Syllabus

Mathematics for the Master Programme "Financial Economics"

Professor, Dr. Mark Levin Autumn 2008
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Description of the Course

The course has been designed to convey to the students how mathematics can be used in the modern micro and macro economic analysis.
Emphasis is placed on the model-building techniques, methods of solution and economic interpretations.
Topics studied comprise the following:
Differential equations, optimal control theory and stochastic processes.
Upon completion an individual will:

- have the ability to solve differential equations and systems of differential equations,
- have acquired the knowledge of the methods of the optimal control theory and its applicability for solving problems in economics,
- have developed skills in working with the Brownian and Wiener stochastic processes and have the idea how Ito's integral is applied.

Attendance Policy

Attendance is strongly encouraged. Attendance on examinations is mandatory. The absence on an examination will be excused if the reasons, such as illness or a similar *force majeure* are documented in writing.

Examinations

There will be three Examinations 120 minutes each. The first of them is scheduled on late September. It follows the completion of the intensive math refreshment.
The rest of examinations are tentatively scheduled on early November and late December, respectively.
Persons absent from Examinations receive unsatisfactory mark unless the absence is excused documentary. The written documentation presented for a missed Examination must be presented to me no later than three days following return to class.
I will give you back each test with a score on it. You can also check with me about your scores at anytime during the semester.

Homework Assignments

Homework will be assigned once a week. Homework will be collected, marked and returned to you.

Marks

Final marks will be determined by weighting work on Examinations as follows:

Examination that follows math refreshment	*15% of final mark*
Midterm Examination	*30% of final mark*
Final Examination	*40% of final mark*
Homework	*15% of final mark*

Since for many students enrolled to the programme calculus and linear algebra were the topics studied on the undergraduate level a good while ago, and taking into account that the cohort of students was drawn from the various institutions thus them having different mathematical background it was suggested to teach students a refresher course whose purpose was to warm up their math aptitude, show their weaknesses (if any). That refresher would tune enrolled students up for a high level mathematics.

Syllabus of the Refresher course on math:

Multidimensional calculus, optimization taught by Prof. M. Levin	Contact hours

Euclidean spaces
- vector
- distance
- open and closed sets
- neighborhood of a point, limiting points, boundary points
- bounded sets, compact sets
Functions
- vector-functions
- limit of a function
- continuity
- arcwise connectedness, path
Multidimensional calculus
- Linear structure
    o linear space
    o linear dependence/independence
    o basis, dimension
    o linear mapping
- Norm. Scalar (dot) product
    o norm
    o scalar product
    o orthogonality
    o angle between vectors
- Total differential
    o partial derivative
    o relation between partial and total derivatives
    o implicit function theorem
Optimization in many variables
    o concept of extrema
    o conditions of extrema
    o least squares method
Gramm's matrix and its properties
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Linear Algebra (that part taught by Associate Prof. K. Bukin) - operations on matrices - matrix multiplication - inverse matrix, its properties - rank of a matrix - linear spaces and subspaces, their properties - Gauss method of solving linear systems - systems of linear equations, Kronecker-Capelli's theorem - projection matrix, its main properties, idempotency, eigenvalues of idempotent matrix - eigenvalues and eigenvectors (definition, relation to the matrix rank, case of a symmetric matrix) - quadratic forms: sign-definiteness of forms - differentiation with respect to a vector - problem of main component - isomorphism - kernel and image of a linear operator - Eucleadean spaces - orthogonalization by Gramm-Schmidt's method - Cauchy-Bunyakovsky's inequality - spur of a matrix and its properties - quadratic form sign-definiteness criterion (by eigenvalues) - Sylvester's criterion - square root of a matrix - reduction of a matrix to a diagonal form	30
Convex analysis and Kuhn-Tucker theorem (taught by Prof. M. Levin) o outset of a non-linear programming program - Convexity o convexity of a set o convex and concave functions, their properties - separability theorem, separating hyperplane - saddle point - Sylvester's criterion - necessary and sufficient conditions of a quadratic form sign-definiteness (n=2) - strict convexity of a function **Unconstrained optimization** - Taylor's expansion in a single variable case - Jacobi's matrix - Jacobi's matrix for a composite function - theorem of existence of inverse operator - sufficient conditions for extrema **Constrained optimization** o Lagrange's classic problem o optimization of a quadratic form on a unit sphere o directional derivative	18

- **Constrained optimization with the inequality constraints** - problem setting, function requirements - necessary and sufficient conditions for extrema and corollaries from that theorem - problem modification for the nonnegative variables - differential characteristics of Kuhn-Tucker conditions - the meaning of Lagrange multiplier - consumer's optimality problem **<u>Textbooks for that part of the course given above</u>** E. Roy Weintraub, Mathematics for economists, 6th edition, Cambridge University Press, 1993 Carl P. Simon, Lawrence Blume, Mathematics for economists, W.W. Norton and company, Inc., 1994 or latest editions	

