Why do dictators prosecute minorities? A model in a two-dimensional policy space.

Arseniy Samsonov

November 18, 2014
Abstract

Under autocracy people are often prosecuted because of their ethnicity, religion or sexual orientation. We present a model in the spirit of “Economic origins of dictatorship and democracy” by Daron Acemoglu and James Robinson that explains why the dictatorship of the rich elite chooses lower levels of minority rights than a democratic regime. In particular, the model shows that even if the rich elite is the most tolerant social stratum, the infringement of rights may still take place, so the phenomenon is not explained by the autocrat’s malevolence. The basic intuition of our model is that because the dictatorship of the rich chooses lower taxes than a democracy, in the first case the poor are more inclined to participate in revolutionary activities. So, a dictator has to give some concessions in other policies, which results in providing lower minority rights, if the poor are sufficiently intolerant.
1 Introduction

The 20th century saw unprecedented repressions carried out by totalitarian regimes. While millions of ordinary citizens were convicted and killed under dictators like Stalin, Hitler and Mao, belonging to a minority often increased one’s chances of falling victim to terror, even if the person’s distinction did not concern his or her income or political views. The Holocaust carried out by Nazi Germany is the most atrocious and well-known example of repression based on nationality, Gypsies and homosexuals being purged alongside Jews (Shirer 1960). Under Stalin, ethnic Poles and Germans were purged in 1930, while the 1950-s saw the “Campaign against cosmopolites”, a series of false trials against Jewish intellectuals (Gregory, Schroeder, Sonin 2006). Even in more recent times, far more moderate autocratic governments continued to discriminate against minorities, though on a lesser scale. For instance, as late as the 1980-s, Jews in the USSR were barred from high-quality education and prestigious jobs (Saul 1999). A common target for modern autocrats are homosexuals. As of 2013, homosexual acts are illegal in 76 countries, e.g. Singapore, Iran, Uganda, etc., being punishable by death penalty in Iran, Yemen and Saudi Arabia (Itaborahy and Jhu 2013). Though not illegalizing homosexual acts, the Russian government has enacted a law that prohibits the advocacy of homosexual lifestyle, and maintains intolerance to sexual minorities as it’s official position (Sperling 2012).

The foregoing anecdotal evidence shows that low income or level of education are not sufficient to explain the prosecution of minorities. Nazi Germany was one of the most educated and rich countries at it’s time (Shirer 1960). The same thing could be said about modern Singapore and Saudi Arabia, which compare to Europe based on per capita GDP and Human Development Index (HDR and The World Bank).

Our model offers an explanation of this tragic phenomenon. Following the standard models from (Acemoglu and Robinson 2006), we assume that the society is divided into several homogenous income groups (for simplify, we consider three of them, the rich, the middle class and the poor). The middle class prefers higher taxes than the rich, while the poor prefer more redistribution than the middle class (Acemoglu and Robinson 2000). Additionally, we assume that these groups differ in their preferences towards minorities and our model predicts which those preferences should be so that minorities would be prosecuted under dictatorship relative to democracy. Suffice it to say for now that in our model minorities can be prosecuted by a dictatorship of the rich elite even in case the rich prefer more minority rights than the middle class or the poor. Again, following Acemoglu and Robinson, we assume that the poor are able to stage a revolution if they are relatively dissatisfied with the policy enacted, while the associated costs are relatively low. We modify the model, however, assuming that the revolution constraint is both present in dictatorship and democracy.

The political process in our model is represented by a five-stage game. At Stage 1, two politicians who only care about electoral victory simultaneously offer a tax rate. On Stage 2, sophisticated voters of the three types vote for one of the proposals. Each group of voters are assumed to have solved a collective action problem, that is, they can collectively decide whether to attend elections and how to vote. Voting is assumed to be costly. On the Stages 3 and 4, the procedure is repeated with the level of rights. On the final stage, the poor decide whether to start a revolution. The revolution is costly for the poor, but is successful if staged and as it’s consequence the ideal point of the poor is selected as a policy point. A revolution is very costly for both politicians.

Under a dictatorship, the dictator simply maximizes the utility of the rich subject to the
revolution constraint, as in (Acemoglu and Robinson 2000). In the paper, we will show which preferences of the voters and group sizes will lead to the decrease of minority rights in dictatorship relative to democracy given such rules of the political process.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 provides the model and the solution. Section 4 contains comparative statics. Section 5 concludes.

