

Shapley-Owen Values for the Political Parties in the Duma 2000-2003

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Abstract

Shapley-Owen values are calculated based on data provided by Faud Aleskerov for political parties in the Duma for each month during which the Duma was in session during the years 2000-2003. These data represent to a two-dimensional ordinal issue space, presenting the parties positions on Liberal/State and Reform/Anti-Reform scales. Although there is significant volatility in the Shapley-Owen values, two parties, Regions of Russia and Narodniy-Deputat parties, consistently achieve the highest values. This result may indicate competition between the Regions of Russia and Narodniy-Deputat to command the political center, i.e., to broker policy outcomes.

Background

The Shapley-Owen value arises as a solution concept in cooperative games, in particular, within the domain of power index concepts.¹ Voting power indices may be broadly classified into two groups, those expressing influence power (i-power) and prize-power (p-power).² Influence power measures the degree to which a member determines the policy upon which a vote is taken. Prize power measures the degree to which a member determines whether the policy is adopted (rejected). Under this classification, the Shapley-Owen Value estimates p-power.³

The Shapley-Owen value is a variant of the Shapley value relevant for spatial voting models, specifically two-dimensional spatial models in which members have Euclidean preferences. The original formulation further assumed that members all had equal votes. It is possible, however, to compute Shapley-Owen values without this assumption provided member ideal points are not spatially coincident.⁴

The Shapley-Owen value of a member equals the fraction of all lines in the two-dimensional issue space for which the member is a median. To formulate this definition precisely requires introduction of a measure space for the set of all lines in the issue space. This can be done for example by using the Lebesgue measure for the set of all ordered pairs consisting of slopes and intercepts representing lines in the issue space.

In practice the Shapley-Owen value can be computed by considering a fixed point and a line passing through this point. The line is spun incrementally around this point and all members projected on to the line. For each increment the median member is determined. The fraction of the total increments for which a member is the median is the computed Shapley-Owen value for the member, subject to an error governed by the number of increments used in the computation. A larger the number of increments (equivalently, smaller angular increment) produces a more precise estimate.

The major difficulty in practice when computing the Shapley-Owen value is that data are often discrete leading to a high likelihood of spatially coincident members. The ideal points of such members need to be separated by some “small” distance, where “small” is determined by natural scale parameters of the issues space, e.g., the separation of other members, the error in ideal point estimation, etc.

Scope, Assumptions, Caveats

The data covers the State Duma of the Russian Federation for the years 2000 through 2003 inclusive. Faud Aleskerov provided these data.⁵

Although the State Duma seats 450 Deputies the actual number voting in any one month was generally lower, averaging 443.

The issue space consists of two dimensions defined as “Liberal-State” and “Reform – Anti-Reform.” The precise definitions of these dimensions are not required for the analysis.

Each dimension is measured using a floating-point scale ranging from 0 to 1000.

For each month during which the Duma was in session, Deputy ideal points are aggregated into their respective party ideal points as described by Aleskerov, et. al.

Party preferences are Euclidean.

The party vote weight equals the number of Deputies who are members of that party.

The decision rule is absolute simple majority rule.

It is unknown whether elections and/or other exogenous factors or internal party management and/or other endogenous factors affected voting by deputies.

Aggregate Analysis

Figure 1 represents the average position and corresponding Shapley-Owen value for each political party in the State Duma from 2000 to 2003. Also shown are several geometric features, namely the Pareto frontier, the yolk, and Copeland Winner (strong point). These features are important in formulating hypotheses offered in this paper. For convenience we recall the definitions of these features

Yolk. The yolk is the smallest sphere intersecting every median hyperplane. It represents a generalization of the median to spaces of more than one dimension. The yolk is a circle and the hyper planes are lines in two dimensions.

Pareto Set. The Pareto set is the set of policies that benefits at least one party without penalizing any other. The convex hull of party ideal points is the frontier of this set.

Strong Point. The policy position that is least vulnerable to defeat, i.e., has the smallest win set.

