Power and preferences: an experimental approach[1]

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Abstract

The paper uses an experimental approach to study the voting power distribution in the context of classical model, as well as in generalized form which takes into account players' preferences to coalesce with each other. Our results extend those of Montero, Sefton & Zhang (2008), confirming their basic findings using independent experimental data, and explain some of their empirical paradoxes. A major result of our experiment is that even small modifications of preferences lead to statistically significant differences in players' shares, justifying the use of generalized power indices over classical ones. Furthermore, we demonstrate that the interplay of preferences significantly affects the process of bargaining and the resulting coalitions.

Keywords: voting power, preferences, experiments, Banzhaf index, symbolic value

JEL codes: C71, C92, D72

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1 Introduction

Voting power studies have evolved consistently since the first classic papers were published in the mid-twentieth century (see Penrose (1946); Shapley & Shubik (1954); Banzhaf (1965); Coleman (1971)). However, the central question - how to measure power possessed by the members of a voting body, - remains largely unsettled. Classical indices of Coleman, Banzhaf, and Shapley and Shubik are known for a long time, and all follow the same logic of determining the voting power of a player by the fraction of instances in which the player is *pivotal*, i.e. her desertion from a coalition will transform it from winning to a losing one. These indices, however, are incomplete in many respects: for instance, they are subject to certain paradoxes (e.g., the Banzhaf index demonstrates the so-called paradox of new members – Brams(1975), Brams & Affuso (1976)), fail to capture the process of coalition formation as well as unevenly distributed preferences of the voters towards each other (Aleskerov, 2006).

In the present paper we explore experimentally the predictive model of classical and extended voting power indices under various treatment conditions and in two generically different settings. The first one is a classical coalitional game with or without veto power, which largely follows the experiment of Montero, Sefton & Zhang (2008), henceforth referred to as MSZ. The second, and novel, part of the experiment measures predictive power of the generalized power indices introduced in Aleskerov (2006). These indices capture the possibility that players may have different preferences towards each other and thus ought to be more inclined to form some coalitions rather than others. We confirm experimentally that such preferences, however small, may produce large effects and change the likelihood of observing different coalitions and different gains accrued to the players.

The paper is organized as follows. Section 2 describes the main theoretical notions and voting indices in both classical and preference-adjusted cases. Section 3 outlines the hypotheses to be checked experimentally and describes our experimental setup. Section 4 deals with the experimental design and procedures. Section 5 contains our main results. Section 6 concludes. Appendix A contains the instructions for the subjects of our experiment.

2 Main notions

A *coalition* S is any subset of N players, $|N| = n$. The set of all possible coalitions is denoted 2^N. Each player $i \in N$ has a certain number of votes w_i (called also voting weights) she may use to support a decision. A *quota* q is the least number of votes required to pass the decision. A coalition $S \subseteq 2^N$ is *winning* if $\sum_{i \in S} w_i \geq q$. Similarly, a *losing* coalition is the one that lacks votes to adopt the decision. A player $i \in S$ is said to be *pivotal* in a coalition S if S is winning while $S\setminus\{i\}$ is losing. Let the payoff of a coalition S be $v(S)$; define $v(S) = 1$ if S is winning and $v(S) = 0$ if S is losing.[5]

To quantify somehow the power the players possess, several power indices have been proposed, of which the Banzhaf index β (Banzhaf (1965)) is perhaps the most intuitive. This (normalized) index shows the relative proportion of winning coalitions in which player i is pivotal with regard to all coalitions in which all players including *i* are pivotal, or

$$\beta_i = \frac{\sum_S (v(S) - v(S \setminus\{i\})}{\sum_{j\in N} \sum_S (v(S) - v(S \setminus\{j\})} \qquad (1)$$

A family of preference-based power indices introduced in Aleskerov (2006) can be defined in a similar manner, and aims at taking into account players' preferences towards coalescing with each other. Assume that such preference for each player i towards player j is given by a real number $p_{ij}, i, j = 1, \dots, n$. We shall refer to these numbers as *modifiers* for they modify each player's attitude towards a given coalition $S \subseteq 2^N$. Next, define a function $f_i(S)$ - *intensity of connections* between a player $i \in N$ and a coalition S as $f_i(S): N \times 2^N \to \mathbb{R}$. For every player i let $\chi_i = \sum_S f_i(S)$ be the sum of these intensities of connections taken over all those coalitions for which player i is pivotal. Finally, define the voting power index of an agent i as

$$\alpha_i = \frac{\chi_i}{\sum_{j\in N} \chi_j} = \frac{\sum_S f_i(S)(v(S) - v(S \setminus\{i\})}{\sum_{j\in N} \sum_S f_j(S)(v(S) - v(S \setminus\{j\})} \qquad (2)$$

The very idea of α_i is similar to that of the Banzhaf index, the difference being that in (1) we evaluate the number of coalitions in which *i* is pivotal, and not the intensity of i's connections within such coalitions.

The following forms of intensity functions may be defined:

a. Product intensity of i's connection with the other members of S:

$$f_i^+(S) = \prod_{j\in S\setminus\{i\}} p_{ij} \qquad (3)$$

b. Product intensity of connection of the other members of $S$with i:

$$f_i^-(S) = \prod_{j\in S\setminus\{i\}} p_{ji} \qquad (4)$$

Naturally, other forms of intensity functions, e.g., additive ones, are possible (see Aleskerov (2006). The intuition behind this extension is quite simple: individual voters in any

[5]A dual notion of *swing* may be used instead: a coalition $S: i \notin S$ is a swing for player i if S is losing, while $S \cup \{i\}$ is winning. For a pivotal player $i \in S$, $v(S) - v(S \setminus\{i\}) = 1$. Alternatively, if a coalition S is a swing for i, $v(S \cup \{i\}) - v(S) = 1$.

committee, parliament etc. may have different preferences towards the identity of potential coalition partners, which are captured by the values of p_{ij} for all i and j. If all p_{ij} are neutral (in our case, equal 1), then the preference-based index equals the classical Banzhaf one; but generally, $\alpha_i \neq \beta_i$.

This extension offers a lot of descriptive flexibility, especially in cases when apparently small differences among the voters can prevent them from coalescing. On the other hand, however compelling are the indices α intuitively, it should be checked whether they can be validated experimentally; and more generally, how robust the influence of pairwise preferences is on players' behaviour in different contexts where the players' voting weights and the quota are varied. It is also interesting to see which of the numerous ways to define the intensity functions is justified in practice; and whether the scale of intensity matters. Finally, quantitative estimates of the effects of modifiers on the voting outcomes and players' payoffs are worth studying. To address these questions, we use laboratory experiments.

3 Experimental research

Experimental investigations of voting power are less numerous than theoretical ones, and sometimes appear to go at odds with theoretical predictions (Maschler (1978), Ulrich (1990)). Selten and Kuon (1993) studied three-person bargaining games that are "semi-structured" in that it combines randomly chosen first-movers with sequential adoptions or rejections of the previous proposals by the subsequent players. In a series of long experiments (lasting for about 4 hours) the authors confirm theoretical predictions of the neutral equilibrium which is a subgame-perfect equilibrium such that «at every information set every locally optimal choice is taken with the same probability» (Selten and Kuon, 1978, p.264).

A recent paper by MSZ studied the *paradox of the new members*: when a new member is added to a voting body, the power indices of the original members may increase even if the voting weights and the decision rule remain the same [6]. In their experiment, participants were able to propose and vote on how to distribute a fixed budget of 120 experimental units among them in three treatments corresponding to the examples from Brams and Affuso (see Table 1).

In all treatments there is a "strong" player (with 3 votes) and two "weak" players (with 2 votes). In addition, in the enlarged game there is also a newcomer – player 4 with 1 vote.

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[6] Brams & Affuso (1976) argued that this is a paradox related to voting power rather than to the mathematical properties of the power index chosen. This conclusion, however, was challenged in the literature (Barry (1980)).

VETO treatment				SYMMETRIC treatment				ENLARGED treatment				
Player#	1	2	3	Player#	1	2	3	Player#	1	2	3	4
Votes#	3	2	2	Votes#	3	2	2	Votes#	3	2	2	1
Banzhaf	72	24	24		40	40	40		50	30	30	10
Quota	5			Quota	4			Quota	5			

Table 1. The three treatments studied by MSZ: veto, symmetric, and enlarged

Whether the game is VETO or SYMMETRIC is defined by the quota: in the VETO game it is set at 5 votes meaning that player 1 has veto power (no decision can be made without her consent), whereas in the SYMMETRIC game powers of each player are *a priori* equal. In the ENLARGED game a new member is added and this setting was used to test against the paradox occurrence with regards to the VETO and SYMMETRIC games. We will refer to these treatments for short as V-game, S-game and E-game, respectively. MSZ confirm experimentally the direction, if not the scale, of theoretical predictions of the classical indices: as expected, player 1 receives significantly higher share in the E- than in the S-game, and players 2 and 3 – significantly more in the E- than in the V-game.

We use the same three games for our control treatment, but the focus of our paper is different: we look at the deviations of the bids from the theoretical predictions, and in particular, at the explanatory potential of the preference-based power indices. To do that, in our experimental treatment we explicitly model a situation where players have different preferences to coalesce, i.e. when modifiers p_{ij} are different from 1 in the same three games plus a FURTHER game, which we label F. We adopt the cardinal preference model for the modifiers (see Aleskerov, 2006), and use multiplicative aggregation: a player's share specified by a proposal is multiplied by her cardinal preferences towards each of those players she is in coalition with[7]. The results we obtain confirm our main hypothesis that modifiers do matter when applied to the players' payoffs, but also suggest a similar effect of the inherent features of the baseline experimental design in both control and experimental treatments. We refer to the former type of modifiers as to explicit, and to the latter – as to implicit modifiers, which will be discussed below in greater details.

