Power distribution in electoral body
with an application to Russian Parliament

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The paper presents several new approaches to evaluate power distribution in an electoral body. The index of consistency of positions of two groups (briefly, the consistency index) is defined which is used to separate possible coalitions in the Parliament. This allows to analyze power distribution with restricted coalition formation. Then we provide several new power indices for the case when the intensity of factions to coalesce is taken into account. The analysis of power distribution model extends the one proposed by Shapley-Owen is given. A new consistency index is given allowing to construct such extension. We illustrate these approaches via the analysis of power distribution among factions in the Russian Parliament (Duma) from 1993 to 2005.

1. Introduction

In legislative bodies decisions are made by voting. The decision accepted if the number of votes for it exceeds some quota, which is defined by the voting procedure. For instance, in the Russian parliament the quota for federal laws is equal to 226 votes (50% +1vote). If a parliament consists of three or more parties, there is a possibility that none of them possesses the votes which exceeds a quota, so to make a decision the parties should coalesce. The coalitions which guarantees the necessary number of votes are of special importance.

Consider two examples.

Example 1. Let three parties *A*, *B* and *C* with 33, 33 and 33 seats, respectively, are presented in a parliament, and the voting rule is the simple majority, i.e., 50 votes for.

Then winning coalitions are *A+B, A+C, B+C, A+B+C* and *A, B* and *C* are pivotal[1] in all but grand coalition. If one measures the power of a party as a number of winning coalitions in which it is pivotal, then all parties in this example has the same power.

Let us change now the distribution of seats. Assume that the parties *A, B* and *C* have 48, 48 and 3 seats, respectively, and the voting rule is the same, i.e., 50 votes for. Then the winning coalitions are the same and parties are pivotal in the same winning coalitions. Thus they have the same distribution of power.

The distribution of power is studied using power indices. Below we will use Banzhaf index [6] which is evaluated as

$$\beta_i = \frac{b_i}{\sum_j b_j},$$

b_i is the number of winning coalitions in which agent *i* is pivotal. This form of Banzhaf index is called the normalized one. Note that non-normalized index of this kind was first introduced in [19].

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[1] An agent *i* is called pivotal for a coalition if agent *i* leaves the coalition it becomes a loosing one.

Example 2. Let us consider now the parliament with 100 seats in which three parties A, B and C are represented with the distributions of seats among them being 50, 49, 1. Let the decision making rule be the simple majority, i.e., 51 votes for. Then the winning coalitions are A+B, B+C, A+B+C. The party A is pivotal in all three coalitions, B is pivotal only in the coalition A+B, and C is pivotal in A+C. Then

$$\beta(A) = \frac{3}{3+1+1} = \frac{3}{5},$$

$$\beta(B) = \beta(C) = \frac{1}{3+1+1} = \frac{1}{5}.$$

Assume now that parties A and B never coalesce in pairwise coalition, i.e., coalition $A+B$ is impossible. Let us, however, assume that the coalition $A+B+C$ can be implemented, i.e., in the presence of `moderator' C parties A and B can coalesce. Then the winning coalitions are $A+C$ and $A+B+C$, and A is pivotal in both coalitions while C is in one; B is pivotal in none of the winning coalitions. In this case $\beta(A) = 2/3$, $\beta(B) = 0$ and $\beta(C) = 1/3$, i.e., although B has almost half of the seats in the parliament, its power is equal to 0.

If A and B never coalesce even in the presence of a moderator C, then the only winning coalition is $A+C$, in which both parties are pivotal. Then, $\beta(A) = \beta(C) = 1/2$.

Such situations are met in real political life. For instance, Russian Communist Party in the second Duma (1997-2000) had had about 35% of seats, however, its power during that period was always almost equal to 0 [1].

We study the problem of power distribution over time of groups and factions of Russian Parliament (Duma) using Banzhaf index. The analysis is made under two main assumptions about coalitions formations. First, we consider the case when all coalitions are admissible, and after we study several scenarios of coalitions formation. To evaluate the possibility of two groups to coalesce we use different versions of the index of consistency of positions of two groups which is based on the similarity of voters in one act of voting or on the closeness of their positions on political map. We study several qualitative scenarios of coalition formation one of which being considered as real.

This paper gives a short description of a study on a distribution of power in Russian parliament during 1994-2005. The complete study can be found in [1-3,5], on the sites www.hse.ru, www.ipu.ru/rcpp and www.indem.ru. It contains complete analysis of power distributions for each electoral period (1994-1995, 1996-1999, 2000-2003, 2004-2005) for two decision rules (simple majority for federal laws and qualified majority (300 out of 450) for constitutional laws) using two power indices – the Banzhaf and Shapley-Shubik indices [21].

The structure of the paper is as follows. Section 2 presents main notions and data used. Section 3 provides an analysis of power distribution in Russian parliament using standard Banzhaf index. In Section 4 we define the index of consistency of positions of two groups (briefly, the consistency index) and analyze power distribution with restricted coalition formation. Section 5 provides several new power indices for the case when the intensity of factions to coalesce is taken into account. We consider an analysis of power distribution in Russian parliament for two out of many intensity functions introduced in [1,2]. Section 6 contains the analysis of power distribution model proposed by Shapley-Owen in a spatial context when the agents have ideal (bliss) points on a political map. We define here a new consistency index and using it propose an extension of Shapley-Owen index. Then we give an analysis of power distribution in Russian parliament using this extension. Section 7 concludes.

2. Main notions and the data used

The set of agents (parties, factions) is denoted as N, $N = \{1,\ldots,n\}$, $n > 1$. A coalition ω is the subset of N, $\omega \subseteq N$. We consider the situation when the decision of a body is made by voting procedure; agents who do not vote `yes' vote against it, i.e., the abstention is not allowed.

Each agent has a predefined number of votes, $v_i > 0$, $i = 1,\ldots,n$. It is assumed that a quota q is predetermined and as a decision making rule the voting with quota is used, i.e., the decision is made if the number of votes for it is not less than q,

$$\sum_i v_i \geq q.$$

The model describes a voting by simple and qualified majority, voting with veto (as in the Security Council of UN), etc.

A coalition ω is called winning if the sum of votes in the coalition is not less than q. An agent i is called pivotal in the coalition ω if the coalition $\omega \setminus \{i\}$ is a loosing one.

For such voting rule the set of all winning coalitions Ω possesses the following three properties:

$$\phi \notin \Omega, \; N \in \Omega, \; \omega \in \Omega, \; \omega' \supseteq \omega \Rightarrow \omega' \in \Omega.$$

Sometimes, one additional condition is applied as well

$$\omega \in \Omega \Rightarrow N \setminus \omega \notin \Omega,$$

implying $q \geq \lceil n/2 \rceil$, where $\lceil x \rceil$ is the smallest integer greater or equal to x.

The system of winning coalitions constitute an n-person simple game in the form of characteristic function, i.e., every coalition $S \subseteq N$ gets a payoff equal to 0 or 1.

The data. During 1993-2007 the Russian Parliament consisted of 450 members one half of them being elected by majority voting and other half by party lists[2]. Factions had been created by electoral blocks which passes by proportional representation scheme. Moreover, there was a possibility to create MPs groups with no less than 35 members (until 2004). Decision making rules are simple majority (226 votes) for federal laws and 2/3 (300) votes for constitutional laws.

We have considered the structure of factions and groups on 16th of each month separately for each of three parliaments (1994-1995, 1996-1999, 2000-2003) and for the part of the 4th Parliament (2004-2005).

Using this structure we calculate the Banzhaf index for federal laws. This evaluations have first been made under the assumption that all coalitions are equally feasible, and then after excluding unfeasible coalitions. As the source of data the one had been used of the foundation INDEM (http://www.indem.ru/indemstat/index.htm)

3. Power distribution in Russian parliament without restrictions on coalition formation

In the case considered in this Section the changes in power distribution are observed only due to the transfers of MPs from one group or faction to the other. Moreover, essential changes will be observed at the moments of huge transfers of MPs which are connected usually with the formation, sometimes, unsuccessful, of new factions or groups.

We have used the following scheme to distribute independent MPs to factions and groups: we distribute them to those factions to which they will transfer afterwards or to which she had belonged

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[2] From 2007 Russian Parliament is elected by party lists only.

before. If none of these situations hold, we studied her political interests and attribute that MP to the group with closest interests.

In fact, we also evaluated the power index for independent MPs separately. The difference between two approaches led to the difference of power indices less than 1%.

Let us now discuss the results. In all three parliaments the following three parties were represented
- Agrarians (Agrarian Group of Russia, APG)
- Communists (Communist Party od Russian Federation, CPRF)
- Liberal-Democrats (Liberal-Democratic Party of Russia, LDPR)
- Yabloko.

Additionally, in the second and third parliaments the group "Regions of Russia" was represented, and in the third Duma pro-presidential parties Edinstvo and OVR and liberal party SPS were also presented.

The changes in power distribution are shown on Fig.1. Communists as well as Yabloko had the maximal of their power in the second Duma, however, the power of Communists had been decreasing from the beginning of the second parliament through the third one. Agrarians in general had the power about 9.3%. Liberal-Democrats had been loosing their power during all those years. In the first parliament it was one of the most powerful parties, while in the third parliament it was one of the weakest parties. The group Regions of Russia had almost stable power about 10%.

The power of groups and the share of their votes for the first and third parliaments were consistent, i.e., there for factions and groups their power values and share of seats were with almost equal.

Another picture can be seen in the second parliament, in which there had been one strong faction (Communists) and 6 small groups. In average the power of Communists exceed its share of votes on 26%, and had the maximum at the beginning of the second parliament when this difference was 30%. For Our Home – Russia this difference was in average 33%, Liberal-Democrats – 19%, for Yabloko –15%, etc. In other words, in the second Duma comparing it with the first and third ones, the distribution of power did not correspond to the distribution of seats.

4. The consistency index

Now we will study the approach allowing to separate admissible coalitions from those which are not. The relation between two groups of MPs are naturally reflected on the results of voting. Groups with similar political positions having common political interests initiate consistent bills and support them in voting. On the contrary, the groups with opposite interests vote in a different way. This point of view is supported by the observation of voting behavior in Russian parliament.

Let q_1 and q_2 be the share of 'ay' votes in two groups in MPs.

Then consistency index is calculated as

$$c(q_1, q_2) = 1 - \frac{|q_1 - q_2|}{\max(q_1, 1 - q_1, q_2, 1 - q_2)}.$$

In other words, if in two groups the share of 'ay' votes is equal, then these groups are considered as consistent. The groups are totally inconsistent ($c = -1$) if one group votes 'ay' while another group votes 'no'. The properties of the consistency index were studied in [7].