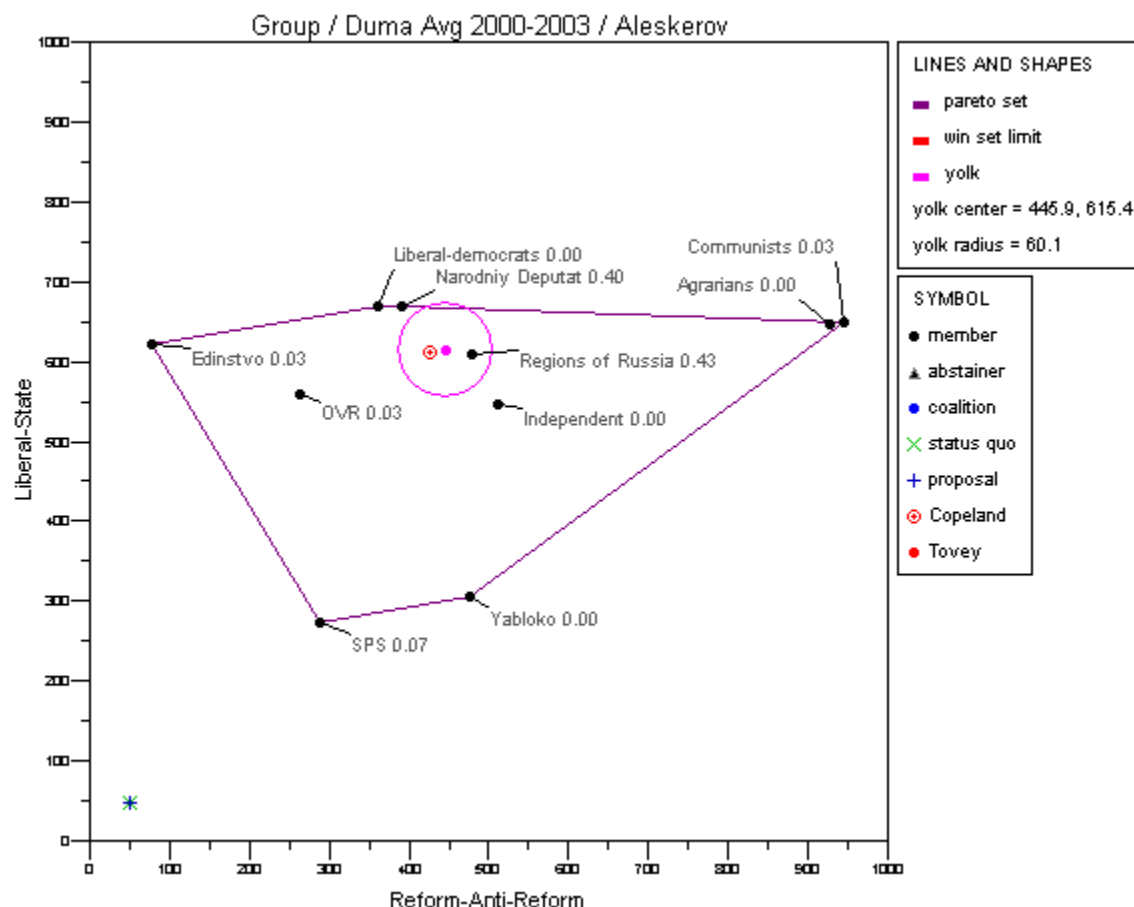


Figure 1 – Ideal Points for Parties in the State Duma Averaged Over 2000-2003 Period

Notice in Figure 1 that parties at or near the Pareto frontier generally have a lower Shapley-Owen value than do parties closer to the yolk. In fact, Feld and Grofman have shown that the Shapley-Owen value is bounded from above with a bound that varies inversely with distance from the yolk center.⁶

In 1998 Felsenthal, Machover, and Zwicker distinguished between two types of voting power: influence power (i-power), the power to achieve policy goals, and prize power (p-power), the power to achieve reward goals.⁷ Recognizing that the significance of this distinction is still being debated, it nevertheless provides a useful conceptual framework through which to understand the of the Shapley-Owen value.⁸ Specifically we will see evidence that parties near the Pareto frontier tend to be concerned with i-power whereas those nearer the yolk tend to be concerned with p-power.

Consider again Figure 1. Each party has staked out a position. In doing so the parties are expressing preferences, but to what end? Parties far from the yolk cannot hope to broker policy outcomes. But they can influence outcomes more favorable to their goals by joining favorable, winning coalitions. Parties near the yolk, on the other hand, function as brokers, determining the location of the adopted proposal. It could be that such outcomes reflect the genuine policy preferences of the brokers, or it could be that brokers behave strategically, positioning

themselves at anticipated policy outcomes. If brokers behave strategically we expect to see greater variance in their policy position as compared to parties that do not behave strategically, in so far as non-strategic parties have fixed policy goals whereas strategic parties do not.

The standard deviations of average party position for 2000-2003 versus distance from the yolk are shown in Figure 2. Observe that parties occupying extreme vertices on the Pareto frontier, namely, the Communist, Agrarian, Edinstvo, and SPS parties exhibit relatively small variances compared to parties near the yolk. The Communist party forms one vertex and was the former ruling party. The Edinstvo party, which strongly supported Putin, forms another vertex. The third major vertex is formed by SPS, a leading liberal opposition party. Other parties also occupy vertices, notably, Yabloko, Liberal-Democrats, and Narodniy-Deputat, but these are weak vertices or “perturbations” of the frontier, compared to the three formerly mentioned.

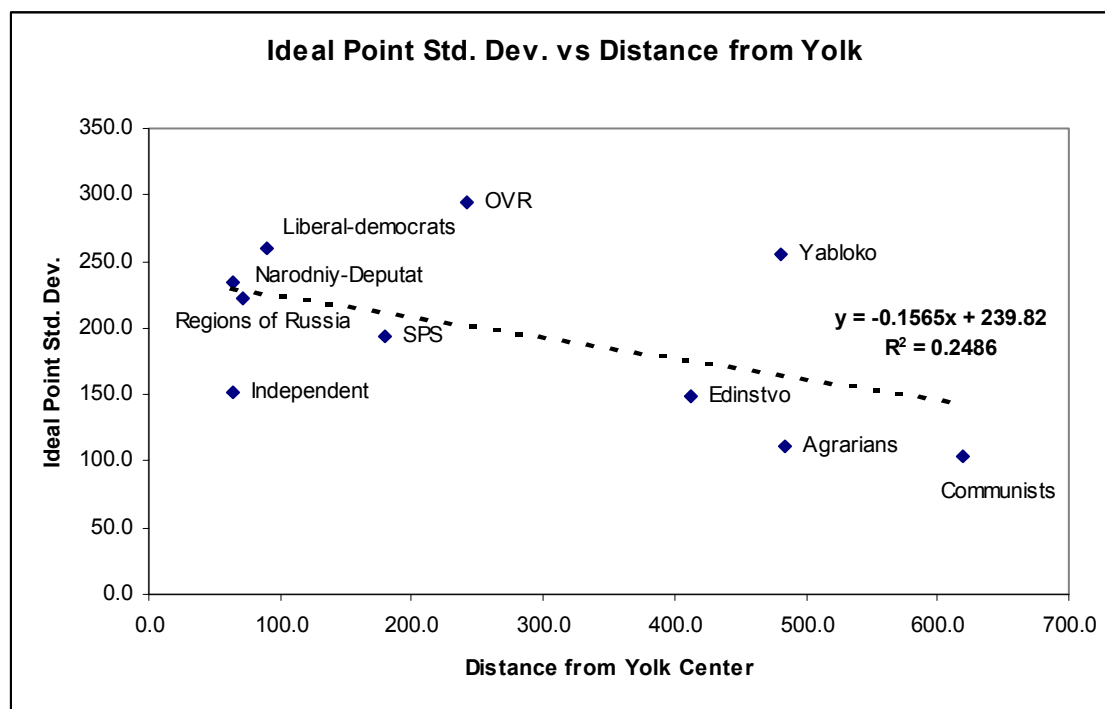


Figure 2 – Standard Deviation of Party Ideal Points Over the Period 2000-2003

These observations do not prove that brokers are behaving strategically. The available data does not document the causes and reasons why parties modify their positions. Nevertheless, this trend suggests that parties near the Pareto frontier are concerned with i-power whereas those near the yolk are concerned with p-power.

The most significant p-power players are two centrist parties, the Regions of Russia and Narodniy-Deputat party. Nearby are the Independents, Liberal-democrats, and OVR.

Yabloko, located on the Pareto frontier with both a large ideal point variance and large average distance from the yolk center, defies this otherwise tidy classification of parties into i-power and p-power groups.

