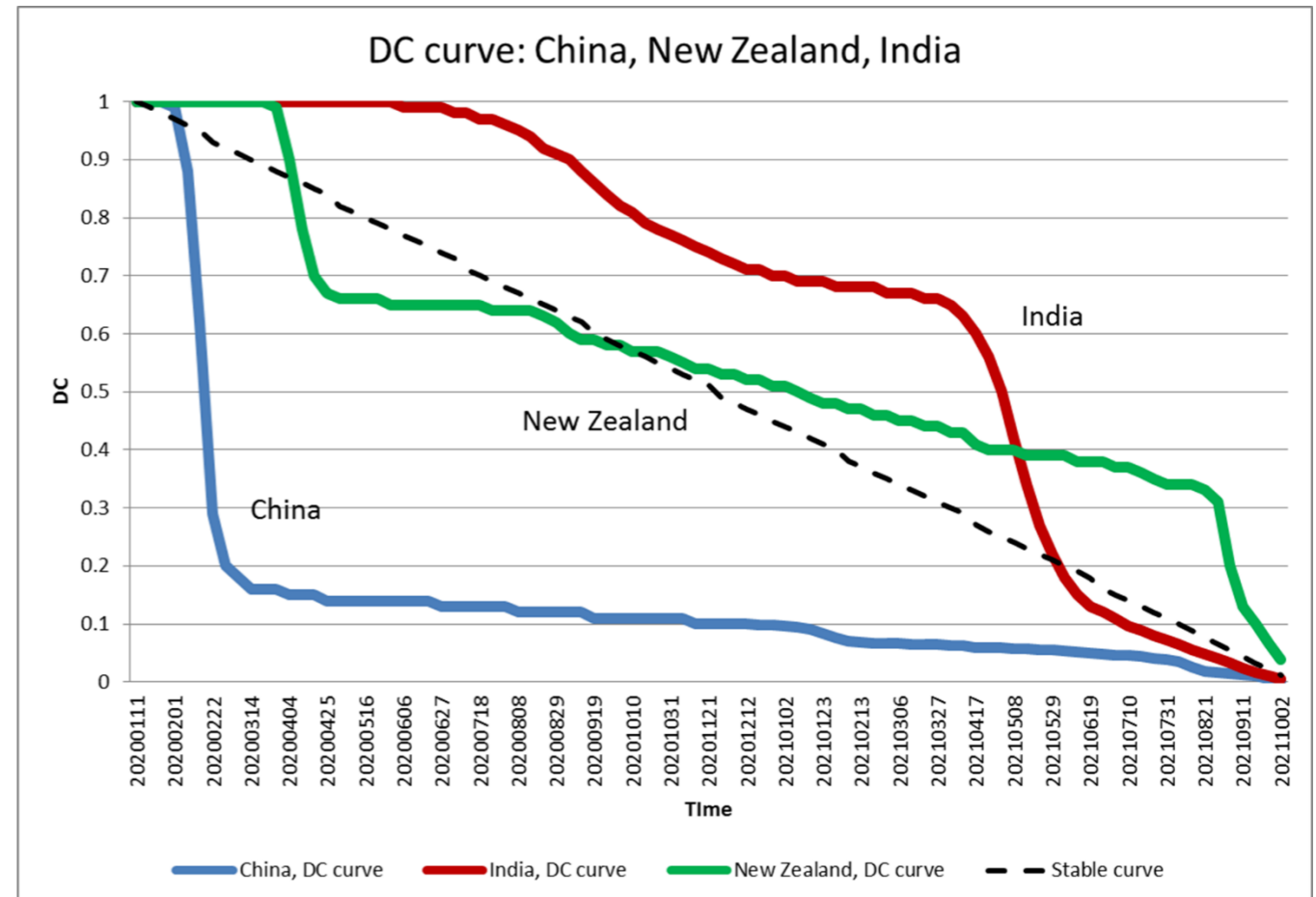
THE MODEL OF TIME-SERIES ANALYSIS OF THE NUMBER OF INFECTED PEOPLE

$$DC(k) = \frac{\sum_{t=k}^n v_t}{\sum_{t=1}^n v_t},$$

where v_t is the value in the period t

$v_t = a_t - a_{(t-1)}$, where a_t is the number of new cases in the moment t (from 1 to n).

Non-stationarity is measured as the deviation from the line



Peak in January – the graph is under diagonal (China)

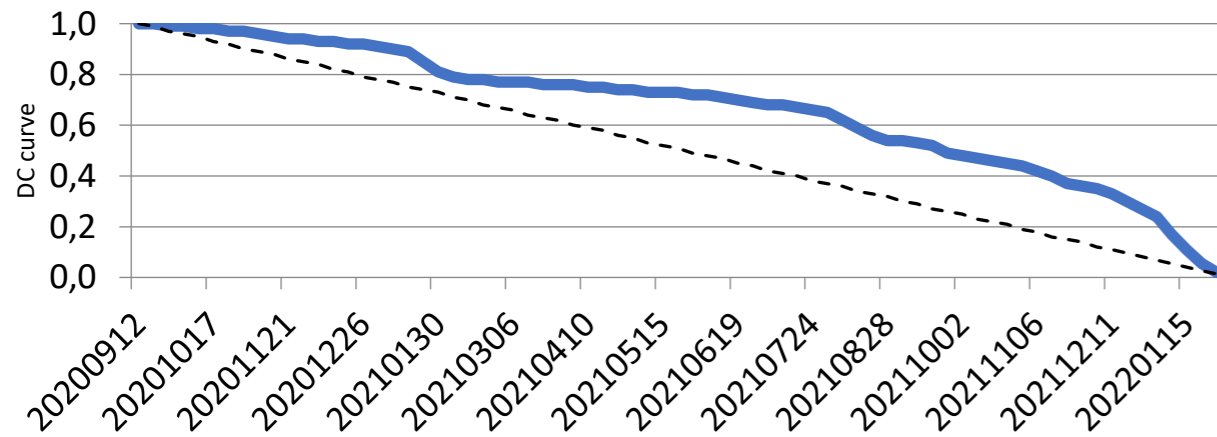
Peak in September – the graph is above diagonal (India)



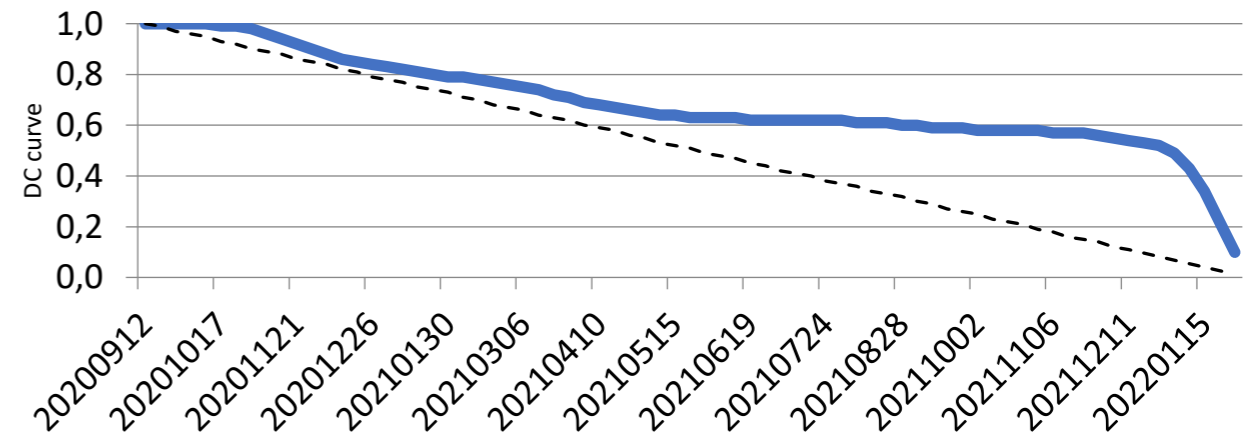
THE MODEL OF NON-STATIONARY TIME-SERIES (INITIAL)

