The implications of the asymmetric price rigidity for the monetary policy in an open economy: A cross-country evidence

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Abstract

In this paper we build a microfounded general equilibrium sticky price model of a small open economy and analyze the optimal monetary policy assuming asymmetric price rigidity. We find that the optimal monetary policy rule is to adjust the interest rate in response to exogenous exchange rate, supply and demand shocks with positive coefficients and that deflationary shocks should be accompanied by a smaller adjustment in the interest rate than inflationary ones of the same size since prices are more sticky downwards. We test the predictions of our model for a set of developed countries. We find that the exchange rate plays a significant role in determining the monetary policy in most countries and we find some evidence of the asymmetry which is in line with our model.

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1. Introduction

The traditional New Keynesian literature on the optimal monetary policy does not assume any non-linearities or asymmetries in its three main building blocks: the IS curve, the Phillips curve and the social loss function. Therefore, the derived optimal monetary policy rules appear to be linear, according to which positive and negative shocks should be accompanied by equal changes in the monetary policy instrument. But empirical evidence of the last decade speaks in favor of different kinds of asymmetries in the behavior of economic agents.

In particular, the Phillips curve is empirically found to be convex (Latxon et al., 1999, and Alvarez Lois, 2000, for the USA; Dolado et. al., 2005, for several European countries) implying asymmetric price rigidity, which means that prices are more sticky downwards than upwards. This results in the Phillips curve being steeper for positive changes in inflation than for negative ones. Therefore, as documented by many authors for many countries (e.g. Cover, 1992), positive demand shocks give rise to inflation without affecting output significantly, while negative ones reduce output without affecting inflation.

There are many explanations for this phenomenon. The most traditional view is that the labour market is the primary source of the asymmetry. Many papers show that a wage cut is a much rare phenomenon than a wage increase (e.g. Holden and Wulfsberg, 2004; Altonji and Devereux, 1999; Holzer and Montgomery, 1993). But an asymmetry is also widely observed in the prices of final goods. For example, Peltzman (2000) studies over 240 markets for consumer as well as producer goods and finds that asymmetries are persuasive, substantial and durable, and exist in periods of low inflation as well as in periods of high inflation. These asymmetries also apply to price indices (Verbrugge, 1998). Among theoretical explanations for the asymmetric price rigidity are consumer search with learning from prices (Benabou and Getner, 1993), consumer search with reference prices (Lewis, 2003), tacit collusion among firms with the past price serving as a focal price (Borenstein et al., 1997), implicit coordination among firms in an industry to rise prices after a positive cost shock while not to reduce prices after a negative one (Bhaskar, 2002), a trend in the marginal costs or desired mark-ups (Dhyne et al, 2006), an overall positive trend inflation in an economy (Ball and Mankiw, 1994) and other.

As claimed by DeLong and Summers (1988) and Ball, Mankiw and Romer (1988), asymmetries in wages and prices may have important implications for the appropriate stabilization policy. Nevertheless, such asymmetries are rarely incorporated into theoretical models of optimal monetary policy. Indeed, such literature emerged only in the 21st century. In this paper we contribute to this stream of literature by studying the implications of the asymmetric price rigidity for the optimal monetary policy.
Orphanides and Wieland (2000) is one of the first studies which analyses the impact of a nonlinear Phillips curve on the optimal monetary policy. In particular, they assume a nonlinear Phillips curve and conclude that monetary policy should be non-linear as well. Dolado et al. (2005) also study the implications of nonlinear Phillips curve for the derivation of the optimal monetary policy rules and find that the policy-maker should “increase the interest rate by a larger amount when inflation or output are above the target than the amount it will reduce them when they are below the target”. Diana and Méon (2005) analyze the optimal monetary policy under asymmetric wage indexation and propose that positive shocks should be absorbed more than negative ones.

The existing literature assumes non-linearities in the Phillips curve ad hoc and does not provide any microeconomic explanations. But in this paper we provide a microfounded derivation of both the IS curve and the Phillips curve. Also the existing literature analyses closed economies, while we build a model for an open economy, which is subject to external shocks as well.

The theoretical prediction that monetary policy is non-linear is tested empirically, but the evidence is mixed. Some papers confirm that monetary policy is asymmetric indeed (Olmedo, 2002, Dolado et al., 2004, 2005, Taylor and Davradakis, 2006). Others do not find any signs of nonlinearities in monetary policy rules (e.g. Bruinshoofd and Candelon, 2004). We also contribute to this empirical literature by studying how the monetary policy in a number of developed countries reacts to positive and negative real exchange rate shocks, which may result from nominal exchange rate shocks, local shocks or foreign shocks.

This paper is structured as follows. In section 2 we lay out the theoretical model and analyze the optimal monetary policy under symmetric and asymmetric price rigidity. In section 3 we perform the empirical tests of the predictions of our theoretical model. Section 4 is devoted to conclusions.

2. The theoretical model

In this section we build a microfounded New-Keynesian open economy model and analyze the optimal monetary policy in response to different kinds of stochastic shocks.

2.1. Demand

The demand side is represented by a New Keynesian IS curve. To derive it we follow Gali and Monacelli (2005).

Consumer choice

We assume that the world consists of an infinite number of symmetric small open economies. Each economy $i$, $i \in [0,1]$, produces an infinite number of goods, indexed by $j$, $j \in [0,1]$. All goods are traded, and subscript $H$ denotes the goods produced domestically while subscript $F$ denotes imported goods. We analyze a typical economy which we call ‘the domestic economy’.
In the domestic economy the representative consumer maximizes the following discounted expected utility function:

$$\max U_t = E_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{C_{t+k}^{1-\sigma} - N_{t+k}}{1-\sigma} \right]$$  (1)

where $\beta$ is the subjective discount factor, $N_t$ is the labor supply and $C_t$ is the following CES consumption index with the elasticity of substitution $\eta > 0, \eta \neq 1$:

$$C_t = \left[ (1-\alpha)^{\frac{1}{\gamma}} (C_{H,t}^{\frac{1-\gamma}{\gamma}} + \alpha^{\frac{1-\gamma}{\gamma}} (C_{F,t}^{\frac{1-\gamma}{\gamma}}) \right]^{\frac{\gamma}{1-\gamma}}$$  (2)

where $\alpha$ is the share of imported goods, $C_{H,t}$ and $C_{F,t}$ are CES consumption indices of domestic and imported goods respectively with the corresponding elasticities of substitution $\epsilon > 0, \epsilon \neq 1$ and $\gamma > 0, \gamma \neq 1$:

$$C_{H,t} = \left( \frac{1}{0} C_{H,t}(j) j^\epsilon \right)^{\frac{1}{\epsilon}}$$

$$C_{F,t} = \left( \frac{1}{0} C_{F,t} j^\gamma \right)^{\frac{1}{\gamma}}$$

where $\gamma$ is the elasticity of substitution of goods imported from different countries and $C_{i,t}$ is the index of consumption of goods, imported from country $i$:

$$C_{i,t} = \left( \frac{1}{0} C_{i,t}(j) j^\epsilon \right)^{\frac{1}{\epsilon}}$$

The utility function (1) is characterized by diminishing marginal utility of consumption ($0 < \sigma < 1$) and increasing marginal disutility of labor supply ($\varphi > 0$). In this model we assume that consumers do not derive utility from holding money, and money serves as a mean of exchange only.

In every period the utility function (1) is maximized subject to the following period budget constraint:

$$\int_0^1 P_{H,t}(j)C_{H,t}(j) dj + \int_0^1 P_{i,t}(j)C_{i,t}(j) dj + E_t \left( \frac{1}{1 + r_t} D_r \right) = D_t + W_t N_t + T_t$$  (3)

where $P_{H,t}(j)$ is the price of the domestic good $j$, $P_{i,t}(j)$ is the domestic price of the good $j$, imported from country $i$, $D_t$ is the value of investment portfolio, $r_t$ is the nominal interest rate, $W_t$ is the wage rate and $T_t$ is the net transfers. So, the left-hand side of the budget constraint describes the consumer’s spending on consumption of domestic and foreign goods and investment, while the right-hand side describes the consumer’s current wealth.