We turn now to the description of our design and state the *a priori* hypotheses of our experiments.

[7] Of course, other models could have been used as well – for instance, modifiers can be ordinal, aggregation procedures can be additive, etc. Given that we obtain the desired effects of the modifiers, we leave these extensions for future research.

3.1. Symmetric game

In the symmetric case (see Table 1), the Banzhaf index, which treats all coalitions as equiprobable, is equal to 1/3 for all players, and predicts that payoffs will be equal to 40 (out of 120 points) for each player[8].

Let us now extend the standard S-game by adding to the following table of explicit modifiers (see Table 2). In terms of this table, if the row player *i* is a member of the winning coalition together with the column player *j*, the payoff accrued to the row player is multiplied by the table entry from the i^{th} row and the j^{th} column, irrespectively of the voting weights and decisions made by either player. This specification reflects the fact that real voters who enter in some coalitions might receive positive or negative "spillovers" from their coalition partners. Such spillovers may take various forms, such as higher transaction costs of negotiations, quality of personal matching idiosyncratic to that pair of players, reputational losses in the eyes of their electorate etc., depending on the context.

The S-game with modifiers from Table 2 represents our experimental treatment, which we refer to as the 1-game. Note that the modifiers from Table 2 affect players' payoffs and preferences only in the coalitions involving players 2 and 3. Further, they are very small ones: if player 2 is in winning coalition with player 3, payoff of the former player is multiplied by 1.01, i.e. becomes higher by 1%. All other modifiers are equal to 1, implying that all players receive the payoff according to the accepted proposal.

Preferences of player *i* towards *j*	Player 1	Player 2	Player 3
Player 1	-	1	1
Player 2	1	-	1.01
Player 3	1	1	-

Table 2. Modifiers for the S-game

Modifier of 1.01 is indeed very small: if we take into account the voter's preference to coalesce, and measure the intensities by the intensity function f^- as defined by (4), the α power index predicts that the pie shares are 39.9338 for players 1 and 2, and 40.1335 for player 3. Arguably, both cases can be viewed as minor changes of the payoffs. Yet we *expect earnings of players 2 and/or 3 to be higher in the 1-games (with modifiers from Table 2) than in the control S-game* (**H1**). In addition, *the coalition {2, 3} is expected to occur significantly more frequently than other coalitions in the 1-game* (**H2**).

[8] We limit attention to the Banzhaf index for two reasons: first, it is the simplest of the classical ones, and second, it has the same structure as the preference-based index α.

If either of these hypotheses is correct, we have evidence that preferences do matter in voting games: small changes in material payoffs make big difference. Moreover, H2 is supported by some recent works in biased preferences, such as Ariely (2008) or Warber et al (2008). These works imply that minor changes in stimuli may have their "symbolic value[9]", leading to visible and significant differences in the observed behaviour. Drawing on these conjectures, we expect the player 1 (the strong player) who is deprived of privileged attitude of the other players, will receive less than her predicted share in the 1-game treatment. Hence, it is possible that, *upon getting experienced, player 1 will try to 'buy' one of the other players' courtesy, offering them more than in the control S-game*. This is our third hypothesis (**H3**). It is hard to predict on the prior grounds whether this will be player 2 or 3, but the size of this payment off player 1's fair share (and the share we observe in the S- game) would give us an idea of what the price of preferences is.

3.2. Veto game

The V-game differs from its symmetric counterpart in that now 5 votes are needed to pass the proposal, which give player 1 the veto power (see Table 1), yet the agreement is still not possible without one of the other players. The Banzhaf index equals 3/5 for player 1 and 1/5 for both players 2 and 3, predicting that player 1 gets 72, while 2 and 3 – 24 points each. Now consider the V-game with the modifiers as defined in Table 3 (referred to as the 2-game).

Preferences of *i* \ towards *j*	Player 1	Player 2	Player 3
Player 1	-	1	1
Player 2	0.99	-	1
Player 3	0.99	1	-

Table 3. Modifiers for the V-game.

Our first hypothesis here (**H4**) is that *earnings of player 1 will be significantly higher in the V- than in S-game*. Furthermore, as players 2 and 3 dislike player 1 just a bit, using the same reasoning as above, we hypothesize that *the agreements reached in the 2-game will invoke significantly higher shares for player 2 and 3 at the expense of player 1 with respect to the control V-games with the neutral modifiers of 1* (**H5**). Finally, we may also hypothesize that (**H6**) *the bargaining time needed to reach an agreement in the 2-games will be significantly higher in the 2- than in the control V-games*.

[9] We could have used larger modifier values, but deliberately chose very small ones. If our hypotheses hold given 1% change in material payoffs, they will most certainly hold for larger changes.

2.3. Enlarged games

The E-games of MSZ is given in Table 1; here the Banzhaf index is $\beta_1 = \frac{5}{12}, \beta_2 = \beta_3 = \frac{3}{12}, \beta_4 = \frac{1}{12}$. Let the multiplicative modifiers be given by Table 4, which define the 3-game:

Preferences of player *i* \ towards *j*	Player 1	Player 2	Player 3	Player 4
Player 1	-	1	1	1
Player 2	0.99	-	1	1
Player 3	1	1	-	1
Player 4	1	1	1	-

Table 4. Modifiers for the E-game

In this case, player 2 dislikes player 1 just a bit, hence hypothesis **H7** is that *in the 3-game the winning coalition* $\{2,3,4\}$ *will be significantly more frequent than in the E-game*, and (**H8**) *the winning coalitions involving both players 1 and 2 will yield significantly higher gains to player 2 in the 3-game than in the E-game*.

This setup leaves a room for more treatment options with the E-game, and of course, for further games. One of these, our F-game, is presented below (see Table 5).

ENLARGED treatment: F-game				
Player#	1	2	3	4
Votes#	3	3	2	2
Banzhaf prediction	40	40	20	20
Quota	6			

Table 5. The F-game

The Banzhaf index is 1/3 for players 1 and 2, and 1/6 for players 3 and 4. Now let the modifiers be as in Table 6.

Preferences of player *i* \towards *j*	Player 1	Player 2	Player 3	Player 4
Player 1	-	0.8	1	1.01
Player 2	0.8	-	1	1.1
Player 3	1	1	-	1
Player 4	1	1	1	-

Table 6. Modifiers for the F-game

In this case, players 1 and 2 can share the pie (presumably equally), but both have incentives to build a larger coalition with players 3 and 4, in which the last two players will be "junior" members, and presumably would get lower share of the pie. Hence, (**H9**) is that *both players 1 and 2 will prefer a large coalition of three players to a smaller one, comprising just 1 and 2*. Further, in this large coalition they have different incentives to attract the weak player 4 because of their payoff modifiers. Hence, (**H10**) *escalation of competition among 1 and 2 for player 4's vote in a large coalition might lead to a more than proportional increase in player 4's gain compared to the F-game*, and (**H11**): *given that player 2's modifier is larger than that of player 1, player 2 is expected to be disproportionately (significantly more than in 1.1/1.01=1.089 times) more generous than player 1 in its offers to player 4.*

4. Experimental design

The experiments were carried out in the autumn 2008 and spring 2009 at the University - Higher School of Economics (HSE), Moscow, Russia, using specially developed software. We conducted 11 experimental sessions, where each session comprised two different games played by the same subjects. Each subject has had no previous exposure to any economic experiments and participated in exactly two games that constituted one experimental session as presented below in Table 7.

In each of the three-player games (with or without modifiers) there were 12 participants playing in 4 groups of 3 players. Each game in these sessions lasted 10 rounds. In each of these games all players were randomly assigned to one of the four groups and to the player's role (1, 2, or 3) in their respective group; both the groups and the roles were re-drawn at random every new round. Subjects did not know the identity of the other members in the group – just that this is someone sitting in the same room. All this has been commonly known before the game began.

In each of the four-player games (again with or without modifiers) there were 16 participants in 4 groups of 4 players; each game in these sessions lasted for 20 rounds, the rest of the procedures being the same. To control for the potential effect of the games' sequencing, both three- and four-player games were repeated twice: once as a first game, and once as a second game within the experimental session. In that way, we have 200 observations (instances of group bargaining) for the S- and 1-games, 80 observations for the V- and 2-games (which turned out to be the most robust), and 160 observations for each of the four-player games. The S-games were played in two design varieties (labeled as S-1 and SC-1C in table 7), in a way that is explained below. Thus, our statistical tests reported below are based on 400 observations for the S-1 games, 320 observations for each of the E-3 and F-4 games, and 160 observations for the V-2 games. All observations are treated as independent within each of these samples. This assumption is further justified below; in particular, we did not obtain any significant time trend, in neither initial nor terminal rounds.

1st Game	2nd Game
S	1
1	V
2	S
SC	1C
1C	2
V	SC
E	3
3	E
F	4
4	F
SC	1C

Table 7. Experimental sessions

Our experimental software and procedures were explicitly aligned with those of MSZ, including the appearance of the game on the computer screen.

The games proceeded as follows. Participants entered the experimental laboratory, signed the registration forms and logged into the game The instructions (see Appendix 1) were read aloud and were made available on the screen should a participant need it at any time. Then, participants were asked if they had any question; upon answering them, the game began.

In each round of any game all groups had to agree on how to divide a pie of the size 120 points by inputting their proposed division in the provided form on a computer screen (see Figure A1 in the appendix for a sample screenshot). Each proposal automatically received the votes of its issuer; any player could also vote for any other proposal available at any time, or submit a new one, which replaced her previous proposal on the screen.