2 Related literature

Our paper relates to the theoretical literature on minority-related politics.

Romer (1998) is close to our study. As in our research, in this paper voters have preferences on two issues, one of them economic (taxation) and the other purely political (religion). A Stackelberg equilibrium of the game between competing political parties is analyzed and it is shown that the existence of a non-economic agenda can change the economic policy chosen by the society relative to the case when only an economic policy has to be selected. Our approach differs from Romer’s, however, as we provide a way to compare the non-economic policy (minority rights) that politicians offer in democracy to the one enacted by a dictatorship. Romer, by contrast, does not extend his model to the case of dictatorship.

Fernandez and Levy (2008) also analyze the two-dimensional policy space, this time both policies being economic (general redistribution taxes and targeted transfers). In addition, they assume that politicians are able to form binding parties, or coalitions, when running for elections. Like Romer, they do not analyze the case of dictatorship. Also, we do not impose the assumption that creating binding coalitions is possible, which makes our model a lot simpler.

Glaeser (2005) offers a model in which politicians whose economic agenda will hurt people of a certain nationality try to make their program more popular by creating an ideology of hatred towards the victimized group. The target audience of those politicians is assumed to receive utility from the fact that the people belonging to the ethnicity they hate will suffer. This approach is applicable to a dictatorship. However, the paper fails to explain the prosecution of minorities like homosexuals that belong to disparate income groups and may react very differently to economic policies. Apart from that, it makes very strong assumptions on voters’ preferences.

Gregory, Sonin and Schroeder (2007) offer a general model of repression against innocent people carried out by dictators. They use Stalin’s terror as an example of such repressions and, in particular, explain his “National operations”, that is, arrests and killings of people belonging to ethnic minorities. The intuitive explanation offered is that people from the nationalities purged were, in Stalin’s expectation, susceptible to collaboration with external enemies. Our approach is different, as we try to explain the general case of minority prosecution. For instance, we try to explain the prosecution of homosexuals, whose difference does not make them more likely to participate in a protest.

3 The model

Voters and their preferences The society chooses a tax rate $\tau$ and level of minority rights $\alpha$, $\alpha, \tau \in [0; \infty]$. There are three types of voters, the rich ($r$), the middle class ($m$) and the poor ($p$). The respective shares are $|r|, |m|, |p|, |m| + |r| + |p| = 1$. Voters of each type
share the same preferences, the ideal point of group i being \((\tau_i, \alpha_i)\) and the utility function defined as:

\[
U_i = -\sqrt{(\tau - \tau_i)^2 + (\alpha - \alpha_i)^2}
\]

where \((\tau, \alpha)\) is the policy chosen. This is simply the negative of the distance from the policy chosen to the voter’s ideal point. The ideal points of groups m, p and r are defined as M, P and R respectively.

Assumption 1: \(\tau_r < \tau_m < \tau_p, \alpha_r > \alpha_m > \alpha_p\). As in (Acemoglu and Robinson 2000), we assume that the rich prefer little redistribution of income, the middle class prefer more taxes that the rich and the poor - more than the middle class. In addition, we assume that the middle class has median preferences over minority rights.

Assumption 2: No group has absolute majority.

Assumption 3: \(|m| > |p| > |r|\).

Assumption 4: If we draw a line between the poor and middle class ideal points, the ideal point of the rich will lie below this line (see Picture 2 for illustration).

We will show that our assumptions are sufficient for democracy to choose higher values of minority rights than a dictatorship.

The political process As we have mentioned before, our model investigates the political process in democracy and under dictatorship.

The political process in democracy The political process in a democracy is described by a five-stage game. On the first stage, two politicians who are running for office propose a tax rate \(\tau\) as their programs. On the second stage, sophisticated voters of the three types p, r and m learn which proposals have been made, and either vote for one of them or abstain from elections. Attending the elections costs \(\epsilon > 0\), where \(\epsilon \to 0\). The politician who won the elections gets a payoff of \(s > 0\). On the third and fourth stages, the same process is repeated for the level of rights, \(\alpha\). If everyone abstains from elections on any stage, all the voters get a payoff of \(-K \to -\infty\) and the game ends.