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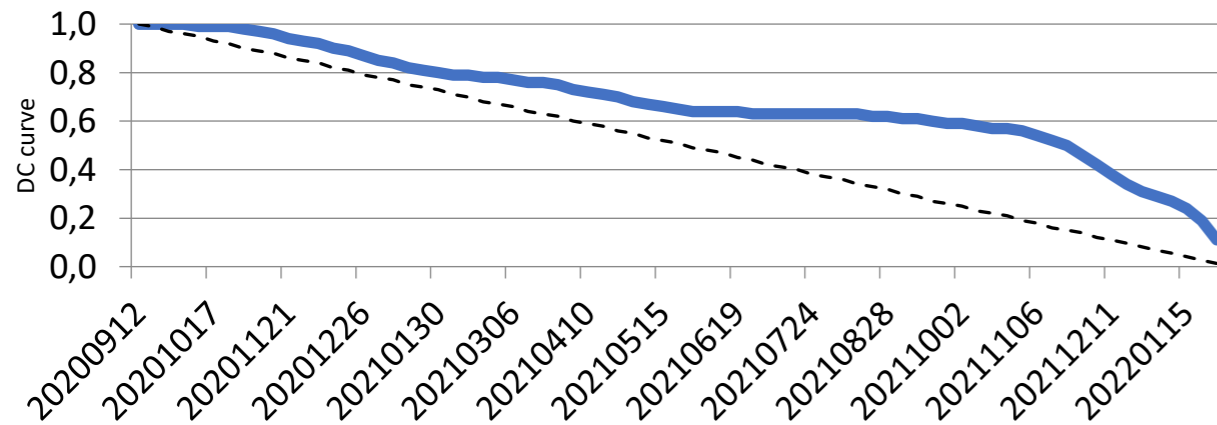
China



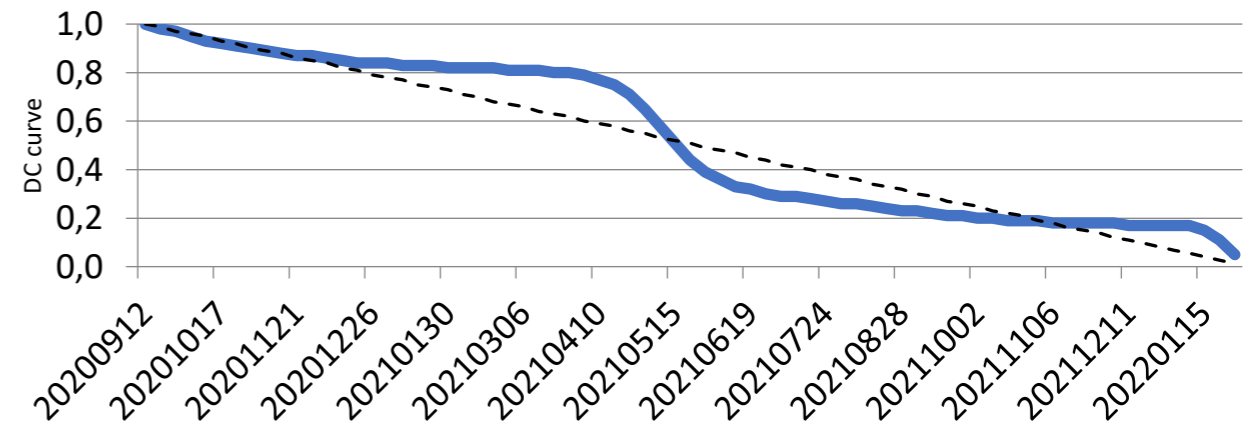
Italy



Germany



India

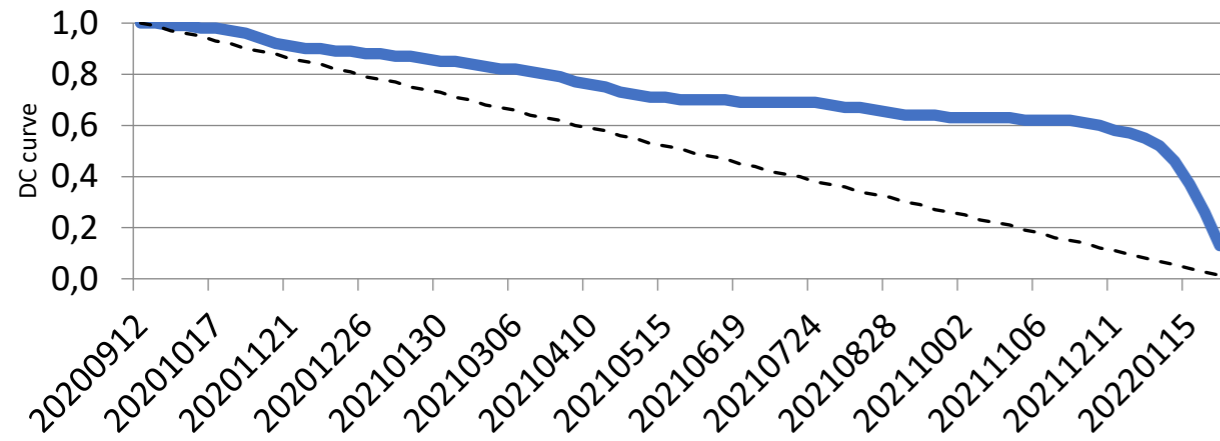




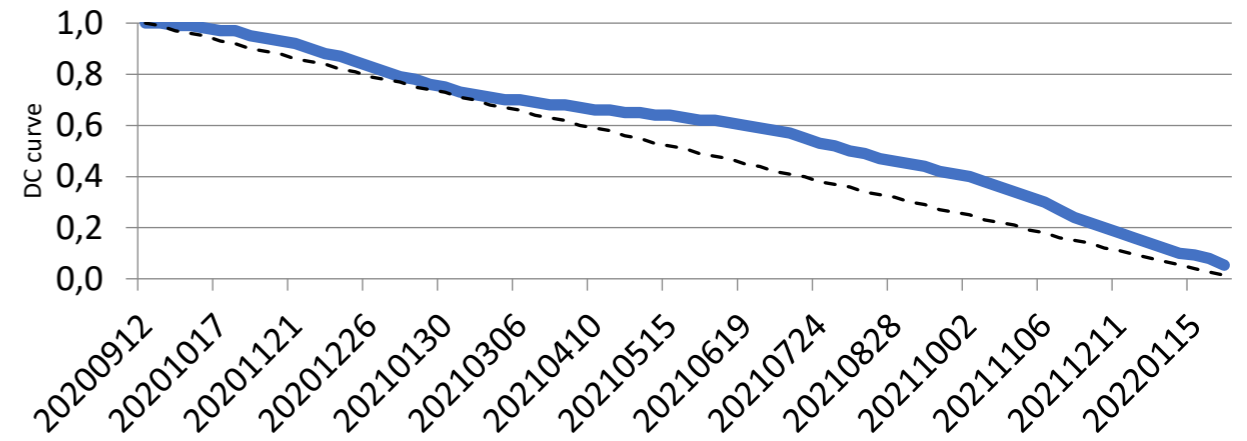
THE MODEL OF NON-STATIONARY TIME-SERIES (INITIAL)