The first (in chronological order) proposal to receive the number of votes matching the quota was enforced, and the group participants received the number of points assigned to them by that proposal. After all four groups completed the round, new groups were formed at random, and the same procedure was repeated again.[10] After the end of the first game participants were introduced to the rules of the second game in a session, and the procedure was repeated again. At the end of the session all the points earned by each participant in both games were summed up,

[10] Inasmuch as the subject pool was the same, some of the players had to wait till the other groups completed their round. In practice, the groups played at a rather similar pace in all games but one.

converted in cash at a flat exchange rate of 1 point = 0.4 Russian rubles (RuR), and paid individually to every participant.

Every round each group had to vote for any proposal in no more than 300 seconds[11]; time counter was shown at the top right corner of the screen. If a group failed to reach an agreement within the allotted time, all players in that group received zero (this happened only five times in all 1120 rounds: three times in the 2-games, and once in the E- and the 4-games).

In the S-1 games, we used the 2 × 2 × 2 design, controlling for the sequence of the games in the session, modifiers, and position of the players on the screen (C), as will be described shortly. In the other pairs of games, namely V-2, E-3 and F-4, we used the 2 × 2 design, controlling for the sequence of the games and modifiers. Inasmuch as game sequencing did not result in significant differences across the treatments, we combine the datasets from different sessions into one for the games of each particular type. In statistical analysis, we treat all observations as independent. The rationale behind this assumption is that by design the same subjects are never matched twice in a single group, and their assigned roles (viz., player 1, player 2, etc.) change randomly every round. Therefore, even if a subject performs the same role several times during the game, she is never matched with her previous partners and thus repeated role effects are negligible.

An interesting dimension of our experiment is the duration of the rounds. In the S games these were quite short, averaging to 25.5 seconds, much below the time allowance of 300. For the 1-games, the average round was just a bit longer (31.4 seconds), presumably reflecting the fact that our subjects had to take mental account of the modifiers. By contrast, the V-2 games were much longer, each round lasting on average over 140 seconds. This fact has its explanation too: in these games the veto players 1 tended to "extract" as much as possible from their opponents, which resulted in longer bargaining[12]. In the four-player games the average length of the round was quite stable – about 100 seconds.

Participants were 148 BA and MA students of the various departments of HSE (mostly in economics, but also in international relations, sociology, political science, psychology, law, and computer science). Subjects were recruited via the university website. Gender composition of the sample was about 50:50; the average age of participants was 19.5 years. The average gain of participants in the 3-player sessions was 340 RuR, minimum[13] – 170 RuR, maximum – 610 RuR

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[11] This feature of our design constitutes the only substantial difference from the experiment design by MSZ. The reason for this difference is purely technical; however, MSZ report that this constraint was not binding in their experiment anyway, and this difference did not cause any significant divergence between our and their results.

[12] In some rounds of this game, subjects were bargaining till the very end, and intended to accept an unfavourable but positive offer on the time of deadline. We observed that zero payoffs in at least some of the five games were due to subjects' failure to do so exactly on time.

[13] Approximately 10 euro at the time when the first sessions of our experiment were conducted (autumn 2008). This amount went down to about 8 euro according to the exchange rate prevailing in the spring 2009. However, inasmuch

per 1- to 1.5-hour session. Corresponding figures for longer 4-player sessions were 485 RuR, with a minimum of 240 RuR and a maximum of 750 RuR. The money was paid in cash at the end of each session. [14]

5 Results

In accordance with the stated hypotheses, we performed several types of statistical tests on our experimental data. Besides comparing the outcomes with those of MSZ and the Banzhaf predictions, we tested for the treatment effects of players' induced preferences towards coalescing each other (explicit modifiers). We do that by looking at several kinds of indicators, such as the shares of the pie at the agreed distribution, the composition of the winning coalition in terms of players' roles, and the average number of offers proposes per round. In another type of analysis, we combined earnings of the players who are *a priori* symmetric (that is, control the same number of votes and have the same modifiers towards coalescing with other players) into a single variable by averaging their payoffs. This procedure (for the case of the players with the same number of votes) was used by MSZ and allowed them, in particular, to deal with the case of *a posteriori* asymmetric "weak" players (see the description of the SC-games below). In our case the applicability of this method was somehow limited to the games without modifiers (with a notable exception for the 2-game, where the modifiers were the same for both "weak" players and hence their earnings could be aggregated). This second type of analysis allowed us to more clearly align our results with those of MSZ, although it showed no pronounced difference with the results of the first analysis and therefore is reported only occasionally in the text.

5.1. S-1 games

Figure 1 represents average (arithmetic means) of the outcomes of all the S-1 sessions aligned at the timeline. On the figure, "Splayer1" labels average payoffs of player 1 across the S games, Splayer2 that of the player 2 in the same type of games, etc.

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as longer (20-rounds) games with four players were ran in 2009, actual earnings of the participants of these games were even higher than those of the shorter (10-rounds) games with three players.

[14] A small show-up fee of 100 RuR was paid to a few extra participants who were invited to the game but were not able to fit in the room.

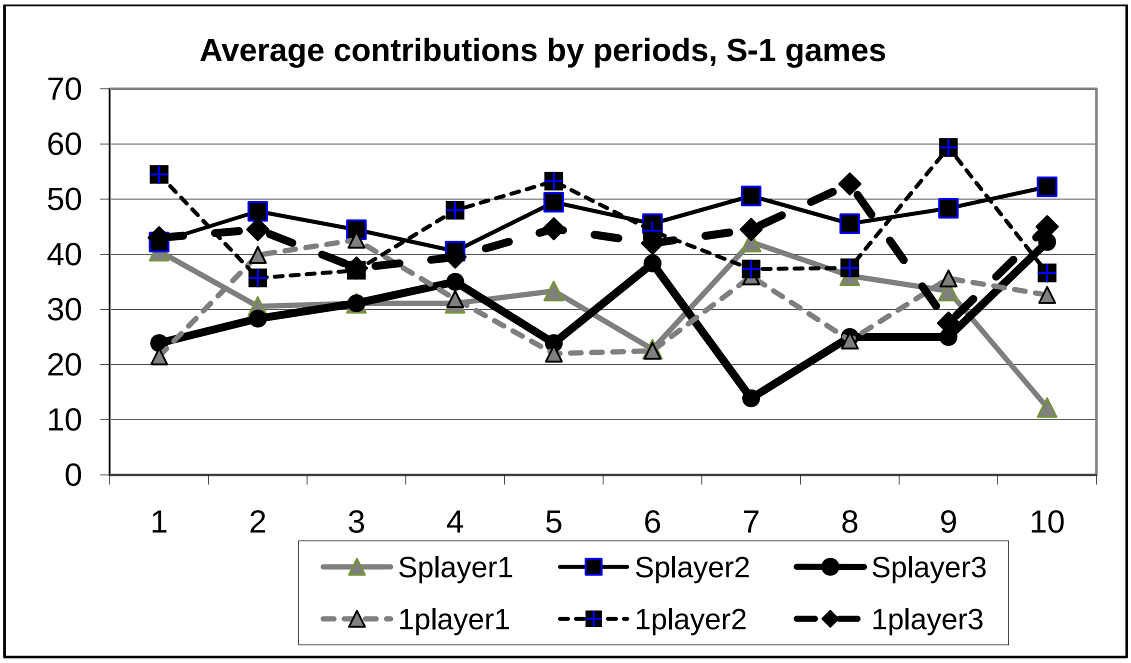

Figure 1. S-1 games, average shares of the players.

As Figure 1 reveals, the mean outcome is about 40, which is pretty much in line with the Banzhaf index prediction, as well as with the results reported by MSZ. As can be seen from that figure, and further confirmed by statistical tests, on average this tendency is robust for players 1 and 2 in S-games vs. 1-games. By contrast, player 3 on average receives systematically more in the 1-treatment (43.85) than in the S-treatment (35.84), which difference is significant (Student t=2.24, two-sided test p<0.0264; Kruskal-Wallis $\chi^2 = 5.89$, p<0.0122). Hence our modifiers do indeed work for player 3: colloquially speaking, "being loved is better than love". This finding is even more notable given that all games in this treatment were played quite quickly, and after the session, many participants reported that they did not pay attention to the modifiers at all. However, both casual observation of Figure 1 and statistics clearly indicate this feeling is wrong: the psychological 'invisible hand' tends to lead their actions in a regular and predicted way, supporting our hypothesis H1.

One more striking feature follows from Figure 1: although in the S-game *a priori* player 2 does not differ from player 3 in any kind, player 2 receives systematically significantly more than player 3 (46.5 vs. 37.66), which difference is significant at any reasonable degree of confidence, and also departs from the Banzhaf prediction. This finding matches the one of MSZ, who attributed it to a "framing effect".

In our view, this effect has a clearer explanation: it is the position of player 2 in the middle of the table. In S-1 games, players quickly realize they all have equal effective bargaining power, and find it more profitable to share the pie with just one of the other players than with two. Given the quick pace, typical for the games of such kind, each player seeks a neighbour[15] to whom she can offer a coalition. Thus, player 2 has two neighbours (1 and 3), and hence may be offered a coalition by either of them, whereas the other two players have just one neighbour (player 2) each. As a result, the supply of offers for player 2 ought to be higher than that for the non-central players. Due to this beneficial situation, player 2 becomes a member of a greater number of winning coalitions in the symmetric game, and her payoff increases.

In contrast to "explicit modifiers" p_{ij}, we call this positioning effect an "implicit modifier" of player 2's payoff. To reduce its effect, we introduced a modification to the original setup by rearranging players on the computer screen: instead of player 2, *each* player in turn is shown in the middle of the table, with clockwise displacements of the other two players. We refer to this treatment as the Standard Centered (SC in Table 7) and 1-Centered (1C) games.