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Budget constraint (3) may be written in a more concise way as:

\[ P_t C_t + E_t \left( \frac{1}{1 + r_t} D_{t+1} \right) = D_t + W_t N_t + T_t \]

where \( P_t \) is the aggregate price index in the economy and \( C_t \) is the aggregate consumption index.

The first-order condition of the consumer’s maximization problem is the Euler equation:\(^1\)

\[ \beta (1 + r_t) E_t \left( \frac{C_{t+1}}{C_t} \right)^{\sigma} \left( \frac{P_t}{P_{t+1}} \right) = 1 \]  

which is log-linearized as follows:

\[ c_t = E_t \{ c_{t+1} \} = \frac{1}{\sigma} \left( \tilde{\pi}_t - E_t [\pi_{t+1}] - \rho \right) \]  

where lower cases denote logarithms, \( \tilde{\pi}_t \equiv \ln(1 + r_t) \), \( \pi_{t+1} \equiv p_{t+1} - p_t = \ln \left( \frac{P_{t+1}}{P_t} \right) \) is the domestic CPI inflation and \( \rho \equiv - \ln \beta = \ln(1 + r) \), where \( r \) is the subjective discount rate.

Euler equation (5) is one of the key equations in our model. It shows that the optimal consumption depends positively on the future expected consumption (consumption smoothing effect) and negatively on the real interest rate (intertemporal substitution effect).

**Exchange rate, terms of trade and inflation**

We use the following notations: \( S \) – terms of trade, \( E \) – nominal exchange rate, \( Q \) – real exchange rate.

The terms of trade of the domestic economy and economy \( i \) equal the ratio of prices of the national goods, expressed in the domestic currency (that is the currency of the economy under consideration):

\[ S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}} \]

Then the effective terms of trade are:

\[ S_t = \left( \int_0^1 S_{i,t}^{-\gamma} \, di \right)^{\frac{1}{\gamma - 1}} = \frac{P_{F,t}}{P_{H,t}} \]  

or in log-linearized form:

\[ s_t = \int_0^1 s_{i,t} \, di = p_{F,t} - p_{H,t} \]  

We assume that the law of one price holds for all individual imported goods:

\(^1\) The derivation of this and the following equations of section 2 is provided in Appendix 1.
\[ P_{i,t}(j) = E_{i,t}P^i_{i,t}(j) \]

where \( E_{i,t} \) is the nominal exchange rate of country \( i \), an increase of which means depreciation of the domestic currency against the currency of country \( i \), and \( P^i_{i,t}(j) \) is the price of good \( j \) denominated in the currency of country \( i \).

Then the price index of goods imported from country \( i \) equals:

\[
P_{i,t} \equiv \left( \int_0^1 P_{i,t}(j)^{1-t} \, dj \right)^{1-t} = E_{i,t}P^i_{i,t},
\]

and the price index of all imported goods equals:

\[
P_{F,i} \equiv \left( \int_0^1 P_{i,t}^{1-t} \, dt \right)^{1-t} = E_iP^*_i
\] (8)

where \( E_i \) is the nominal effective exchange rate and \( P^*_i \) is the world price index.

Equation (8) in log-linear form looks as follows:

\[
p_{F,i} = e_i + p^*_i
\] (9)

Substituting equation (9) into equation (7) we arrive at the following expression for the terms of trade:

\[
s_i = e_i + p^*_i - p_{H,i}
\] (10)

The real exchange rate of economy \( i \) equals the ratio of consumer price indices, expressed in the domestic currency:

\[
Q_{i,t} \equiv \frac{E_{i,t}P^i_{i,t}}{P_i}
\]

and in log-linear form:

\[
q_i = e_i + p^i_{i,t} - p_i
\]

Then the logarithm of the real effective exchange rate equals:

\[
q_i \equiv \frac{1}{0} q_{i,t} \, dt = \frac{1}{0} \left[ e_{i,t} + p^i_{i,t} - p_i \right] \, dt = e_i + p^*_i - p_i
\] (11)

Substituting equation (10) into equation (11) and making use of the following expression for the consumer price index:

\[
p_i \equiv (1 - a)P_{H,i} + aP_{F,i} = P_{H,i} + a s_i
\]

we arrive at the following expression of the logarithm of the real exchange rate:

\[
q_i = (1 - a)s_i
\] (12)
Equilibrium in the goods market

In equilibrium the world demand for a domestically produced good \( j \) (i.e. the demand for good \( j \) of the consumers living in the domestic economy and all other economies) should be equal to its production \( Y_t(j) \):

\[
Y_t(j) = C_{it, j}(j) + \int_0^1 C'_{it, j}(j) \, di = \left( \frac{P_{H,i,j}(j)}{P_{H,t}} \right)^{\tau} \left[ (1 - \alpha) \left( \frac{P_{H,i,j}}{P_t} \right)^{\gamma} C_t + a \int_0^1 \left( \frac{P_{H,i,j}}{E_{i,t}^i P_{F,t}^j} \right)^{\gamma} \left( \frac{P_{F,i,j}^j}{P_t^i} \right)^{\gamma} \right] C_i \, di \quad (13)
\]

Then the aggregate output of the domestic economy equals:

\[
Y_t \equiv \int_0^1 Y_t(j) \, \frac{e^{-\gamma t}}{(1 - \gamma) t} \, dj = \left[ (1 - \alpha) \left( \frac{P_{H,i,j}}{P_t} \right)^{\gamma} C_t + a \int_0^1 \left( \frac{P_{H,i,j}}{E_{i,t}^i P_{F,t}^j} \right)^{\gamma} \left( \frac{P_{F,i,j}^j}{P_t^i} \right)^{\gamma} \right] C_i \, di = \quad (14)
\]

\[
= S^{\alpha \gamma}_i C_i \left[ (1 - \alpha) + a \int_0^1 \left( S_{i,t} S_i \right)^{\gamma} \frac{1}{P_t} \, di \right]
\]

where \( S_i \) are effective terms of trade, \( S_{i,t} \) are terms of trade with country \( i \) and \( S_i^j \) are effective terms of trade of economy \( i \).

Taking into account that \( \int_0^1 s_t \, di = 0 \), equation (14) is log-linearized as follows:

\[
y_t = c_t + a \gamma s_t + a \left( \eta - \frac{1}{\sigma} \right) q_t \quad (15)
\]

Substituting the expression for \( s_t \) from equation (12) into equation (15) we get:

\[
y_t = c_t + a \gamma q_t - a \left( \eta - \frac{1}{\sigma} \right) q_t = c_t + \nu q_t \quad (16)
\]

where \( \nu \equiv a \left( \frac{1}{\sigma} - \frac{\gamma}{1 - \alpha} - \eta \right) \).

Finally, we substitute the expression for \( c_t \) from equation (16) into equation (5) and arrive at the following version of the IS curve for the economy:

\[
y_t = \frac{\rho}{\sigma} + E_t[y_{t+1}] - \frac{1}{\sigma} \left( \tilde{\gamma}_t - E_t[\pi_{t+1}] \right) + \nu E_t[\Delta q_{t+1}]
\]

We assume that the potential output of the economy, \( \bar{Y} \), is constant and given exogenously. Then the IS curve may be written in terms of the output gap, as it is common in the literature:

\[
\hat{y}_t = \frac{\rho}{\sigma} + E_t[\hat{y}_{t+1}] - \frac{1}{\sigma} \left( \tilde{\gamma}_t - E_t[\pi_{t+1}] \right) + \nu E_t[\Delta q_{t+1}] \quad (17)
\]

where \( \hat{y}_t \equiv y_t - \bar{Y} \) is the output gap.

The above derivation of the IS curve is similar to Gali and Monacelli (2005) except for the fact that they model the output gap as a function of the expected inflation of the domestically
produced goods only, while in our model the output gap depends on the expected CPI inflation and the expected real exchange rate depreciation.

We assume that the economy is subject to an exogenous stochastic demand shock \( \xi_t \), \( \xi_t \sim N(0, \sigma^2_\xi) \). Such a demand shock may be either a government spending shock, which we do not model here and, hence, assume fully exogenous, or a consumption shock caused, for example, by a change in the subjective discount factor \( \beta \).