National Research University
Higher School of Economics

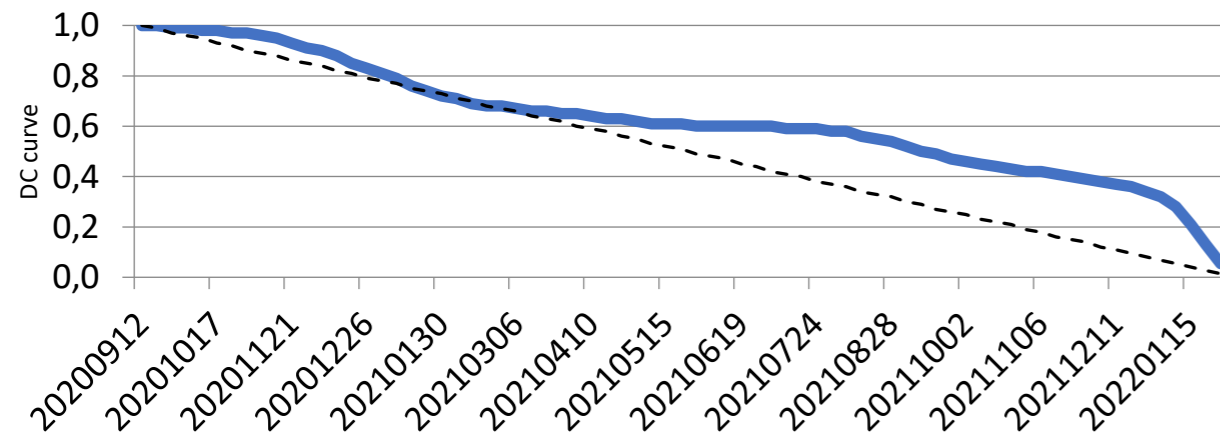
France



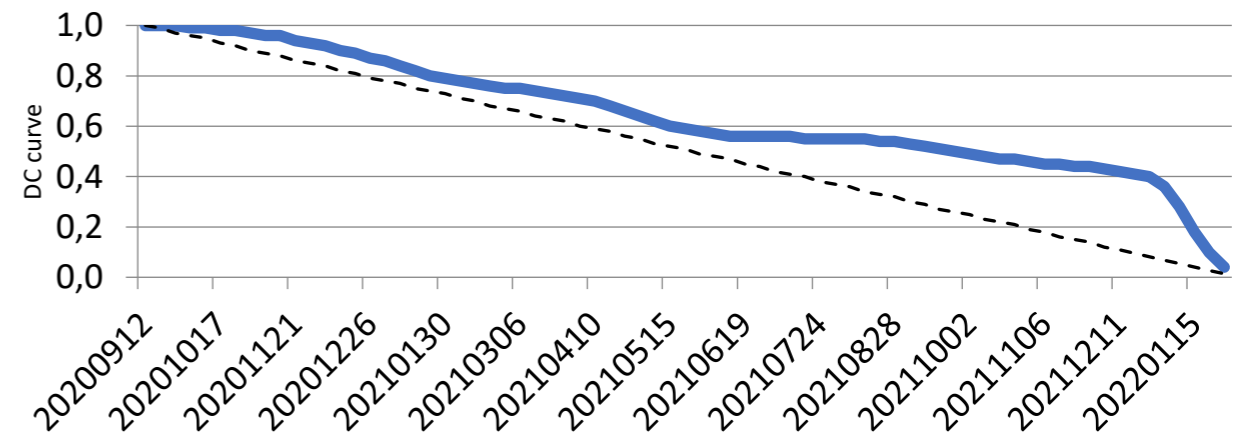
Russia



USA



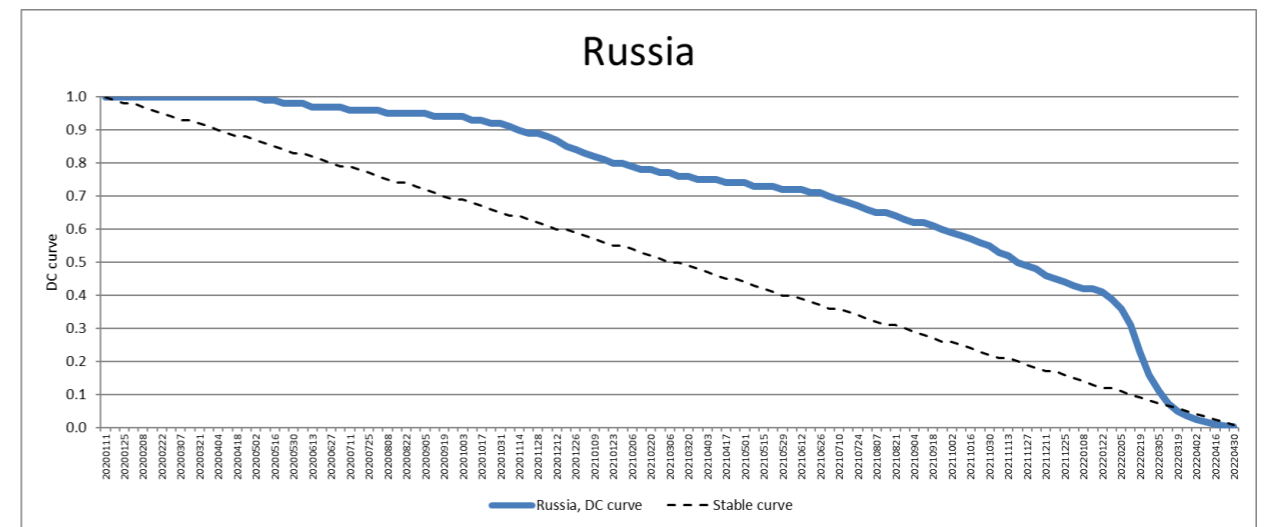
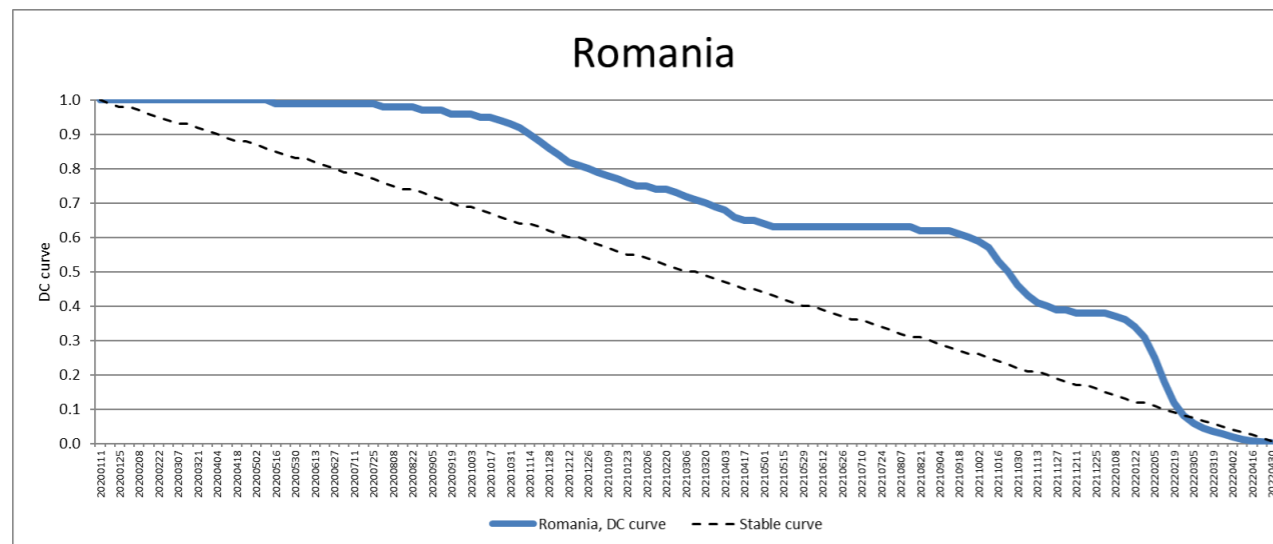
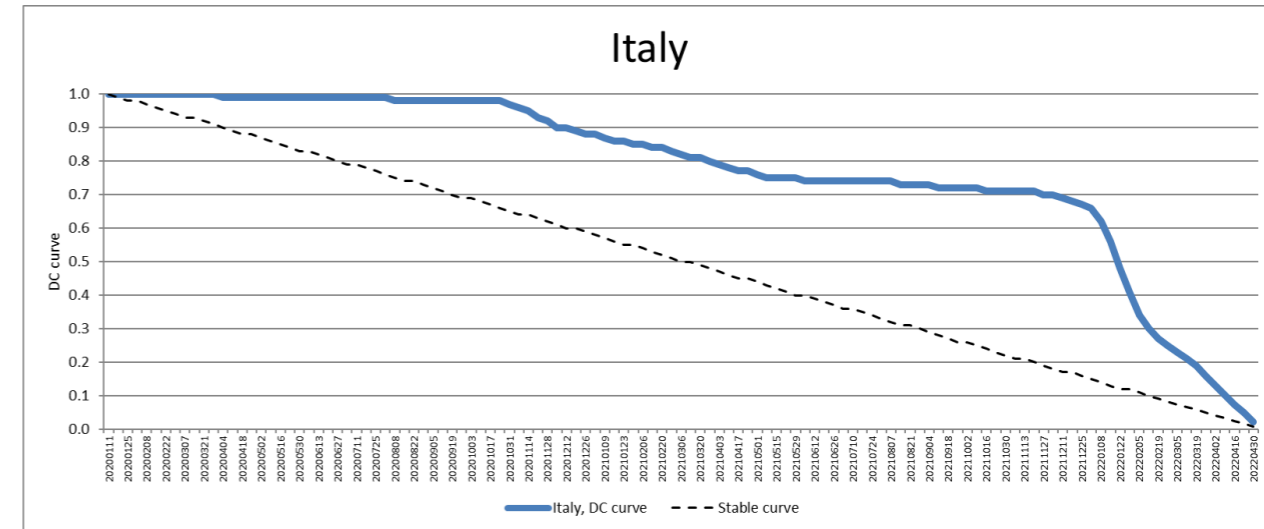
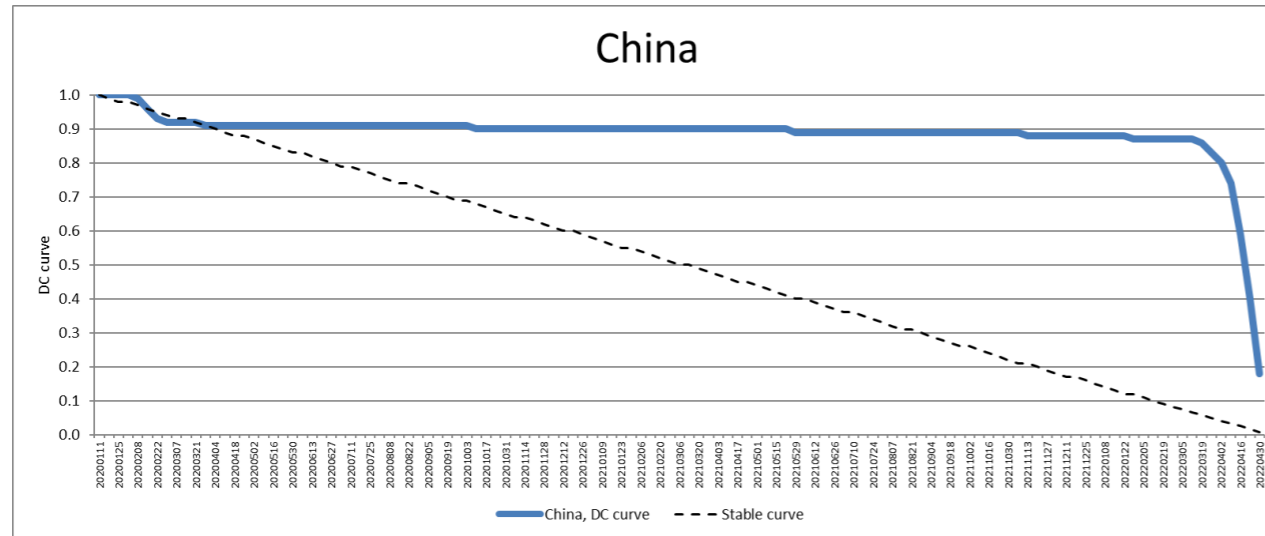
Canada