On the final stage, when both tax and rights have been selected, the poor decide whether to stage a revolution. If a revolution is staged, it is successful. As it’s result, the policy chosen is the ideal point of the poor, while each politician who proposed a winning policy at any period gets a payoff of \(-k \to -\infty\).

The political process in a dictatorship As in (Acemoglu and Robinson 2000), in a dictatorship everyone but the rich are disenfranchised. The dictator simply maximizes the utility of the rich subject to the revolution constraint.

Solution Now, we will find the subgame-perfect Nash equilibria in games describing the political process in dictatorship and democracy.

The solution for dictatorship A dictator maximizes the utility of the rich subject to the revolution constraint. So, she picks the closest point to R, which is the ideal point of the rich, among those in which the revolution doesn’t happen. Consider the circle with radius c and center P. For points inside the circle,

\[
\sqrt{(\tau - \tau_p)^2 + (\alpha - \alpha_p)^2} < c \to -\sqrt{(\tau - \tau_p)^2 + (\alpha - \alpha_p)^2} > -c \to U_p(\tau_p, \alpha_p) - c < U_p(\tau, \alpha)
\]
So, the poor are better off if they don’t stage a revolution if any point \((r, \alpha)\) within the circle is chosen. So, they will not stage a revolution if such a policy is implemented and vice versa. We assume for simplicity that the poor don’t stage a revolution if the policy lies on the circumference of the circle. So, the dictator has to choose from points within and on the border of the circle with center P. Obviously, the closest point to R is the point of tangency between the circles with centers R and P. This will be the point picked by the dictator, as illustrated on Picture 1.

**Picture 1.** Point C represents the policies selected by the dictator. It is the closest point to the ideal point of the rich in which the revolution does not happen.

**The solution for democracy.** Our main result is that a democratic process will select a point of tangency between the circle with center P and the circle with center M, as shown on Picture 2. This explains why a dictatorship yields lower minority rights, \(\alpha\), than democracy. In the beginning, we have assumed that if we draw the line MP, point R will lie below this line. The tangent point of two circles is the one where a line drawn through their centers crosses their borders. Thus, the point chosen in democracy lies higher on the border of the circle with center P than the one picked by a dictator. See Picture 2 for illustration.
The point chosen under dictatorship is C, which is the tangency point of circles with centers P and R, while democracy yields point B, which is the tangency point between circles with centers M and P. Because point lies below the line MP, the whole line RP lies below MP. Thus, RP crosses the circle with center P at a lower point than PM does. This means that the lower value of minority rights, $\alpha$, is selected under dictatorship.

In the rest of the section, we will prove the result for democracy. No new results will be provided, so if one is encouraged to go to the next section (comparative statics) if not interested in the proof.

Before proving our main result, we will prove a lemma concerning the voting outcomes and their geometric interpretations which is due to (Feld and Grofman 1987) and also appear in their paper as Lemma 1.

Lemma 1 (Feld and Grofman 1987): Let’s draw a perpendicular from the voter’s ideal point to an arbitrary line and call the point where the perpendicular crosses the line the projection. Then each voter prefers points on the line that are closer to his projection to those further.

Proof: From the Pythagorean Theorem it follows that the further the points on the line are from the projection, the further they are from the ideal point. Because our utility function is the negative of the distance from the ideal point, points further from the voter’s projection provide her less utility. See Picture 3 for illustration.
3. Point A is preferred to point B as it is closer to the projection.

Now, we can move on to proving our result that concerns democratic politics.

Proposition 1: If the policy space is limited to points within the circle with center P and \( \alpha_m \geq \alpha_p + \kappa \), the point chosen in equilibrium will be the point of tangency between the circles with centers M and P, as shown on Picture 4.

Picture 4. Point C will be the policy chosen in democracy if the policy space is limited to points within the circle with center P.

Because we assume that points are chosen within the circle with center P, the poor do not revolt at the final stage regardless of what happens in the previous stages. So, we can analyze a game that consists of four stages of the original game (the last one is excluded) and the feasible policy set is limited to the points within the circle with center P.

We will solve the game backwards to find the subgame-perfect Nash equilibrium.

Definition: Let \( \bar{\alpha} \) be the value of \( \alpha \) that minimizes \( |\alpha - \alpha_m| \) subject to \( \sqrt{(\alpha - \alpha_m)^2 + (\tau - \tau_m)^2} \leq c \). In other words, it is the level of rights that is closest to the optimum level of \( m \) in the set of feasible policies.