Then the IS curve (17) with simplified notations looks as follows:

\[
\hat{y}_t = a + E_t[\hat{y}_{t+1}] - b[E_t[\pi_{t+1}] + \nu E_t[\Delta q_{t+1}] + \xi_t,
\]

where \( a \equiv \frac{\rho}{\sigma} \) and \( b \equiv \frac{1}{\sigma} \). This New Keynesian IS curve shows that the output gap depends positively on the expected future output gap due to the consumption-smoothing effect and negatively on the real interest rate due to the intertemporal substitution effect. The dependency on the expected real exchange rate change distinguishes this open-economy IS curve from a standard closed-economy IS curve, popularized by Clarida, Gali and Gertler (1999).

2.2 Supply

The supply is represented by a New Keynesian Phillips curve. To derive it, we distinguish between domestic goods’ pricing and foreign goods’ pricing.

**Domestic goods’ pricing**

We assume that the domestic producers set prices in a staggered fashion a la Calvo (1983). In every period a producer receives a signal with probability (1-\( \theta \)) that it should re-set its price. Therefore, its price will stay intact with probability \( \theta \). The value of \( \theta \) is assumed to be constant, so that the probability of changing the price in a given period does not depend on whether the price was changed in the previous period or not. The higher is the parameter \( \theta \), the more sticky are domestic goods’ prices in the economy.

Having received the signal to adjust the price, the producer of \( j \)-th good sets the new price \( P_{H,j}(j) \), which minimizes his expected discounted losses from log-deviations of this price from an optimal price \( \tilde{P}_{H,j}(j) \):

\[
\min \frac{1}{2} \sum_{k=0}^{\infty} \beta^k \left( P_{H,j}(j) - \tilde{P}_{H,j}(j) \right)^2
\]

Since the price will remain unchanged for \( k \) periods with probability \( \theta^k \), the function of the expected discounted losses from deviation of the price \( P_{H,j}(j) \) from the optimal price looks as follows:

\[
\text{Recall that } \rho \equiv - \ln \beta = \ln(1 + r), \text{ where } r \text{ is the subjective discount rate. If } r \text{ is subject to stochastic shocks, then these shocks will be translated into the demand shock } \xi.
\]
Minimizing function (18) with respect to \( p_{H,t}(j) \) we get the following recursive first-order condition:

\[
p_{H,t}(j) = (1 - \theta \beta) \tilde{p}_{H,t}(j) + \theta \beta E_t \tilde{p}_{H,t+1}(j)
\]

Equation (19) means that the current price equals the average of the desired price and the following period price, weighted with the probability of changing the price.

Since in every period the share of firms (1-\( \theta \)) adjust their prices while the share of firms \( \theta \) keep their prices constant, the aggregate price index equals:

\[
\hat{P}_{H,t} = \left[ \theta \hat{P}_{H,t-1}^{1 - \epsilon} + (1 - \theta) \hat{p}_{H,t}^{1 - \epsilon} \right]^{1/\epsilon}
\]

or in log-linear form:

\[
p_{H,t} = \theta p_{H,t-1} + (1 - \theta) p_{H,t}
\]

Equation (20) is iterated forward and transformed, after which we arrive at the expression for the expected price in the future period:

\[
E_t \hat{p}_{H,t+1}(j) = \frac{1}{1 - \theta} E_t \hat{\pi}_{H,t+1} + p_{H,t}
\]

which is then substituted into equation (19):

\[
p_{H,t}(j) = (1 - \theta \beta) \tilde{p}_{H,t}(j) + \frac{\theta \beta}{1 - \theta} E_t \hat{\pi}_{H,t+1} + \theta \beta p_{H,t}
\]

The optimal price of the producer is found from the profit maximization problem and can be described by the following expression:

\[
\tilde{p}_{H,t} = p_{H,t} + \chi \hat{y}_t
\]

where \( \chi \) is a positive constant.

Substituting this expression for the optimal price (23) into the equation for the price of good j (22), and then into the expression for the general price index (20), after some transformations, we arrive at the New-Keynesian Phillips curve for the inflation of the domestic goods:

\[
\hat{\pi}_{H,t} = \beta E_t \hat{\pi}_{H,t+1} + \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \chi \hat{y}_t
\]

With simplified notations and a stochastic cost shock \( \omega_t, \omega_t \sim N(0, \sigma^2) \), the Phillips curve for the domestic goods looks as follows:

\[
\pi_{H,t} = \beta E_t \pi_{H,t+1} + d\hat{y}_t + \omega_t
\]

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3 See Gali and Monacelli (2005) for the derivation in a similar model set-up.
where \( d = \frac{(1-\theta)(1-\theta \beta) \chi}{\theta} \) is the slope of the Phillips curve.

It should be noted that the slope of the Phillips curve depends negatively on parameter \( \theta \): the higher is \( \theta \), the fewer firms adjust their prices every period and the more is price stickiness among the domestic goods.

**Import goods’ pricing**

As it was already mentioned, we assume that the law of one price holds for all individual imported goods. Recall, that the logarithm of the import price equals:

\[
p_{F,t} = e_t + p^*_t
\]  

(9)

Equation (9) means that there is complete exchange rate pass-through onto import prices.

Then the inflation of the imported goods equals:

\[
\pi_{F,t} = \Delta e_t + \pi^*_t
\]  

(25)

where \( \Delta e_t \equiv e_t - e_{t-1} \) and \( \pi^*_t \) is the world inflation.

From equation (11) for the real exchange rate it follows that:

\[
\Delta q_t = \Delta e_t + \pi^*_t - \pi_t
\]  

(26)

Combining equations (25) and (26) we get the following expression for the import goods’ inflation in terms of the real exchange rate depreciation and the domestic CPI inflation:

\[
\pi_{F,t} = \Delta q_t + \pi_t
\]  

(27)

**Aggregate supply**

Recall the following equation for the log-linearized consumer price index:

\[
p_t \equiv (1-\alpha)p_{H,t} + \alpha p_{F,t}
\]

Then the aggregate inflation is the following:

\[
\pi_t \equiv (1-\alpha)\pi_{H,t} + \alpha \pi_{F,t}
\]  

(28)

Substituting the expressions for the domestic goods’ inflation (24’) and import goods’ inflation (27) into equation (28) we get the following expression for the overall inflation:

\[
\pi_t = \beta E_t \pi_{H,t+1} + d\tilde{y}_t + \frac{\alpha}{1-\alpha} \Delta q_t + \omega_t
\]  

(29)

Writing equation (28) for the period t+1, transforming it and making use of equation (27) we find the equation for the next period domestic goods’ inflation:

\[
\pi_{H,t+1} = \pi_{t+1} - \frac{\alpha}{1-\alpha} \Delta q_{t+1}
\]  

(30)

Finally, substituting equation (30) into equation (29), we arrive at the aggregate New Keynesian Phillips curve for the economy:
\[
\pi_t = \beta E_t \pi_{t+1} + d \hat{y}_t + \frac{a}{1 - \alpha} (\Delta q_t - \beta E_t \Delta q_{t-1}) + \omega_t
\]  
(31)

The obtained specification of the Phillips curve is different from the standard specification in that the inflation depends explicitly on the real exchange rate change with a positive coefficient \(\frac{a}{1 - \alpha}\), which measures the degree of pass-through of the real exchange rate onto CPI inflation. The higher is the share of the import goods \(\alpha\) in the consumption basket, the higher is the pass-through in the economy. If \(\alpha = 0\) the consumer prices do not react to exchange rate changes implying zero pass-through\(^4\). If \(\alpha = 1\) then \(\pi_t = \pi_{F,t} = \Delta e_t + \pi^*_t\) implying the purchasing power parity \((q_t = 0)\). Here we assume some intermediate value of \(\alpha\) and, hence, incomplete pass-through and possibility of real exchange rate fluctuations.

### 2.3. Equilibrium

The equilibrium in the economy is described by the system of equations \((17')\) and \((31)\) holding simultaneously.

To complete the model we assume that the real exchange rate follows a random walk:

\[
\Delta q_t = \psi_t
\]

where \(\psi_t\) is a exogenous real exchange rate shock, \(\psi_t \sim N(0, \sigma^2_\psi)\).

From equation (26) it follows that shocks to the nominal exchange rate, domestic inflation and foreign inflation will all affect the real exchange rate. Therefore, shock \(\psi\) is an aggregate shock, which we analyze. Also, the real exchange rate behavior as in equation (32) is often observed empirically in developed countries. Hence, we assume such behavior, which would simplify our model.