Time-Series Analysis

Figure 3 presents the average Shapley-Owen value for all parties though out the four-year period.

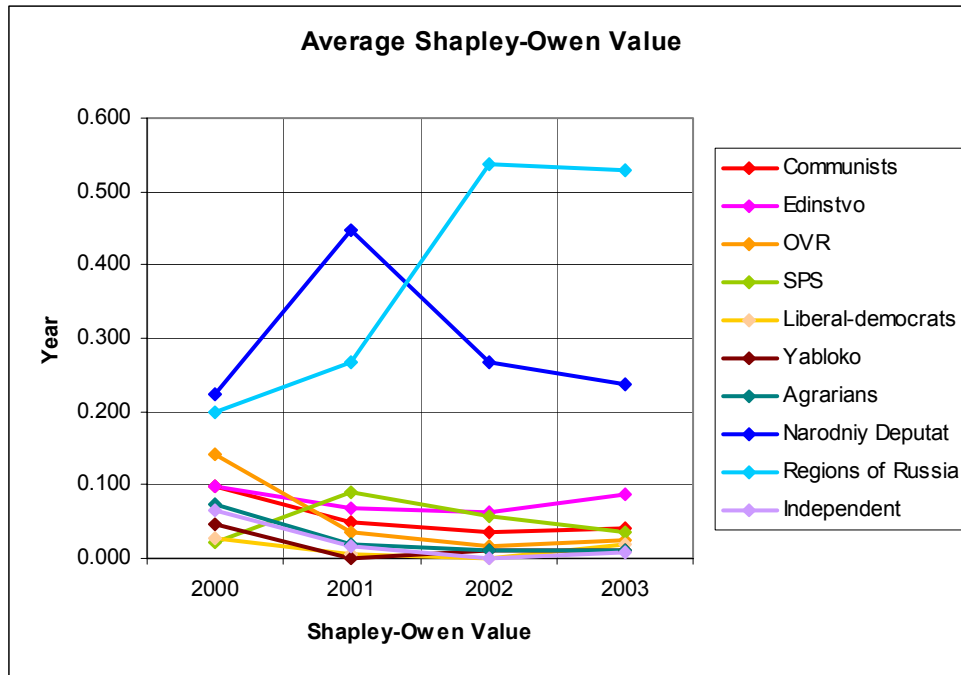


Figure 3 – Shapley-Owen Values for Parties Averaged Over the 2000-2003 Period

Two parties, Regions of Russia and Narodniy-Deputat, stand out as having consistently achieved the largest Shapley-Owen values. During any one month, however, there could be substantial variation among the Shapley-Owen values of parties near the yolk. As already noted, we cannot determine from these data whether or not there was a deliberate strategy to seek prizes, as opposed to influence outcomes.

Using the i-power/p-power model and the interpretation of Shapley-Owen values as expressing p-power, the variation in ideal points can be explained as if those parties whose variance is small have fixed policy goals and are seeking to influence outcomes. Those with large variances, by contrast, have fluid policy goals and are maneuvering for strategic advantage, presumably for p-power, but possibly i-power. Those with moderate variance could be explained as parties have good information about likely policy outcomes. Such information would enable them to define positions near the yolk and strong point, and thereby receive the largest prize.

If parties are engaging in strategic behavior, one might expect that policy outcomes would not stable, i.e., that cycling might be observable.⁹ Examining the time-series behavior of the yolk for the Duma for 2000-2003 indeed suggests cycling. Figure 4 plots the position of the yolk center throughout the four-year period. Observe that the yolk center moves throughout the average Pareto set. However, as we noted earlier in our discussion of strategic behavior, without access to the policies adopted, amended, and/or rescinded a conclusive case for cycling cannot be affirmed.

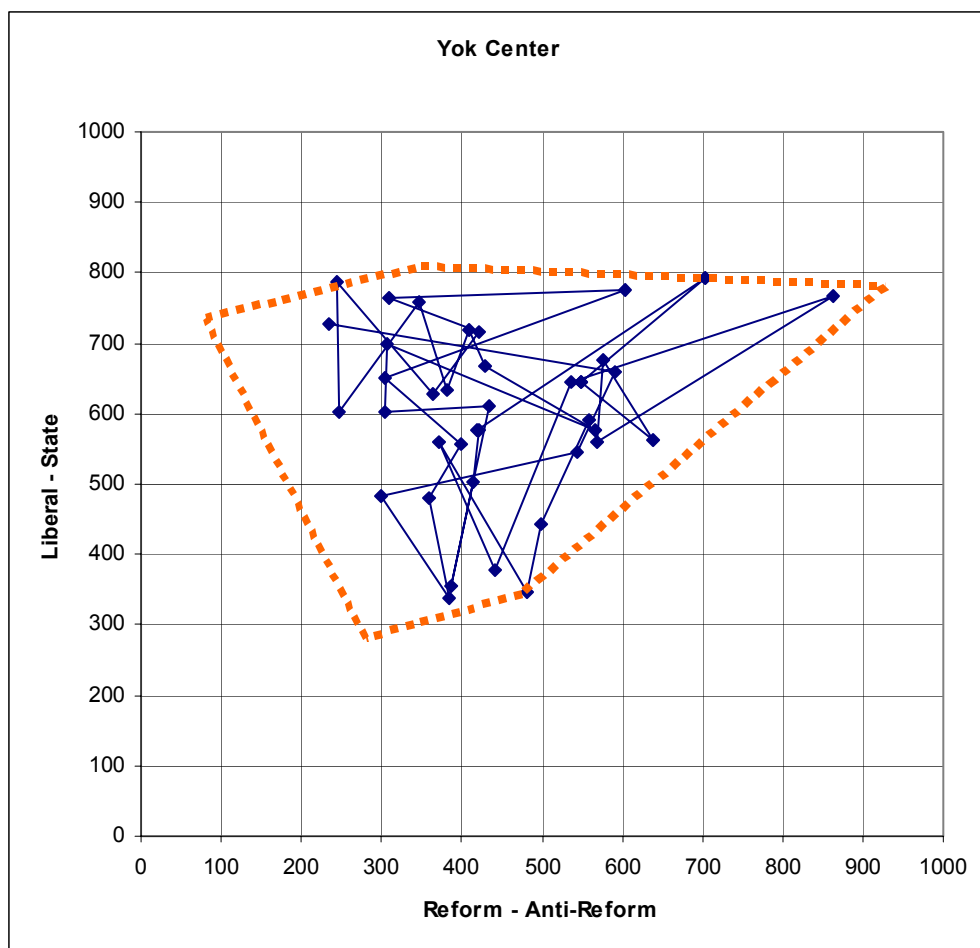


Figure 4 – Center of the Yolk for Individual Months During the Period 2000-2003 (Average Pareto Frontier Shown as Dashed Line)

Using the yolk center to estimate the status quo, observe that the status quo ranges over 50% the policy space during the four-year period. In terms of area, the yolk center movement is bounded by the Pareto frontier, covering an area roughly 20% the area of the entire policy space (and well over 50% the “central” area of the policy space). The strong point, being the least vulnerable policy position, might also be used as an estimator for the status quo. The strong point exhibits essentially the same behavior (not shown) over this period, tracking the yolk center closely.