Suppose that the tax rate \( \tau' \) has been chosen on the second stage. Then on the third stage, choosing the value of \( \alpha \) with the given tax rate \( \tau' \) is tantamount to choosing the
point on the line $\tau = \tau'$, as shown on Picture 4. Because the line $\tau = \tau'$ is parallel to the $\alpha$-axis, the projections of R, M and P on this line coincide with the lines $\alpha_r$, $\alpha_m$ and $\alpha_p$ respectively. Consequently, the most-preferred value of $\alpha$ for $m$ is $\bar{\alpha}$, $p$ prefer lower values, if they are feasible and $r$ prefer higher values, if they are feasible.

**Lemma 3**: In a unique Subgame-perfect equilibrium of the subgame beginning at Stage 3, both politicians offer $\alpha = \bar{\alpha} (\tau')$, where $\tau'$ is the tax rate chosen at Stage 2, after which $m$ attend elections and vote for one of the identical proposals while $p$ and $r$ abstain.

For the proof see Appendix, Part 1.

So, if tax $\tau$ is selected at stage 2, at stage 3 both politicians will offer level of rights $\bar{\alpha} (\tau)$ and this level of rights will be selected.

Consider the case when $\alpha_m > \alpha_p + c$ (this case is presented on Picture 5). Then $(\tau', \bar{\alpha} (\tau'))$ is the point of intersection between the line $\tau = \tau'$ and the circle with center P.

From the previous statement and Lemma 3 it follows that the game on the first two stages is reduced to choosing between the points on the upper half of the circumference of the circle with center P, as shown on Picture 6.
6. Choosing between tax rates $\tau_1$ and $\tau_2$ on Stage 2 means choosing between points A and B, since for any selected tax rate $\tau'$, the level of rights chosen on the final stage will be $\bar{\alpha}(\tau')$, which corresponds to the intersection point of the line $\tau = \tau'$ and the border of the circle with center P.

So, from now on we can analyze the actions of politicians at Stage 1 as offering points on this semi-circumference.

Lemma 4: In a unique equilibrium, at Stage 1 both politicians will offer point C of tangency between circles with centers P and M.

Obviously, this is the closest point to the ideal point of M among those feasible.

Proof: By Lemma 4, all points that politicians can offer lie on the border of the upper semicircle with center P. So, they are on the same distance from P. Because of that voters of type p are indifferent between them. So, regardless of the actions of other players in any other periods, they are strictly better-off if they don’t go to the elections at Stage 2.

We have already proven that it is not an equilibrium when no-one votes, because $\forall \alpha, \tau, i : -K < U_i(\tau, \alpha) - \epsilon$. So, in equilibrium only m, only r or both will attend elections at Stage 2. Now assume that one politician offers point B and the other offers point C. Of course, $|BM| > |CM|$, so $C \succ M B$. If $C \succ_R B$, there are two equilibria, in which either r or m goes to the elections and votes for C. If $C \prec_R B$, than if both m and r attend elections, C wins, since $|m| > |r|$. This is not an equilibrium, because if r abstain, they do not change the outcome of voting but get $\epsilon$ more. If only r attend elections, they vote for B, which is the least preferred outcome of m. So, m can profitably deviate by attending elections and voting for C. Finally, if only m attend elections, they vote for C and do not deviate to avoid getting $-K < U_m(C)$. On the other hand, r will not deviate by attending elections since their presence does not change anything, but they lose $\epsilon$ if they vote. So, given policies B and C, only C can be chosen in a Nash equilibrium at Stage 2. The politician that offered B will lose the election if the other one offered C. So, $(C, C)$ is part of a subgame-perfect equilibrium. Obviously, it is the only action profile possible in equilibrium, because if some points A and B are offered, any politician can profitably deviate to C.

We have thus proven Proposition 1.

Proposition 2: If $\alpha_m \leq c + \alpha_p$ and if the feasible policy set is limited to points within the circle $\sqrt{(\alpha - \alpha_p)^2 + (\tau - \tau_p)^2} < c$, the point selected by democracy will be C.
Proof: In this case $\bar{\alpha}(\tau', \tau')$ is either the point of intersection between $\tau = \tau'$ and the arc of the circle with center P that lies between lines $\alpha = \alpha_m$ and $\alpha = \alpha_p$ (arc GC at Picture 6) or the point within the circle with center P and on the line $\alpha = \alpha_m$ (as point I on Picture 6 that lies on the line segment CB).