In our study we use the mean value of consistency index taken from m monthly observations, i.e.

$$\overline{c} = \frac{1}{m} \sum_{i=1}^{m} c(q_{1i}, q_{2i}).$$

To evaluate that mean value we select votings on the basis of several criteria which express different information about voting and division among factions and MPs. We assume that the abstention of MP means usually her disagreement with the bill. In general, the selection of the result of voting is made in two stages. First, we select those results in which even with very few votes against a bill one can obtain the essential difference of votes for and against in at least two factions. For each voting result we calculate the difference between maximal and minimal over factions share of "ay" votes, and then choose those results for which this measure exceeds some predetermined threshold. Then from the list of results the 'non-important' votings are excluded, for instance, those for which the bill is supported by no less than 30 votes or by at least 320 votes "ay". Finally, those results are excluded from the consideration for which the difference in voting among factions is caused by some technical reasons, and further such bills are voted anew.

The analysis of the parliament when not all coalitions are feasible. The assumption that all coalitions are feasible is too strong. For instance, in the first parliament the coalition between Choice of Russia (main pro-presidential party) and Communists was hardly possible. Obviously, the real power distribution was not stable in all three parliaments in the light of many changes that had been happening during all those years. At that time many bills were approved dealing with most important changes in the country, from constitutional reforms to the reforms of natural monopolists.

To construct a power distribution more adequate to real situation, it is necessary to measure the possibility of coalition formation depending on the relations among groups of MPs. We construct the model of coalition formation depending on a threshold value of the consistency index.

At the beginning by the introduction of different threshold values of the consistency index the impossible coalitions were excluded. The consistency index introduced above had been calculated for all pairs of groups and factions in the parliament from 1994 to 2005 for all results of voting described above. According to the approach used, a coalition is considered to be impossible if the value of the consistency index for two groups in the coalition is less than the threshold value of that index.

It is assumed that under some threshold level the evaluated power index should be close to its real value. We consider three values of threshold for consistency index and, thus, three different distributions of power index. Those three values for threshold are[3] $c \geq 0.4$, $c \geq 0.5$, $c \geq 0.6$.

The choice of the threshold 0.5 is obvious and does not need any additional explanations. The choice of other levels need some explanation. The evaluation of consistency index for explicit ideological contestants shows that for this case the value of c does not exceed 0.4. The value of consistency index between 0.5 and 0.7 corresponds to the relations from potential allies relations to full allies relations. The threshold value 0.6 gives from one point of view a minimal level of allies relations, and, from the other point of view preserves enough possibilities for coalition formation.

So, the key question is which value of the consistency index generates power distribution reflecting real power distribution in Russian parliament? The answer to this question has been given on the basis of scenario approach applying to coalition formation mechanisms.

To construct scenarios the scale was suggested for evaluation of relations among groups and factions in the parliament. This scale includes four grades: explicit "contestants", potential contestants, potential allies, explicit allies. Using these grades three scenarios were constructed:
- "mild" scenario (coalition are excluded with explicit contestants);
- "average" scenario (coalition are excluded with explicit and potential contestants);
- "rigid" scenario (only coalition with closest allies are allowed).

The scenario when all coalitions are admissible can be called as null-scenario.

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[3] In [3] the analysis for the threshold values less than 0.4 is made as well.

Mild scenario is by definition a real one. Indeed, the strategy when in a coalition potential contestants can be included seems to be in some sense optimal. It leaves enough freedom for coalition formation but excludes uncompensated losses a party could meet if she coalesce with explicit contestant which cannot be forgiven from the point of view of the electorate. One may expect that experienced politicians managing political factions and groups in the Russian parliament follow this optimal strategy.

The average scenario is interesting in that it allows to evaluate the abilities of the participants of political process. For heavy players which fill the extreme position it is an ability to attract the majority of voters; for the players at center of political field it is an ability to participate in winning coalitions.

For rigid scenario situations are of special importance in which coalition are formed only with closest allies.

The change in consistency index for pair of party factions over time for the third Duma is given on Fig.2.

As it can be seen, Communists and Agrarians are the closest allies, their consistency index is about 0.85. On the contrary, the relation between Communists and Edinstvo are worsening over time achieving the level about 0.1. It is important to note that the minimum of the index for this pair is seen at the moments when the most crucial decisions are made in July 2001.

The dynamics of consistency index for Edinstvo and OVR reflects the process of organization of the largest party in the parliament - Edinaya Rossiya. After January 2002 both parties are most closest allies, their consistency index is greater than 0.8.

We show the dynamics of power indices for large parties on Figs. 4-6 below for the scenarios $c=0.4$, 0.5, and 0.6, respectively. At the Figs. 4-5 the standard Banzhaf index values are given as well.

The share of votes for the faction of Communists was in average 18% while her value of Banzhaf index was much smaller and from July 2001 it did not exceed 3%. The factions Edinstvo and Narodnyi Deputat had the power greater than their share of votes.

In general, the following conclusions can be derived from the obtained results:

In scenario $c=0.4$ the centrist factions increases their power since they do not have explicit contestants. Thus, they possess the same possibilities as in the null-scenario (all coalitions are admissible). On the contrary, groups expressing extreme positions and having large contestants should expect serious losses.

In scenario $c=0.5$ maximal losses should expect those factions which coalesce with potential contestants. The groups which can create majority using explicit and potential allies preserve or even increase their power.

In scenario $c=0.6$ those groups can preserve their power which can create majority leaning only to explicit allies.

The most close to real power distribution is the distribution observed for the scenario $c=0.4$.

5. Intensity functions, ordinal and cardinal power indices

We introduce here new indices based on the idea similar to Banzhaf power index, however, taking into account agents' preferences to coalesce.

In these indices the information is used about agents' preferences over other agents. These preferences are assumed to be linear orders. Since these preferences may not be symmetric, the desire of agent 1 to coalesce with agent 2 can be different than the desire of agent 2 to coalesce with agent 1. The indices take into account in a different way such asymmetry of preferences and are constructed on the following basis: the intensity of connection $f(i,\omega)$ of the agent with other members of ω is defined. Then for an agent i the value χ_i is evaluated as

$$\chi_i = \sum_{\omega} f(i,\omega),$$

i.e., the sum of intensities of connections of i over those coalitions ω in which i is pivotal. Naturally, other functions instead of summation can be considered.

Then the power indices are constructed as

$$\alpha(i) = \frac{\chi_i}{\sum_j \chi_j}.$$

The very idea of $\alpha(i)$ is the same as for Banzhaf index, with the difference that in Banzhaf index we evaluate the number of coalitions in which i is pivotal, i.e., $f(i,\omega)$ in the definition of Banzhaf index is equal to 1, on the contrary, in our case it can be less than 1.

The main question now is how to construct the intensity functions $f(i,\omega)$. Below we give two different forms of these functions.

Each agent i is assumed to have a linear order[4] P_i revealing her preferences over other agents in the sense that i prefers to coalesce with agent j rather than with agent k if P_i contains the pair (j, k). Obviously, P_i is defined on the Cartesian product $(N \setminus \{i\}) \times (N \setminus \{i\})$.

Since P_i is a linear order, the rank p_{ij} of the agent j in P_i can be defined. We assume that $p_{ij} = |N|$ -1 for the most preferable agent j in P_i.

The value p_{ij} shows how many agents less preferable than j are in P_i. For instance, if N ={A,B,C,D} and $P_A : B \succ C \succ D$, then $P_{AB} = 3$, $P_{AC} = 2$ and $P_{AD} = 1$.

Using these ranks, one can construct different intensity functions.

A second way of construction of $f(i,\omega)$ is based on the idea that the values p_{ij} of connection of i with j are predetermined somehow. In general, it is not assumed $p_{ij} = p_{ji}$. Then the intensity function can be constructed as above.

Below we give three different ways how to construct $f(i,\omega)$ in ordinal case and only one way of construction of cardinal function $f(i,\omega)$. Other forms of intensity functions can be found in [1].

Ordinal indices. For each coalition ω and each agent i construct now an intensity $f(i,\omega)$ of connections in this coalition. In other words, f is a function which maps $N \times \Omega$ $(= 2^N \setminus \{\emptyset\})$ into R^1, $f : N \times \Omega \to R^1$. This very value $f(i,\omega)$ is evaluated using the ranks of members of coalition. Three different ways to evaluate f using different information about agents' preferences are provided:

a) Intensity of i's preferences. In this form only preferences of i's agent over other agents are evaluated, i.e.,

$$f^+(i,\omega) = \sum_{j \in \omega} \frac{p_{ij}}{|\omega|}$$

b) Intensity of preferences for i. In this case consider the sum of ranks of i given by other members of coalition ω

$$f^-(i,\omega) = \sum_{j \in \omega} \frac{p_{ji}}{|\omega|}$$

c) Average intensity with respect to i's agent

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[4] i.e. irreflexive, transitive and connected binary relation. We often denote it as $\succ$.

$$f(i,\omega) = \frac{f^+(i,\omega) + f^-(i,\omega)}{2}$$

Consider now several examples.

Example 3. Let $n=3$, $N=\{A,\ B,\ C\}$, $v(A)=33$, $v(B)=v(C)=33$, $q=50$. Consider two preference profiles given in Tables 1 and 2.

P_A	P_B	P_C
C	C	A
B	A	B

Table 1. First preference profile

P_A	P_B	P_C
B	C	A
C	A	B

Table 2. Second preference profile

For both preference profiles there are three winning coalitions in which agents are pivotal. These coalitions are $A+B$, $A+C$ and $B+C$.

Let us calculate the functions f as above for each agent in each winning coalition. The preferences from Tables 1 and 2 can be re-written in the matrix form as

$$\|p_{ij}\| = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} A\ \ B\ \ C \\ \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \end{array}, \quad \|p'_{ij}\| = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} A\ \ B\ \ C \\ \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \end{array}$$

Now, for the profile given in Table 1 one can calculate the values of intensities a)-c) obtained by each agent i in each winning coalition ω. Using these intensity functions one can define now the corresponding power indices $\alpha(i)$. Let i be a pivotal agent in a winning coalition ω. Denote by χ_i the number equal to the value of the intensity function for a given coalition ω and agent i. Then the power index is defined as follows

$$\alpha(i) = \frac{\displaystyle\sum_{\substack{\omega, \\ i \text{ is pivotal in } \omega}} \chi_i}{\displaystyle\sum_{j \in N} \sum_{\substack{\omega, \\ j \text{ is pivotal in } \omega}} \chi_j}$$

The indices $\alpha(i)$ will be denoted by $\alpha_1(i),\ldots,\alpha_3(i)$.

