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A tool for quantitatively analyzing the chances of a university to compete in the world market of new students

Abstract

Two groups of problem associated with the economics of distance learning are the subject of this presentation.

The first group is associated with developing sets of blended courses for colleges/universities by including in these courses fragments of lectures of professors from distinguish universities in the world. For a particular college/university (the University further on), the first problem within the group consists of maximizing the minimal percentage of the total number of students who are expected to succeed in studying a particular course from a set of blended courses when all these courses a) are considered by the University as equally important and are expected to be offered to the students within the next few years, and b) are to be offered within a limited yearly budget. The second problem within the group (for the same University) consists of minimizing the yearly budget allocated by the University for running the offered blended courses from the set, provided that a percentage of the students expected to succeed in studying each course from all the offered blended courses will not be smaller than a particular number. A mathematical model is proposed to formalize both problems, and, based on the proposed model, both problems are formulated as Boolean programming ones, which can effectively be solved by standard software packages. The formulated problems can be considered as a tool helping all the interested Universities negotiate corresponding financial problems with both federal authorities and private sponsors on providing competitiveness in the world market of new potential students. The current status of the research results in this field is also discussed.

This part of the presentation is based on the author's paper "Developing and Running a Set of Competitive College/University Blended Courses" published in the book "Data Analysis and Optimization. In Honor of Boris Mirkin's 80^th Birthday / Ed. by B. Goldengorin, S. Kuznetzov," in 2023.

The second group of the problems, which are the subject of consideration in this presentation, is associated with choosing electives to study by a University student. These electives are to be chosen from a set of those which the University offers to study, along with a set of core courses in each specialization for which it's authorized to award degrees. The problems from this group concern every public administration caring about the quality of both public and private higher education in the area (district) of its responsibilities.

This part of the presentation is based on the results developed by the author together with Dr. Tamara Voznesenskaya; these results have been reflected in the paper "Finding optimal sets of electives to study by a college/university student", which has been submitted by the authors to one of International Journals on Education.

THE FIRST GROUP OF THE PROBLEMS

Problem statements and their mathematical formulations

A University in a country (for instance, in one of Eastern European countries) intents to start teaching blended courses in one of foreign languages, for instance, in English, and the University is to decide how to organize this activity. The total number of blended courses in English that are planned to be offered to the students there equals K, and the number of teachers-native English speakers—who a) can cover all the K blended courses, and b) can be hired by the University from abroad—equals (or doesn't exceed) L. Besides hiring teachers from abroad to run the above

K blended courses, the University considers a possibility to offer to teach some courses from this set (of K blended courses) or even all these K blended courses to teachers-native speakers of the country's language who speak reasonable English. To this end, the University considers potential candidates to teach blended courses from among those who are currently employed either by the University or by other universities in the country.

The University plans to buy (or take from the open sources) recorded online courses that are taught by distinguished professors from leading universities in the world and to use these online recordings in designing all the above K blended courses, no matter who will finally be chosen to teach these courses. Each of the teachers who a) invited from abroad, b) currently employed by the University, and c) invited from other universities in the country, are to teach any particular course from the set of K blended courses in one and the same manner. That is, they are to teach each such course in the form of lectures and seminars, and all these lectures and seminars are to be based on (or substantially use) the above materials from the recorded online courses (which are to be acquired by the University).

Each of the teachers considered by the University to be invited to teach courses from the set of K blended courses (from among those who can be chosen from professors currently working at the University, or can be hired from other universities in the country, or can be invited from abroad) can teach no more than a certain number of these blended courses. If a teacher from the University is assigned to teach some courses from the set of K blended courses, her/his existing assignments for teaching courses in the country's language are to be covered by other teachers either currently working at the University or by those to be hired from other universities in the country. Each potential candidate considered to be offered to teach any particular course from the set of K blended courses is tested by the experts recognized by the University. Such experts are to estimate particularly a percentage of the students who are likely to succeed in studying this course should this candidate be selected to teach the course.

The University is interested in estimating two numbers associated with organizing and running the above K blended courses (see the Introduction):

1) What is the minimal budget sufficient to organize and run all these K blended courses in English to secure the percentage of the University students (who choose to study courses from this set) who are expected to succeed in studying each particular course (from these K blended courses) to be not lower than a certain desirable percentage?
2) What is the maximal percentage of the students (who are to study courses from this set of K blended courses) expected to succeed in studying each particular course from these K courses under a particular budget that the University can afford to spend to organize and to run these courses?