In the SC games, the difference in the earnings of players 2 and 3 mitigates completely: in particular, player 2's average share is 40.91 vs. 39.20 for player 3, which difference is clearly insignificant[16] . The effect is qualitatively similar, albeit less pronounced for the 1C games, and goes in the other direction (average gains of 39.23 for player 2 vs. 43.17 for player 3, which is not significant ($t=-0.95$, $p<0.3413$). Thus, the difference between the weak players 2 and 3 disappears[17], and we conclude that the effect of an implicit modifier vanishes in our symmetric treatment. Further confirmation of this new hypothesis comes from the composition of the winning coalitions which are reported in Table 8 for all our games.

The table entries list the players who entered (received positive payoffs in) the winning coalitions[18], most of which were efficient (minimal), although in some cases nonminimal coalitions (due to the order of votes in the round) were observed too. In the S-games (first column of Table 8), player 2 entered the winning coalition in over 90% of cases (58 cases out of 64), whereas in the SC game, all three winning coalitions were approximately equally likely. To further support this conclusion, we count the number of offers made in the 80 games of the S and the 120 games of the SC treatment conditions. In the former, players 2 were made an offer in

[15] By a neighbour here we mean a player who is the closest in the sense of both location on the screen and natural order. One might also think of a more subtle effect, such as preference of player 2 for player 1 over player 3 because 1 is a "more prominent" number than 3.

[16] Here and in what follows, our comparisons are based not on the *agreed*, but on the *accrued* gains of players, including the explicit modifier effect. Given that these are typically quite small, there is not much difference between the two.

[17] There may be the "remainder" of implicit modifiers due to the natural order of player numbers, as discussed in the footnote 15. We believe that this effect could have been destroyed completely had we used random placement of all three players on the screen. However, this would come at the expense of extra burden imposed on subjects' attention to the new position of themselves and their opponents in *every* round, which would have been challenging given the time constraint. Our design was chosen as a compromise which was acceptable according to the subjects' reports, and especially stressed in the experimental instructions.

[18] Note that these players are not necessarily those who did vote for the respective coalition, although the two numbers correspond quite closely. In particular, in all our data sample there were only 6 inefficient coalitions (votes of the players who shared the pie did not met the quota, so these outcomes are probably attributable to players' mistakes), and 5 instances of no agreement.

39.1% of all offers (159 times out of 407), while players 1 and 3 were made an offer in 32.1% and 28.7% of cases (131 and 117 times, respectively). In the latter, the difference disappeared: players 1, 2 and 3 were offered coalitions 184, 186 and 180 times, respectively, implying that the centered treatment indeed made all players behaviourally symmetric.

	S-1 games						V-2 games	
	S	SC	S all	1	1C	1 all	V	2
1&2	29	35	64	17	30	47	41	40
1&3	6	32	38	11	35	46	27	26
2&3	29	38	67	30	41	71	-	-
nonminimal	16	15	31	22	14	36	12	10
none	0	0	0	0	0	0	0	3
inefficient	0	0	0	0	0	0	0	1

	E-3 games	
	E	3
1&2	73	74
1&3	57	51
2&3&4	13	26
nonminimal	16	8
none	1	0
inefficient	0	1

	F-4 games	
	F	4
1&2	82	64
1&3&4	38	31
2&3&4	33	56
nonminimal	6	7
none	0	1
inefficient	1	1

Table 8. Numbers of winning coalitions by treatments.

The explicit modifiers' effect for player 3 is less pronounced in the centered sessions; however, the nonparametric (Kruskall-Wallis) test suggests that the null hypothesis that observations for all but player 3 were drawn from the same distribution is valid over the entire range of observations. On these grounds, we pull together our data for the S and SC games, and compare them with our data for the 1 and 1C games for player 3 (see Table 9). The combined data confirm the overall treatment effect: average payoffs of player 3 go up from 36.42 in the S-game to 42.74 in the 1-game, meaning that 1% of preferential change brings to that player about a fair 17% increase in revenues. This increase is quite substantial, and significant at less than 1%. (Student t= 2.32, two-sided test p<0.0204; Kruskal-Wallis $\chi^2 = 7.28$, p<0.0070), confirming our main hypothesis (H12) for player 3, but not for player 2. All in all, this means that if voters have non-uniform preferences towards different coalitions they might be part of (and however small such preferences are!), it is more appropriate to use preference-based power indices than the standard Banzhaf ones. Moreover, preferences of player 2 in the 1-game treatment significantly affect the payoff of player 3 but neither her own nor that of player 1. This indicates that for a given player in the symmetric context, preferences of *other* players towards coalescing with *her* (probably) matter more than *her* own preferences to coalesce with the *other* players, suggesting the intensity function of the f^- type.

By contrast, the effect of player 2's payoff being higher than that of player 3 in non-centred games, documented in both MSZ and in our S-1 treatments, is largely due to the unintended "implicit modifiers", and should hardly be important outside the specific setup of laboratory experiment. This might be one of the explanations why our second hypothesis (H2) receives much less support, whether on the grounds of the aggregate data (see Table 8), or on the grounds of the centered games alone. Despite the mitigated effect of implicit modifiers, the share of the winning coalitions involving players 2 and 3 did not differ significantly between the SC and the 1C games, going up from 32.7 to 35.8%. By contrast, our hypothesis H3 receives empirical support. Despite player 3 was not observed to enter in significantly higher fractions of the winning coalition in the 1-games in comparison to S-games, in the former case it enjoys much more attention of player 1. The overall share of offers made by player 1 in all S-games and all 1-games is almost the same (126 vs. 125), yet in the former player 3 received positive offer in 61 instances, in the latter, this number increases to 85. Average offer of player 1 to player 3 went up as well, but not significantly, which is not surprising because this treatment effect is not only very small, but also indirect: it is the reaction of player 1 on increased popularity of player 3 in the eyes of player 2. The interpretation is further confirmed by the decreased number of positive offers of player 1 to player 2 (105 in the S-games vs. 82 in the 1-games) which also slightly decreased on average. . This redistribution of interest of player 1 to player 3 at the expense of player 2 remains robust when restricted to centered games.

A summary of all S-1 and SC-1C games is presented on Figure 2 and Table 9.

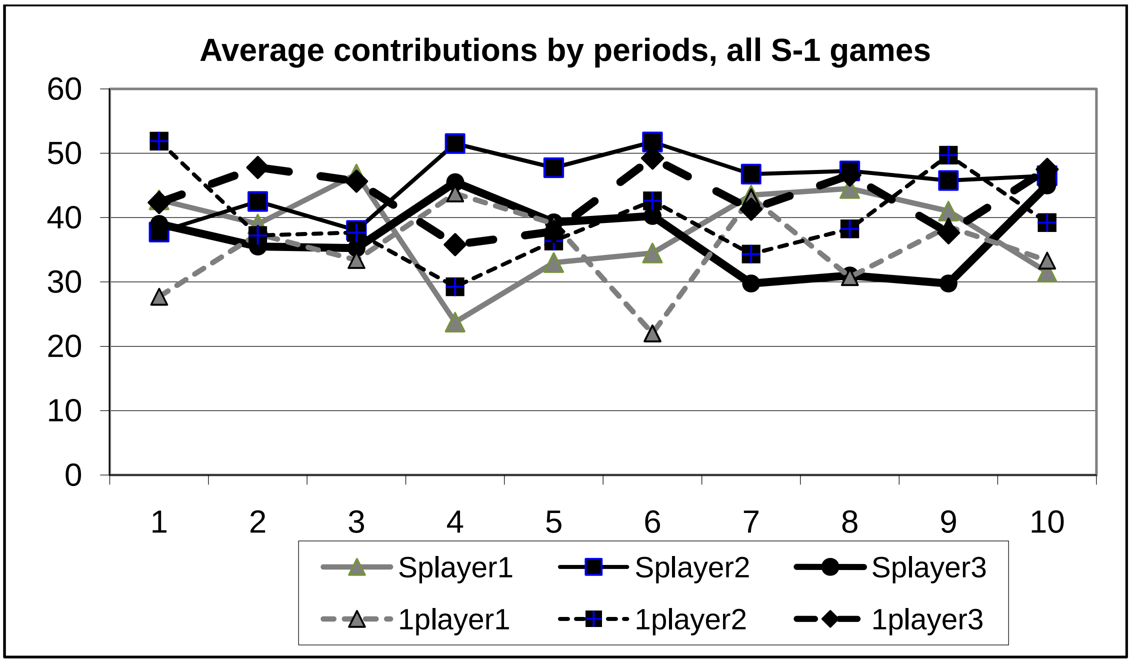

Figure 2. All S-1 and SC-1C games, average contributions by periods.

All (N = 400)	mean	s.d.	min	max
Player 1	36.47	28.71	0	80
Player 2	44.06	24.89	0	100
Player 3	39.58	27.26	0	111
Game S (N=200)				
Player 1	38.02	28.80	0	80
Player 2	45.55	23.97	0	100
Player 3	36.42	27.17	0	80
Game 1 (N=200)				
Player 1	34.91	28.61	0	80
Player 2	42.57	25.76	0	99.99
Player 3	42.74	27.06	0	111

Table 9. S-1 games, summary statistics.

5.2. V-2 games

Results of the veto sessions are presented in Figure 3 and in Table 10.