Let us evaluate now the values $\alpha_1(\cdot),\alpha_2(\cdot)$ for all agents for the preference profile from Table 1.

The agent A (as well as agents B and C) is pivotal in two coalitions; the sum of the values $f^+(i,\omega)$ for each i is equal to 3/2. Then

$$\alpha_1(A) = \frac{3/2}{3/2+3/2+3/2} = \frac{1}{3} = \alpha_1(B) = \alpha_1(C).$$

The value $\alpha_2(\cdot)$ is evaluated differently. The sum of values $f^-(i,\omega)$ from Table 3 for all i and ω is equal to 9/2. However, for A $\sum_\omega f(A,\omega) = 3/2$, $\sum_\omega f(B,\omega) = 1$ and $\sum_\omega f(C,\omega) = 2$. Then

$$\alpha_2(A) = \frac{3}{9} = \frac{1}{3}; \ \alpha_2(B) = \frac{2}{9} \ \text{and} \ \alpha_2(C) = \frac{4}{9}.$$

The values of the indices $\alpha_1 - \alpha_3$ for both preference profiles are given in Table 3 as well as the values of Banzhaf index β

	First profile (Table 1)			Second profile (Table 2)		
	A	B	C	A	B	C
α_1	1/3	1/3	1/3	1/3	1/3	1/3
α_2	1/3	2/9	4/9	1/3	1/3	1/3
α_3	1/3	5/18	7/18	1/3	1/3	1/3
β	1/3	1/3	1/3	1/3	1/3	1/3

Table 3. Power indices values

Consider now another example.

Example 4. Consider the case when 3 parties A, B and C have 50, 49 and 1 seats, respectively. Assume that decision making rule is the simple majority, i.e. 51 votes. Then the winning coalitions are A+B, A+C and A+B+C. Note that A is pivotal in all three coalitions, B and C are pivotal in one coalition each. Then $\beta(A) = 3/5$, $\beta(B) = \beta(C) = 1/5$.

Consider now the case with the preferences of agents given below: $P_A : C \succ B$; $P_B : C \succ A$ and $P_C : A \succ B$.

Then the values of α_1 and α_2 (constructed by $f^+(i,\omega)$ and $f^-(i,\omega)$) are as follows

$$\alpha_1(A) = 5/12, \quad \alpha_1(B) = 1/4, \quad \alpha_1(C) = 1/3,$$

$$\alpha_2(A) = 5/12, \quad \alpha_2(B) = 7/36, \quad \alpha_2(C) = 7/18.$$

Consider another preference profile: $P_A : C \succ B, P_B : C \succ B; P_C : B \succ A$, i.e., only agent C changes her preferences. Then one can easily evaluate $\alpha_1(A) = 5/11, \quad \alpha_1(B) = 3/11, \quad \alpha_1(C) = 3/11$; $\alpha_2(A) = 10/33$, $\alpha_2(B) = 3/11$, $\alpha_2(C) = 14/33$.

In the second type of power index the information about the intensity of preferences is taken into account as well, i.e., we extend the former type of power index to cardinal information about agents' preferences.

Cardinal indices. Assume now that the desire of party i to coalesce with party j is given as real number p_{ij}, $\sum_j p_{ij} = 1$, $i, j = 1, \ldots, n$. In general, it is not assumed that $p_{ij} = p_{ji}$.

One can call the value p_{ij} as an intensity of connection of i with j. It may be interpreted as, for instance, a probability for i to form a coalition with j.

As in the previous case we define now several intensity functions

a) average intensity of i in connection with other members of coalition ω

$$f^+(i,\omega) = \frac{\sum_{j\in\omega} p_{ij}}{|\omega|};$$

b) average intensity of connection of other members of coalition ω with i

$$f^-(i,\omega) = \frac{\sum_{j\in\omega} p_{ji}}{|\omega|};$$

c) average intensity for i

$$f(i,\omega) = \frac{1}{2}(f^+(i,\omega) + f^-(i,\omega)).$$

Using the consistency index defined above as the cardinal intensity function one can construct the power distribution for Russian Parliament. The value of the index α_1 for the third Duma is given on the Fig. 6. One can see that the graphs are more smooth than in the previous case when the 'threshold' model of coalition formation was used. In fact, such model can be used here as well.

In [1,2] an axiomatic construction of the first cardinal intensity function is given. In an analogous way other intensity functions can be constructed.

6. Extended Shapley-Owen index and power distribution in Russian Parliament

In Shapley-Owen index the power of an agent depends not only on the voting rule of decision making, but on the position of agents in spatial context, or, in political space [20], i.e., on ideology, as well. This index (the Shapley-Owen value, for short SOV [17]) gives a special role to an ideology in coalition formation, i.e., only the ideologically close agents will coalesce.

Let each player has its own ideal point $P_i \in \Re^m$ in m-dimensional Euclidean space. The ideal points reflect the preferential political outcomes of each player. Let $\Psi \subseteq \Re^m$ be a set of all the outcomes of voting. Each outcome is a vector $x \in \Psi$.

Assume that function $u_i(x)$ such as $u_i : \Psi \to \Re^m$ exists for each player and measures the level of player's attitude to outcome x. Using the values of this function a strict order $\succ$ can be defined on the set N, thus, $j \succ i$, if $u_j(x) - u_i(x) \geq 0$. This relation indicates that player j likes the outcome x more than i.

Define $Y_{ij} = u_i(x) - u_j(x)$. If $Y_{ij} \leq 0$, then j joins to a coalition of players supporting the outcome x more willingly than i. Owen and Shapley introduced the player's power index in the spatial context. They consider unit vectors $x \in \Re^m$ on the unit-sphere H^{m-1}, $\forall x \in \Psi$ $\langle x, x \rangle = 1$. Each vector defines a direction in the space. It was proposed that the function values are determined by the inner product $u_i(x) = \langle x, P_i \rangle$. Then each unit vector x randomly chosen from uniform distribution induce an order relation $\succ$ as $i \succ_U j \Leftrightarrow \langle U, P_i \rangle \geq \langle U, P_j \rangle$.

So, the power index for player i can be written as ratio

$$SOV_i = \frac{q_i}{n!},$$

where q_i is the number of orderings, for which player i is pivotal, $n!$ is the total number of all possible orderings. Effective computational scheme for evaluation of SOV is given in [11].

We now extend the Shapley-Owen index using the notion of the consistency of the players positions.

Let d_{ij} be Euclidean distance between ideal points of players i and j in normalized two-dimensional political space. Consider an index of consistency of two players

$$k_{ij} = \frac{1}{\sqrt{2}}\left(\frac{1+\sqrt{2}}{1+d_{ij}} - 1\right) \qquad (1)$$

In Shapley-Owen model a player is pivotal if she occupies the median position in the linear order, obtained on each step, i.e. pivotal player splits the set of players N to two disjoint sets, where one of them is a winning coalition.

Denote the coalition located on the left of the pivotal player in linear order obtained on some step as S, and one on the right of the pivotal player as T (see Fig. 7). The pivotal player can make winning each of these coalitions after joining them.

Then we introduce the weight of player i, which is pivotal,

$$w_{im} = \frac{1}{l}\sum_j k_{ij}, \ i \neq j, \qquad (2)$$

It is computed as the sum of indices of consistency for each step $m=1,...,t$, i.e., for each increment of angle of line rotation about origin of considered political space. Summation in (2) includes that parties j enter to this coalition, which i can make winning, and l is the number of players of this coalition.

So, two numbers of the weight value are computed, both by the sum of index of consistency of pivotal position and players of coalition S positions and by the sum of index of consistency of pivotal position and players of coalition T positions. The largest weight is chosen meaning that the pivotal player enters to that coalition with players being more consistent with him.

Then the average value of i's weight is computed, here t is the number of steps:

$$v_i(t) = \frac{\sum_{m=1}^{t} w_{im}}{t} \qquad (3)$$

The power index of player i is determined as

$$PI_1(i) = \frac{v_i(t) \cdot \lambda_i}{\sum_{j=1}^{n} v_j(t) \cdot \lambda_j}, \qquad (4)$$

where $\lambda_i = n_i \Big/ \sum_j n_j$ is the share of votes, and n_i is the number of votes of party i.

Two more indices can be constructed based on this very idea: the one based on the consistency of the players positions without taking into account the share of votes of each agent and another one based on the consistency of the player's ideal point to the system of players 'center of mass' [5]

Let us compute now the power for the political parties in the III State Duma of the Russian Federation using the power index introduced above. The data about player's preferences covers the State Duma of the Russian Federation for each month from 2000 to 2003.

The issue space consists of two dimensions defined as "Liberal – State oriented" (horizontal axis) and "Antireform – Pro-reform oriented" (vertical axis). Each dimension is measured using a floating-point scale ranging from -1 to 1. The preferences of players are Euclidean. The decision rule is the simple majority rule. These political map (issue space) has been obtained using factor analysis of votings in that Duma [1].

Fig. 8-11 represents the average distribution of power PI_1 of all parties for the period under study. As we for instance can see on Fig. 9, the party "Narodny deputat" was the leader at the beginning of 2000, its $\overline{PI_1} \approx 0.29$. But to 2001 the value of its power became lower, $\overline{PI_1} \approx 0.25$. In 2002 the average value of power index of "Narodny deputat" declined, $\overline{PI_1} \approx 0.16$. This effect can be explained by the fact that in the beginning of the third Duma this party 'started' like centrists, its motion pass occupied the considerable area [1]. But to 2002 it is noted that ideal points of this party migrated from center to top left corner of political map and area of motion path is decreased leading to the reduction of frequency of events in which this party was pivotal.

Party "Regions of Russia" in 2000 had $\overline{PI_1} \approx 0.16$, it was the second with respect of power distribution, and the average value of its power constantly grew during all period under study. In 2003 its $\overline{PI_1} \approx 0.33$ and as we can see this value was twice large to the end of 2003. This shows that the frequency of event, when this party was pivotal, had increased. The political map of "Regions of Russia" motion paths shows that party ideal points movement was active, and ideal points area was considerably wide.

Agrarians were in the third place with $\overline{PI}_1 \approx 0.14$, but the average value of the index $\overline{\overline{PI}}_1$ strongly decreased to 2002 ($\overline{PI}_1 \approx 0.058$), and to the end of the period it was almost the same ($\overline{PI}_1 \approx 0.078$). Communists and OVR were the next in our rating with $\overline{PI}_1 \approx 0.12$.

Communists had almost constant average value of index during all period of 2000-2003.