All the teachers-native speakers of the country's language who are potentially capable of teaching courses in English from the set of K blended courses (both from the University and from other universities in the country) are called course developers further in the paper. If any course from the K blended courses is to be taught by a teacher-native speaker of English (who is invited from abroad), the students chosen to study this course are a) to take an advanced course in English language, and b) to take corresponding tutorials to be prepared to understand all the fragments from the corresponding recorded courses, prior to the commencement of the course.

The mathematical model (I)

Let

K be the number of blended courses that the University plans to teach in English, say, in the next T years,

B be the yearly budget that the University plans to allocate to cover all the expenses associated with developing and running the set of K blended courses within T years,

B_0 be the cost of the recorded online courses that the University plans to acquire to be use in developing the set of K blended courses,

M be the number of potential course developers who are currently employed at the University,

R be the number of teachers currently working at other universities in the country, who are interested in working at the University and are considered by the University as potential course developers of blended courses from the above set of K courses,

L be the number of teachers-native speakers of English from abroad who can cover the needs of the University in teaching courses from the K blended courses and who the University can financially afford to hire,

c_i be the basic yearly salary of course developer $i \in \overline{1,M}$ (who is a teacher currently employed by the University), chosen (assigned) to teach courses from the set of K blended courses,

∇_{ik} be the additional yearly salary of course developer $i \in \overline{1,M}$ from the University, assigned to teach courses from the set of K blended courses, for developing and teaching course $k \in \overline{1,K}$,

b_r be the basic yearly salary of course developer $r \in \overline{1,R}$ from another university in the country, invited to teach courses from the set of K blended courses,

δ_{rk} be the additional yearly salary of course developer $r \in \overline{1,R}$, invited from another university in the country to teach courses from the set of K blended courses, for developing and teaching course $k \in \overline{1,K}$,

g_r be the relocation cost associated with the invitation of course developer $r \in \overline{1,R}$ from another university in the country to teach courses from the set of K blended courses,

a_l be the basic yearly salary of teacher-native speaker of English $l \in \overline{1,L}$, invited from abroad to teach courses from the set of K blended courses in English,

Δ_{lk} be the additional yearly salary of teacher-native speaker of English $l \in \overline{1,L}$, invited from abroad to teach courses from the set of K blended courses in English, for teaching course $k \in \overline{1,K}$,

h_l be the relocation cost associated with the invitation of teacher-native speaker of English $l \in \overline{1,L}$ from abroad to teach courses from the set of K blended courses in English,

d_i be the per hour salary of a teacher hired from another university in the country to substitute course developer $i \in \overline{1,M}$ (assigned to teach courses from the set of K blended courses) to teach the courses "vacated" by course developer $i \in \overline{1,M}$ (if there are such teachers),

t_{ik} be the number of hours per year that course developer $i \in \overline{1,M}$ "vacates" (as a result of switching to teaching courses from the set of K blended courses) that are to be covered by other teachers (either by those who are currently employed by the University or by those to be hired from other universities in the country),

α_{ik} be the expert estimate of a percentage of the students expected to succeed in studying blended course $k \in \overline{1,K}$ if it's taught by course developer $i \in \overline{1,M}$ from the University,

β_{rk} be the expert estimate of a percentage of the students expected to succeed in studying blended course $k \in \overline{1,K}$ if it's taught by course developer $r \in \overline{1,R}$ invited from another university in the country,

γ_{lk} be the expert estimate of a percentage of the students expected to succeed in studying blended course $k \in \overline{1,K}$ if it's taught by teacher-native speaker of English $l \in \overline{1,L}$, invited from abroad,

ω_k be the (targeted by the University) minimal desirable percentage of the students expected to succeed in studying blended course $k \in \overline{1,K}$ at the University,

u_k be a Boolean variable that equals 1 if blended course $k \in \overline{1,K}$ is to be taught by a teacher-native speaker of English invited from abroad (so that a special English language course and corresponding tutorials are to be run by the University for the students chosen to study course k), and equals 0, otherwise,

f_k be the cost of running a special English language course and corresponding tutorials for the students chosen to study blended course $k \in \overline{1,K}$,

x_{ik} be a Boolean variable that equals 1 if course developer $i \in \overline{1,M}$ from the University is assigned to teach blended course $k \in \overline{1,K}$, and equals 0, otherwise,

s_r be a Boolean variable that equals 1 if course developer $r \in \overline{1,R}$ from another university in the country is invited to teach courses from the set of K blended courses and equals 0, otherwise,

y_{rk} be a Boolean variable that equals 1 if course developer $r \in \overline{1,R}$ from another university in the country is invited to teach blended course $k \in \overline{1,K}$, and equals 0, otherwise,

w_l be a Boolean variable that equals 1 if teacher-native speaker of English $l \in \overline{1,L}$ from abroad is invited to teach courses from the set of K blended courses and equals 0, otherwise,

z_{lk} be a Boolean variable that equals 1 if teacher-native speaker of English $l \in \overline{1,L}$ from abroad is invited to teach blended course $k \in \overline{1,K}$, and equals 0, otherwise,