Figures 5-8, on the subsequent pages, plot the Shapley-Owen value and corresponding yolk radius month by month over the 2000-2003 period. Months during which data are not provided are imputed with the values from the previous month. Observe that the yolk radius varies with the Shapley-Owen value of either the Regions of Russia or the Narodniy-Deputat parties. This is due to the large Shapley-Owen value of two parties. These parties are effectively median voters. The yolk, representing a generalized median, therefore correlates strongly with these two parties.

Note that a large yolk and no party commanding a large Shapley-Owen value characterized opening months of the year 2000, suggesting a period of particular uncertainty.

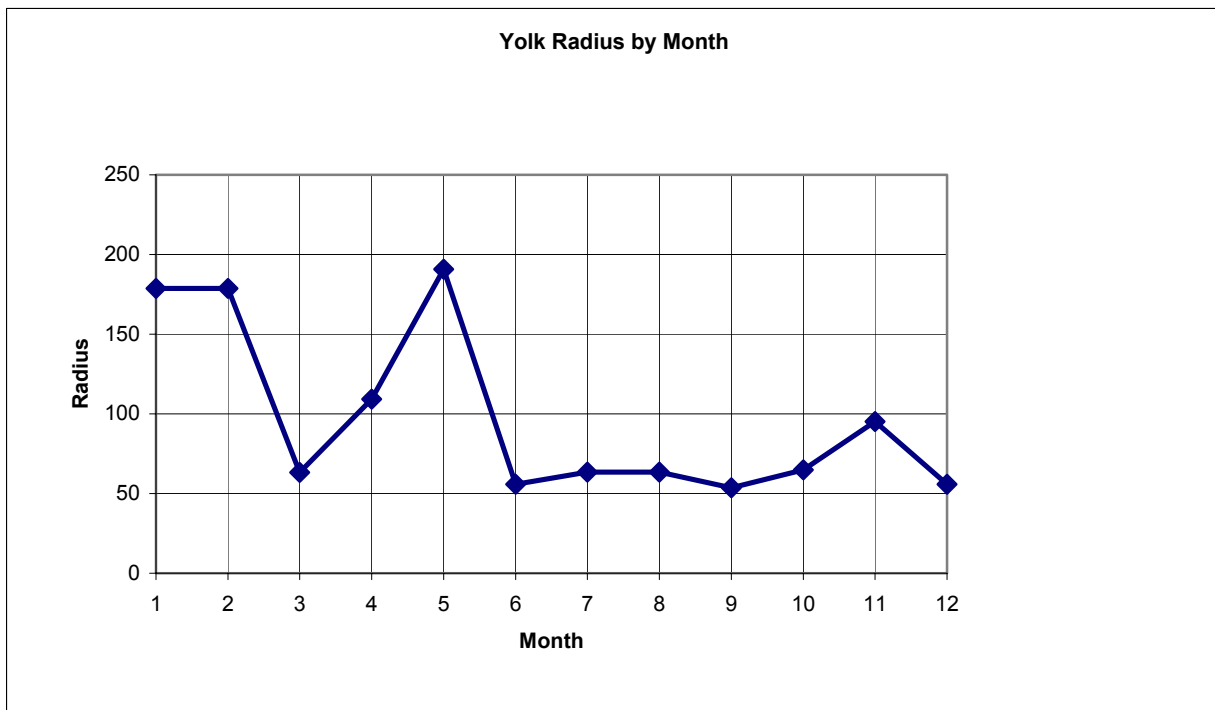
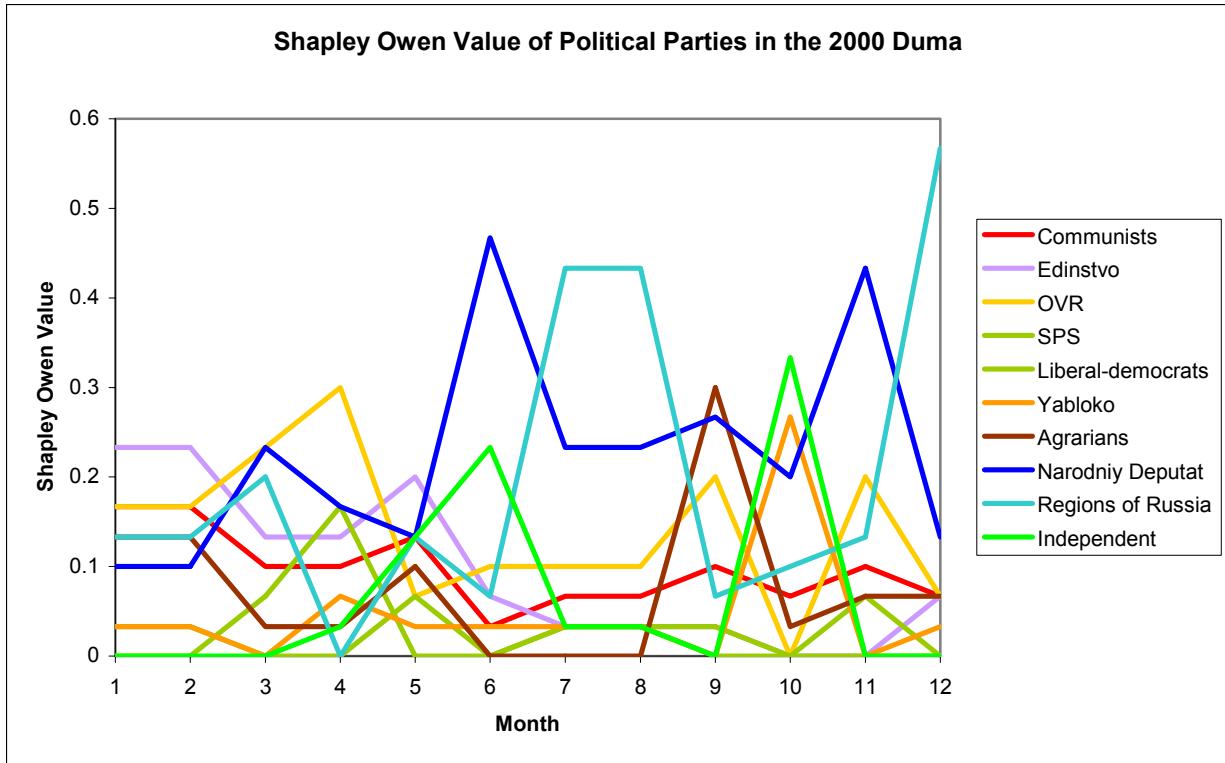