### 2.4. Discretionary monetary policy

**Symmetric price rigidity**

Assume that in every period the monetary authority chooses the value of its monetary policy instrument, the interest rate, which minimizes the following loss function:

\[
\min L_t = (\pi_t - \pi^T)^2 + \lambda \hat{y}^2_t
\]

where \(\pi^T\) is the target inflation rate.

We assume that the monetary authority cares only about the current deviations of inflation and output from their targets. As it will be seen later, in the absence of shocks the inflation in our

\(^4\) It should be noted that in this case the IS curve \((17')\) will transform into a standard closed-economy IS curve since parameter \(\nu\) will become zero.
model equals the target one and the output gap is zero. Since the expectation of all shocks is zero, the expected future deviations of inflation and output from their targets are zero as well. Therefore, there is no sense in putting them into the loss function.

Substituting the Phillips curve (31) into the loss function (33) and making use of (32) and minimizing the losses with respect to $\pi$, we obtain the following reaction function of optimal inflation to expected inflation:

$$\pi_t = \frac{d^2}{d^2 + \lambda} \pi^* + \frac{\lambda}{d^2 + \lambda} E_t[\pi_{t+1}] + \frac{a}{(1-a)(d^2 + \lambda)} \psi_t + \frac{\lambda}{d^2 + \lambda} \omega_t,$$

(34)

Taking expectations of the both sides of (34), solving for the expected inflation and substituting it back into the reaction function (34), we obtain the expression for the equilibrium inflation, which minimizes the losses of society:

$$\pi_t = \pi^* + \frac{a \lambda}{(1-a)(d^2 + \lambda)} \psi_t + \frac{\lambda}{d^2 + \lambda} \omega_t,$$

(35)

We see that in order to minimize the social losses, domestic inflation should be adjusted to exchange rate and supply shocks. It should be noticed that there is no dynamic inconsistency here since the expected inflation equals the target inflation and there is no incentive to deviate from this target unless there are some unexpected shocks.

Substituting the equation (35) into the Phillips curve (31) we find the equilibrium output gap:

$$\hat{y}_t = -\frac{a d}{(1-a)(d^2 + \lambda)} \psi_t - \frac{d}{d^2 + \lambda} \omega_t,$$

(36)

From equation (36) it follows that positive exchange rate and supply shocks lead to the output being lower than the potential output.

The final step is to derive the optimal instrument rule where the real interest rate serves as the instrument for the monetary policy. To do this we substitute equation (36) into the IS curve (17') and solve for $r$:

$$\tilde{r}_t^{opt} = \frac{a}{b} + \pi^* + \left[ \frac{a d}{b(1-a)(d^2 + \lambda)} \right] \psi_t + \frac{d}{b(d^2 + \lambda)} \omega_t + \frac{1}{b} \xi_t,$$

(37)

which can be expressed in a general form as:

$$\tilde{r}_t^{opt} = r_0 + A \psi_t + B \omega_t + K \xi_t,$$

(37')

where $A$, $B$ and $K$ are positive coefficients. This rule states that in order to minimize the social losses the real interest rate should respond to all types of shocks in the economy: real exchange rate shocks, cost shocks and demand shocks – with positive coefficients. This means, for example, that positive shocks, being inflationary, should be accompanied by a contractionary monetary policy.

\footnote{We set $\beta=1$ to simplify the derivation since this parameter will not affect the results significantly.}
leading to negative output gap and lower equilibrium inflation, than what would be without intervention.

**Proposition 1.** The monetary policy should react to exogenous exchange rate shocks and should smooth the exchange rate effect on the domestic inflation. The adjustment in the interest rate should be more significant if:

- the share of import goods and the pass-through effect are higher (parameter \( a \) is higher)
- prices are less sticky (parameter \( d \) is higher, provided that \( d < \sqrt{\ell} \))
- the elasticity of consumption with respect to the interest rate is lower (parameter \( b \) is lower)
- the government cares less about the output gap (parameter \( \lambda \) is lower)

**Proof.** To prove this we differentiate \( A \) with respect to each of the parameters and determine the signs of the corresponding derivatives. The expressions for the derivatives and their signs are presented in Appendix 3.

The finding that the higher is pass-through effect on the aggregate inflation the higher should be the optimal adjustment in the interest rate is in line with Devereux and Engel (2000) who claim that although under low pass-through freely floating exchange rate (a monetary policy in which exchange rates are not a consideration) may be optimal in some circumstances, this is never true in case of producer currency pricing leading to full pass-through.

**Proposition 2.** The optimal interest rate response to a cost shock should be more significant if:

- prices are less sticky (parameter \( d \) is higher, provided that \( d < \sqrt{\ell} \))
- the elasticity of consumption with respect to the interest rate is lower (parameter \( b \) is lower)
- the government cares less about the output gap (parameter \( \lambda \) is lower)

**Proof.** We differentiate \( B \) with respect to each of the parameters and determine the signs of the corresponding derivatives. The expressions for the derivatives and their signs are presented in Appendix 3.

So, an exchange rate shock and a cost shock, both being supply shocks, affect the economy in a similar way. Therefore, the monetary policy should react to them similarly with the only difference that the degree of exchange rate pass-through should be taken into account when reacting to an exchange rate shock.

The analysis of a demand shock \( \varepsilon_t \) is straightforward. Since such a shock does not create any trade-off between the targeted parameters the task of the optimal monetary policy is simply to adjust the interest rate along the IS curve in order to bring the economy back to the target. Therefore, the
magnitude of the interest rate adjustment does not depend on the parameters of the model except for the elasticity of consumption to the interest rate $b$ (the slope of the IS curve).

**Asymmetric price rigidity**

The above analysis assumed symmetrically rigid prices and, hence, a linear interest rate rule. But if prices are asymmetrically rigid, this would be captured by the slope of the Phillips curve, parameter $d$. In particular, the value of $d$ will be higher if prices rise (lower price rigidity) than if they fall.

We assume that more firms will adjust their prices upwards due to an inflationary shock, than will cut prices due to a deflationary shock. Then the exogenous parameter $\theta$ takes two values:

$$\theta = \begin{cases} 
\theta_1, & \text{if } p_{H,t} > p_{H,t-1} \\
\theta_2, & \text{if } p_{H,t} < p_{H,t-1} \\
\theta_1 < \theta_2 
\end{cases}$$

Recall from equation (24') that $d \equiv \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \chi$ depends negatively on $\theta$. Then parameter $d$ takes two values:

$$d = \begin{cases} 
\hat{d}_1 = d(\theta_1), & \text{if } p_{H,t} > p_{H,t-1} \\
\hat{d}_2 = d(\theta_2), & \text{if } p_{H,t} < p_{H,t-1} \\
\hat{d}_1 > \hat{d}_2 
\end{cases}$$

In such a case the Phillips curve becomes kinked at the zero level of inflation:

$$\pi_t = \begin{cases} 
\beta E\pi_{t+1} + d_1^* \hat{y}_t + \frac{a}{1-a}(\Delta q_t - \beta E\Delta q_{t+1}) + \omega_t, & \pi_t > 0 \\
\beta E\pi_{t+1} + d_2^* \hat{y}_t + \frac{a}{1-a}(\Delta q_t - \beta E\Delta q_{t+1}) + \omega_t, & \pi_t < 0 \\
\hat{d}_1 > \hat{d}_2 
\end{cases} \quad (38)$$

Now the equilibrium in the economy is described by the IS curve (17') and the Phillips curve (38) holding simultaneously. Minimizing the social loss function (33) subject to the Phillips curve (38), after similar derivations as in the symmetric case, we obtain the following interest rate rule:

$$\bar{r}_{t}^{opt} = \begin{cases} 
\left[ \frac{a}{b} + \pi^* \right]^+ + \frac{a d_1}{b(1-a)(d_1^2 + \lambda)} \phi_t \left( \frac{d_1}{b(d_1^2 + \lambda)} \right)^+ \omega_t + \frac{d_1}{b} \xi_t, & \pi_t > 0 \\
\left[ \frac{a}{b} + \pi^* \right]^+ + \frac{a d_2}{b(1-a)(d_2^2 + \lambda)} \phi_t \left( \frac{d_2}{b(d_2^2 + \lambda)} \right)^+ \omega_t + \frac{d_2}{b} \xi_t, & \pi_t < 0 
\end{cases} \quad (39)$$

Figure 1 analyses the optimal monetary policy in cases of a positive and a negative exchange rate or cost shocks of the same size under asymmetric price rigidity assuming zero target inflation.
Figure 1. Monetary policy reaction under asymmetric price rigidity
(supply shocks)

A negative supply shock shifts the kinked Phillips curve downwards by the same magnitude as a positive one shifts it upwards. If prices were symmetrically rigid (represented by a hypothetical PC*), the optimal points in cases of a positive and a negative shock would be points C and E respectively. In order to reach these points, the interest rate should be adjusted by the same absolute value. But since prices are assumed to be more sticky downwards than upwards, the optimal point in case of the negative supply shock is point D, which corresponds to a flatter part of the Phillips curve and, hence, lies to the left of point E. This means that the optimal interest rate rule is to adjust less in response to a negative supply shock than in response to a positive one.