$M\theta_i \subset \overline{1,K}$ be a subset of courses from the set of K blended courses that course developer $i \in \overline{1,M}$ from the University can't teach,

$R\theta_r \subset \overline{1,K}$ be a subset of courses from the set of K blended courses that course developer $r \in \overline{1,R}$ from another university can't teach,

$L\theta_l \subset \overline{1,K}$ be a subset of courses from the set of K blended courses that teacher-native speaker of English $l \in \overline{1,L}$ from abroad can't teach,

ν_i be the maximal number of courses from the set of K blended courses that course developer $i \in \overline{1,M}$ from the University can teach yearly,

μ_r be the maximal number of courses from the set of K blended courses that course developer $r \in \overline{1,R}$ from another University in the country can teach yearly,

π_l be the number of courses from the set of K blended courses that teacher-native speaker of English $l \in \overline{1,L}$ from abroad can teach yearly, and

q_k be the yearly budget that can be spent for running blended course $k \in \overline{1,K}$ to cover all the additional salary to all the teachers who are a) assumed to teach course k, b) may run an additional course in English (associated with running course k), c) may develop and run any tutorials for course k, d) may prepare schoolbooks aimed at better understanding course k, and e) expected to substitute any teacher assigned to teach blended course k (if need be),

The mathematical model (II)

The following system of constraints

$$\sum_{i=1}^{M} x_{ik} + \sum_{r=1}^{R} y_{rk} + \sum_{l=1}^{L} z_{lk} = 1, \quad k \in \overline{1,K},$$

$$\sum_{i=1}^{M} x_{ik} \le 1, \quad k \in \overline{1,K}, \quad \sum_{k=1}^{K} x_{ik} \le \nu_i, \quad i \in \overline{1,M},$$

$$x_{ik} = 0, \quad k \in M\theta(i) \subset \overline{1,K}, \quad i \in \overline{1,M},$$

$$y_{rk} \le s_r \le \sum_{k=1}^{K} y_{rk}, \quad r \in \overline{1,R}, \quad k \in \overline{1,K},$$

$$\sum_{r=1}^{R} y_{rk} \le 1, \quad k \in \overline{1,K}, \quad \sum_{k=1}^{K} y_{rk} \le \mu_r, \quad r \in \overline{1,R},$$

$$y_{rk} = 0, \quad k \in R\theta(r) \subset \overline{1,K},$$

$$\sum_{l=1}^{L} z_{lk} - u_k = 0, \quad k \in \overline{1,K}, \qquad (1)$$

$$z_{lk} \le w_l \le \sum_{k=1}^{K} z_{lk}, \quad l \in \overline{1,L}, \quad k \in \overline{1,K},$$

$$\sum_{l=1}^{L} z_{lk} \le 1, \quad k \in \overline{1,K}, \quad \sum_{k=1}^{K} z_{lk} \le \pi_l, \quad l \in \overline{1,L},$$

$$z_{lk} = 0, \quad k \in L\theta(l) \subset \overline{1,K},$$

$$\sum_{l=1}^{L} \Delta_{lk} z_{lk} + \sum_{r=1}^{R} \delta_{rk} y_{rk} + \sum_{i=1}^{M} \nabla_{ik} x_{ik} + \sum_{i=1}^{M} d_i t_{ik} x_{ik} + f_k u_k \le q_k, \quad k \in \overline{1,K},$$

$$\sum_{l=1}^{L} w_l (a_l + h_l) + \sum_{k=1}^{K}\sum_{l=1}^{L} \Delta_{lk} z_{lk} + \sum_{r=1}^{R} s_r (b_r + g_r) + \sum_{k=1}^{K}\sum_{r=1}^{R} \delta_{rk} y_{rk} +$$
$$\sum_{k=1}^{K}\sum_{i=1}^{M} \nabla_{ik} x_{ik} + \sum_{k=1}^{K}\sum_{i=1}^{M} d_i t_{ik} x_{ik} + \sum_{k=1}^{K} f_k u_k \le B - B_0 \qquad (2)$$

is to hold beginning from the first year in the set of T years.