The average value of the index $\overline{PI}_1$ of group OVR changed strongly, in 2001 $\overline{PI}_1 \approx 0.183$, in 2002 this value was 0.077. "Edinstvo" had strong changes in its power for all the period of 2000-2003. In 2000 the average value of its power was approximately 0.08, but to 2002 power of this party increased strongly, more than to 50 percent, $\overline{PI}_1 \approx 0.189$. All of these four factions have strong party discipline.

Till then the OVR group unified with "Edinstvo" in December of 2001. Factions SPS, LDPR, "Yabloko" were tiny groups and they were at the end of power distribution rating. The average values of their power were less than 0.05 and there were not any important changes in these values during all this period.

Thus, it may be concluded that both the greatest power values and strong power changes in time have shown by those parties which change their political position permanently. It means that these groups did not have fixed political position, they could maneuver in order to receive strategic advantages. Such power is called payoff-power (or P-power) [9]. Therefore, our power index measures the degree of player's ability to predict and adjust to outcome. This hypothesis is confirmed by political maps of ideal point motion paths, presented month by month for each year [1]. The ideal points of such group migrate throughout the political space.

One of the findings obtained in the work is that if we construct a trajectory of factions positions on the political map, each point corresponds to the position of a faction in a chosen month, we can see to which extent the overall behavior of a party was volatile or stable. The graphs on Figs. 12-15 presents these very trajectories for the third Duma. As one can see, Communists and Edinstvo have rather tight positions during all these years. On the other hand, one of the main liberal parties in Russia – Yabloko – during half a year before elections in 2003 drifted to Communist position. Narodny deputat passed through almost all plane during these 4 years.

That parties for which power index is small have exact political views, firm politics, they try to find the way to effect the outcome of voting. Such power is called influence-power (or I-power) [9].

Let us come back to Figs. 8-11. As we can see on Fig. 8 there is a peak of Communists PI_1 in autumn of 2000. This peak is associated with their behavior under discussion in the Parliament the bill for children benefits supported by Communists, Agrarians (the power value of APG is also increased to 0.2), Narodny deputat and Regions of Russia. Majority (263 votes) voted for this bill, but it was not passed because of the Federation Council veto (300 votes were necessary to get override the veto).

The next power value peak observed in May of 2002 corresponds to the alternative military service federal law. The leftists assemble the majority to defeat this bill. From May to September of 2003 the power of CPRF declined. It can be explained by some bills adoption, for example, the bill of local government reform or federal budget bill, as well as by non-confidence vote to government. In all these votes CPRF was always in minority.

On Fig. 8 power distribution of Edinstvo is presented. Examining the most important power value changes, it should be noted that there are repeated falls in power to the zero value during all over the period. First such fall is observed in December of 2000 and January-February of 2001, when laws concerning nuclear exhausts problem had been considered. In 2001 the amendments to these acts allowed to import nuclear exhausts for technological storage as well as for waste disposal in Russia. Edinstvo and LDPR were consolidated for law to pass (Fig. 11) and had power value fall at the same time,. In addition the decrease of power value for Edinstvo and LDPR can be explained by the fact that they were in majority when amendments to pension federal law have been considered since they voted against those amendments. The next fall in Edinstvo power value is observed in November-December of 2001, when the questions of judicial authority reform have been discussed.

Centrists had to reconcile the viewpoints with leftists and liberal factions for adoption of this bill. The left liberal factions took an advantage of the law liberalization. Thus, there are observed increments of power value of SPS, LDPR, APG (power value for SPS increased to 0.107, this value was the highest for SPS, APG had $PI_1 = 0.138$, LDPR had $PI_1 = 0.05$). In spring of 2002 the power value peak of Edinstvo is observed, $PI_1 = 0.4$. This peak can be aligned with the break of the so-called package agreement, accepted at the beginning of 2000. The package agreement break was initiated by centrist factions and had been supported by SPS and Yabloko. Edinstvo was a key player in that voting. The next more important power value peak of Edinstvo is observed in September-November of 2002, it can be explained by adoption of the federal law of referendum of the Russian Federation. This draft law proposed by Edinstvo was supported by all factions except Communists and Agrarians.

On Fig. 9 changes of centrist factions power distribution, namely, Narodny deputat and OVR, are given. There are strong changes of power observed for these factions. In January-February of 2001 the power value peak is observed that can be explained by alteration in federal law of pensions. Narodny deputat was the pivotal player in this voting when the veto had been negotiated. There is the power growth observed for Narodny deputat and OVR in September-November of 2001. Narodny deputat had $PI_1 = 0.45$ and OVR had $PI_1 = 0.266$, one of the most important peaks of this party. It can be associated with the adoption of the most important bill of 2001, the Russian Federation labor code. All factions except Communists and Agrarians voted for this law, and Narodny deputat was pivotal in that voting. Faction OVR had another peak observed in December of 2000. There is one of the highest values at that period, when the Russian Federation national symbol legislative package was supported by all parties except SPS and Yabloko. The reason of this peak appearance is that OVR was decisive in that voting.

On Fig. 10 power distribution curves of small groups, namely, Regions of Russia and APG, are represented. Regions of Russia had the most interesting and important results in May-June of 2001, when its power increased to 0.42. This was the highest value at this period. This peak can be explained by the law of political party consideration. Edinstvo, OVR, LDPR, Narodny deputat, Yabloko and Regions of Russia voted for this law. Votes of Regions of Russia were decisive in this voting, and that was the reason of the power value growth. There is one more peak of Regions of Russia power value observed in January-February of 2002, its value $PI_1 \approx 0.52$. At this time interval some bills had been considered, namely, the act of nationalization, the termination of broadcasting of TV6 act, nationality law, the act of electric and heat energy rate management. Regions of Russia was in majority in voting for these laws, and power value peak can be explained that Regions of Russia was the pivotal player in these votes. The next peak of Regions of Russia power value is observed in March-April of 2003, $PI_1 \approx 0.51$, when problems of housing and communal services reform had been examined. The act of housing and communal services reform was accepted in third reading after some amendments to this act, and Regions of Russia was a pivotal player.

As one can see the extended power index differs from SOV results. The most important changes are observed for CPRF and its ally APG. The power value of these parties is higher than SOV. On the contrary, for parties Narodny deputat and LDPR this value is lower than SOV.

The obtained results of extended Shaopley-Owen power index and SOV computed for political parties of the III State Duma correlate badly with the results computed in Section 3. Those results shows that the most powerful groups were the parties Edinstvo and CPRF, Narodny deputat and OVR took the third and forth places accordingly, and the last were Regions of Russia, APG and SPS, respectively.

Results coincide for tiny groups of III Duma, namely, for SPS, LDPR and Yabloko. Power analysis on the basis of both standard Banzaf and extended Shapley-Owen indices shows that power of these groups is very low.

Similarly. the results coincide for OVR and APG, both analyses point out that these parties took average positions in power rating.

7. Conclusion

We conclude the paper with several remarks.

Remark 1. The suggested approach is valid when the following assumption holds. First, factions vote homogeneously. This assumption seems to be true for the French parliament, but not for the Russian one. Then the assumption above should be substituted by another assumption – the deviation of faction discipline is the same in all factions with respect to the member of MPs in each faction. This assumption seems to be strong as well.

Then our assumption can be re-formulated as follows: the deviation of homogeneous behavior in each faction is small, contingent and independent from factions and votings. Then the balance of power among opposing coalitions will be stable in average over time.

On can expect that this latter case is most close to real behavior of parties. On the other hand, obtained results can be used as indirect proof of this assumption.

Remark 2. In the fourth and already in the newly elected fifth Duma in December 2007 there is the only power holder, the party Edinaya Rossiya, which possesses the majority sufficient for constitutional laws passage. However, it is well known that these party consists of several 'wings', representing different opinions, from liberal to centrist and even conservative. On the other hand, the regional interests of groups in this party are also different. One of the directions of research in the analysis of power distribution in the fifth Duma is to study the power distribution among regional groups of different parties. It is very complex computational problem which may be solved by a special approach.

Remark 3. One of the routes to overcome high complexity of evaluation of power indices for large societies is usage of generating functions – a special type polynomials which are widely used in combinatorial theory. It has been shown how to use these functions for evaluating power indices for the case of unrestricted coalition formation [8] as well as for restricted coalition formation [25] and for the coalition formation taking into account agents' preferences to coalesce [23]. For some cases it turns out that this technique allows to obtain results which can not be even thought about using direct algorithms.

Remark 4. The obtained results and technique allow us to study power distribution in large organizations such as International Monetary Fund and United Nations Organizations as well as many other institutions. For European Union and IMF several works have been done [10,12-15, 18,22,24] including the one in which we studied some models of coalition formation on the basis of their regional proximity, economic relations, etc. [4] However, this work makes only a first step in this direction.

Remark 5. Another interesting direction of research seems to be an analysis of power distribution surveying MPs on their desire to coalesce with their colleagues on different issues. We are going to start such surveys in one of the regional parliaments of Russian Federation.

Remark 6. In [16] another index was introduced taking into account agents' preferences to coalesce. It will be interesting to compare the results produced by these approaches on the same data.