But this asymmetry in monetary policy reaction would disappear had the target inflation been high enough to overweight the negative supply shock. In particular, it follows from equation (35) that if $|\omega| \leq \frac{d^2 + \frac{\lambda}{\lambda}}{\lambda} \pi^T$, the optimal inflation is non-negative and the economy is still on the steeper part of the Phillips curve6.

**Proposition 3.** If the supply shocks are large enough (e.g. $|\omega| > \frac{d^2 + \frac{\lambda}{\lambda}}{\lambda} \pi^T$) the optimal monetary policy should be asymmetric depending on the sign of the shock. The optimal degree of the interest rate adjustment should be higher in case of positive shocks than in case of negative ones due to higher downward price rigidity and lower downward pass-through, provided that $d < \sqrt{\lambda}$.

---

6 But if the kink of the Phillips curve was at the target level of inflation rather than at the zero inflation, as in Diana and Méon (2005), then for any level of the target inflation and supply shocks the optimal monetary policy should be asymmetric.
**Proof.** Recall from the proofs of propositions 1 and 2 that the derivatives of A and B with respect to parameter d are positive, provided that $d < \sqrt[4]{\lambda}$. Since prices are more flexible upwards than downwards, the adjustment in the interest rate should be more significant in case of an inflationary shock than in case of a deflationary one of the same size.

So, our theoretical model prescribes that the monetary policy should be non-linear with higher reaction to inflationary shocks than to deflationary ones of the same size due to the asymmetric price rigidity.

### 3. The empirical evidence

In this part we test the prescriptions of the model empirically for a set of countries. We concentrate our analysis on exchange rate shocks.

**Data**

Our sample is formed from developed countries with either clean or dirty floating exchange rate regimes according to Reinhart and Rogoff (2002) de facto classification: the USA, Canada, Australia, the United Kingdom, Euro area, Norway, Sweden, Czech Republic and Poland.

We study the following periods: 1990-2006 for the USA, Canada, Australia and the UK, 1998-2006 for Euro area and 1999-2006 for Norway, Sweden, Czech Republic and Poland.

We use the following quarterly time series:

- **Interest rate (r)** – the federal funds rate (USA), money market rate (Canada, Australia, Czech Republic), interbank rate (UK, Euro area), discount rate (Norway, Poland), repurchase rate (Sweden). All interest rates are annual.

- **Real effective exchange rate (reer)** – exchange rate against a trade-weighted basket of currencies in logarithm. An increase in reer means appreciation of the domestic currency.

- **Consume price index (p)** - in logarithm.

- **Real GDP (y or y_sa)** – GDP volume (2000=100) in logarithm, seasonally adjusted for Norway, Sweden, Czech Republic and Poland. In other countries seasonality was not observed.

The source of all data is the IMF’s International Financial Statistics.

**Methodology**

To take into account the endogenous nature of the above variables we estimate the following VAR model:
\[
\begin{align*}
    r_t &= x_{11}r_{t-1} + x_{12} \text{reer}_{t-1} + x_{13} \text{dummy}^* \text{reer}_{t-1} + x_{14} \pi_{t-1} + x_{15} y_{t-1} + x_{16} t + x_{17} + \zeta_{11} \\
    \text{reer}_{t} &= x_{21}r_{t-1} + x_{22} \text{reer}_{t-1} + x_{23} \text{dummy}^* \text{reer}_{t-1} + x_{24} \pi_{t-1} + x_{25} y_{t-1} + x_{26} t + x_{27} + \zeta_{12} \\
    \pi_{t} &= x_{31}r_{t-1} + x_{32} \text{reer}_{t-1} + x_{33} \text{dummy}^* \text{reer}_{t-1} + x_{34} \pi_{t-1} + x_{35} y_{t-1} + x_{36} t + x_{37} + \zeta_{13} \\
    y_t &= x_{41}r_{t-1} + x_{42} \text{reer}_{t-1} + x_{43} \text{dummy}^* \text{reer}_{t-1} + x_{44} \pi_{t-1} + x_{45} y_{t-1} + x_{46} t + x_{47} + \zeta_{14}
\end{align*}
\]

where the dummy variable equals 1 if reer goes up and 0 otherwise. We include the trend variable into the model since some of the variables are trended.

In general, the above VAR model looks as follows:
\[
\Omega_t = \Xi_\text{reer} r_{t-1} + X_3 \text{dummy}^* \text{reer}_{t-1} + X_6 t + X_7 + Z_t
\]  
(40')

where \(\Omega\) is a vector of endogenous variables \((r, \text{reer}, \pi, y)\), \(X\) is a 4*4 coefficient matrix, \(X_3\) is the dummy coefficient vector, \(X_6\) is the trend coefficient vector, \(X_7\) is a vector of intercepts and \(Z\) is a vector of residuals.

To test whether the exchange rate is an important variable in a country’s monetary policy we perform the variance decomposition test. This test shows us the percentage of the interest rate variance explained by real exchange rate shocks. Our theoretical model prescribes that this variance should be higher in countries with higher exchange rate pass-through effect on consumer prices (proposition 1).

**Hypothesis 1.** The percentage of variance of the interest rate explained by exchange rate shocks is higher in countries with higher pass-through effect.

To test the first hypothesis we need to know the degree of pass-through in the studied countries. Although there exist numerous empirical literature on the pass-through effect, their samples of countries are different from ours, and the estimates would be incomparable. Therefore, we estimate pass-through elasticities ourselves and compare them across our countries.

To estimate the pass-through elasticities we first estimate the following Vector Error Correction Model which takes into account long run adjustments:
\[
\Delta \Omega_t = \Xi_\text{reer} \Omega_{t-1} + X_7 + X_8 Z_{t-1} + Z_t
\]  
(41)

where \(\Omega\) is a vector of endogenous variables \((r, \text{reer}, \pi, y)\), \(\Delta\) means the first difference, \(Z_{t-1}\) represents the lagged residuals from cointegration equation among the endogenous variables, \(Z_t\) represents residuals, \(\chi\) is a 4*4 coefficient matrix, \(X_7\) is a vector of intercepts.

Then we build an impulse-response function to trace the effect of an exchange rate shock on consumer prices. We use the following Cholesky ordering:
\[
\text{reer} \to r \to p \to y
\]  
(42)

For example, real GDP is a trended variable, but we do not apply HP filter to avoid ad hoc de-trending procedure.

Usually pass-through effect is measured in response to nominal exchange rate changes, but in our case we can use the real exchange rate since the correlation between the nominal and the real exchange rates was close to 1 in the studied countries during the studies period.
i.e. we assume that the monetary authority reacts to an exogenous exchange rate shock by adjusting the interest rate which affects prices and output.

To test how monetary policy in the studied countries reacts to exchange rate shocks we test the sign and significance of coefficient $x_{12}$ in (40). According to the theoretical model the domestic currency appreciation should be accompanied by a reduction of the interest rate while its depreciation – by an increase in the interest rate, ceteris paribus (proposition 1).