Figure 5 – Year 2000

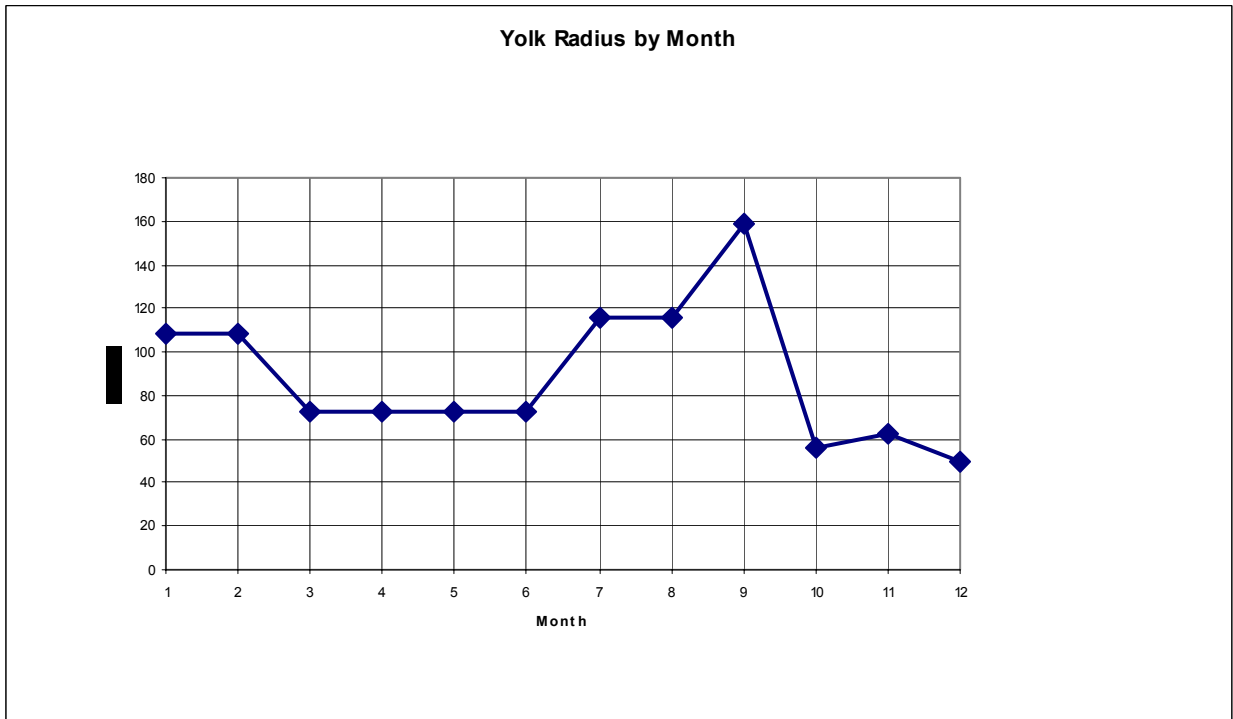
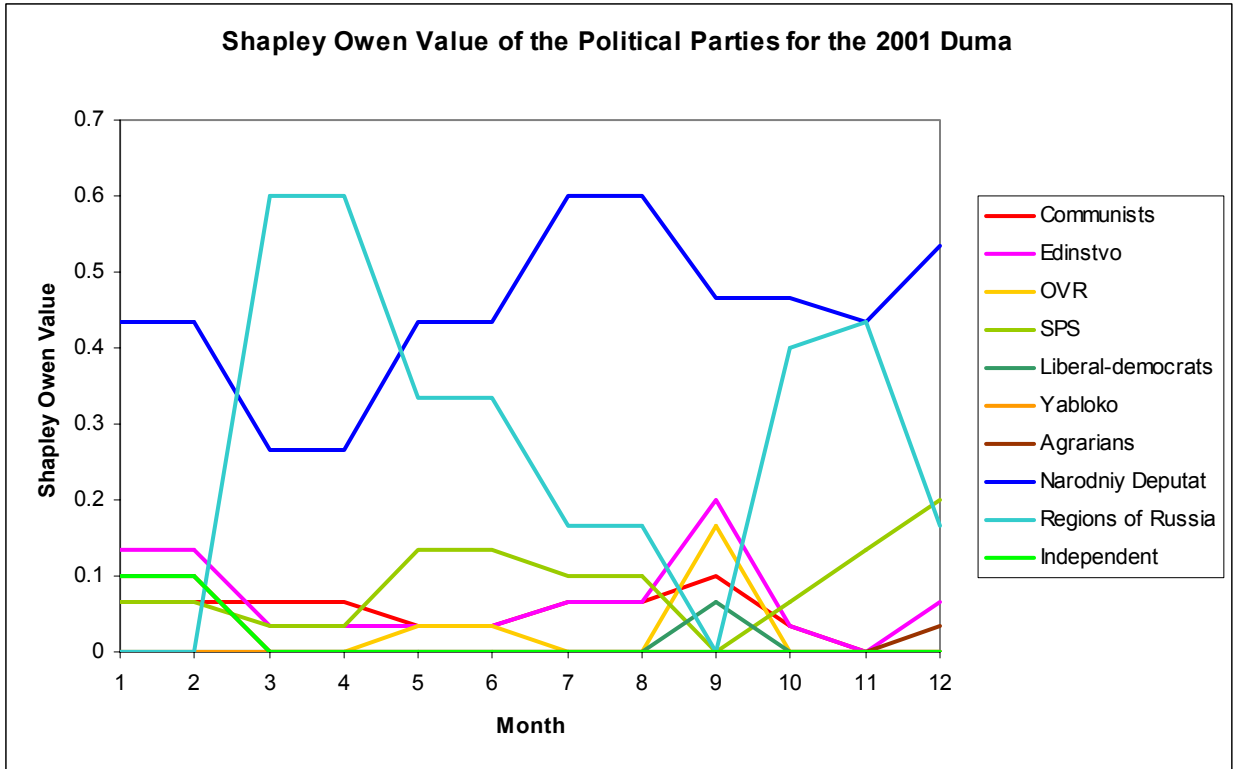