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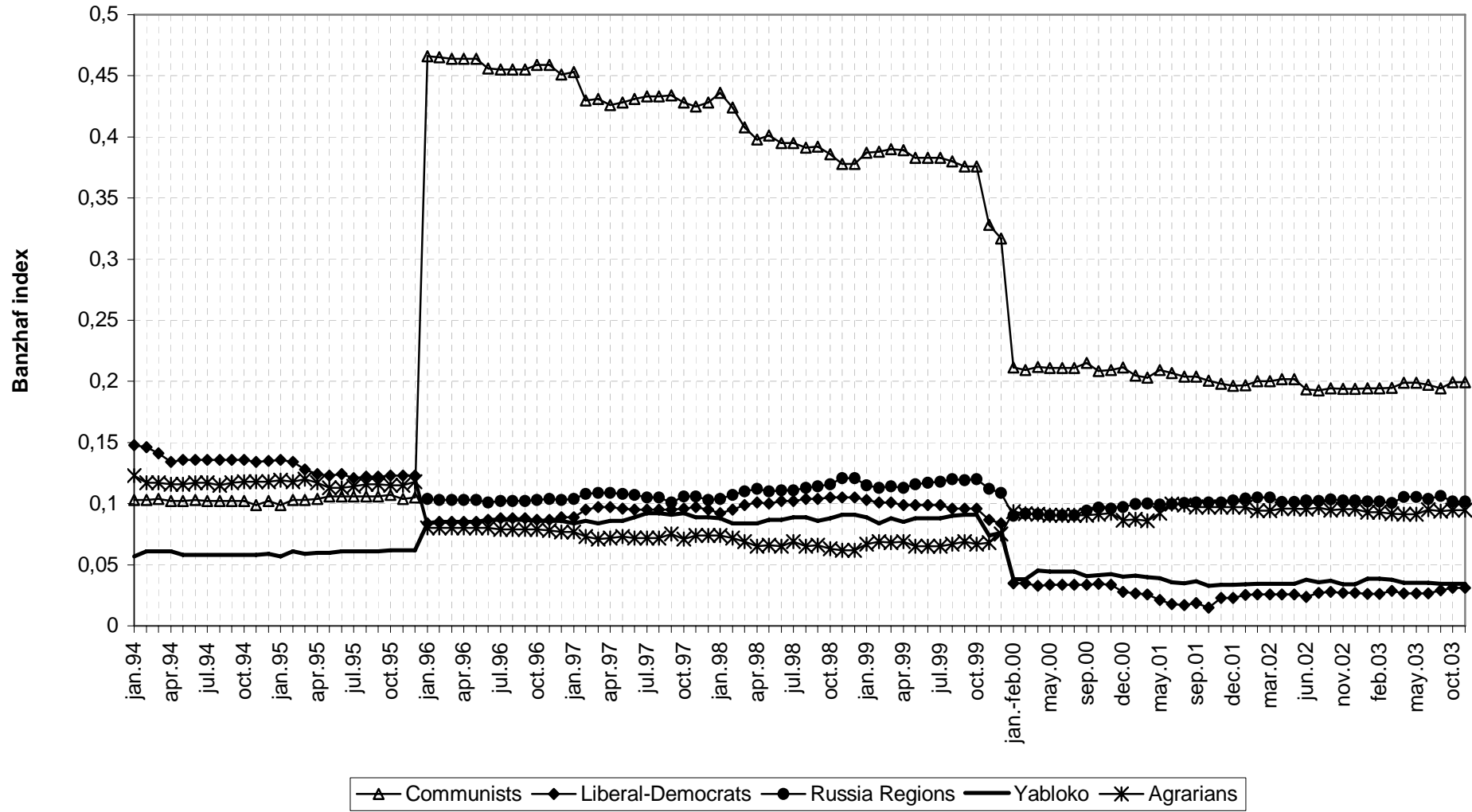# Power distribution of some parties in Russian parliament (1994-2003)

Figure 1.

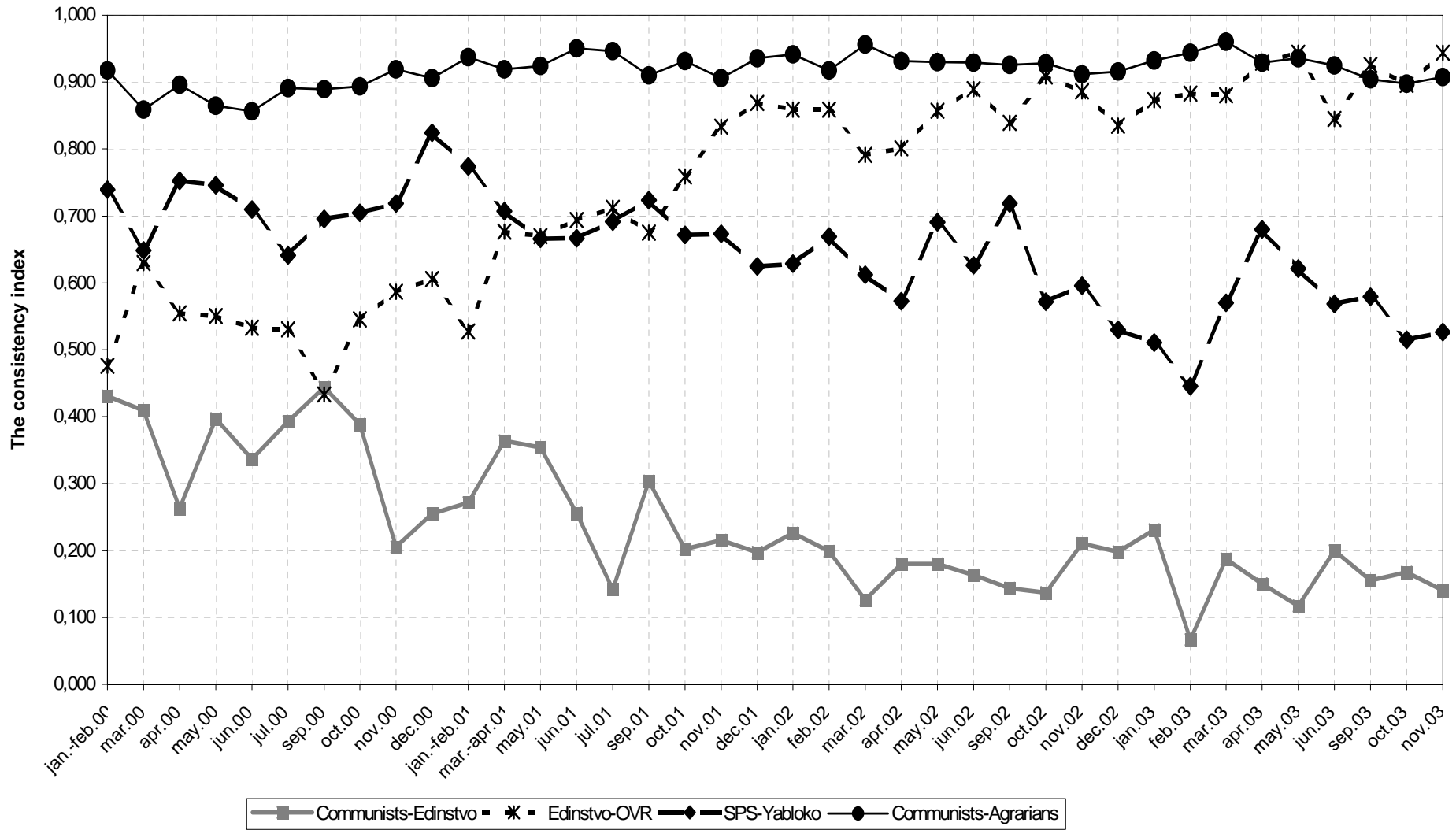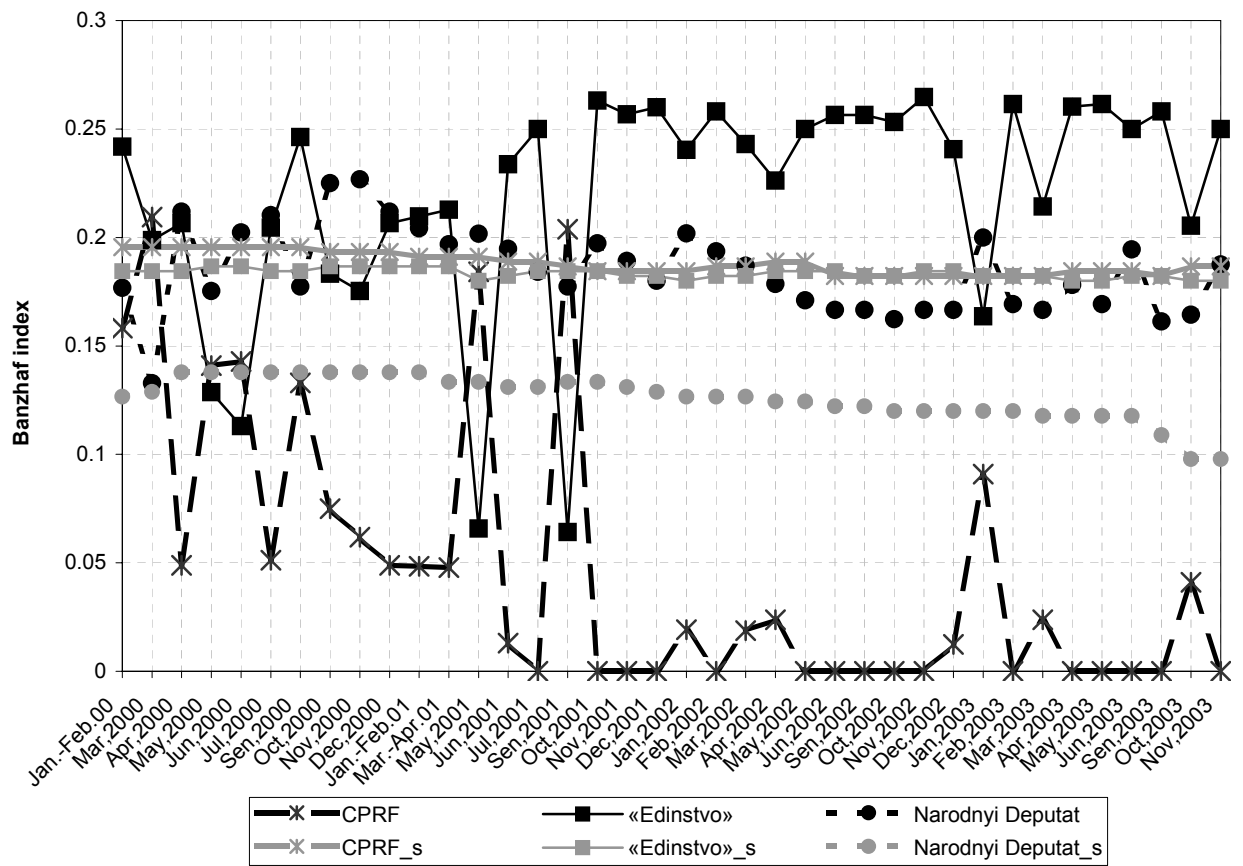# The dynamics of the consistency index for the "key" pairs of fractions in the third parliament
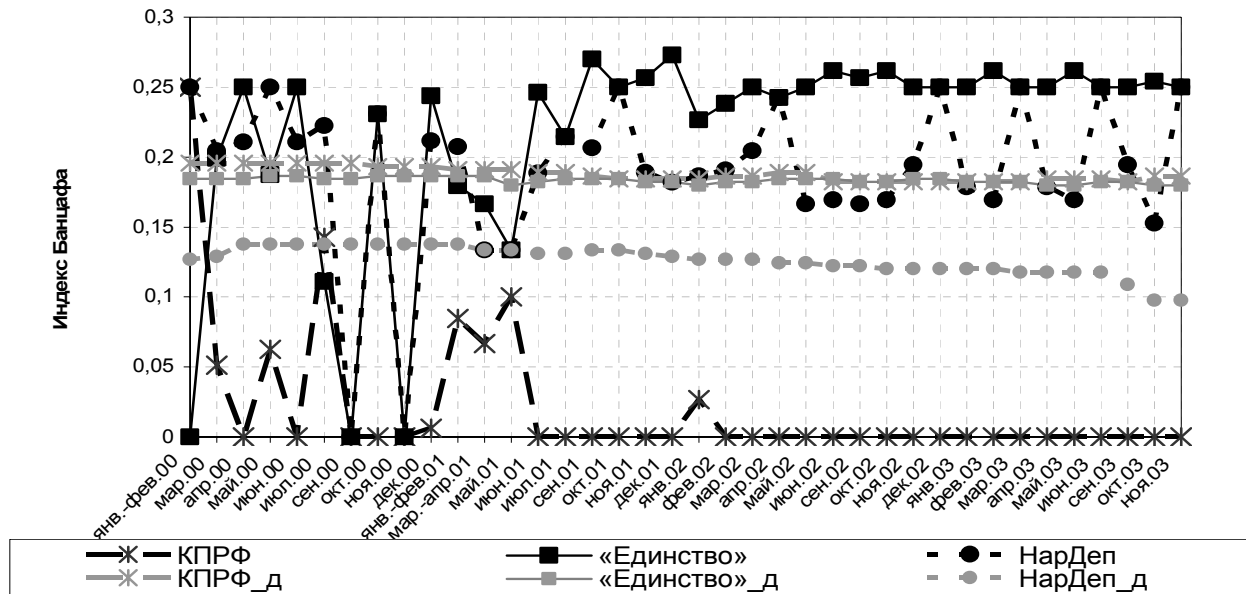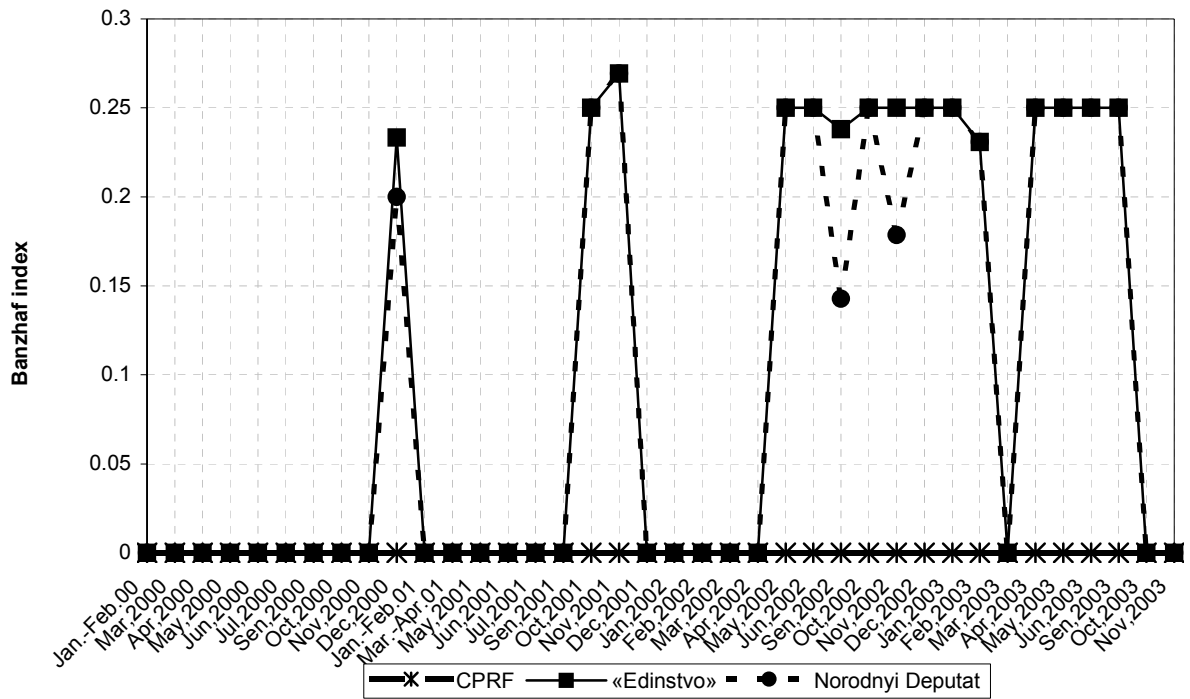