**Picture 7:** The points selected at the final stage can lie either on the arc GC on the line segment BC.

Now, let’s consider stage 1 when politicians are offering points in this set. Consider the profile $(C, C)$. Suppose that one of the politicians deviates and offers point H on the arc GC. This is not a profitable deviation, since $P$ are indifferent between H and C, $m$ prefer C, $|m| > |r|$. So, $m$ will attend elections and vote for C, while the other groups will abstain. Now, consider the case when one of the politicians deviates to point I that lies on the segment CB. Because it lies inside the circle, p prefer I to C. However, r prefer C to I for the following reason.

$|PM| > c$, so M lies outside the circle. Because $|RP| > c$, $\tau_r < \tau_m$. R also lies outside the circle and to the left of M. So, the projection J of R on $\alpha = \alpha_m$ lies outside of the circle. Consequently, $|JC| < |JI|$, where $JC$ is the distance from the projection of R on $\alpha = \alpha_m$ to C and $JI$ is the distance from this projection to C. By Lemma 1, point C is preferred by r to point I. So, if points I and C are offered, point C will win (For the complete proof see Appendix, Part 2).

We have proven that if politicians don’t pick policies which cause revolution in the final stage of the original game, point C of tangency between circles with centers P and M will be chosen. Now, we will prove that even if they are allowed to do so, such policies will be chosen in equilibrium.

**Proposition 3:** Politicians do not offer values of $\tau$ at stage 1 s.t. $\tau < \tau_p + c$ and values of $\alpha$ at stage 3 s.t. $\sqrt{(\alpha - \alpha_p)^2 + (\tau - \tau')^2} > c$, where $\tau'$ is the tax rate selected by majority vote at stage 2.

**Proof:**

Let’s first prove the last part of the statement, namely that at stage 3 no politician will offer such a level of rights, which, given the previously selected tax rate, will cause a revolution on the final stage. As we remember from the proof of Proposition 1, picking values of $\alpha$ at stage 3 is the same as picking points on the vertical line $\tau = \tau'$, where $\tau'$ is the
pre-established tax. Consider an action profile when one politician picks point A outside the circle and the other - point B inside the circle. If A beats B (if the voters employ random strategies, this can happen with positive probability), the winning politician bears the cost $k, k > s$, so she has an incentive to deviate and offer such $\alpha$ that the corresponding point is inside the circle, or at least such that she does not get elected. If B beats A, this can be an equilibrium, in which point A outside of the circle is not selected.

Suppose now that both politicians offer points outside of the circle. Same as in the previous case, the politician who wins has an incentive to deviate in order not to get elected.

Thus, there are no equilibria in which values of $\alpha$ corresponding to points outside the circle are selected.

We have already proven in Proposition 1 that there are no equilibria in which two points inside the circle other than $(\tau', \tilde{\alpha})$ are selected.

So, profiles other than $(\tilde{\alpha}, \tilde{\alpha})$ can not be equilibrium actions. Now we have to prove that $(\tilde{\alpha}, \tilde{\alpha})$ would still be an equilibrium in the subgame beginning at stage 3 if all policies are allowed. Indeed, deviations to points inside the circle are not profitable (see the proof of Proposition 1). Deviations to points outside are not profitable either, since the politician who deviates can win and get $s - k < 0$ or lose and get at least the same as she would have without deviating. This completes the proof.

Now, let’s prove that values of $\tau$ s.t. $\tau < \tau_p + c$ will not be offered by politicians at stage 1. The proof is analogous. Suppose that both politicians offer such tax rates. Then, there will inevitably be a revolution in the final stage, so the politician who won at stage 2 will lose $k$ in the final period and has an incentive to deviate at stage 1 so as not to get elected. If one offered a tax rate $\tau_1$ s.t. it lies outside of the circle and the other offered a tax rate $\tau_2$ such that it lies inside the circle, if $\tau_1$ wins, the politician who offered it has an incentive to deviate so an not to get elected. If it loses, the tax outside of the circle is not selected. Finally, if both offer the tax rate within the circle, each has incentive to deviate to the one corresponding to C (see the proof of Proposition 1). It follows from the foregoing statements and from the proof of Proposition 2 that if both offer the tax rate corresponding to C, no-one will have an incentive to deviate.