Figure 6 – Year 2001

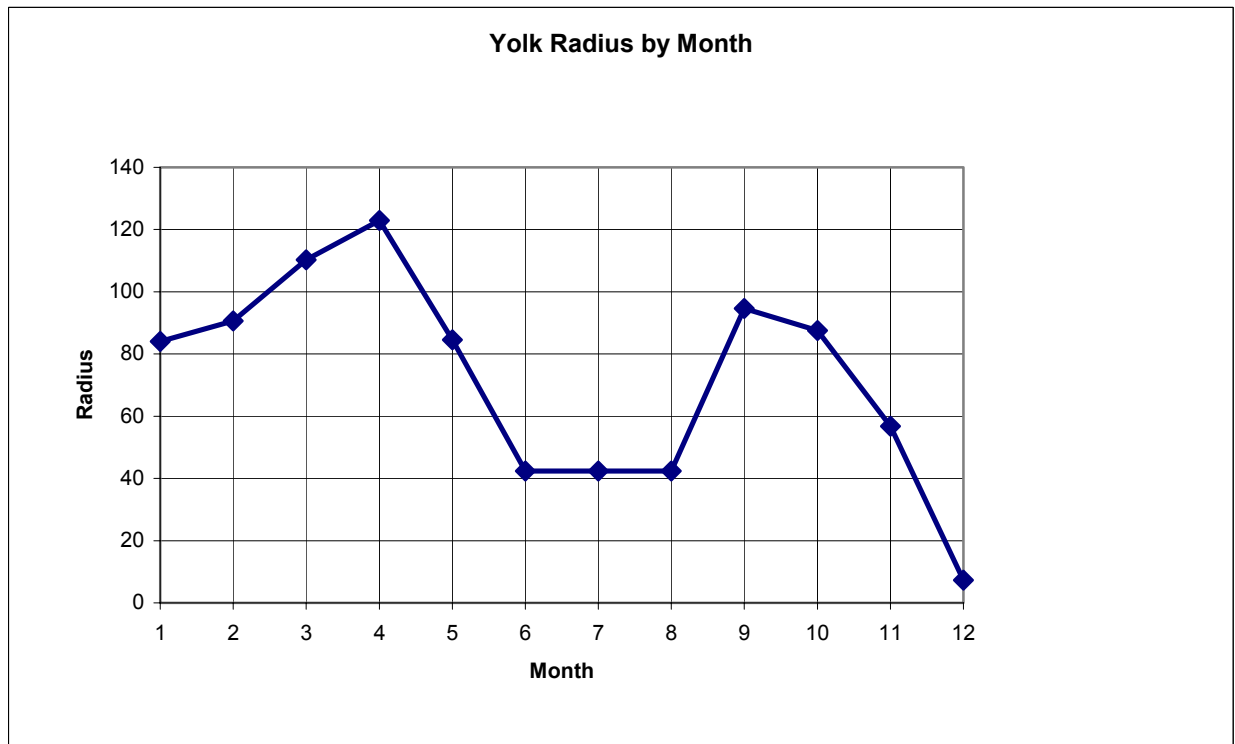
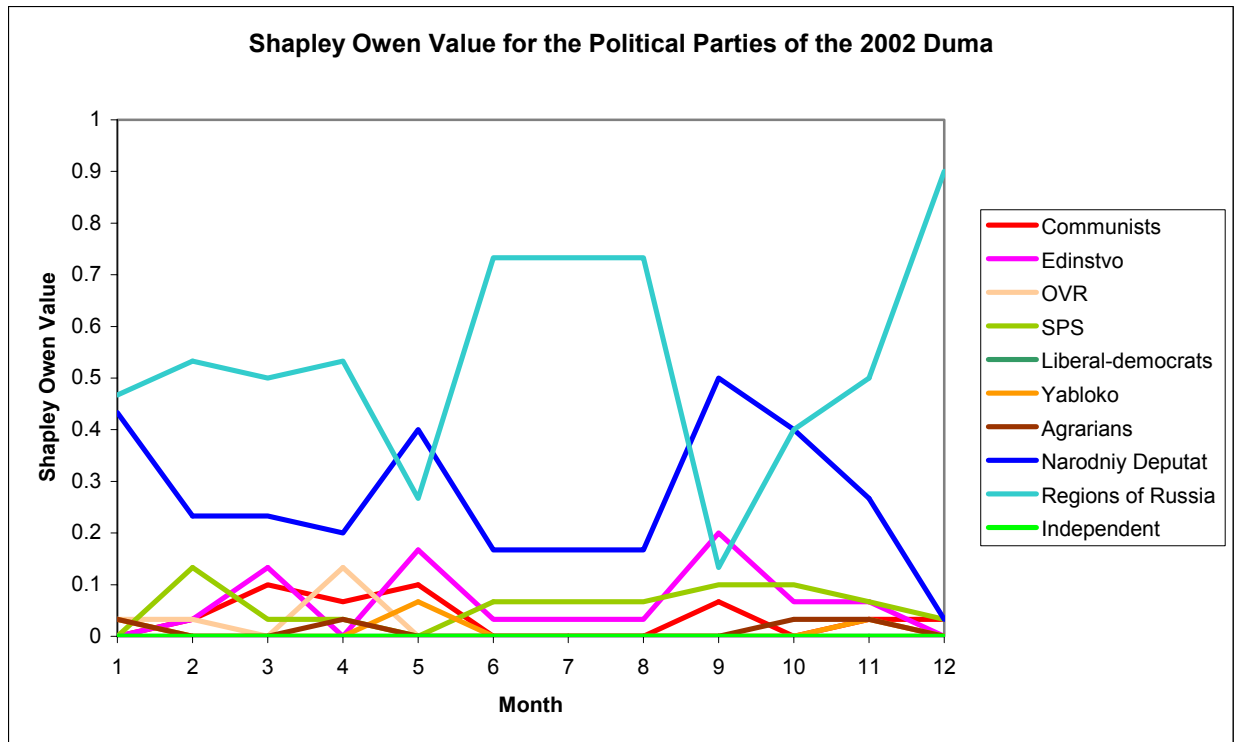