Figure 2

Distribution of power for large factions *(CPRF, «Edinstvo», Narodnyi Deputat)*

at the III State Duma (scenario 0,4)

Figure 4

Distribution of power of large factions *(CPRF, «Edinstvo», Narodnyi Deputat)*

in the III State Duma (scenario 0,5)

Figure 5

Distribution of power of large factions *(CPRF, «Edinstvo», Narodnyi Deputat)*

in the III State Duma (scenario 0,6)

Figure 6

Distribution of power of groups and faction in Russian parliament (index alpha1)
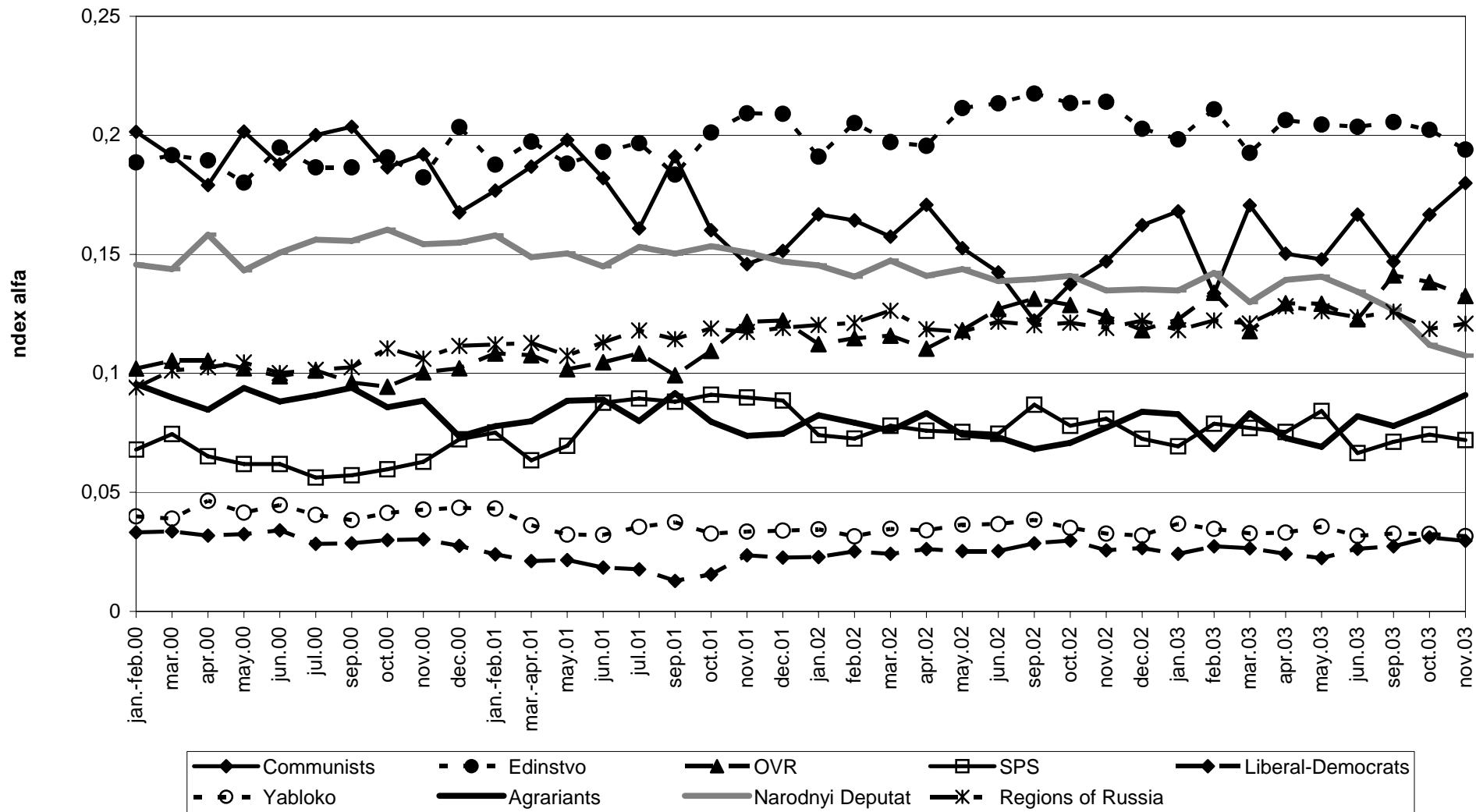
Figure 6.
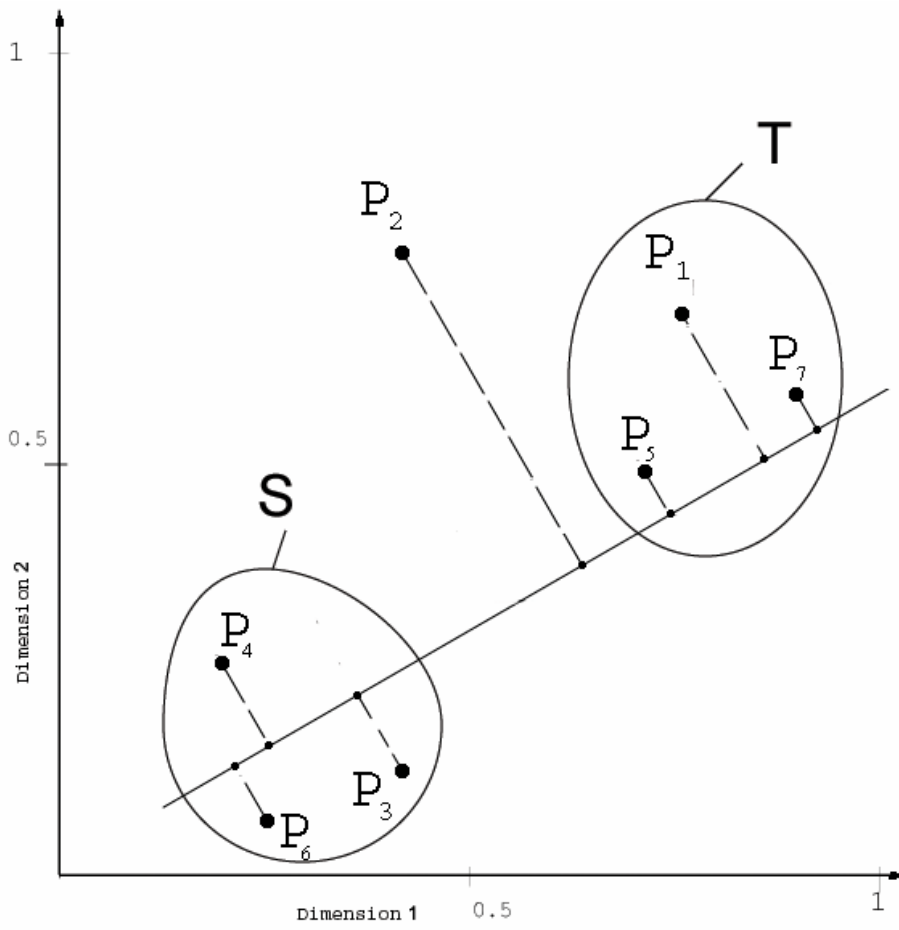
Figure 7.
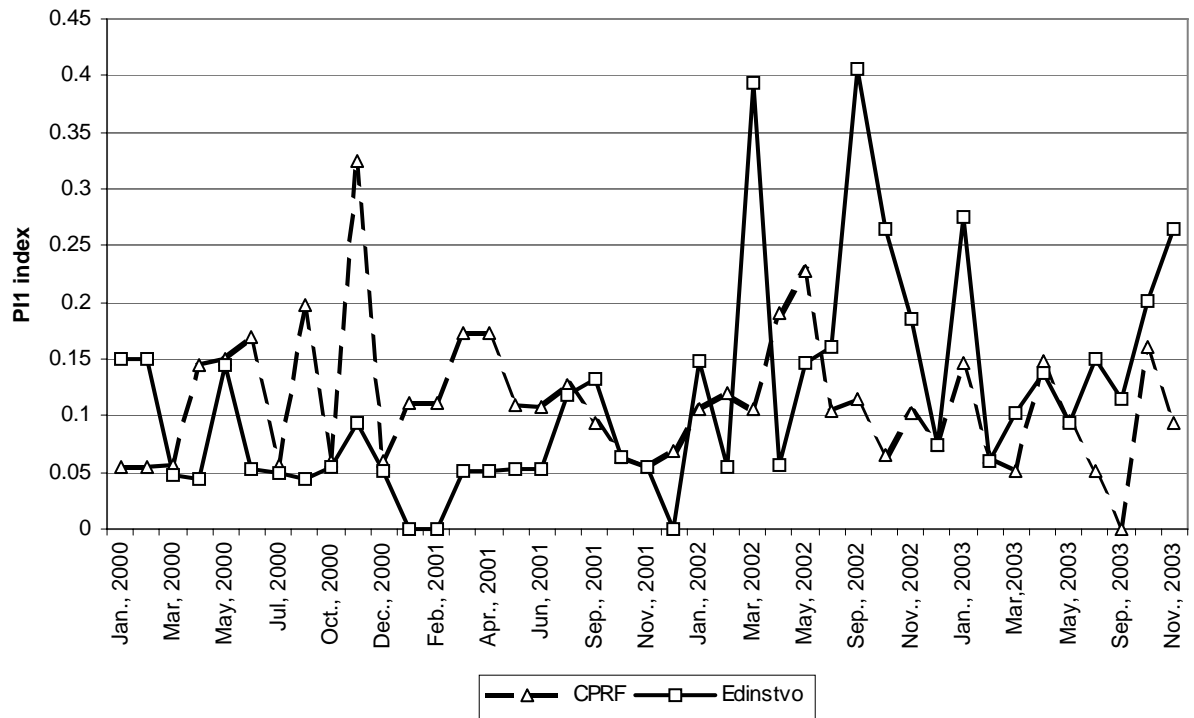