Thus, we have proven that the democratic process indeed selects the most preferred policy point of the middle class among those within the revolution constraint. Graphically, it is the point of tangency between the circles with centers P and M. As we remember from the solution for dictatorship, the dictator picks a point that is the tangent point between the circles with centers P and R, as she maximizes the utility of the rich subject to the revolution constraint. Thus, the dictator picks a point that lies lower in terms of $\alpha$, the level of rights.

Provided with these results, we are now going to do comparative statics.

4 Comparative statics

Mathematically, the policy in democracy is chosen in the same way as under dictatorship. So, we can simply fix the ideal point of one of the groups, say the middle class, and then shift the preferences of the poor.

We have hitherto only shown that a dictator picks lower levels of $\alpha$ than democracy. Now, let’s compute their exact values.
At Picture 8, the equilibrium policy points are represented graphically. Consider $\triangle MAP$ and $\triangle BPL$. Because $BL, MA \perp AP$, those triangles are similar. So, $\frac{|BP|}{|MP|} = \frac{|PL|}{|AP|}$ $\implies |PL| = \frac{|BP||AP|}{|MP|}$. $BP = c$, $AP = \tau_m - \tau_p$, $MP = \sqrt{(\alpha_m - \alpha_p)^2 + (\tau_m - \tau_p)^2}$. Thus, we can compute $|PM|$. The equilibrium tax rate equals $\tau_p - |PL|$. Thus, the equilibrium tax rate equals:

$$\tau = \tau_p - \frac{c}{\sqrt{(\alpha_m - \alpha_p)^2 + (\tau_m - \tau_p)^2}}(\tau_p - \tau_m).$$

From the similarity of the same triangles, we can compute the equilibrium $\alpha$:

$$\alpha = \alpha_p + \frac{c}{\sqrt{(\alpha_m - \alpha_p)^2 + (\tau_m - \tau_p)^2}}(\alpha_m - \alpha_p)$$

Substituting $c = 1$, $\alpha_p = 1$, $\tau_m = 1$, $\alpha_m = 2$, we get the following graphs:

The effect of tax preferences of the poor on equilibrium tax:

![Graph](image)

**Picture 7.** The y-axis represents the equilibrium tax rate, the x-axis represents the tax preferences of the poor.

The effect of tax preferences on equilibrium rights:
We are interested in the values of $\tau_p > \tau_m$. For these graphs, which means $\tau_p > 1$. Thus, the more taxes the poor prefer, the greater tax rate is set in equilibrium and the less rights are provided.

Now we fix $\tau_p = 2$ and start changing $\alpha_p$. We get the following graphs:

**Picture 8.** The y-axis represents the equilibrium tax rate, the x axis represents the tax preferences of the poor.

The effect of rights preferences on tax rate:

**Picture 9.** The y-axis represents the equilibrium level of minority rights, while the x axis represents the minority rights preferences of the poor.

The effect of rights preferences on rights:

**Picture 9.** The y-axis represents the equilibrium level of minority rights, while the x axis represents the minority rights preferences of the poor.

We are interested in values of $\alpha_p$, such that $\alpha_p < \alpha_m \rightarrow \alpha_p < 2$. So, making the poor more tolerant decreases taxation and increases rights.
5 Conclusion

We have provided a framework for analyzing a dictatorship and democracy in a two-dimensional policy space. Our treatment of dictatorship is the same as that of (Acemoglu and Robinson 2000), but our contribution is extending the approach to the case when the dictator has to choose two independent policies. Our analysis of democracy required a number of assumptions, since generally a Condorcet winner does not exist in a two-dimensional policy space, so there is no alternative that is selected in a pure-strategy equilibrium. Still, given our assumptions and the procedure of collective choice in democracy we are describing, the winning policy exists in pure strategies.