Figure 7 – Year 2002

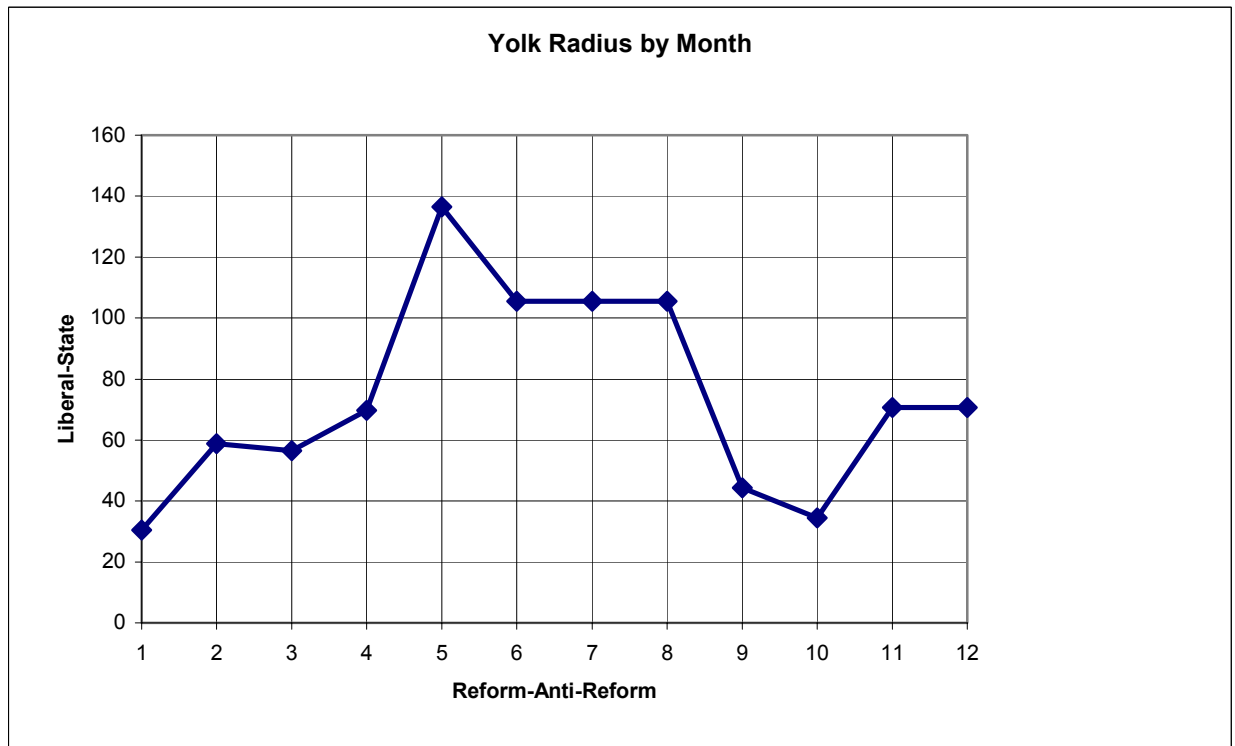
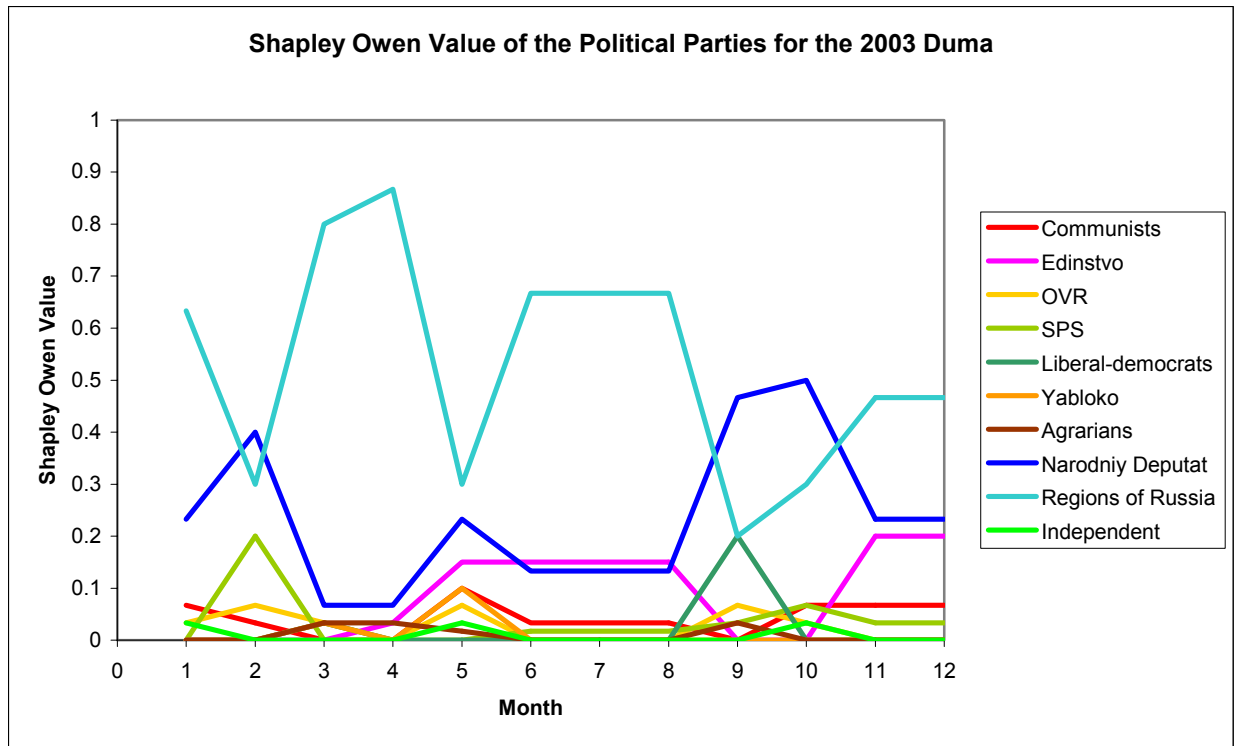


Figure 8 – Year 2003

Conclusion

The Shapley-Owen value arises as measure of a voter's ability to influence the outcome of a policy position, i.e., likelihood to be a median voter. Using the Shapley-Owen value to analyze the Russian Duma during the period 2000-2003 we identified two centrist parties, the Regions of Russia and Narodniy-Deputat as competing for the largest share of the game prize.