Figure 8. Extended power index values PI_1 for the III State Duma (Edinstvo, CPRF)
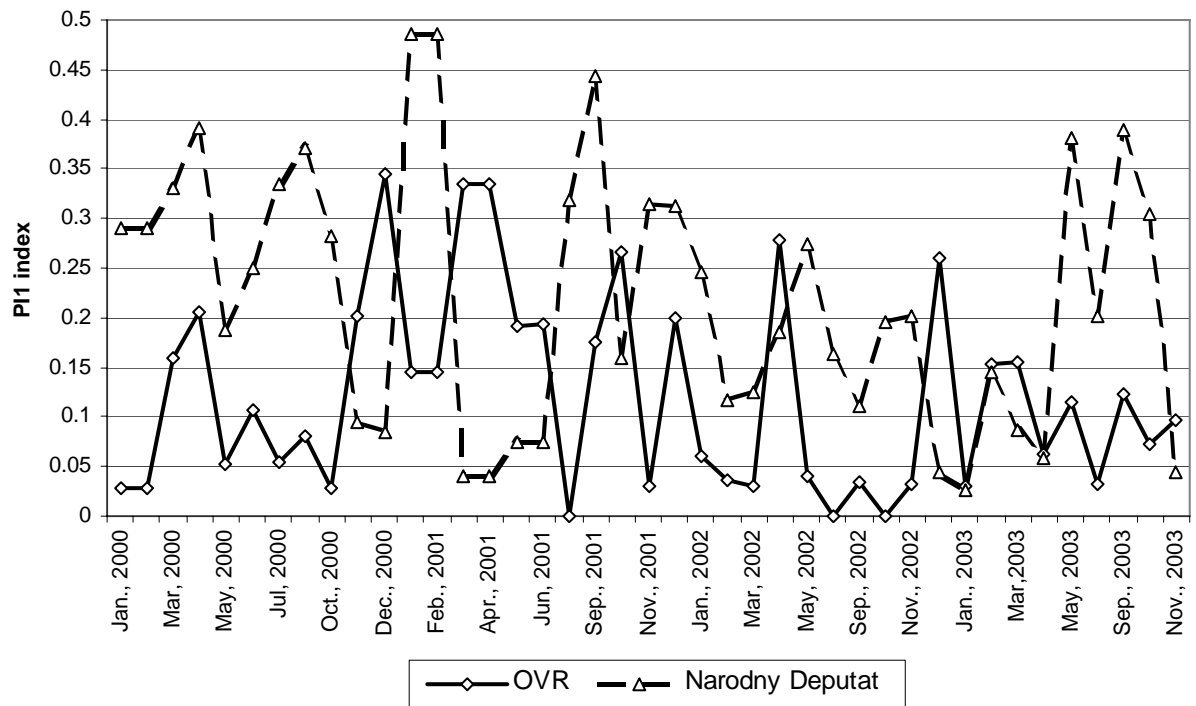
Figure 9. Extended power index values PI_1 **for the III State Duma**
(Narodny Deputat, OVR)
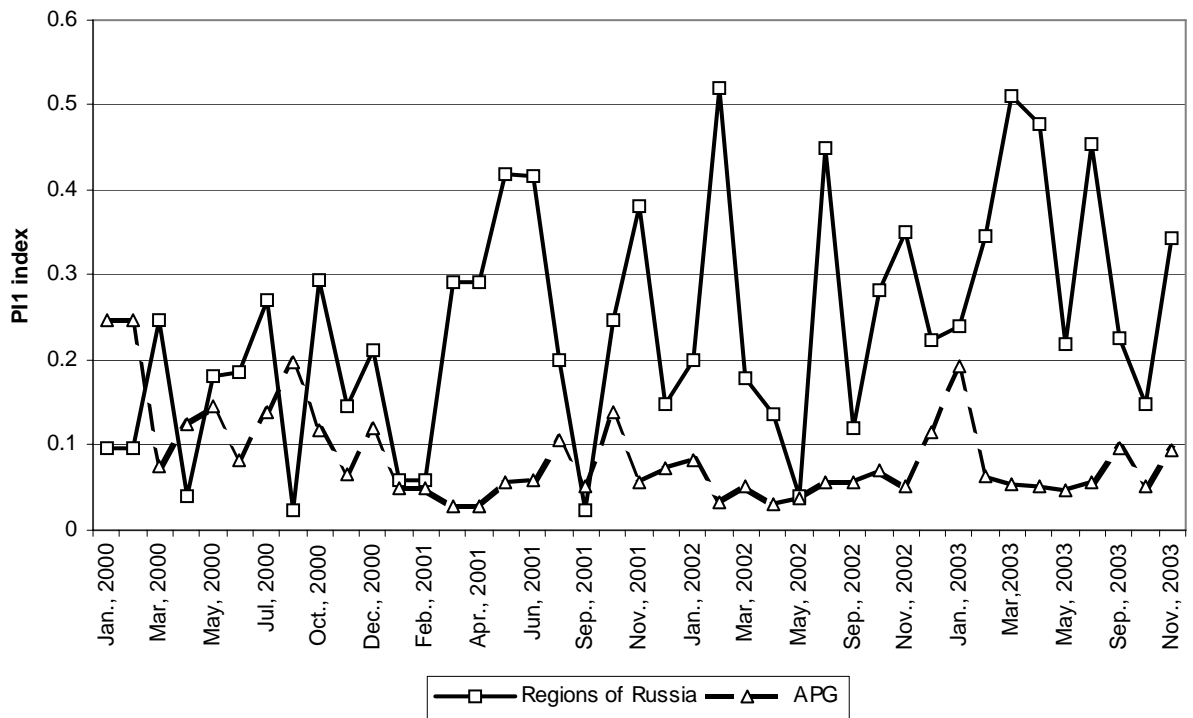
Figure 10. Extended power index values PI_1 **for the III State Duma (Regions of Russia, APG)**
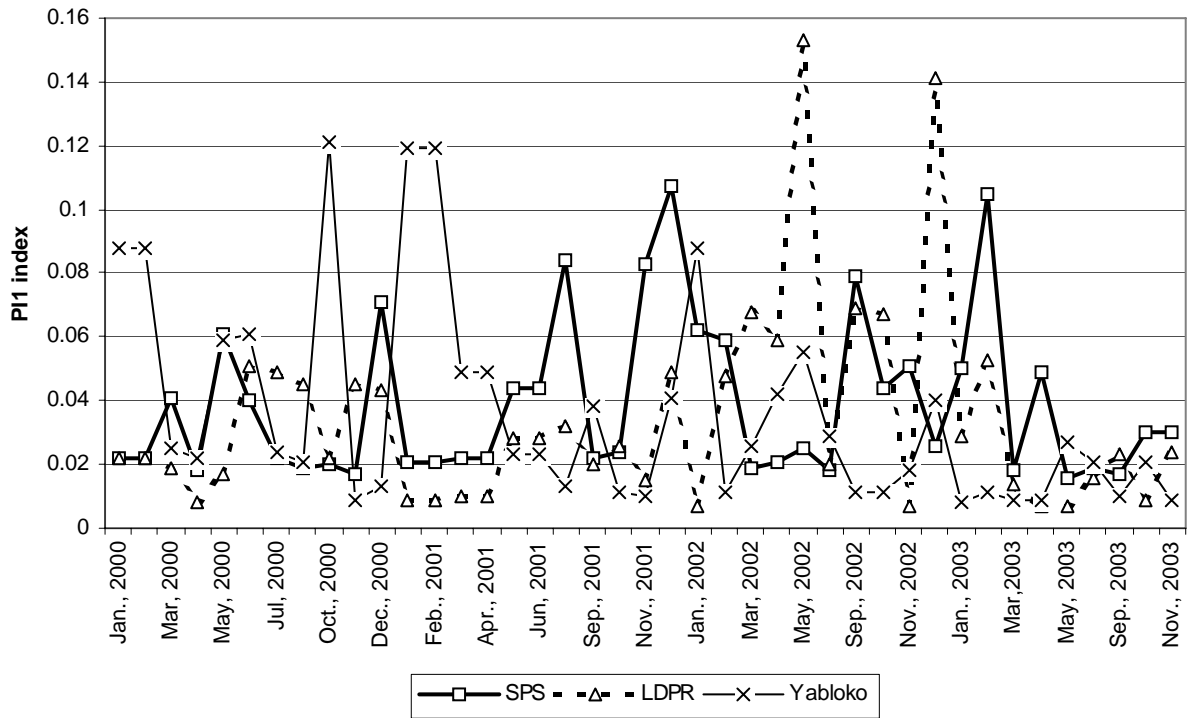
Figure 11. Extended power index values PI_1 **for the III State Duma (SPS, LDPR, Yabloko)**
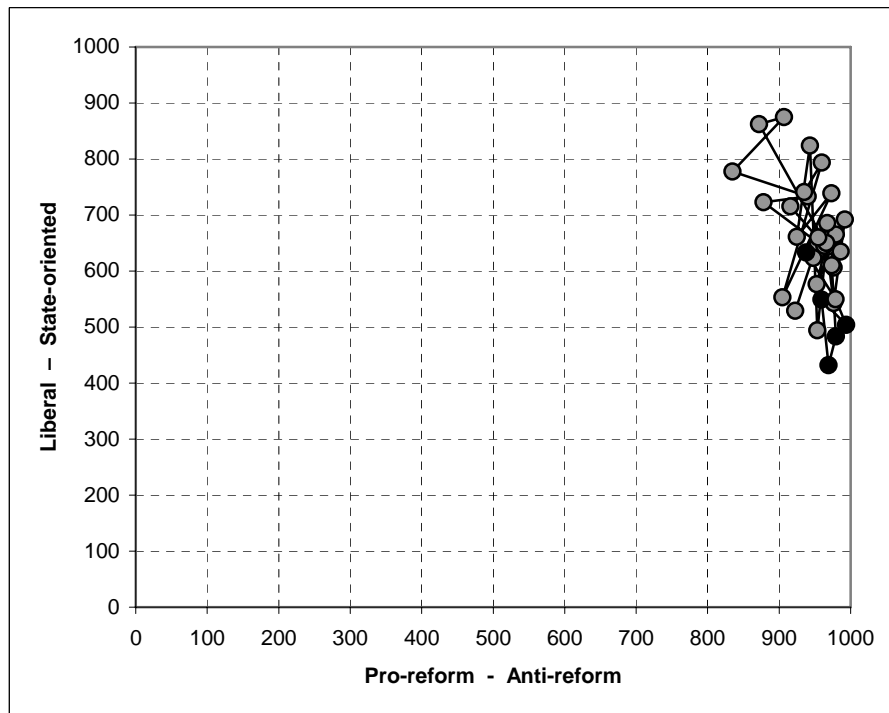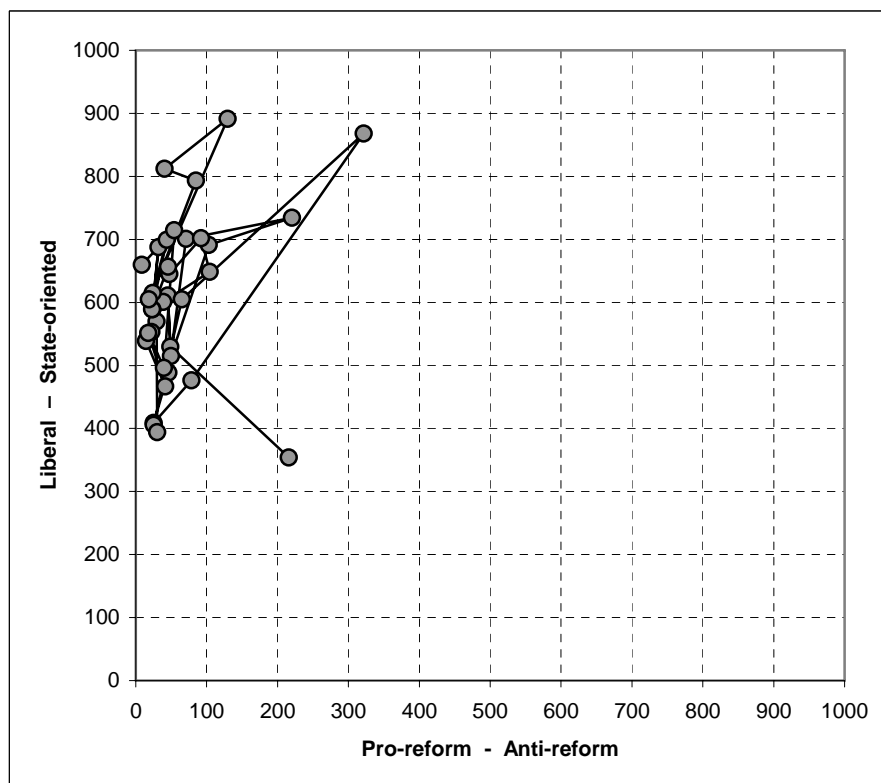
Figure 12. Dynamics of political positions (CPRF)