**Hypothesis 2.** Coefficient $x_{12}$ in model (40) is negative.

Coefficient $x_{12}$ shows immediate reaction of the monetary policy to an exchange rate shock. But in fact the interest rate may adjust gradually. Indeed, as noted by Woodford (1999), gradual adjustment in the interest rate to changes in economic conditions is optimal as small but consistent interest rate changes have greater impact on long rates and, hence, on economic activity. To estimate such gradual adjustment we build impulse-response functions for the interest rates for the model (40) using the ordering (42).

**Hypothesis 3.** The response of the interest rate to an exchange rate shock is negative, cumulative over time and significant.

Finally, to test whether the monetary policy is indeed asymmetric in response to positive and negative exchange rate shocks we test the sign and significance of coefficient $x_{13}$ in model (40). According to our model, the domestic currency appreciation should be accompanied by a smaller by the absolute value reduction in the interest rate than the increase in the interest rate after the domestic currency depreciation of the same size (proposition 3).

**Hypothesis 4.** Coefficient $x_{13}$ in model (40) is positive.

**Results**

Table 1 presents selected results of estimation of models (40) and (41). Columns 3 and 4 report the estimates of the coefficients which are most important for our analysis, columns 5 and 6 show characteristics of the model as a whole, columns 7-9 show the average values of the endogenous variables and columns 10-11 show the variance decomposition results. Sign “+” in column 11 means that the exchange rate explains the highest percentage of the interest rate variance in comparison with the other variables (inflation and output). Column 12 reports the estimated degree of exchange rate pass-through over 2 years.

---

9 Actually changing the ordering does not alter the estimation results.

10 The negative value of the pass-through elasticity is expected since the domestic currency depreciation should lead to an increase in prices and visa versa.
Table 1. Results of estimation of models (40) and (41)

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$R^2$</th>
<th>F</th>
<th>Mean $r$ pa, %</th>
<th>Mean reer</th>
<th>Mean infl pa</th>
<th>Var* for 2 years (%)</th>
<th>Max VAR?</th>
<th>PTE for 2 years</th>
</tr>
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<tbody>
<tr>
<td>USA</td>
<td>1990Q3</td>
<td>-4.35</td>
<td>0.04</td>
<td>0.96</td>
<td>223.21</td>
<td>4.20</td>
<td>4.50</td>
<td>0.02</td>
<td>62.28</td>
<td>+</td>
<td>-0.022</td>
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<td></td>
<td>2006Q2</td>
<td>[-5.38]</td>
<td>[1.74]</td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Canada</td>
<td>1990Q3</td>
<td>1.45</td>
<td>0.00</td>
<td>0.92</td>
<td>116.22</td>
<td>4.85</td>
<td>4.64</td>
<td>0.02</td>
<td>0.125</td>
<td>-</td>
<td>0.006</td>
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<tr>
<td></td>
<td>2006Q2</td>
<td>[0.77]</td>
<td>[0.06]</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Australia</td>
<td>1990Q3</td>
<td>1.00</td>
<td>0.01</td>
<td>0.97</td>
<td>311.96</td>
<td>6.11</td>
<td>4.70</td>
<td>0.03</td>
<td>8.04</td>
<td>+</td>
<td>-0.018</td>
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<tr>
<td></td>
<td>2006Q2</td>
<td>[1.22]</td>
<td>[0.51]</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>UK</td>
<td>1990Q3</td>
<td>-0.31</td>
<td>0.07</td>
<td>0.97</td>
<td>357.55</td>
<td>6.26</td>
<td>4.47</td>
<td>0.03</td>
<td>4.80</td>
<td>-</td>
<td>0.007</td>
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<tr>
<td></td>
<td>2006Q1</td>
<td>[-0.33]</td>
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<tr>
<td>Euro Area</td>
<td>1998Q3</td>
<td>-3.72</td>
<td>-0.03</td>
<td>0.93</td>
<td>52.73</td>
<td>3.02</td>
<td>4.71</td>
<td>0.02</td>
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<td>-0.006</td>
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<tr>
<td></td>
<td>2006Q1</td>
<td>[-1.92]</td>
<td>[-1.04]</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>1999Q1</td>
<td>-6.89</td>
<td>0.06</td>
<td>0.97</td>
<td>126.60</td>
<td>6.60</td>
<td>4.73</td>
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<tr>
<td>Sweden</td>
<td>1999Q1</td>
<td>-4.86</td>
<td>0.02</td>
<td>0.94</td>
<td>57.13</td>
<td>2.98</td>
<td>4.56</td>
<td>0.01</td>
<td>1.67</td>
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<td></td>
<td>2006Q2</td>
<td>[-1.89]</td>
<td>[1.10]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Czech Republic</td>
<td>1999Q1</td>
<td>-7.59</td>
<td>0.00</td>
<td>0.98</td>
<td>247.51</td>
<td>3.75</td>
<td>4.73</td>
<td>0.02</td>
<td>52.44</td>
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<td>-0.053</td>
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<tr>
<td></td>
<td>2006Q2</td>
<td>[-4.19]</td>
<td>[-0.19]</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Poland</td>
<td>1999Q1</td>
<td>-8.04</td>
<td>0.11</td>
<td>0.97</td>
<td>147.94</td>
<td>11.08</td>
<td>4.62</td>
<td>0.04</td>
<td>39.39</td>
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<td></td>
<td>2006Q2</td>
<td>[-1.88]</td>
<td>[1.06]</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

* - The percentage of variance of the interest rate explained by the exchange rate

The variance decomposition test shows that the exchange rate is a significant variable in explaining the interest rate behavior in 5 countries out of 9: the USA, Euro Area, Norway, Czech Republic and Poland. Furthermore, in all these countries the exchange rate explains the highest percentage of the interest rate variance. And in most of these countries (the USA, Czech Republic and Poland) the estimated pass-through effect is the greatest among all countries in the sample, what can explain the high percentage (over 40%) of the interest rate variance explained by the exchange rate, according to our model.

Although pass-through effect in the Euro Area and Norway is close to zero, the exchange rate nevertheless plays a significant role in the monetary policy of these countries. We see the following explanations for this. Norway, being a resource exporter, has been experiencing the real appreciation of its currency after 1999 due to the rising prices of resources. And it is after 1999 when the real exchange rate started to play a role in its monetary policy: the interest rate is increased to smooth the currency appreciation. And the estimated pass-through effect during this period is zero (even positive) because the currency was mainly appreciating, but there is significant empirical evidence in favor of asymmetric pass-through effect\(^\text{11}\). Concerning the Euro Area, since Euro is one

\(^\text{11}\) It was observed empirically that the domestic prices rise more as a result of the domestic currency depreciation, than they fall as a result of the domestic currency appreciation (e.g. Goldberg, 1995; Pollard & Coughlin, 2004, for the USA;
of the most influential currencies, it is important to take its fluctuations into account in designing the optimal monetary policy, while the pass-through effect may be small due to a low share of imported consumer goods.

The variance decomposition test shows that in Canada, Australia, the UK and Sweden the exchange rate does not play a significant role in the long run in determining the monetary policy. And if we look at the pass-through elasticities in these countries, they are either positive and almost zero (Canada and the UK) or have the correct sign (negative) but very low (less than 2% over two years in Australia and Sweden).

Figure A4.1 in Appendix 4 plots the percentage of the interest rate variance explained by the exchange rate fluctuations against pass-through elasticities. We can observe the following relationship: the higher is the pass-through by the absolute value, the higher is the variance, and the more significant role is played by the exchange rate in determining the interest rate fluctuations in the long run. This supports our first hypothesis.

Concerning the immediate reaction of the interest rate to a real exchange rate shock, it is significant for the same countries plus Sweden. In other words, coefficient $x_{12}$ turns out to be significantly negative for all countries except Canada, Australia and the UK. This means that in these countries the domestic currency appreciation is followed by a reduction in the interest rate, while its depreciation – by an increase. Therefore, we cannot reject our second hypothesis for the countries in which the exchange rate indeed plays a role.

Moreover, the reaction of the interest rate to an exchange rate shock is greater (coefficient $x_{12}$ is greater by the absolute value) the higher is the pass-through effect, what supports our first hypothesis again. Figure A4.2 in Appendix 4 clearly demonstrates this relationship.