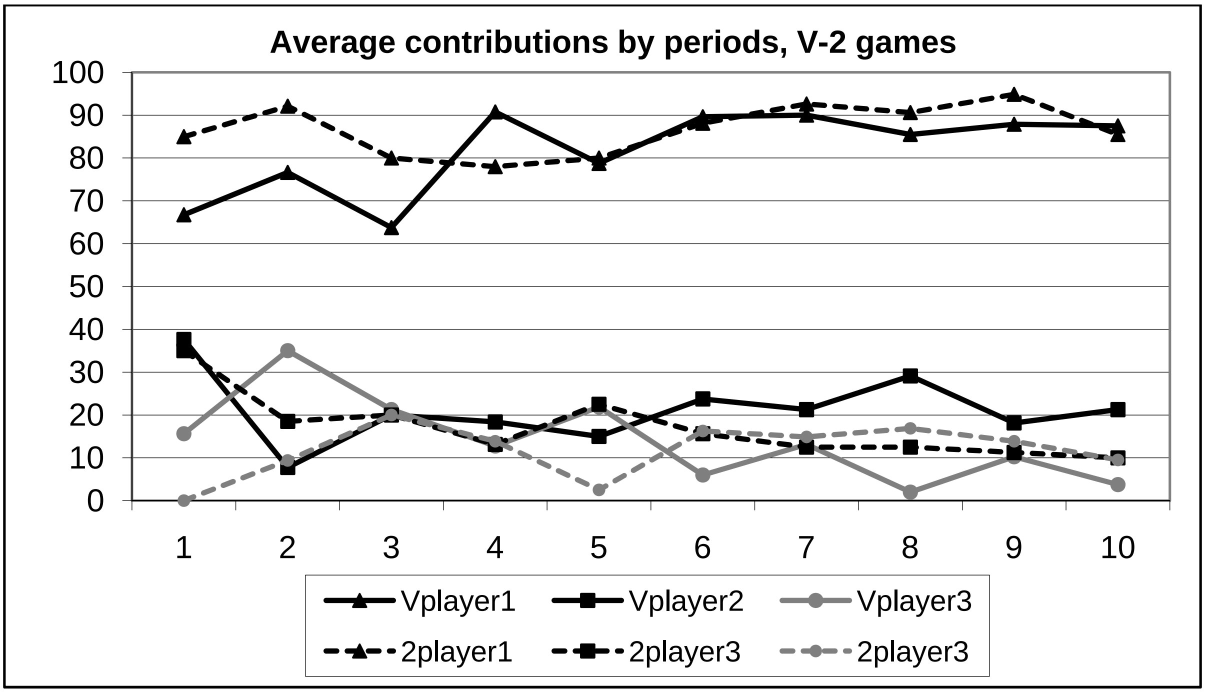

Figure 3. V-2 games, average shares of the players.

Figure 3 is largely self-explanatory: the veto player 1 gets a lion's share of the pie, even more than both the Banzhaf index prediction and the MSZ results); all statistical tests confirm this at any reasonable degree of confidence. Thus H4 cannot be rejected. As for players 2 and 3, this time, although their gains fall in the 2-games vs the V-games, none of the differences is statistically significant for the whole dataset, neither the fraction of particular coalitions is (see Table 8). If we consider the last 7 rounds only (omitting the initial learning stage), then player 2 gets a slight but significant decrease in average payoff in the 2-game compared to the V-game (13.81 vs. 20.98; Student t=2.14, two-sided test p<0.0344; Kruskal-Wallis $\chi^2 = 4.401$, p<0.0359). The same effect (significant decrease of "weak" players' payoffs in the 2-game vs the V-game) holds for the whole dataset if we average out the earnings of the (*a priori* symmetric) players 2 and 3.

These results mean that a small distaste (of 0.99) of players 2 and 3 towards player 1 clearly cannot overturn the fact that player 1 is crucial for any positive gain. It rejects our hypothesis H5 about the role of the modifiers in the veto context. On the contrary, the negative modifiers of the "weak" players towards the "strong" player 1 result in a significant decrease in their payoffs as soon as the players gained sufficient experience. This suggests that in the veto context only players' *own* preferences to coalesce with *other veto* players may matter, - in other words, the intensity function of the f^+ type is more appropriate in this context to describe the power of the "weak" players. Taken together with the S-1 games, this finding implies that the intensity of connection functions are context-dependent and therefore in is hard to think of its "universal" version.

A distinct feature of this treatment is the bargaining strategy. Participants have quickly realized the crucial role of the veto player: in many games, players 1 were simply asking a bulk of the pie, leaving some bit to one weak player and nothing to the other – and were waiting for the offered player to accept. Occasionally this strategy failed; yet in most cases this resulted in the most intensive bargaining procedure (on average, 5.78 offers per session in the V-2 games vs. 2.28 offers per session in the S-1 sessions), and five times longer rounds. In relation to this, we checked hypothesis H6 by a fit of a duration model on the time of reaching an agreement, but were unable to observe any significant effect.

All (N = 160)	mean	s.d.	min	max
Player 1	85.65	23.23	0	120
Player 2	19.09	19.39	0	60
Player 3	12.88	17.91	0	60
Game V (N=80)				
Player 1	84.61	21.26	40	119
Player 2	21.23	20.63	0	60
Player 3	14.16	19.16	0	60
Game 2 (N=80)				
Player 1	86.69	25.14	0	120
Player 2	16.96	17.94	0	60
Player 3	11.60	16.59	0	59.4

Table 10. V-2 games, summary statistics.

5.3. E-3 games

Summaries of the E-3 games are presented in Figure 4 and Table 11 below.

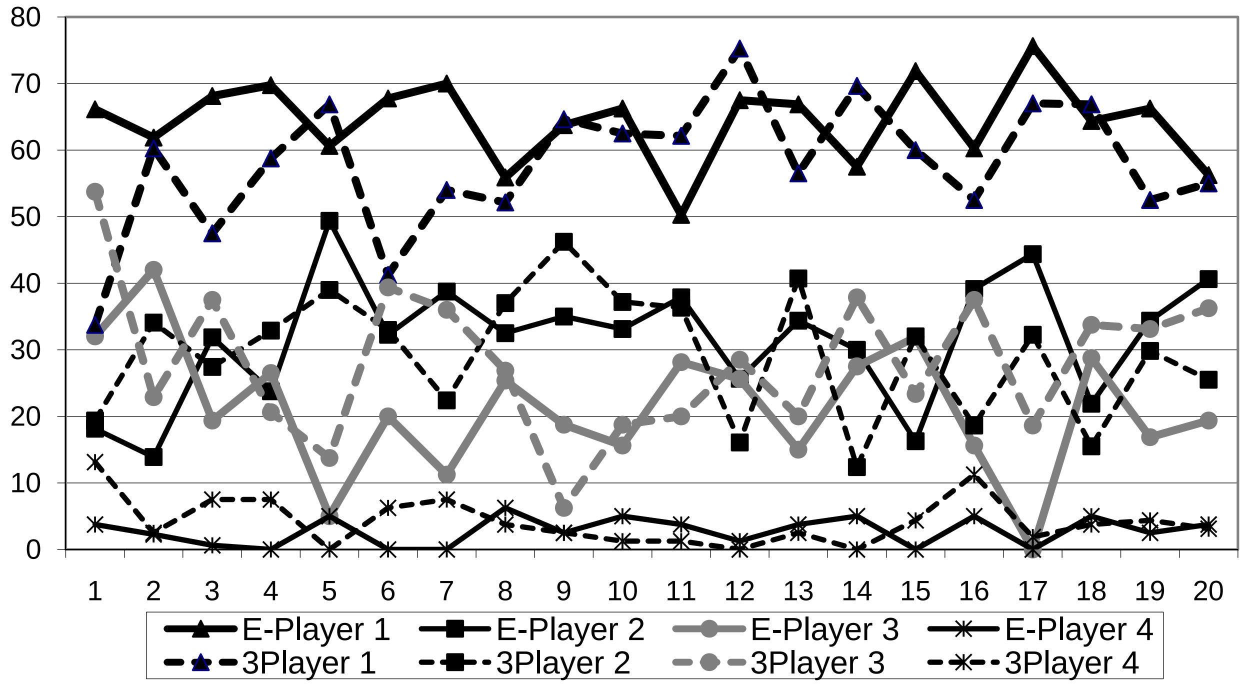

Figure 4. E-3 games, average shares of the players

Player 1 in both of these games receives significantly and systematically more than the Banzhaf index prediction of 50, and clearly does so at the expense of other players (especially, player 4), while gains of players 2 and 3 are generally smaller than the index prediction. Payoffs of player 1 significantly increase in comparison to the S-1 games, while those of players 2 and 3 systematically increase in comparison to the V-2 games (cf. Tables 10 and 11), confirming the paradox of new members. All these facts are in line with the MSZ findings, although our effects are somewhat sharper than theirs.