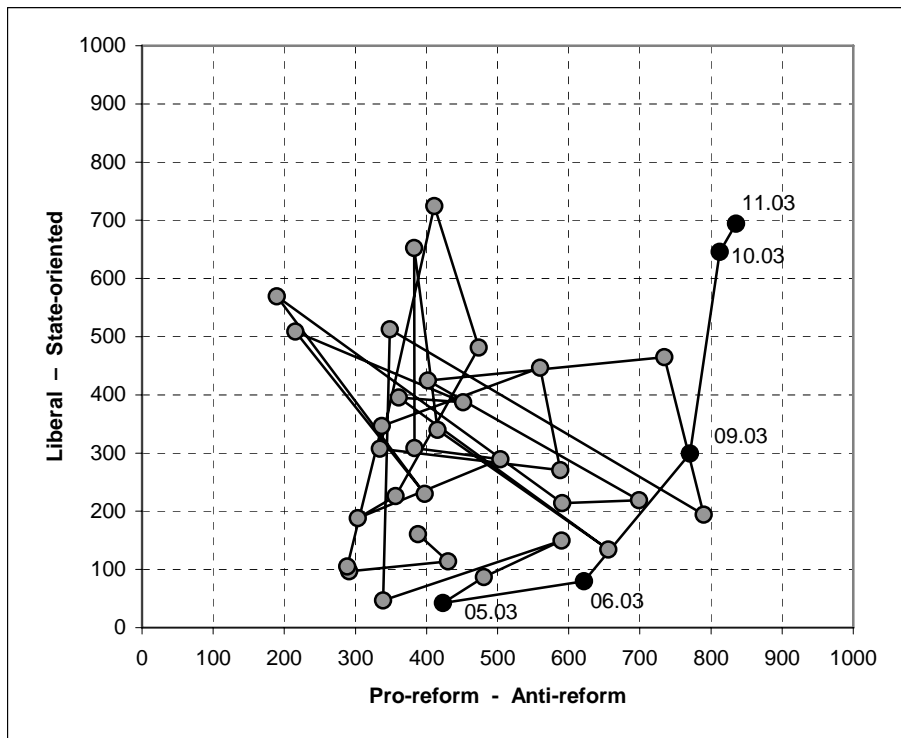**Figure 13. Dynamics of political positions (Edinstvo)**
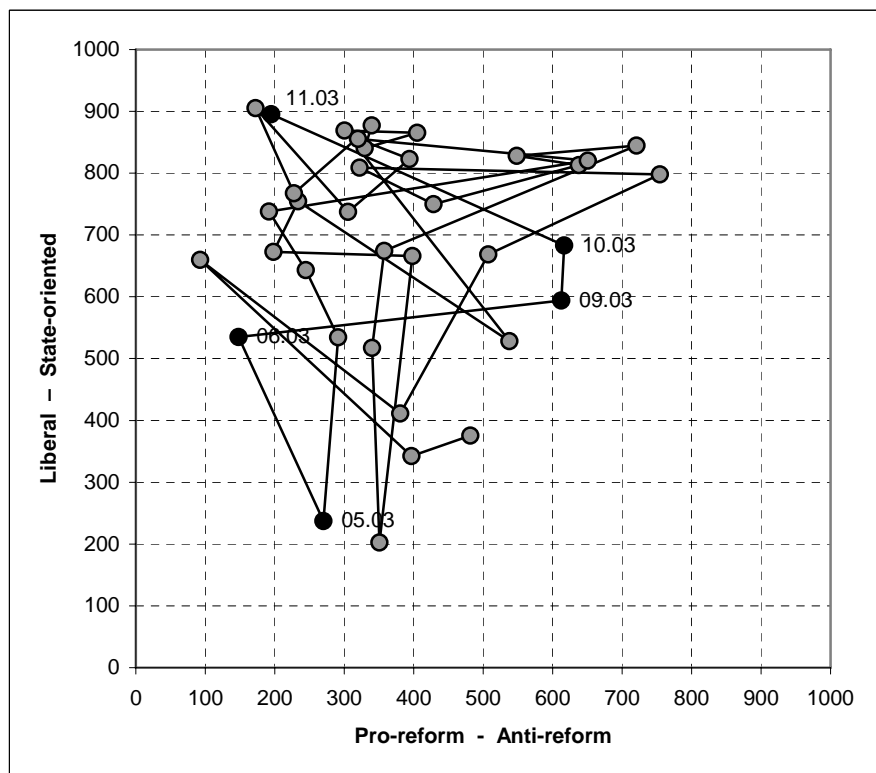
Figure 14. Dynamics of political positions (Yabloko)

Figure 15. Dynamics of political positions (Narodny Deputat)