THE MODEL OF NON-STATIONARY TIME-SERIES (NOW)

National Research University
Higher School of Economics





GROUPS OF COUNTRIES WITH SIMILAR STRUCTURE OF EDUCATION LEVEL, LAW-ABIDINGNESS, MEASURES AND INFECTED PEOPLE

Initial data:

66 countries

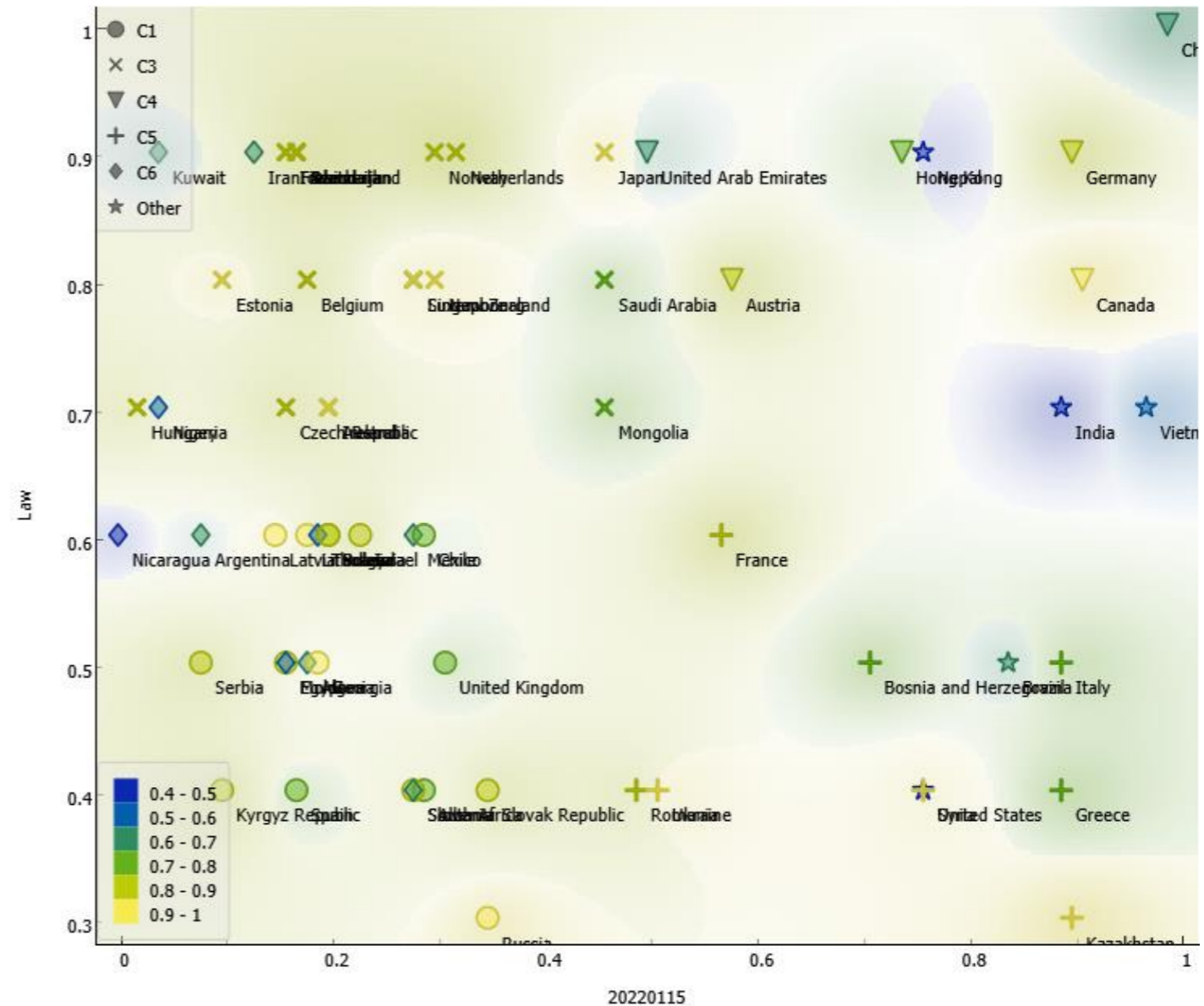
Basic system of indices

Period under study:

February 1, 2020 – May 18, 2022

Methods used: Cluster analysis

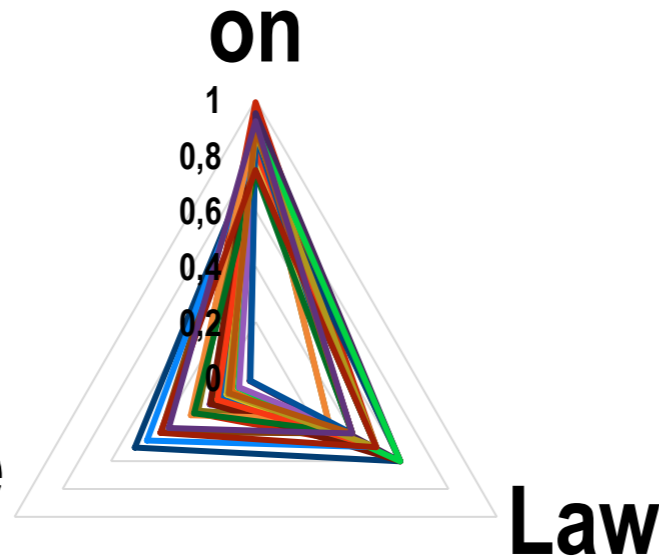
Example 1: 15 January 2022



$$N_t(\text{Cluster 1}) = 0,015 + 0,523N_{t-1} - 0,715N_{t-2} - 0,009N_{t-3} + 0,868N_{t-4} + 0,078N_{t-5}$$