Two more tendencies peculiar to our design extend the above story. As follows from Table 11, the share of player 1 who has the highest bargaining power falls from 64.34 in the E-games to 57.95 in the 3-games, change that is significant over all 20 rounds (Student t-statistic 2.23, p<0.0262; Kruskal-Wallis $\chi^2 = 3.07$, p<0.073) albeit it is more explicit in the first rounds of the game. By contrast, player 3's average payoff increases from 21.23 to 28.23, which effect is quite strong (Student t-statistic 2.57, p<0.0104; Kruskal-Wallis $\chi^2 = 6.2$, p<0.0128) and holds for the entire 20-rounds range. That is, in this setting the effect of a small negative modifier of player 1 against player 2 indirectly benefits player 3, whose gain in the 3-game increases by a quarter in comparison to the E-game. Another interesting feature of the E-game compared to the 3-game is that the frequency of coalitions involving three players (the coalition {2, 3, 4}) is twice as high in the 3-games (26 instances) as in the E-games (13 instances - see Table 8), which effect is especially pronounced if the E-game is followed by the 3-game. By contrast, the number of winning coalitions consisting of player 1 and 3 falls down from 57 in the E-game to 51 in the 3-game, A conjunction of these findings admits the following natural interpretation. Players 2, realizing that they dislike player 1, attempt to build a larger coalition with players 3 and 4, even though building it is clearly more difficult technically, and requires sharing of the pie with three players instead of two. To support further this interpretation, consider changes in offers made by player 2 to the other players in the E-games vs 3-games. In the E-games, player 1 was offered positive stakes 133 times out of 149, with the average stake of 59.42; in the 3-games, the number of offers falls to 107 out of 134, with the average stake of 52.21. By contrast, the number of offers to player 3 went up from 37 to 43, and the average offer – from 21.23 to 28.23. Player 4 benefited too, but at a much lower extent: the number of offers to that player increased from 24 to 33, and the average offer – from 2.76 to 4.21. This is exactly our hypothesis H7, with an important addition that it is player 3, as the strongest of the remaining two, who benefits most from this situation. Benefits being restricted to player 3, our hypothesis H8 was not confirmed: average payoffs of player 2 in the coalitions with player 1 differs insignificantly across treatments (Student t=1.27, p<0.2026; Kruskall-Wallis $\chi^2 = 0.521$, p<0.5184).

.

All (N = 320)	Mean	s.d.	min	max
Player 1	61.15	25.76	0	100
Player 2	30.52	23.74	0	70
Player 3	24.73	24.54	0	70
Player 4	3.49	9.01	0	70
Game E (N=160)				
Player 1	64.34	22.36	0	95
Player 2	31.66	23.17	0	70
Player 3	21.23	23.72	0	70
Player 4	2.77	7.41	0	40
Game 3 (N=160)				
Player 1	57.95	28.47	0	100
Player 2	29.39	24.32	0	69.3
Player 3	28.24	24.91	0	65
Player 4	4.21	10.33	0	70

Table 11. E-3 games, summary statistics.

5.4. F-4 games

This case is a novel feature of our experiment; it cannot be compared to MSZ. We included it for the sake of completeness, as well as an additional check for the effect of induced preferences: large negative modifiers of players 1 and 2 towards each other are expected to have more significant effect on the outcomes than that of the small modifiers of 1% in the previous games.

This expectation is fully confirmed by the experiment, albeit again in a specific and sometimes indirect way. As can be seen from Figure 5 and Table 12, a large negative modifier for player 1 significantly lowers her earnings: on average these fell down from 48.43 in the F-game to 31.73 in the 4-game (Student t-statistic 5.9, p=0.000; Kruskal-Wallis $\chi^2 = 56.505$, p<0.0001). This decrease of original payoff by one third is striking: average gains of player 1 descend from the level somewhat above the Banzhaf share of 40 to the level below it. At the same time, there is no similar treatment effect for player 2 whose accrued gains are on average indistinguishable across treatments. There is, however a uniform significant shift of gains of players 3 and 4 from the average shares of 12.95 (12.66 for player 4) in the F-game to 18.03 (18.41 for player 4) in the 4-game[19]; gains of these players in the modified game 4 are much closer to the Banzhaf share of 20 than in the benchmark game F.

Thus we observe a rather complicated indirect effect of the modifiers: a sharp decrease of the gains of player 1 results in an increase in the gains of the two weak players; and the fact that player 1 prefers player 4 to a smaller extent than player 2 backfires on the former but not on the latter. Indeed, player 2 is much more inclined to form a grand coalition with the weak partners: the number of coalitions of players 2, 3 and 4 almost doubles from treatment E (33 cases) to treatment 4 (56 cases), which decrease is matched by a decline of the number of coalitions of players 1 and 2. This tends to confirm our hypotheses H9 and H10, which hold for both players 3 and 4, but not our hypothesis H11. Although offers of player 2 to player 4 indeed increased dramatically (from 10.23 in the F-games to 18.41 in the 4-games), this move was almost exactly matched by the player 1 whose respective offers went up from 9.77 to 17.77. Furthermore, this move was matched by that one of player 3, and all differences are significant at any degree of confidence. These facts again suggest that the interaction of explicit and possibly implicit modifiers is rather complicated, and its classification and explanation requires further research.

[19] Corresponding test statistics for player 3 are: Student t-statistic=3.065, p<0.0024; Kruskal-Wallis $\chi^2 = 14.047$, p<0.0002; for player 4: Student t-statistic=3.4288, p<0.0007; Kruskal-Wallis $\chi^2 = 16.144$, p<0.0001.

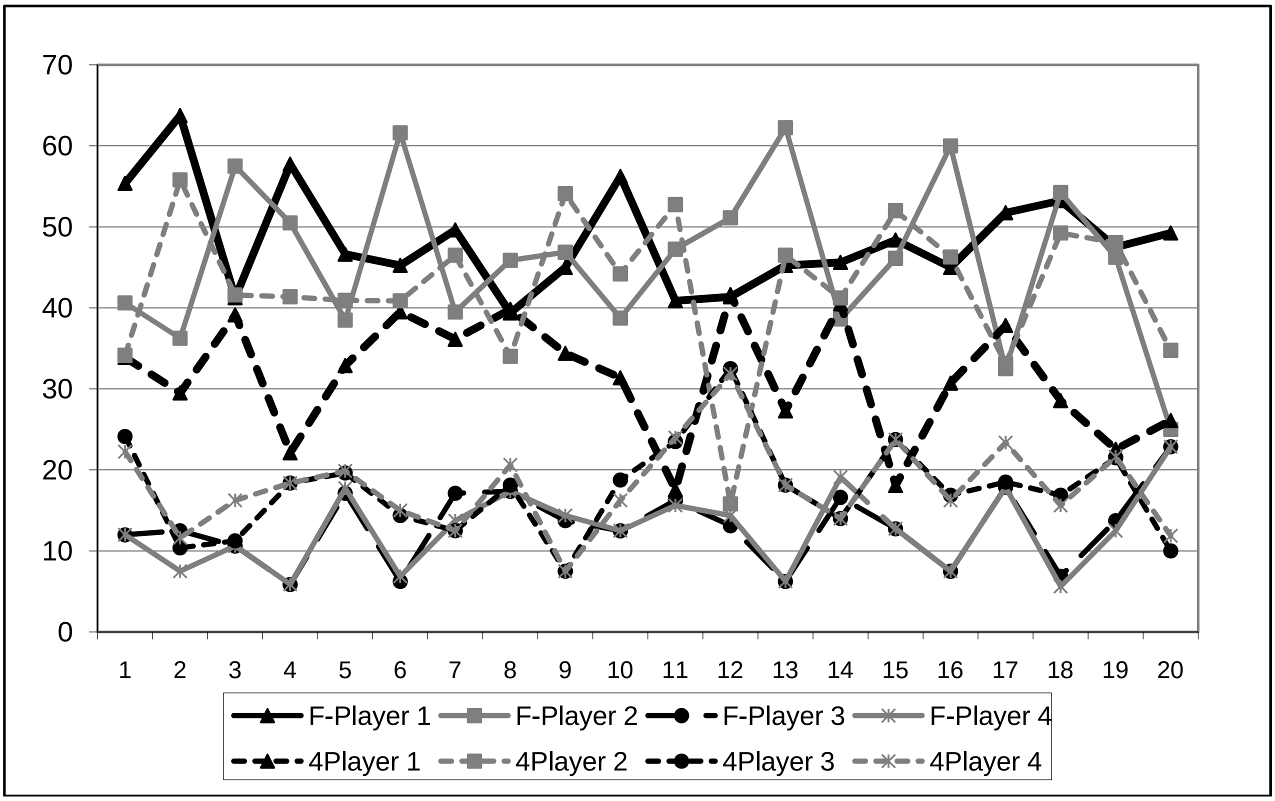

Figure 5. F-4 games, average shares of the players

All (N = 320)	mean	s.d.	min	max
Player 1	40.08	26.60	0.00	80.80
Player 2	44.52	25.70	0.00	80.00
Player 3	15.49	15.02	0.00	45.00
Player 4	15.53	15.27	0.00	50.00
Game F (N=160)				
Player 1	48.43	25.82	0.00	80.00
Player 2	45.97	26.94	0.00	80.00
Player 3	12.95	13.86	0.00	40.00
Player 4	12.66	13.69	0.00	40.00
Game 4 (N=160)				
Player 1	31.73	24.76	0.00	80.80
Player 2	43.08	24.39	0.00	77.00
Player 3	18.03	15.73	0.00	45.00
Player 4	18.41	16.23	0.00	50.00

Table 12. F-4 games, summary statistics.

5.5. Discussion

Our results presented above can be seen from two points of view. In general, they are in line with both the empirical evidence reported by MSZ, and with the theoretical predictions of the Banzhaf index. At the same time, they also confirm descriptive adequacy of the generalized power index by Aleskerov (2006), clearly suggesting that preferences among the players do matter for the outcome of the vote, and in practice should be taken into account. The power of explicit modifiers is quite strong, and operates through creation of the "focal points" in the set of all winning coalitions, albeit sometimes in a tricky way which hardly could have been expected on prior grounds. Alongside with the explicit modifiers, we observe the effect of implicit modifiers, which were destroyed in our centered treatment, but probably not in the other games.

Disentangling the effects of the explicit and implicit modifiers could be important to understand the subtle nature of "symbolic" factors that affect behavior to a significant extent in both the laboratory settings and in real life. But our results imply more than that. Table 8 already suggests that minimal coalitions are much more frequent than the nonminimal ones, yet both kinds of coalitions are used in the definitions of the classical Banzhaf and Shapley-Shubik as well as generalized power indices. Our evidence discards this logic, arguing for restricting attention to minimal coalitions.