It's also natural to assume that a) the University, which intends to offer its students blended courses from the set of K courses, has teachers who are capable of developing and teaching at least some of the courses from the set $\overline{1,K}$ (i.e., the University has teachers whom it considers as potential developers of blended courses from the set $\overline{1,K}$), which is reflected by the absence of certain restrictions for the variables x_{ik} that are present in the corresponding restrictions for the variables y_{rk} and z_{lk} in the system of restrictions (1), and b) the basic salary of each developer $i \in \overline{1,M}$ (currently employed by the University) is not a part of either budget B or budgets $q_k, k \in \overline{1,K}$, and this is the case, regardless of the number of blended courses (not exceeding ν_i) that this developer may be assigned to teach.

The problems

Problem 1

The problem consists of maximizing the minimum percentage of the students expected to succeed in studying a blended course from the set $\overline{1,K}$ (when these courses are considered to be equally important from the University viewpoint), and this problem is formulated proceeding from the fixed budget B. The problem is the Boolean programming one

$$\min_{k \in \overline{1,K}} \left(\sum_{i=1}^{M} \alpha_{ik} x_{ik} + \sum_{r=1}^{R} \beta_{rk} y_{rk} + \sum_{l=1}^{L} \gamma_{lk} z_{ik}\right) \to \max_{(x_{ik}, y_{rk}, z_{lk}, u_k, s_r, w_l)} \quad (3)$$

under the systems of constraints (1), (2).

Problem 2

The problem consists of minimizing the total yearly budget for organizing and running courses from the set of K blended courses, provided the desirable (targeted) percentages of the students expected to succeed in studying courses from this set, determined by the numbers ω_k, $k \in \overline{1,K}$, are attained. The problem is the Boolean programming one

$$\sum_{l=1}^{L} w_l (a_l + h_l) + \sum_{k=1}^{K} \sum_{l=1}^{L} \Delta_{lk} z_{lk} + \sum_{r=1}^{R} s_r (b_r + g_r) + \sum_{k=1}^{K} \sum_{r=1}^{R} \delta_{rk} y_{rk} +$$
$$\sum_{k=1}^{K} \sum_{i=1}^{M} \nabla_{ik} x_{ik} + \sum_{k=1}^{K} \sum_{i=1}^{M} d_i t_{ik} x_{ik} + \sum_{k=1}^{K} f_k u_k \to \min_{(x_{ik}, y_{rk}, z_{lk}, u_k, s_r, w_l)} \quad (4)$$

under the system of constraints (1) and the additional system of constraints

$$\sum_{i=1}^{M} \alpha_{ik} x_{ik} + \sum_{r=1}^{R} \beta_{rk} y_{rk} + \sum_{l=1}^{L} \gamma_{lk} z_{ik} \geq \omega_k, \quad k \in \overline{1,K} \quad (5)$$

In both problems, it is assumed that the systems of constraints (1), (2) (in Problem 1) and (1), (2), (5) (in Problem 2) are compatible, which can be verified with the use of the technique proposed, in particular, in
A.S. Belenky, Analyzing the potential of a firm: an operations research approach, Mathematical and Computer Modelling, **35** (13), 1405–1424 (2002).

THE SECOND GROUP OF THE PROBLEMS

This group includes three problems, which concern every public administration caring about the quality of both public and private higher education in the area (district) of its responsibilities.
The first problem consists of finding a set of electives that should look optimal to a particular University student (or to a group of any particular students) from the viewpoint of the University administration. The second problem consists of finding subsets of this set of electives that may look optimal to a University student herself/himself. The third problem consists of finding the totality of electives offered in all the specializations in which a particular University awards its degrees while a) requiring the minimal cost of teaching all these electives, and b) letting their graduates for each of such specializations receive the expected GPAs not lower than certain achievable numbers.

A methodology to determine what set of electives may look optimal in each of these three problems is proposed. Three new mathematical models to formalize two of these problems as discrete optimization ones with Boolean variables and the third one as a bilinear programming problem with mixed variables are also proposed. It's demonstrated that the proposed approaches to modeling all these three problems let solve them using effective standard software packages. This feature makes these models a helpful tool that can be used in negotiations between the University administration and public administrations of all the levels on substantiating the level of financing of any corresponding education projects there.