To analyze how the interest rates adjust to exchange rate shocks over time, we estimate the impulse-response functions, which are presented in Figure A4.3 in Appendix 4 together with the corresponding confidence intervals. We can observe a significantly negative long run reaction of the interest rates in the USA, Euro Area, Czech Republic and Poland. Indeed, the interest rates in these countries adjust gradually to exogenous exchange rate shocks, as was predicted by the third hypothesis. And again, these are the countries with the highest pass-through.

The long run reaction of the interest rate to an exchange rate shock in Norway is insignificant, although negative, while coefficient $x_{12}$ is significantly negative and the percentage of the interest rate variance explained by the exchange rate is significant and equals to 30%. Therefore, it can be concluded that the exchange rate plays a role in determining the monetary policy in Norway only in

Webber, 2000, for a set of Asian countries; Ohno, 1990, for Japan; Dobrynskaya & Levando, 2005, for Russia)
the short run, and in general this role is not so significant as in the USA, Euro Area, Czech Republic and Poland\(^{12}\).

In Sweden, the exchange rate plays even smaller role, and only in the short run, as the long run impulse-response function and the variance decomposition tests give statistically insignificant results.

We do not find any evidence that the monetary policies in Canada, Australia and the UK take into account their exchange rate fluctuations. Having also estimated impulse-response functions for the interest rate in response to inflation and output shocks, we find that the only significant variable in Canada is the output, in the UK – inflation, and in Australia nothing is significant in explaining the interest rate behavior. The same conclusion follows from the variance decomposition test: in Canada the highest percentage of the interest rate variance is explained by the output (20\%), in the UK – by inflation (10\%), while in Australia each of the variables explains not more than 8\% of the variance.

Our findings go in line with the conclusions of Nogueira Junior (2006), who studies monetary policies in Canada, Sweden, the UK, Czech Republic, Brasil, Mexico, South Africa and South Korea and does not find any evidence of foreign exchange interventions (including adjustments in the interest rate in response to exchange rate shocks – so-called “interest rate defense of exchange rate”) in Canada, Sweden and the UK, while confirming significant interventions in Czech Republic and other countries.

What concerns the monetary policy asymmetry, coefficient \(x_{13}\) turns out to be significant for only three countries: the USA, the UK and Norway. This coefficient is estimated to be positive and much less by the absolute value, than the corresponding coefficient \(x_{12}\). This means that while, for example, in the USA the interest rate rises by 4.35 percentage points in response to 1\% depreciation of the dollar, it falls by 4.31 percentage points in response to a 1\% appreciation of the dollar. A similar conclusion is valid for Norway. Such interest rate behavior corresponds to the prescriptions of our model and supports hypothesis 4.

Dolado, Pedrero and Ruge-Murcia (2004) also analyze the US monetary policy and find non-linearity in its interest rate rule for the period 1983-2000. They conclude that when inflation is above the target the interest rate is adjusted more than when inflation is below the target, and they explain this by asymmetric central bank preferences. But the authors study the US as a closed economy, and do not analyze the exchange rate impact, which is significant according to our findings.

The situation in the UK is intriguing. Since coefficient \(x_{12}\) turns out to be insignificant while coefficient of the asymmetry \(x_{13}\) is significant, we can conclude that the monetary authorities behave

\(^{12}\) The same can also be concluded from the variance decomposition test since the percentage of variance in Norway is the smallest among these five countries.
asymmetrically, although without paying much attention to the exchange rate in general. Taylor and Davradakis (2006) also find asymmetries in the UK monetary policy, but further research is needed here.

For the other countries in the sample we do not find significant asymmetries in the reaction of the monetary policy to exchange rate shocks. Nevertheless, the estimates of coefficient $x_{13}$ are positive in most cases, what corresponds to the prescriptions of our model.

We see two explanations for our finding that the monetary policy is symmetric on most countries.

First, according to our model, under some conditions the optimal monetary policy is indeed symmetric. For example, if the target inflation is rather high while the variance of shocks is rather low so that even deflationary shocks only cause disinflation, then there is no reason for the asymmetric monetary policy.$^{13}$

Second, even if the optimal monetary policy should be asymmetric indeed, Central Banks might not know this since this is a new idea in the literature. In such a case the monetary policy rules should be revised in order to minimize social losses.

4. Conclusion

In this paper we build a microfounded general equilibrium sticky price model of a small open economy. Using a quadratic loss function as an approximation of the social utility losses, we find that the optimal monetary policy rule is to adjust the interest rate in response to exogenous exchange rate, supply and demand shocks with positive coefficients. We claim that the optimal degree of such adjustment depends positively on pass-through effect and negatively on price stickiness in an economy.

Since the numerous empirical evidence speaks in favour of asymmetric price rigidity, in our theoretical model we assume lower price flexibility in case of downward adjustments, resulting from deflationary exchange rate or supply shocks. Under this assumption, the optimal monetary policy should be different in cases of positive and negative shocks. We claim that deflationary shocks such as the domestic currency appreciation or a reduction in raw materials prices should be accompanied by a smaller adjustment in the interest rate than inflationary ones of the same size since prices are more sticky downwards.

This analysis is new to the Keynesian literature, it is interesting from the theoretical point of view and has important practical implications for the conduct of monetary policy. It predicts that in order to minimise the social losses, the monetary authority should determine not only the direction

$^{13}$ According to our model, the only reason the monetary policy asymmetry is the asymmetric price rigidity. But, in general, there may be other reasons, e.g. asymmetric central bank preferences (Dolado, Pedrero and Ruge-Murcia, 2004) or asymmetric exchange rate pass-through effect (Dobrynskaya, 2008).
of the required policy instrument change, but also its magnitude depending on the sign of a shock. If the monetary policy rule is specified so that it does not take into account such asymmetries, then following this rule may result in the equilibrium inflation and output gap, which are far from optimal. For example, if there is significant downward price rigidity in an economy, while it may be optimal to increase the interest rate significantly as a result of a sharp depreciation of the domestic currency, it may also be optimal not to respond to the domestic currency appreciation at all. Then, following a symmetric rule of an adjustment in the interest rate due to an exchange rate shock will lead to higher social losses in case of an appreciation of the domestic currency than would be without monetary policy reaction.

We test the predictions of our model for a set of developed countries. We find that the exchange rate plays a significant role in determining the monetary policy in most countries, and that the domestic currency appreciation is generally accompanied by a reduction of the interest rate while the domestic currency depreciation leads to an increase in the interest rate. We also find some evidence of the asymmetry in the monetary policy reaction to positive and negative exchange rate shocks, which is in line with our model, although this asymmetry is quite weak in most countries.

References

Appendix 1. Derivation of the IS curve

Derivation of the Euler equation (4):

\[
\max L_t = E_t \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{t+k}^{1+\epsilon}}{1 - \sigma} - \frac{N_{t+k}^{1+\phi}}{1 + \phi} \right) - \lambda E_t \sum_{k=0}^{\infty} \left( \frac{1}{1 + r_{t+k-1}} \right)^k \]

\[
\begin{pmatrix}
D\tau_k + W_{\tau+k}N_{\tau+k} + T_{\tau+k} - P_{\tau+k}C_{\tau+k} - E_t \left( \frac{1}{1 + r_{\tau+k}} D_{\tau+k+1} \right)
\end{pmatrix}
\]

\[
\frac{\partial L}{\partial C_i} = C_i^{\gamma} + \lambda P_i = 0 \quad \Rightarrow \quad \lambda = - \frac{C_i^{\gamma}}{P_i}
\]

\[
\frac{\partial L}{\partial C_{i,t+1}} = E_t \left( \beta C_{i,t+1}^{\gamma} + \frac{\lambda}{1 + r_t} P_{i,t+1} \right) = 0 \quad \Rightarrow \quad \lambda = - E_t \left( \beta (1 + r_t) C_{i,t+1}^{\gamma} \right)
\]

Equating \(\lambda\), we get:

\[
\beta (1 + r_t) E_t \left( \frac{C_{t+1}}{C_t} \right)^{\gamma} \frac{P_t}{P_{t+1}} = 1
\]

Derivation of (13):