Using the yolk, Pareto set, Feld-Grofman theorem, the notions of i-power and p-power, and observations about the volatility of party ideal points, we used a party's Shapley-Owen value along with the party's presence on the Pareto frontier to hypothesize that the Communist, Agrarian, Edinstvo, and SPS parties are more concerned with influencing policy direction (i-power) than deriving benefits from policy outcomes (p-power).

We also observed evidence of cycling by tracking the trajectory of the yolk center.

¹ Shapley L S, Owen G (1989), *Optimal Location of Candidates in Ideological Space*, International Journal of Game Theory, p 125-142.

² Felsenthal, D S, Machover M, (2004) *A Priori Voting: What's It All About*, Political Studies Review: Vol 2, p.1-23
<http://www.politicalstudies.org/pdf/psr/felsenthal.pdf>

³ Godfrey, J, Computation of the Shapley-Owen Power Index in Two Dimensions, VPP4
http://www.lse.ac.uk/collections/VPP/VPPpdf_Wshop4/godfrey.pdf

⁴ Godfrey, J, op. cit.

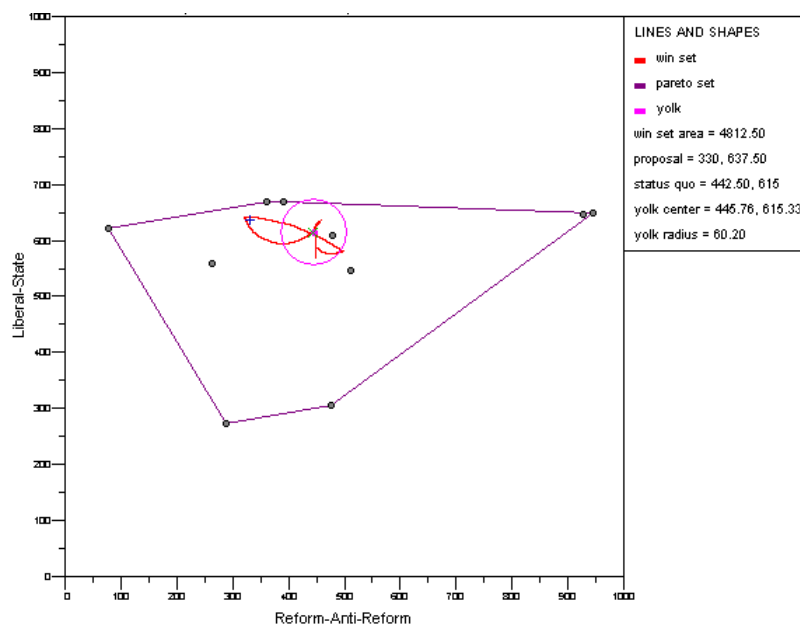
⁵ Aleskerov, F, Blagoveschenski, N, Satarov, G, Sokolova, A, Yakuba, V, *Power Distribution Among Groups and Fractions in Russian Parliament (1994 -2003)*, VVP3
http://www.lse.ac.uk/collections/VPP/VPPpdf_Wshop3/Aleskerov.pdf

⁶ Feld S L, Grofman B (1990), *A Theorem Connecting Shapley-Owen Power Scores and the Radius of the Yolk in Two Dimensions*, Social Choice and Welfare (7), p 71-74

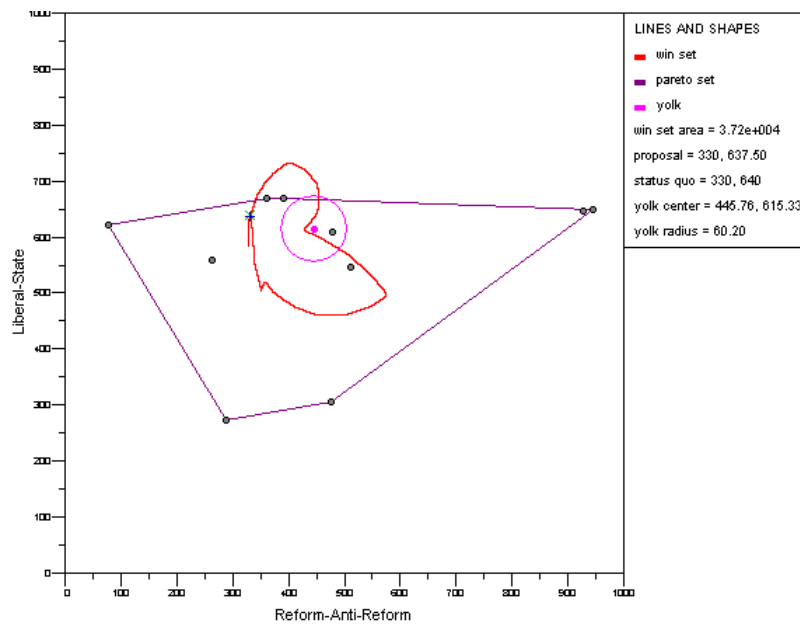
⁷ Felsenthal, D S, Machover M (1998) *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*, Cheltenham: Edward Elgar

⁸ Turnovec, F, Mercik, J W, Mazurkiewicz, M (2005) *Power Indices: A Unified Approach*, ECPS 2005
http://www.dur.ac.uk/john.ashworth/EPCS/Papers/Turnovec_Mercik_Mazurkiewicz.pdf

⁹ The following pair of images illustrate how starting from a status quo within the Duma Pareto set, located at the center of the status quo a proposal can be adopted that results in a win set that reaches outside the Pareto set



Submitting a proposal in the portion of the win set outside the Pareto set further expands the win set's reach outside the Pareto set. So, by following this sequence, eventually all points in the policy space can be reached, in a phenomenon known as McKelvey's "Chaos" Theorem.



In particular, it is possible to return to the initial status quo, resulting in a cycle.