C1

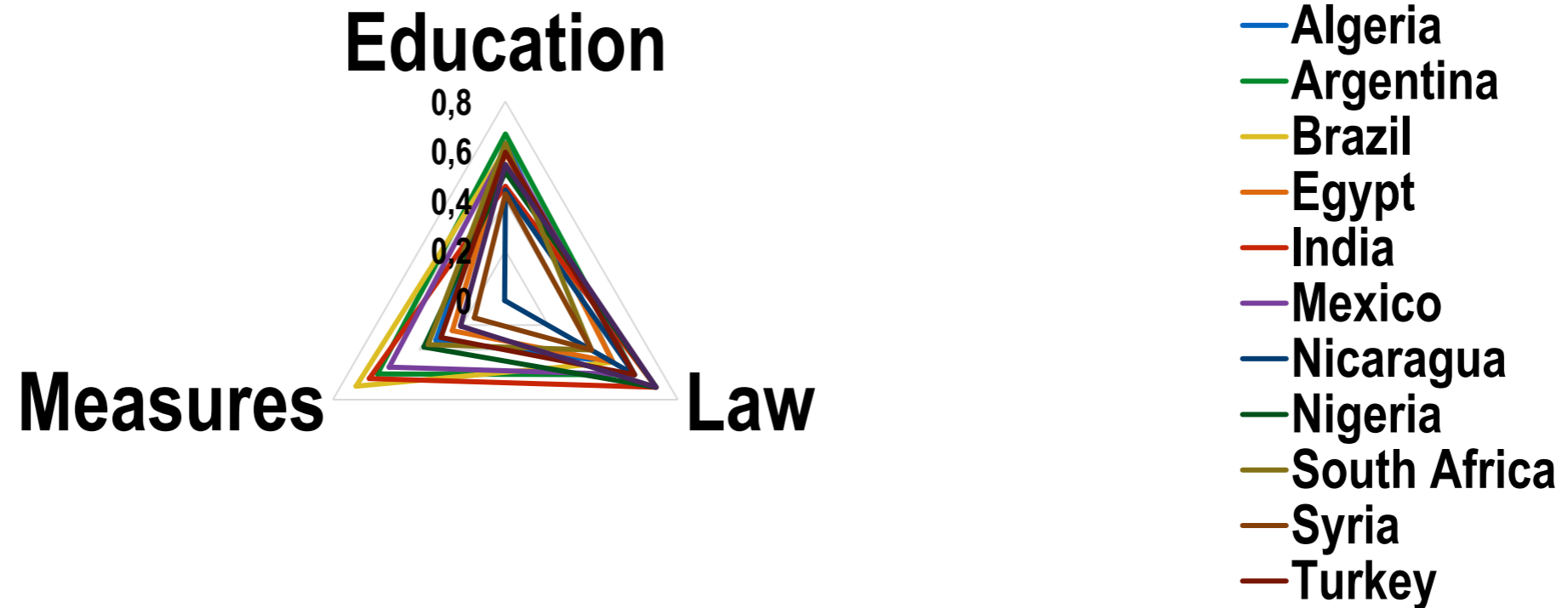
**Measure
S**



- Albania
- Bosnia and Herzegovina
- Bulgaria
- France
- Georgia
- Greece
- Israel
- Italy
- Kazakhstan
- Kyrgyz Republic

$$N_t(\text{Cluster 2}) = 0,036 - 0,356N_{t-1} - 0,328N_{t-2} - 0,461N_{t-3} - 0,44N_{t-4} + 1,737N_{t-5}$$

C2





REGRESSION EQUATIONS

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$$N_t(C1) = \begin{array}{l} \text{Intercept} \\ +0,23 \end{array} \quad \begin{array}{l} t-1 \\ +0,53 \end{array} \quad \begin{array}{l} t-2 \\ -0,01 \end{array} \quad \begin{array}{l} t-3 \\ -0,69 \end{array} \quad \begin{array}{l} t-4 \\ -0,48 \end{array} \quad \begin{array}{l} t-5 \\ +0,66 \end{array} \quad \begin{array}{l} t-6 \\ -0,95 \end{array} \quad \begin{array}{l} t-7 \\ +1,08 \end{array} \quad \begin{array}{l} t-8 \\ -0,54 \end{array} \quad \begin{array}{l} t-9 \\ +0,48 \end{array} \quad \begin{array}{l} t-10 \\ -0,38 \end{array} \quad \begin{array}{l} t-11 \\ -1,21 \end{array} \quad \begin{array}{l} t-12 \\ +1,00 \end{array}$$

$$N_t(C2) = \begin{array}{l} \text{Intercept} \\ +0,02 \end{array} \quad \begin{array}{l} t-1 \\ +1,10 \end{array} \quad \begin{array}{l} t-2 \\ -0,93 \end{array} \quad \begin{array}{l} t-3 \\ +0,80 \end{array}$$

$$N_t(C3) = \begin{array}{l} \text{Intercept} \\ +0,33 \end{array} \quad \begin{array}{l} t-1 \\ -0,09 \end{array} \quad \begin{array}{l} t-2 \\ +0,08 \end{array} \quad \begin{array}{l} t-3 \\ -0,37 \end{array} \quad \begin{array}{l} t-4 \\ -1,54 \end{array} \quad \begin{array}{l} t-5 \\ +0,27 \end{array} \quad \begin{array}{l} t-6 \\ +2,50 \end{array} \quad \begin{array}{l} t-7 \\ -1,74 \end{array} \quad \begin{array}{l} t-8 \\ -1,12 \end{array} \quad \begin{array}{l} t-9 \\ +0,53 \end{array} \quad \begin{array}{l} t-10 \\ +0,05 \end{array} \quad \begin{array}{l} t-11 \\ -0,63 \end{array}$$

$$N_t(C4) = \begin{array}{l} \text{Intercept} \\ +0,88 \end{array} \quad \begin{array}{l} t-1 \\ +2,55 \end{array} \quad \begin{array}{l} t-2 \\ -0,71 \end{array} \quad \begin{array}{l} t-3 \\ -3,64 \end{array} \quad \begin{array}{l} t-4 \\ +1,54 \end{array}$$

$$N_t(C5) = \begin{array}{l} \text{Intercept} \\ +0,67 \end{array} \quad \begin{array}{l} t-1 \\ -0,88 \end{array} \quad \begin{array}{l} t-2 \\ +1,65 \end{array} \quad \begin{array}{l} t-3 \\ -0,98 \end{array} \quad \begin{array}{l} t-4 \\ -2,85 \end{array} \quad \begin{array}{l} t-5 \\ +0,83 \end{array} \quad \begin{array}{l} t-6 \\ +2,97 \end{array}$$

$$N_t(C6) = \begin{array}{l} \text{Intercept} \\ +0,09 \end{array} \quad \begin{array}{l} t-1 \\ +0,05 \end{array} \quad \begin{array}{l} t-2 \\ -2,95 \end{array} \quad \begin{array}{l} t-3 \\ +1,35 \end{array} \quad \begin{array}{l} t-4 \\ +3,38 \end{array} \quad \begin{array}{l} t-5 \\ -3,62 \end{array} \quad \begin{array}{l} t-6 \\ -2,78 \end{array} \quad \begin{array}{l} t-7 \\ +2,79 \end{array} \quad \begin{array}{l} t-8 \\ +2,95 \end{array}$$



RESULTS

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Higher School of Economics

Cluster	All	Average	1	2	3	4	5	6
Multiple R	0,765	0,959	0,992	0,996	0,793	0,980	0,999	0,994
R Square	0,585	0,925	0,984	0,992	0,630	0,961	0,997	0,988
Adjusted R Square	0,455	0,769	0,921	0,969	0,047	0,807	0,980	0,889
Standard Error	0,318	0,176	0,068	0,053	0,539	0,187	0,040	0,167
Observations	64	10,667	16	5	19	6	8	10