Further evidence is given in Table 13, which lists all offers made by all players in all games by the minimal winning coalitions (recall from Table 8 that these constitute an overwhelming majority of all offers made). Table 13 suggests very strong patterns of offer structure in all four games (without great differences for the treatments with explicit modifiers).

Thus, in S-1 games any two players in a minimal coalition split the pie among themselves in almost equal proportions, although offers to form coalition {2, 3} are more frequent than others (an explicit modifier effect!).

In the minimal coalitions of the V-2 games, player 1 is offered about 90 points, while either of the weak players – about 30 points; in nonminimal coalitions players are inclined to share the pie in more equal proportions.

In E-3 games, in a small coalition the strong player is offered about 70 points and the weak one – about 50, while in a large coalition about 50 points accrue to the each of the weak players (having two votes) and 20 – to the player having one vote.

Finally, in the F-4 games an equal split is common to both members of the small winning coalition {1, 2}; and in the large winning coalition the strong player receives about 60 points whereas both weak players get about 30 points each.

In all these instances except nonminimal offers in the V-2 games, the standard deviations for the shares offered are much smaller than shares themselves, suggesting that these tendencies are very robust. And indeed, the best predictors of the offers (and accepted divisions of the pie) can be obtained by regressing the size on the identity of the winning coalitions, which alone explain 70 to 90% of variation of the dependent variables.

Games S-1

Coalition	Offer to	Obs	Mean	Std.Dev.	Min	Max
{1,2}	Player 1	460	57.43	13.99	2	80
	Player 2	460	59.66	6.47	40	90
	Player 3	460	0	0	0	0
{1,3}	Player 1	481	52.17	19.10	1	80
	Player 2	481	0	0	0	0
	Player 3	481	61.23	7.25	40	100
{2,3}	Player 1	636	0	0	0	0
	Player 2	636	59.02	7.18	20	80
	Player 3	636	60.97	7.18	40	100

Games V-2

Coalition	Offer to	Obs	Mean	Std.Dev.	Min	Max
{1,2}	Player 1	734	88.09	19.33	30	119
	Player 2	734	31.90	19.33	1	90
	Player 3	734	0	0	0	0
{1,3}	Player 1	596	91.33	18.49	30	119
	Player 2	596	0	0	0	0
	Player 3	596	28.67	18.49	1	90
{1,2,3}	Player 1	525	47.67	25.79	10	118
	Player 2	525	35.35	16.95	1	100
	Player 3	525	36.97	18.13	1	90

Games E-3

Coalition	Offer to	Obs	Mean	Std.Dev.	Min	Max
{1,2}	Player 1	847	71.31	6.97	40	95
	Player 2	847	48.69	6.97	25	80
	Player 3	847	0	0	0	0
	Player 4	847	0	0	0	0
{1,3}	Player 1	675	70.45	8.21	30	90
	Player 2	675	0	0	0	0
	Player 3	675	49.54	8.21	30	90
	Player 4	675	0	0	0	0
{2,3,4}	Player 1	589	0	0	0	0
	Player 2	589	50.60	6.49	30	75
	Player 3	589	49.84	6.45	10	65
	Player 4	589	19.55	10.75	5	55

Games F-4

Coalition	Offer to	Obs	Mean	Std.Dev.	Min	Max
{1,2}	Player 1	580	60.25	3.71	50	80
	Player 2	580	59.74	3.71	40	70
	Player 3	580	0	0	0	0
	Player 4	580	0	0	0	0
{1,3,4}	Player 1	276	60.65	10.30	30	80
	Player 2	276	0	0	0	0
	Player 3	276	29.83	5.17	20	45
	Player 4	276	29.52	5.40	20	45
{2,3,4}	Player 1	356	0	0	0	0
	Player 2	356	61.73	8.07	40	80
	Player 3	356	29.01	4.42	20	40
	Player 4	356	29.26	4.60	20	45

Table 13. Summaries of the offers by proposed coalitions

The above observations suggest that, notwithstanding the Banzhaf indices, as well as many other solution concepts based on the overall structure of the cooperative game, the outcomes of the vote in small groups with three to four players that we considered, depends on two very simple phenomena: first, the role of the player, together with her number of votes vs. the quota, and second, the composition of the winning coalition.

For us this result was somewhat unexpected, and led us to reject our original intention to use theoretical equilibrium-based model from either cooperative solution concepts, such as the bargaining set (Aumann & Maschler, 1964), or the noncooperative ones, such as subgame perfect equilibrium, to predict bidding behavior. Our evidence convincingly speaks in favour of a much simpler model, such as the one outlined above. Moreover, the outcomes of our experiment do not correspond to the logic of many of the available solution concepts: e.g., the offers and outcomes made to the large coalitions in the F-4 games do not even belong to the bargaining set, and hence, to the kernel and the nucleolus. This evidence suggests that a positive theory of coalition formation, even if grounded on the existing theoretical solution concepts (Greenberg, 1994) require re-thinking from an empirical perspective.

6. Conclusions

Results of our work may be summarised as follows. Explicit modifiers work in all treatments of the S-1 games, and increase the payoff of player 3 by about 22%; effects for the other players are not significant. This fact can be interpreted in two ways. Verbally, "*being loved is better than love*", at least in terms of material payoffs. Formally, in terms of the model of Aleskerov (2006), this finding supports the use of f^- intensity function, at least for a symmetric context. We have also found (and ruled out) the nuisance effect of implicit modifiers in the S-game, and feel confident to attribute it to the peculiar visual representation of the game.

The veto games are different: in this case, explicit modifiers do not matter at all for the veto player (the fact that one player has veto power overrules personal attitudes), but are important for the "weak" players, influencing their payoffs. In particular, in the veto context only players' *own* preferences to coalesce with the *other veto* players matter, which supports the use of the f^+ intensity function, at least for estimating power of the "weak" players. This also implies that the effect of preferences exists in the veto games, but is of secondary importance to a player's number of votes and the quota.

We conclude that the intensity functions are context-dependent and therefore different versions should be used for different contexts.

In the four-player games, the effect of modifiers is more complicated, and often is manifested indirectly. Negative modifiers do affect the frequencies of particular coalitions, so that the players affected by them prefer larger coalitions with those players they do not dislike, even though the need to share the pie among more members swallows most of their own gain.[20] This fact suggests that pairwise preferences not only matter, but also might affect the outcomes of the games in a rather unexpected manner, which hardly can be figured out on prior grounds (and resulted in rejection of many of our prior hypotheses).

Our experiment confirms validity of the preference-based approach, which in some cases turns out to be descriptively superior to the (unadjusted) Banzhaf index (e.g., see the gains to the weak players in our 4–game). At the same time, it leads to a number of new questions. First, our results call for a more thorough study and classification of different contexts under which a particular intensity-of-preference function *f* is to be used. Second, it calls for a specific theory of bargaining process in the voting games such as the one used in this experiment. Finally, on the grounds of the above results, one would like to have a general theory whose predictive power could facilitate understanding the properties of various voting mechanisms when voters' preferences are nonuniform. We intend to address these and related issues in the near future.

[20] In particular, players 2 who initiate the bigger coalitions in both games 3 and 4, tend to receive less than in the preference-neutral games E and F, respectively (see Tables 11 and 12), even though this difference is not significant statistically.

Appendix A: Instructions for S-games and 1-games

A1. Introduction (common to all treatments)

You are a participant of an economic experiment in group decision making. During the experiment, you as well as other participants in this room will be making decisions.

We ask you to comply with the rules of the experiment and obey the instructions of the experimenter. Any communication with other participants except by means of your computer terminal is strictly prohibited. During the experiment you must not talk, exchange notes, watch other participants' actions, use the Internet for purposes not related to the experiment, leave the room and use mobile devices (cell phones, players, etc.) which must be switched off for the whole time of the experiment.

At the end of the experiment you will be paid in cash, the amount of money you earn being dependent on the decisions that you and other participants make. Your earnings are measured in points and will be converted into cash with the exchange rate 1 point=0.4 Russian roubles. Thus, a total payoff of 300 points means that at the end of the experiment you will receive 120 roubles, a total payoff of 500 points – 200 roubles, etc.

A2. Description of Game S.

There are ten rounds in this game. In each round you will be making a decision in a group with two other people, but you will not know who they are because the people in your group will change randomly every round.

At the beginning of each round the computer will randomly assign you one of the numbers: 1, 2 or 3. This number may change from round to round. Each player has a certain number of votes, namely:

- Player 1 has 3 votes;
- Player 2 has 2 votes;
- Player 3 has 2 votes.

This information is also shown in the table in the top-left corner of the screen.