Theory of probability and statistics (taught by Prof. G. Kantorovich) - random variable, sample space - cumulative distribution function and its density - uniform distribution - normal distribution, reduction of the Gaussian variable to a standard variable - expectation $E(X)$, $E(f(X))$ - initial and central moments - joint distributions of the random variables - conditional distributions - iterated expectations formula - limiting densities - covariation and correlation - standard normal vector and its properties - marginal and conditional normal distributions - quadratic forms in a standard normal vector - χ^2 distribution and its properties - Student's distribution and its properties - Fisher's distribution and its properties - point estimation of parameters - unbiasness and efficiency of estimators - elements of large-sample distribution theory - convergence in probability and convergence in distribution - asymptotic distribution - interval estimation - hypothesis testing - errors of the first and second type - critical region of the test, decision rule **<u>Textbook for that part of the refresher course</u>** Newbold Paul, Carlson William L., Thorne Betty, Statistics for business and economics, 5th edition, Pearson Education, Inc., Upper Saddle River, NJ, 2003	**30**

Topics studied in the course and recommended texts	Contact hours
1. Differential Equations 1.1. First-Order Ordinary Differential Equations 1.1.3. Stability. 1.1.4. Analytical Solutions. 1.1.5. Linear, first-order differential equations with constant coefficients. 1.1.6. Linear, first-order differential equations with variable coefficients.	4
1.2. Systems of Linear Ordinary Differential Equations 1.2.1. Phase Diagrams. 1.2.5. Analytical Solutions of Linear, Homogeneous Systems. 1.2.6. The Relation between the Graphical and Analytical Solutions. 1.2.7. Stability. 1.2.8. Analytical Solutions of Linear, Nonhomogeneous Systems. 1.2.9. Linearization of Nonlinear Systems. The Time-Elimination Method for Nonlinear Systems.	6
2. Dynamic Optimization in Continuous Time 2.1. The Typical Problem. 2.2. Heuristic Derivation of the First-Order Conditions. 2.3. Transversality Conditions. 2.4. The Behavior of the Hamiltonian over Time. 2.5. Sufficient Conditions. 2.6. Infinite Horizons. Example: The Neoclassical Growth Model. 2.7. Transversality Conditions in Infinite-Horizon Problems. 2.8. Summary of the Procedure to Find the First-Order Conditions. 2.9. Present-Value and Current-Value Hamiltonians. Multiple Variables. **3. Finite-Horizon Dynamic Programming** 3.1 Examples of the Dynamic Programming Problems 3.2 Histories, Strategies and the Value function 3.3 Markovian Strategies 3.4 Existence of an Optimal Strategy 3.5 The Bellman Equation 3.6 Stationary Strategies 3.7 Example: the Optimal Growth Strategy **Textbooks for that part of the course** Алексеев В.М., Тихомиров В.М., Фомин С.В. «Оптимальное управление». – М.: Наука. Главная редакция физико-математической литературы, 1979. Kamien, M.I., Schwartz, N.L. Dynamic optimization: the calculus of variations and optimal control in economics and management, 2nd ed. New York: North-Holland, 1991. Rangarajan K. Sundaram. A first course in optimization theory, Cambridge University Press, 1996, 11th printing in 2007	22

4. **Uncertainty, information, and stochastic calculus** — 28

4.1 Probability essentials:

Sigma-algebras. Basic properties of sigma-algebras. Borel sigma algebras. Measurable functions. Probability as measure. Expectation.

4.2 Conditional expectation:

Definition. Calculation of conditional expectation. Properties of conditional expectations.

4.3 Discrete-time stochastic processes:

Filtration, Adapted process, Predictable process, Markov process, Markov chains, Examples,

4.4 Martingales:

Definitions of martingales. Properties. Examples. Random walk.

4.5 Continuous-time stochastic process:

Arithmetic and geometric Brownian motion, martingales in continuous time, multi-dimensional processes.

4.6 Ito calculus:

Stochastic integral, Ito's lemma, SDE

4.7 Change of measure:

Girsanov theorem, solution of Black-Scholes model via Girsanov theorem

4.8 Introduction to Matlab:

Basic matrix operations, functions, scripts, graphs, flow control

Textbooks for that part

Sprenger, Carsten, Lecture Notes in Financial Economics, Part 1.

Brzezniak, Zastawniak, (2006), Basic Stochastic Processes, Spinger

Jeffrey S. Rosenthal. (2007), A first look at rigorous probability, World Scientific Publishing

Cvitannic and Zapatero, Introduction to the Economics and Mathematics of Financial Markets, MIT Press, 2004 – Chapters 3 and 16.

Additional readings

Shreve S., (2004), Stochastic Calculus for Finance I, II,. Springer-Verlag

Fima C. Klebaner, (2006), Introduction to stochastic calculus with applications, Imperial College Press

Munk, Claus, Financial Asset Pricing Theory, mimeo, available at http://www.sam.sdu.dk/~cmu/cmu_pub3.htm - Chapters 2 and Appendix.

Neftci, Salih N., An Introduction to the Mathematics of Financial Derivatives, 2nd edition, San Diego Academic Press, 2000, Chapters 3,5,6,9,10.