Albania	Brazil	Australia	Austria	Bosnia and Herzegovina	Algeria
Bulgaria	India	Azerbaijan	Canada	France	Argentina
Chile	Nepal	Belgium	China	Greece	Egypt
Georgia	Syria	Czech Republic	Germany	Italy	Iran
Israel	Vietnam	Denmark	Hong Kong	Kazakhstan	Kuwait
Kyrgyz Republic		Estonia	United Arab Emirates	Romania	Mexico
Latvia		Finland		Ukraine	Nicaragua
Lithuania		Hungary		United States	Nigeria
Moldova		Iceland			South Africa
Poland		Ireland			Turkey
Russia		Japan			
Serbia		Luxembourg			
Slovak Republic		Mongolia			
Slovenia		Netherlands			
Spain		New Zealand			
United Kingdom		Norway			
		Saudi Arabia			
		Singapore			
		Switzerland			



REGRESSION MODELS

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Higher School of Economics

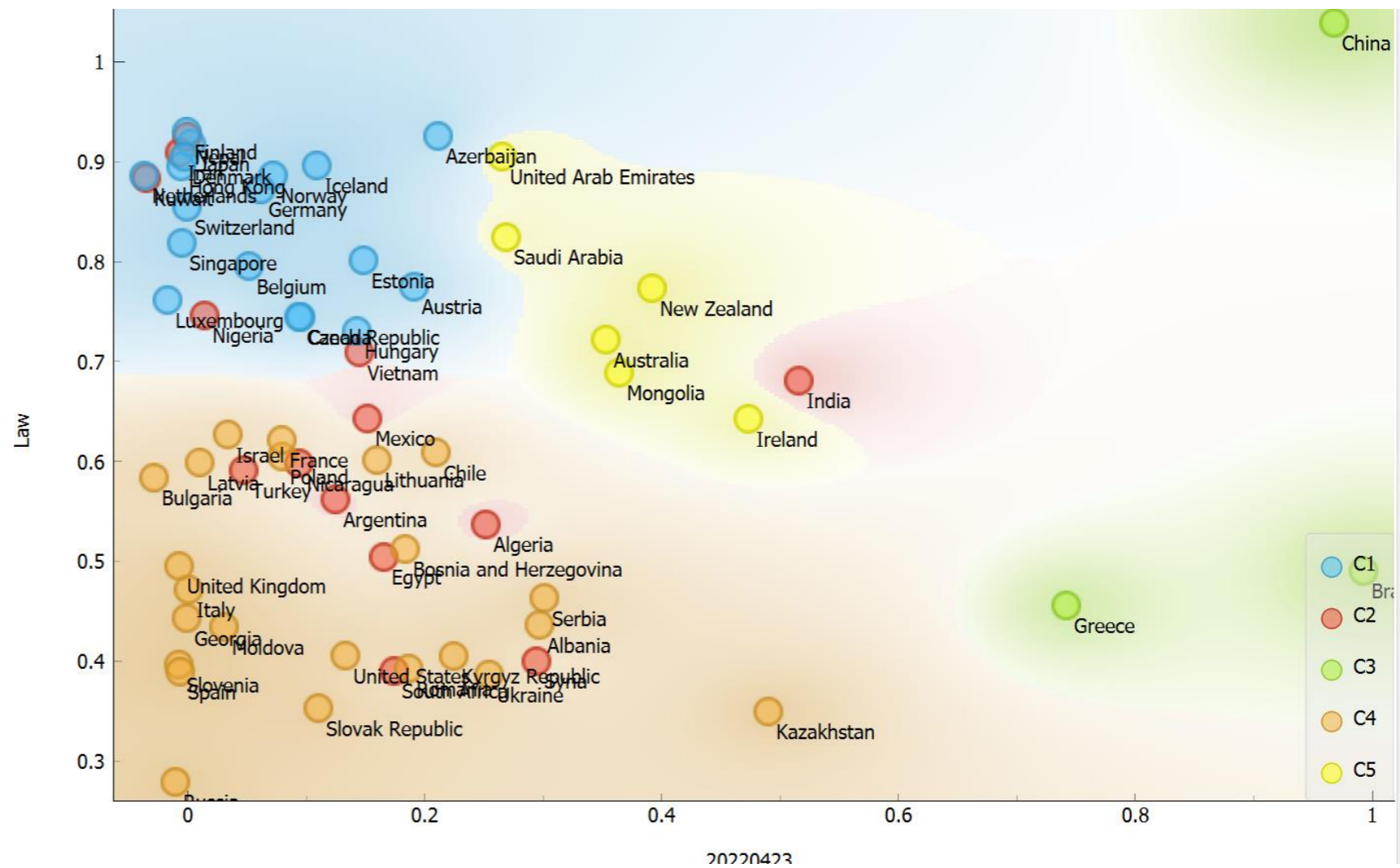
Comparison of the results as of July 25, 2020 for Cluster 1 with the entire sample

Number of weeks	R-squared for Cluster 1	R-squared for all countries
1	0.487	0.346
2	0.893	0.644
3	0.896	0.655
4	0.896	0.679
5	0.911	0.678

Example 2: 23 April 2022

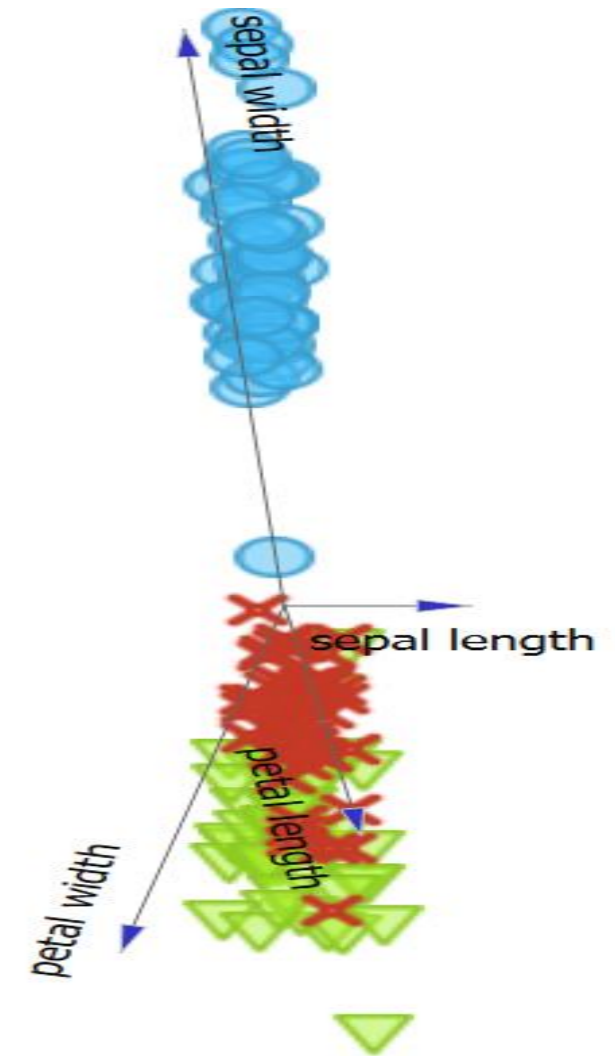
Initial data:
66 countries
Basic system of indices