We have applied this model to minority politics. It is a well-known and little-explained phenomenon that dictators tend to prosecute people based on their skin color, religion, sexual orientation, etc., while in democracies such policies are less common and more mild. Our explanation is based on the fact that the agenda on another policy, the tax rate in the model, causes this shift in minority rights. Democracy chooses a tax rate that is close to what is ideal for the middle class, while a dictator, who cares only about the rich, sets lower levels of redistribution. So, a dictator’s tax policy is further from what the poor want than a tax rate that emerges in democracy. If the poor are relatively intolerant towards the minorities, the dictator has an incentive to decrease minority rights so as to appeal to the poor, which would allow her to set a lower tax rate without causing a revolution.

For further exploration, it would be interesting to see how different democratic electoral rules change the outcome. In addition, it would be worth to design a more complicated and realistic model of dictatorial behavior that would take into account a revolution threat from the middle class or, possibly, imperfect information of the different sides in the conflict.

References

Lemma 3: In a unique Subgame-perfect equilibrium of the subgame beginning at Stage 3, both politicians offer $\alpha = \bar{\alpha}(\tau')$, where $\tau'$ is the tax rate chosen at Stage 2, after which $m$ attend elections and vote for one of the proposals while $p$ and $r$ abstain.

Proof: First, let’s notice that $(\tau', \bar{\alpha})$ is the closest point on the line $\tau = \tau'$ to the projection of $M$. So, by Lemma 1, it is the point on this line and in the set of feasible policies that is most preferred by $m$. It is also the median projection, so one of the remaining groups prefers values of $\alpha$: $\alpha > \bar{\alpha}$, while the other prefers $\alpha < \bar{\alpha}$. Suppose that at Stage 3 both politicians offered $\bar{\alpha}$. Let’s assume for simplicity that $\exists \alpha > \bar{\alpha} : \sqrt{(\alpha - \alpha_p)^2} + (\tau' - \tau_p)^2 \leq c, \exists \alpha < \bar{\alpha} : \sqrt{(\alpha - \alpha_p)^2} + (\tau' - \tau_p)^2 \leq c$. That is, there exist possible deviations within the set of feasible policies. Consider the case when both politicians offer $\bar{\alpha}$ at stage 3. This yields 6 different equilibria which all have the same outcome: only one group of voters attends election and votes for one of the politicians, thus selecting $\bar{\alpha}$ because no other alternative exists. Suppose that two or more groups came to the elections. Than any one of them can profitably deviate and abstain from elections, since in case of their deviation $\bar{\alpha}$ will be chosen anyway, but they will not have to lose $\epsilon$, which is the cost of attending the elections. If no group goes to the elections, each group has an incentive to deviate and vote, because $\forall \alpha, \tau, i : U_i(\tau, \bar{\alpha}) - \epsilon > -K$, where $-K$ is the cost that each group bears in case no-one attends elections. Finally, if only one group attends elections, no matter which, they will vote for the only alternative $\bar{\alpha}$. They have no incentive to deviate to avoid paying very large $K$, while other groups don’t deviate to avoid paying $\epsilon$.

Consider Stage 3. Suppose that one of the politicians deviates and offers $\alpha' > \bar{\alpha}$. Let’s prove that $\bar{\alpha}$ will beat $\alpha'$ on the next stage, so the politician will not make such a deviation. Let’s notice that it is sufficient for either $m$ or $p$ to be present at elections for $\bar{\alpha}$ to win, since $\bar{\alpha} > m, \alpha', \bar{\alpha} > p, \alpha'$ and $|m| > |p| > |r|$. So, in case only $m$ or $m$ and $p$ go to the elections, they will vote for $\bar{\alpha}$ and outvote $r$ in case the latter go to elections. Because of this, there are no equilibria in which both $r$ and some other group go to the elections since $r$ will have an incentive to deviate, because their presence does not change the result of voting, but they lose $\epsilon$ in case they vote.

If no-one or only $r$ go to the elections, $m$ or $p$ will deviate and also go to the elections, since otherwise they will get $-K < U_i(\tau', \bar{\alpha}) - \epsilon$, if no-one attends elections, or $U_i(\tau', \alpha') < U_i(\tau', \bar{\alpha}) - \epsilon$, if only $r$ do, $i \in \{p, m\}$. So, there’re also no equilibria in which only $r$ go to the elections or no-one goes to the elections.