The following demand functions correspond to consumer price index (2):

\[
C_{H,i} = (1 - \alpha) \left( \frac{P_{H,i}}{P_t} \right)^{-\eta} C_i \tag{A1}
\]

\[
C_{F,i} = \alpha \left( \frac{P_{F,i}}{P_t} \right)^{-\eta} C_i \tag{A2}
\]

\[
C_{i,t} = \left( \frac{P_{i,t}}{P_{i,t}} \right)^{-\eta} C_{F,i} \tag{A3}
\]

\[
C_{H,i}(j) = \left( \frac{P_{H,i}(j)}{P_{H,i}} \right)^{-\eta} C_{H,i} \tag{A4}
\]

\[
C_{i,t}(j) = \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\eta} C_{i,t} \tag{A5}
\]

Substituting (A1) into (A4), we obtain the domestic demand for good j:

\[
C_{H,i}(j) = \left( \frac{P_{H,i}(j)}{P_{H,i}} \right)^{-\eta} (1 - \alpha) \left( \frac{P_{H,i}}{P_t} \right)^{-\eta} C_i \tag{A6}
\]
Since all economies are characterized by the same preferences, we make use of equations (A1)-(A5) to derive the demand of economy $i$ for good $j$, keeping in mind that good $j$ is an import good for them:

$$C^i_{H,j}(j) = \left( \frac{P_{H,j}(j)}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{H,j}}{P_{H,t}} \right)^{\gamma} a \left( \frac{P_{F,j}}{P_{i,t}} \right)^{-\gamma} C^i_j \tag{A7}$$

Substituting (A6) and (A7) into the aggregate demand function for good $j$, we get:

$$Y_i(j) = C_{H,j}(j) + \int_0^1 C_{H,j}(j) di = \left( \frac{P_{H,j}(j)}{P_{H,t}} \right)^{-\gamma} (1 - a) \left( \frac{P_{H,j}}{P_{t}} \right)^{-\gamma} C_i + \int_0^1 \left( \frac{P_{H,j}(j)}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{F,j}}{P_{i,t}} \right)^{-\gamma} C_i di \tag{A8}$$

Derivation of (14):

From (A8) follows:

$$Y_t = \int_0^1 Y_t(j)^{-1} dj = \left( 1 - a \right) \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} C_i + a \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_{i,t}} \right)^{-\gamma} C_i di = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} (1 - a) C_t + a \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_{i,t}} \right)^{-\gamma} C_i di = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} \left( 1 - a \right) C_i + a \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_{i,t}} \right)^{-\gamma} C_i di = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} \left( 1 - a \right) C_i + a \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{P_{F,t}}}{P_{i,t}} \right)^{-\gamma} C_i di = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} \left( 1 - a \right) C_i + a \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{P_{F,t}}}{P_{i,t}} \right)^{-\gamma} C_i di = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma} \left( 1 - a \right) C_i + a \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\gamma} Q_{i,t}^i C_i di$$

Since all economies are symmetric, and also assuming that the uncovered interest parity holds:

$$1 + r_i' = (1 + r_t)E_{t'} \frac{E_{i}}{E_{i,t'}}$$

the optimal consumption in economy $i$ is characterized by the Euler equation similar to (4):
\[ \beta (1 + r_t) E_t \left\{ \frac{C_{i,t+1}}{C_i} \right\}^{-\sigma} \left( \frac{P^i_{t+1}}{P^i_t} \right) \left( \frac{E^i_{t+1}}{E^i_t} \right) = 1 \] (A10)

From equations (4) and (A10) follows:

\[ C_s = C_i^\frac{1}{Q_{i,s}^r} \] (A11)

We substitute (A11) into (A9):

\[
Y_t = \left( \frac{P_{H,s}}{P_t} \right)^{-\frac{1}{\tau}} C_t \left[ (1 - \alpha) + \alpha \int_0^1 \left( \frac{P_{E,s}^{i,t}}{P_{E,s}^{i,t}} \right)^{\frac{1}{\tau}} Q_{i,s}^{\frac{1}{\tau}} di \right] = \\
= \left( \frac{P_{H,s}}{P_t} \right)^{-\frac{1}{\tau}} C_t \left[ (1 - \alpha) + \alpha \int_0^1 \left( \frac{P_{E,s}^{i,t}}{P_{E,s}^{i,t}} \right)^{\frac{1}{\tau}} Q_{i,s}^{\frac{1}{\tau}} di \right] = \\
= \left( \frac{P_{H,s}}{P_{E,s}^{i,t}} P_{E,s}^{i,t} \right)^{-\frac{1}{\tau}} C_t \left[ (1 - \alpha) + \alpha \int_0^1 \left( S_{i,s} S_t^{i,t} \right)^{\frac{1}{\tau}} Q_{i,s}^{\frac{1}{\tau}} di \right] = \\
= S_t^{\frac{1}{\tau}} C_t \left[ (1 - \alpha) + \alpha \int_0^1 \left( S_{i,s} S_t^{i,t} \right)^{\frac{1}{\tau}} Q_{i,s}^{\frac{1}{\tau}} di \right]
\]

Derivation of (17):

From equation (16) follows:

\[ c_t = y_t + v q_t \]

We substitute this into equation (5):

\[
y_t + v q_t = E_t[y_{t+1} + v q_{t+1}] - \frac{1}{\sigma} (\tilde{r}_t - E_t[\pi_{t+1}] - \rho) \quad \Rightarrow \quad \\
y_t = E_t[y_{t+1}] - \frac{1}{\sigma} (\tilde{r}_t - E_t[\pi_{t+1}] - \rho) + v E_t[q_{t+1} - q_t] = \\
= \frac{\rho}{\sigma} + E_t[y_{t+1}] - \frac{1}{\sigma} (\tilde{r}_t - E_t[\pi_{t+1}]) + v E_t[\Delta q_{t+1}] \]

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Appendix 2. Derivation of the Phillips curve

Derivation of (19):

\[ p_{H,t}(j) \sum_{k=0}^{\infty} \theta^k \hat{\beta}^k - E_t \sum_{k=0}^{\infty} \theta^k \hat{\beta}^k \tilde{p}_{H,t+k}(j) = 0 \]

\[ p_{H,t}(j) \frac{1}{1 - \theta \hat{\beta}} = E_t \sum_{k=0}^{\infty} \theta^k \hat{\beta}^k \tilde{p}_{H,t+k}(j) \]

\[ p_{H,t}(j) = (1 - \theta \hat{\beta}) \tilde{p}_{H,t}(j) + (1 - \theta \hat{\beta}) E_t \sum_{k=1}^{\infty} \theta^k \hat{\beta}^k \tilde{p}_{H,t+k}(j) \]

\[ p_{H,t}(j) = (1 - \theta \hat{\beta}) \tilde{p}_{H,t}(j) + \theta \hat{\beta} E_t p_{H,t+1}(j) \]

Derivation of (21):

We iterate forward (20):

\[ E_t p_{H,t+1} = \theta p_{H,t} + (1 - \theta) E_t p_{H,t+1}(j) \]

We add and subtract \( p_{1,t} \):

\[ (1 - \theta) E_t p_{H,t+1}(j) = E_t p_{H,t+1} - \theta p_{H,t} + p_{H,t} - p_{H,t} \]

(A12)

Since \( E_t \pi_{H,t+1} \equiv E_t p_{H,t+1} - p_{H,t} \), equation (A12) turns into:

\[ (1 - \theta) E_t p_{H,t+1}(j) = E_t \pi_{H,t+1} + (1 - \theta) p_{H,t} \]
**Appendix 3**

**Table A3.1. Proof of proposition 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha/(1-\alpha)$</th>
<th>$d$</th>
<th>$b$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta A$</td>
<td>$d$</td>
<td>$\frac{a(\lambda - d^2)}{(1-\alpha)b(d^2 + \lambda)^2}$</td>
<td>$-\frac{a}{(1-\alpha)b(d^2 + \lambda)^2}$</td>
<td>$-\frac{a}{(1-\alpha)b(d^2 + \lambda)^2}$</td>
</tr>
<tr>
<td>Sign</td>
<td>$+$</td>
<td>$+$ if $d &lt; \sqrt{\lambda}$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>
Appendix 4

Figure A4.1. The relation between the percentage of the interest rate variance explained by the exchange rate and the degree of pass-through

Figure A4.2. The relation between the degree of reaction of the interest rate to exchange rate and the degree of pass-through
Figure A4.3. Response of the interest rate to a exchange rate impulse over 2 years