Average R^2 : 0.8



Reducing dimensionality in Data

1. PCA
2. MDM;
3. Expert Analysis;
4. Neural Networks;
5. tSNE.

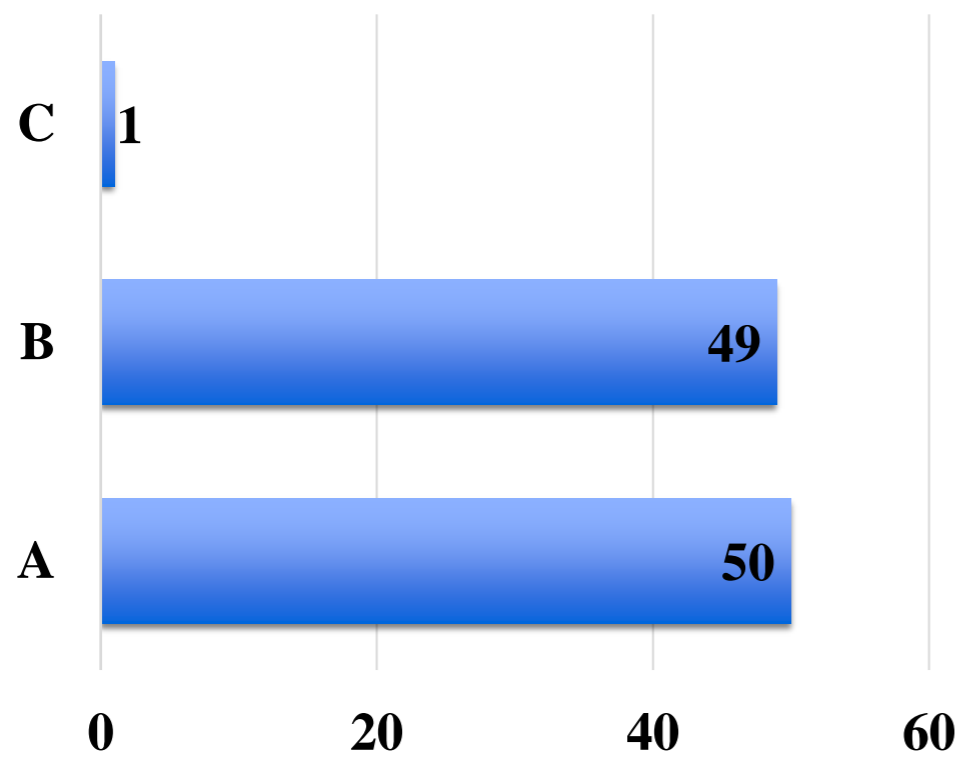


Power Indices

- N – Set of parties;
- w_i – number of votes;
- q – quota;
- **Wining coalition:**
 $S \subseteq 2^N$, if $\sum_{i \in S} w_i \geq q$;
- **Pivotal paryt:**
Party $i \in S$ is pivotal if:
 1. S – *wining coalitions*;
 2. $S \setminus \{i\}$ *is not wining coalition*.