Consider the case when only $m$ and $p$ go the elections. Both then and if one of the groups abstains $\bar{\alpha}$ will be selected. So, each group has a profitable deviation to abstain.
The only candidates for equilibrium left are those in which only p or only m go to the elections. In this case, any group that abstains will not gain anything by going to the elections, while the one that goes to the elections will not find it profitable to abstain since \( U_i(\tau', \bar{\alpha}) - \epsilon > -K, i \in \{p, m\} \). So, these profiles are Nash equilibria. There are only two Nash equilibria in the final stage and in both \( \bar{\alpha} \) wins. Knowing that, no politician will deviate and set \( \alpha > \bar{\alpha} \).

Suppose instead that at Stage 3 one of the politicians deviates and offers \( \alpha' < \bar{\alpha} \). The proof is analogous. It is not an equilibrium when everyone abstains. Since \( |m| > |p| > |r| \), \( \alpha' \prec_m \bar{\alpha}, \alpha' \prec_r \bar{\alpha} \), it is sufficient for m to attend elections for \( \bar{\alpha} \) to win. So, it is not an equilibrium when two or more players attend the elections. Obviously, if one of those groups are r, the smallest group, they find it profitable to deviate since their presence does not change the outcome. If only m and p attend elections, p can profitably deviate for the same reason. So, we are left with candidates for equilibrium in which only one group attends the elections. If this group is p, the outcome will be \( \alpha' >_p \bar{\alpha}, \alpha' \prec_m \bar{\alpha} \). Since \( |m| > |p| > |r| \), m can profitably deviate by attending elections and voting for their preferred alternative, \( \bar{\alpha} \). Profiles when only m attend elections and vote for \( \bar{\alpha} \) is a Nash equilibrium, because their presence is sufficient for \( \bar{\alpha} \) to win, while if they deviate, they will get \( -k < U_m(\tau', \bar{\alpha}) - \epsilon \). If one of the groups that abstain decides to attend elections, it will not change the outcome, but will additionally lose \( \epsilon \).

Thus, if one of the politicians at Stage 3 offers \( \bar{\alpha} \) and the other offers \( \alpha' \neq \bar{\alpha} \), the deviator will lose the elections. So, profile \((\bar{\alpha}, \bar{\alpha})\) can be part of a subgame-perfect equilibrium.

Further, it is the only possible profile. If both politicians set \( \alpha \neq \bar{\alpha} \), any politician can deviate to \( \bar{\alpha} \) and win the elections (we have already proven that if one politician offers \( \bar{\alpha} \) and the other offers \( \alpha' \neq \bar{\alpha} \), the one who offered \( \alpha' \) loses). So, no other profile but \((\bar{\alpha}, \bar{\alpha})\) is possible in the subgame-perfect equilibrium.

Part 2

For your convenience, we, again, place here Picture 7 that illustrates stages 1 and 2 when \( \alpha_m \leq \alpha_p + c \).

![Picture 7](image)

We are to prove that if at stage 1 one of the politicians offers point C and the other offers point I on the segment CB, point C will win the majority vote at stage 2. We have already
proven in the main text that r and m prefer C while p prefer I. So, it is sufficient for m to attend elections for C to win. Let’s define \((i, j, k)\) as the situation when groups \(i, j\) and \(k\) attend elections. If we consider a profile in which at least two groups attend elections and the result of the elections will not change if group \(i\) abstains, this is not an equilibrium, because group \(i\) has a profitable deviation to abstain. The result will be the same, but they will spare the cost of voting, \(\epsilon\).

\((m, r)\) cannot happen in equilibrium since both groups prefer C, so if one of them abstains, C is going to win anyway.

\((m, p)\) is not an equilibrium, because since \(p < m\), C wins regardless of the presence of p. So, p prefer to deviate.

\((p, r)\) is not an equilibrium because p prefer I, r prefer C, \(p > r\). So I is selected regardless of the presence of r, r decide to deviate.

It is not an equilibrium when no-one votes due to our assumption that all the players pay a very high cost in this case.

Finally, \((m, r, p)\) is not an equilibrium because p or r can deviate without changing the result.

\((p)\) is not an equilibrium because m can attend the elections and thus select their most-preferred alternative, while if they don’t I will be selected.

\((r)\) is not an equilibrium, because p can deviate by attending elections and choosing their most-preferred alternative, I, otherwise r would have selected C.

\((m)\) is an equilibrium. They cannot deviate because otherwise they will pay the large cost \(K\). Moreover, p and r will not deviate because their presence does not change the result.