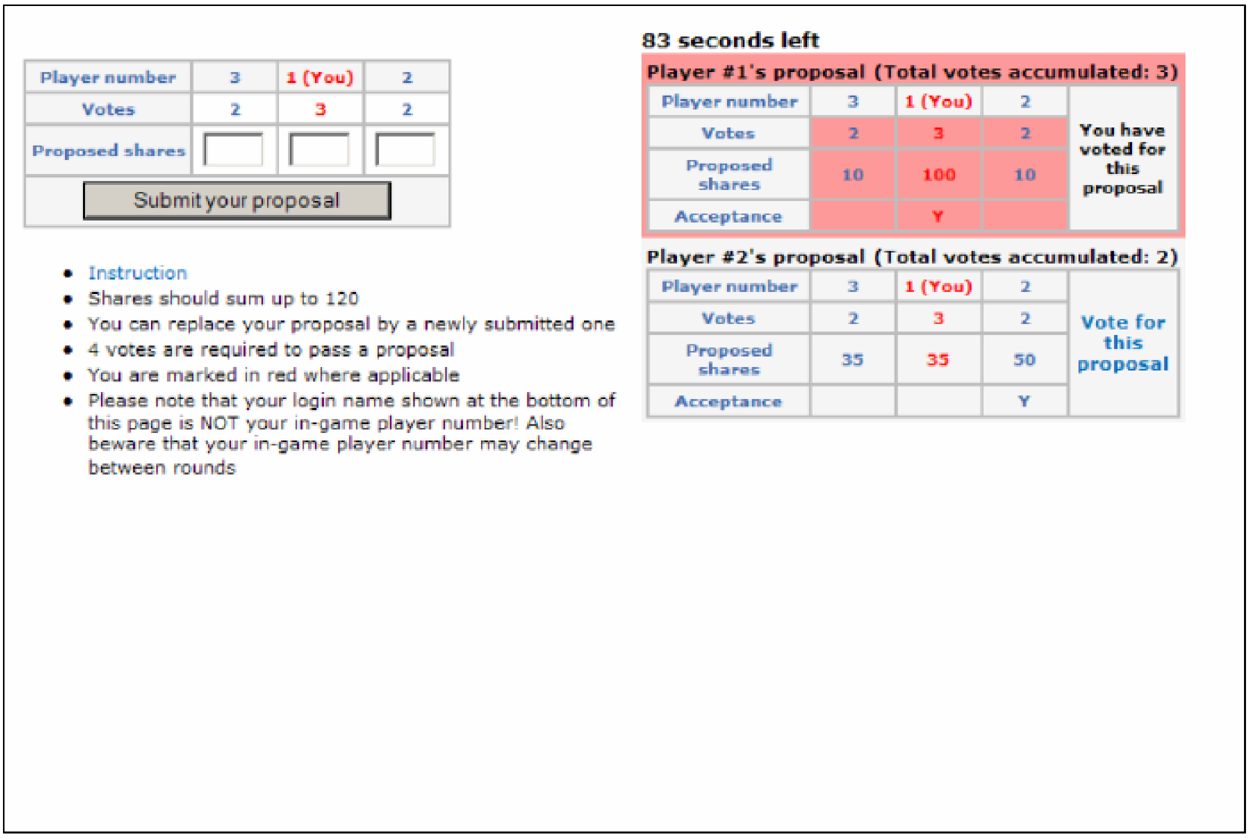

Figure A1. Screenshot of the S-game

In each round three players in each group should agree on how to divide 120 points among them. Any member of a group at any moment may make a public proposal about how this amount should be divided. In addition, any member of a group may also vote for any of the already submitted proposals. The first proposal that receives FOUR votes out of total 7 votes will be enforced and each of your group members will receive the number of points specified in that proposal.

There is a time limit of 300 seconds for each round. If during that time you and other people of your group have not accepted a proposal, which requires the support of at least 4 votes out of 7, then you and other members of your group receive zero points in the current round.

At the beginning of the next round you will be again randomly assigned one of the numbers: 1, 2 or 3. You are always positioned in the centre of the table and your number in the current round is highlighted in red and has the label ‘You’ next to it.

Your total payoff is determined as the sum of points that you have earned in all 10 rounds.

A3. Description of Game 1.

There are ten rounds to this game. In each round you will be making a decision in a group with two other people, but you will not know who they are because the people in your group will change randomly every round.

At the beginning of each round the computer will randomly assign you one of the numbers: 1, 2 or 3. This number may change from round to round. Each player has a certain number of votes, namely:

- Player 1 has 3 votes;
- Player 2 has 2 votes;
- Player 3 has 2 votes.

This information is also shown in the table in the top-left corner of the screen.

In each round three players in each group should agree on how to divide 120 points among them. Any member of a group at any moment may make a public proposal about how this amount should be divided. In addition, any member of a group may also vote for any of the already submitted proposals. The first proposal that receives FOUR votes out of total 7 votes will be enforced.

The payoff of each player also depends on MODIFIERS that are specified in the table in the bottom-left corner of the screen. Elements of this table change (modify) the *nominal* payoffs of the players defined according to a proposal. If a proposal is enforced, your *calculated* payoff in the current round equals your share specified in the proposal multiplied by your "modifiers" towards all other members who voted for the proposal along with you.

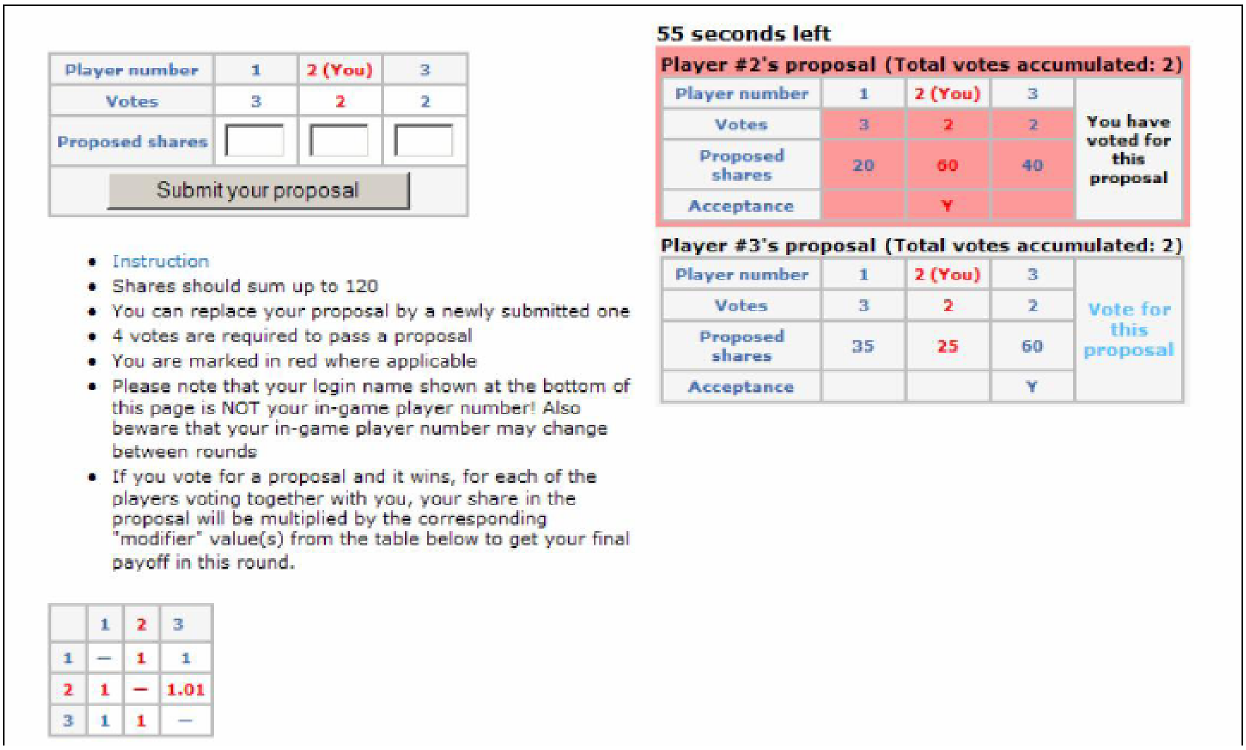

Figure A2. Screenshot of the 1-game

For example, the payoff modifier of player 2 for a proposal where player 2 and player 3 vote together is 1.01; therefore, if this is the case, then player 2's nominal payoff is multiplied by 1.01. On the other hand, if player 1 and player 2 vote together for a proposal, then, as player 2's modifier towards coalescing player 1 equals 1, player 2's nominal payoff equals her calculated payoff (and the same holds for player 1 as she has a modifier of 1 towards coalescing player 2).

At the end of each round, all group members will receive the number of points in accordance with their calculated payoffs (thus taking into account their modifiers).

There is a time limit of 300 seconds for each round. If during that time you and other people of your group have not accepted a proposal, which requires the support of at least 4 votes out of 7, then you and other members of your group rec eive zero points in the current round.

At the beginning of the next round you will be again randomly assigned one of the numbers: 1, 2 or 3. You are always positioned in the centre of the table and your number in the current round is highlighted in red and has the label 'You' next to it.

Your total payoff is determined as the sum of points that you have earned in all 10 rounds.

A3. Rules of voting and making proposals (common to all treatments)

To propose a new division of 120 points you need to fill in the form in the top-left corner of the screen, specifying for each player the number of points (an integer number from 0 to 120) that you would like to offer to her. The sum of the proposed payoffs must equal 120 points; otherwise the computer will not allow your proposal to be submitted. As soon as you press the button "Submit proposal", your proposal is displayed in the right-hand side of the screen. Your proposal will automatically receive the number of votes you control in the current round. Any other participant of your group can support your proposal pressing "Vote for this proposal", as well as you can vote for a proposal of any other member of your group.

At any moment you may also make a new proposal using the same form in the top-left corner of the screen. As soon as you submit a new proposal, it replaces your previous proposal and is shown in the bottom of the list of all proposals in the right hand side of the screen.

As soon as one of existing proposals in your group receives the minimum required number of votes for a proposal to pass (or, alternatively, the length of the round exceeds the time limit for decision making process), the round is finished and the distribution of the payoffs in your group is shown on the screen.

In order to continue the game and move to the next round, press "Go to next round". A new round starts after all the participants finish the previous, which may take some time. Please, take this fact into consideration and wait patiently for the beginning of the next round. You can update your screen using the F5 key.

Main game rules are also shown in the list under the form for making proposals in the left side of the screen.

Do you have any questions?

If everything is clear to you, then please press "I am ready" and start the game.

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