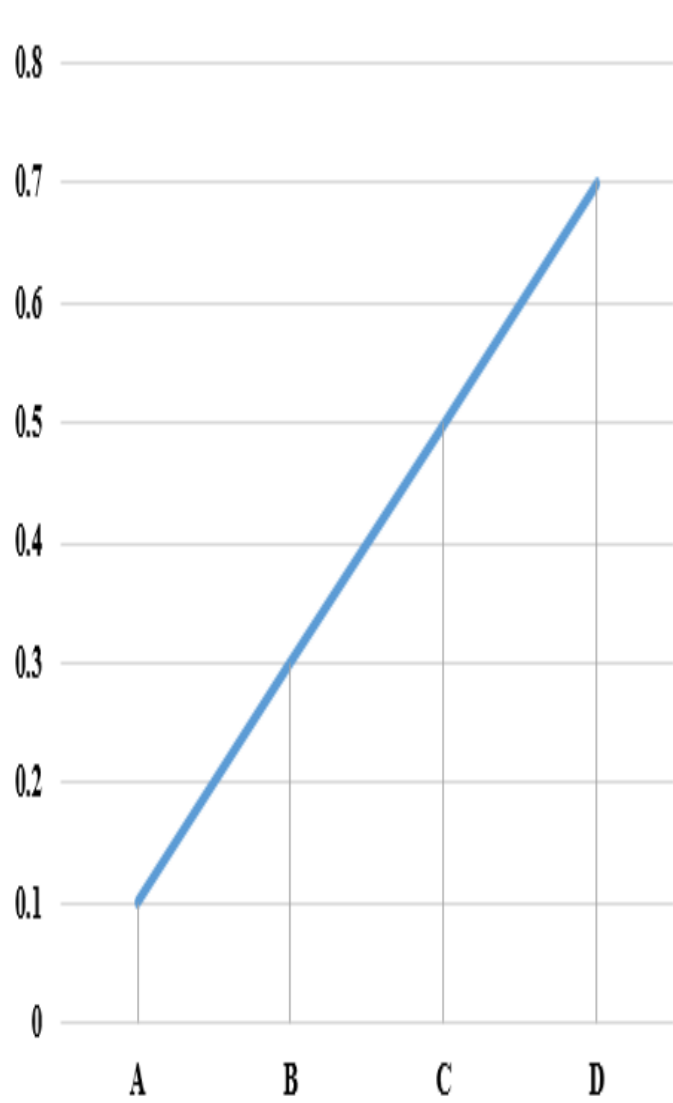


Banzhaf Index

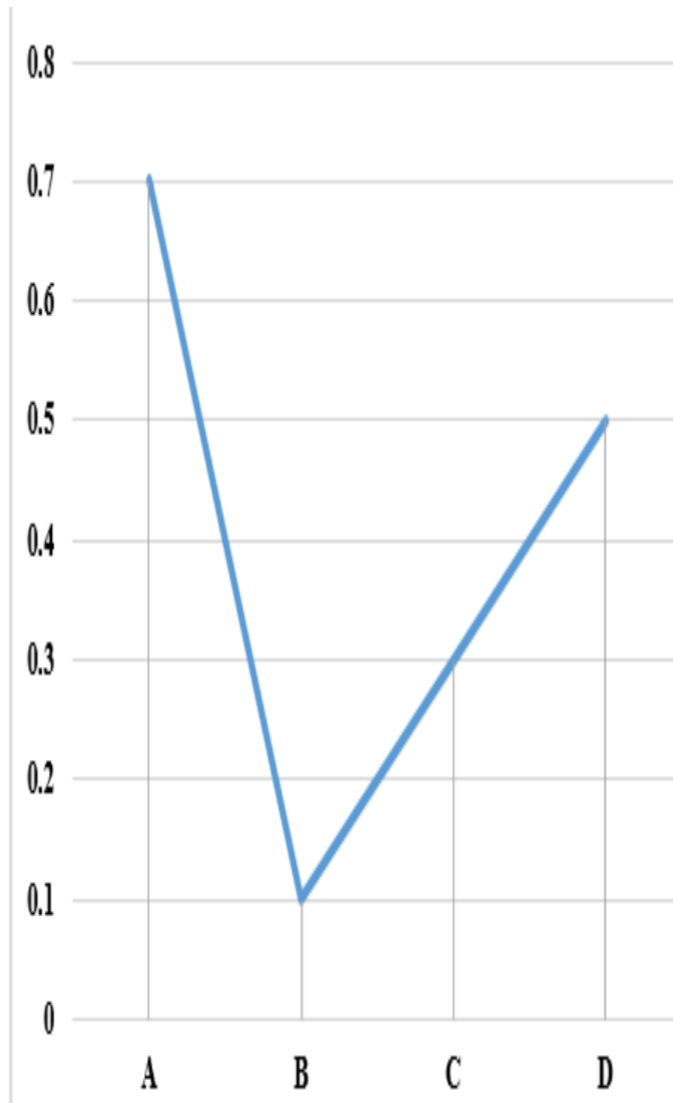


$$\beta(i) = \frac{b_i}{\sum_j b_j}$$

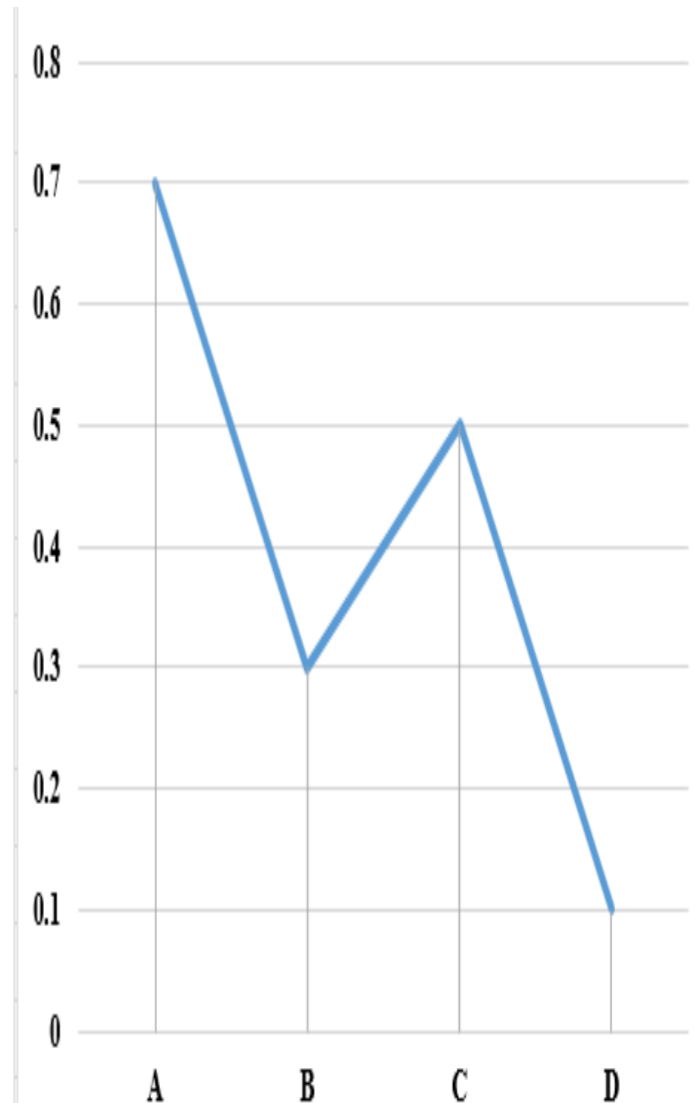
$$\beta(A) = \frac{3}{5}; \beta(B) = \frac{1}{5}; \beta(C) = \frac{1}{5}.$$



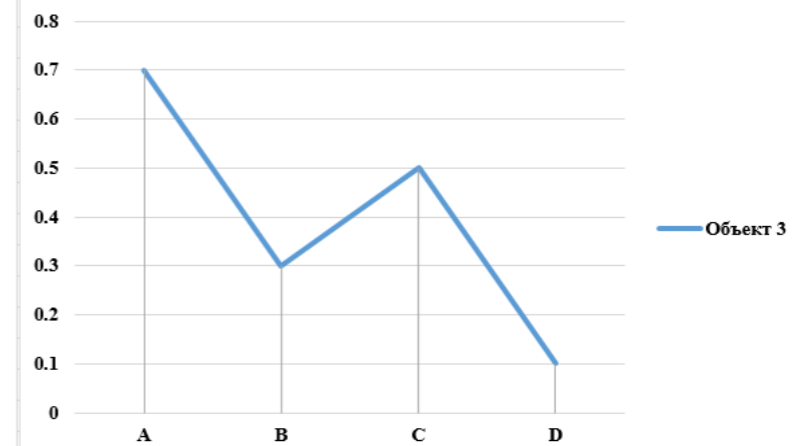
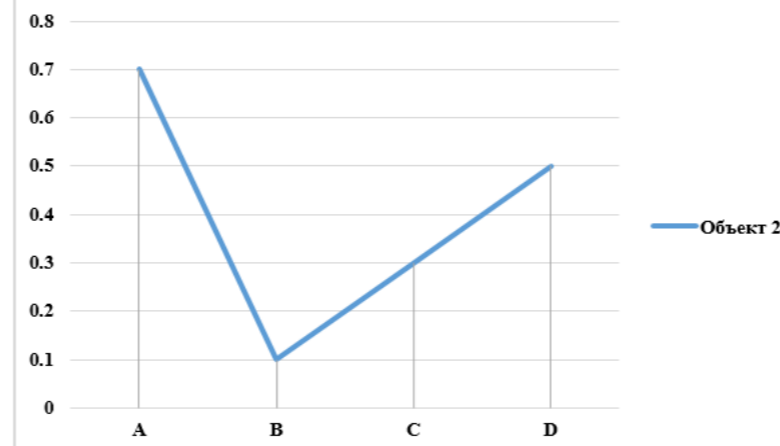
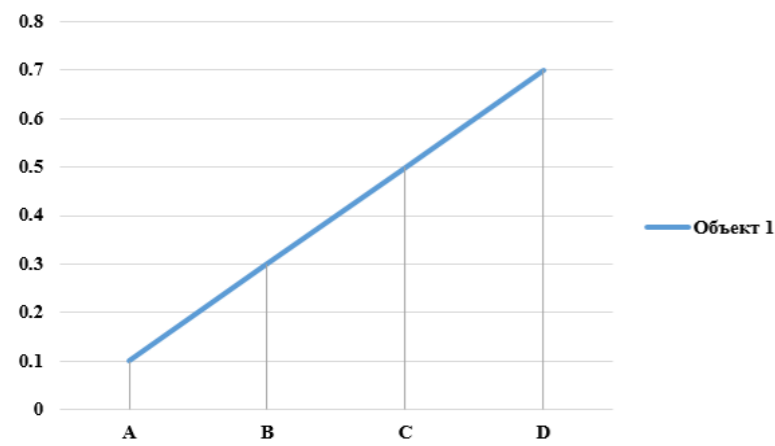
Объект 1



Объект 2



Объект 3



AB: $\{x_1\}, \{x_2, x_3\}$;

AC: $\{x_1\}, \{x_2, x_3\}$;

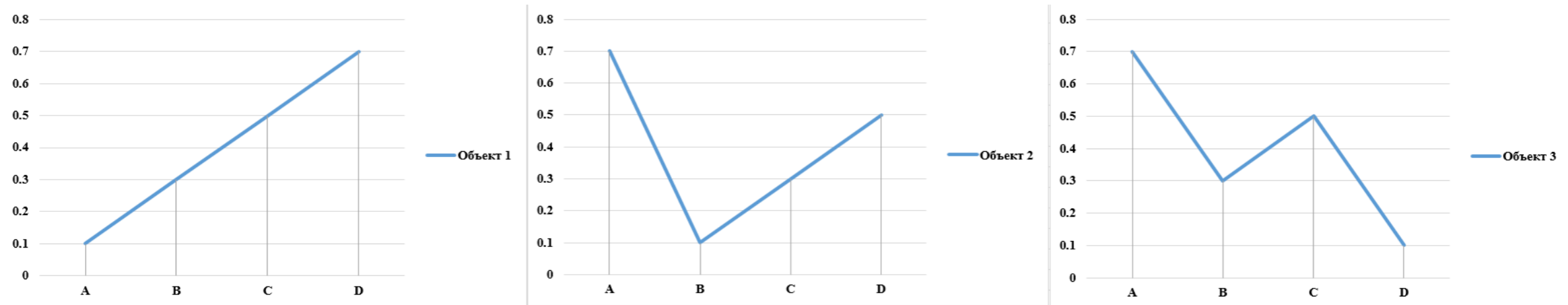
AD: $\{x_1\}, \{x_2, x_3\}$;

BC: $\{x_1, x_2, x_3\}$;

BD: $\{x_1, x_2\}, \{x_3\}$;

CD: $\{x_1, x_2\}, \{x_3\}$;

ABC: $\{x_1\}, \{x_2, x_3\}$;
 ABD: $\{x_1\}, \{x_2\}, \{x_3\}$;
 ACD: $\{x_1\}, \{x_2\}, \{x_3\}$;
 BCD: $\{x_1, x_2\}, \{x_3\}$;
 ABCD: $\{x_1\}, \{x_2\}, \{x_3\}$.



AB: $\{x_1\}, \{x_2, x_3\}$; BC: $\{x_1, x_2, x_3\}$;
 AC: $\{x_1\}, \{x_2, x_3\}$; BD: $\{x_1, x_2\}, \{x_3\}$;
 AD: $\{x_1\}, \{x_2, x_3\}$; CD: $\{x_1, x_2\}, \{x_3\}$;

ABC: $\{x_1\}, \{x_2, x_3\}$;
 ABD: $\{x_1\}, \{x_2\}, \{x_3\}$; $\mu(A) = \frac{6}{20}$ $\mu(C) = \frac{4}{20}$
 ACD: $\{x_1\}, \{x_2\}, \{x_3\}$;
 BCD: $\{x_1, x_2\}, \{x_3\}$;
 ABCD: $\{x_1\}, \{x_2\}, \{x_3\}$. $\mu(B) = \frac{4}{20}$ $\mu(D) = \frac{6}{20}$

THANK YOU!



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