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# THE IMPLICATIONS OF THE ASYMMETRIC PRICE RIGIDITY FOR THE MONETARY POLICY IN AN OPEN ECONOMY AND A CROSS-COUNTRY EVIDENCE

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We build a New Keynesian model of a small open economy and analyze the optimal monetary policy assuming symmetric and asymmetric price rigidity. We find that, in the presence of asymmetric price rigidity, inflationary and deflationary shocks should be treated asymmetrically, and the optimal direction of the asymmetry depends on the price rigidity and the social preferences. In particular, if prices are sufficiently flexible and/or the output gap is not very important (e.g. strict inflation targeting), then inflationary shocks should be contracted more severely than deflationary ones of the same size should be accommodated. But in the opposite case the optimal asymmetry is reversed. We test the predictions of our model for a set of developed countries, concentrating on the exchange rate policy. We find that the exchange rate plays a significant role in the monetary policy in most countries, and we find some evidence of the asymmetry which is in line with inflation targeting according to our model.

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# 1. Introduction

In the modern literature it is common to analyze the monetary policy in a New Keynesian framework where the equilibrium in an economy is described by linear IS and Phillips curves, and the monetary authority minimizes a symmetric loss function. In such setting the derived monetary policy rule (the Taylor rule) is symmetric, according to which the interest rate adjusts by the same magnitude to positive and negative shocks of the same size.

But recently different kinds of non-linearities and asymmetries have been observed in the behavior of economic agents and macro-economy as a whole. In particular, the Phillips curve is observed to be convex (Latxon et al., 1999, and Alvarez Lois, 2000, for the USA; Dolado et. al., 2005, for several European countries) because of the asymmetric price rigidity[1]. Such Phillips curve is steeper for positive changes in inflation or output gap than for negative ones leading to asymmetric reaction of the economy to positive and negative shocks.

Indeed, as found by several authors for several countries (e.g. Cover, 1992), positive demand shocks affect inflation more and output less, than similar negative ones do. Therefore, the asymmetric price rigidity should have important implications for the appropriate stabilization policy. Nevertheless, such asymmetries are rarely incorporated into theoretical models of the optimal monetary policy, especially into New Keynesian models which are linear by nature.

Orphanides and Wieland (2000) is one of the first studies which analyses the impact of a nonlinear Phillips curve on the optimal monetary policy. In particular, they assume zone-linear Phillips curve and conclude that monetary policy should be non-linear as well. Diana and Méon (2005) analyze the optimal monetary policy under asymmetric wage indexation when wages are indexed upwards but not downwards, and propose that positive supply shocks should be absorbed more than negative ones. Dolado at al. (2005) assume a

[1] Asymmetric price rigidity means that prices react more and faster to inflationary shocks than to deflationary ones of the same size (e.g. Peltzman, 2000).

Also asymmetries are documented in the social loss function (e.g. Ruge-Murcia, 2004; Dolado, Pedrero, and Ruge-Murcia, 2004), in the monetary transmission mechanism and interest rate pass-through (e.g. Hoffman and Mizen, 2004, and de Bondt et al., 2005, for asymmetric pass-through on borrowing and lending rates; Ellingsen and Söderström, 2001, for asymmetric pass-through on short and long rates).

more general setting and study the implications of a convex Phillips curve for the optimal monetary policy rules and find that the policy-maker should "increase the interest rate by a larger amount when inflation or output are above the target than the amount it will reduce them when they are below the target".

Generally speaking, the papers confirm that the monetary policy should react asymmetrically to shocks of different signs, and they propose a certain direction of the asymmetry. But the empirical evidence of the behavior of Central banks is mixed. Some papers find that the monetary policy is asymmetric indeed. For example, Dolado et al. (2005) conclude that their non-linear interest rate rule describes the dynamics of the monetary policy rate in the Euro area better than a standard linear Taylor rule. Dolado et al. (2004) also find asymmetries in the US monetary policy during Volcker-Greenspan period, but not during Burns-Miller period. Asymmetric interest rate defense of the exchange rate in the US was also documented by Dobrynskaya (2008a). Taylor and Davradakis (2006) analyze the interest rate behavior in the UK and find that while it can be described by a Taylor rule when inflation is above the target, it follows a random walk when inflation is below the target. So, they conclude, that the UK monetary policy is asymmetric, and the Taylor rule is applied once the inflation hits the target. Interestingly, while inflationary shocks seem to be contracted more than deflationary ones in the USA and European countries, the asymmetry is found to be the opposite in Russia (Dobrynskaya, 2008b). Other papers do not find any signs of non-linearities in monetary policy rules (e.g. Bruinshoofd and Candelon, 2004).

Although the actual behavior of Central banks cannot be regarded as a proof of the theory, it can reveal the country's preferences towards inflation and output. Therefore, the observed different directions of the monetary policy asymmetry still need to be explained in terms of the social preferences. The existing literature does not shed light on this issue since it proposes a definite direction of the asymmetry. But the existing theoretical literature has two general shortcomings. First, the theoretical models are not micro-founded, the non-linearity in the Phillips curve is assumed ad hoc and the authors do not provide a theoretical rationale for using a particular functional form of the Phillips curve. Secondly, the existing models analyze closed economies, and hence may overlook some important implications coming from international transmission of shocks.

In this paper we propose a New Keynesian open economy model with micro-foundations. First, we derive a linear version of it, we show that the equilibrium in an economy can be described by IS and Phillips curves which are different from their closed-economy counterparts in terms of the coefficients, and we are able to obtain a closed-form solution for the optimal monetary policy rule. This allows us analyze the impact of different parameters of an economy (such as price stickiness, exchange rate pass-through, weight of the output gap in the social loss function, and others) on the optimal interest rate response to a shock (e.g. demand, cost, exchange rate, or foreign inflation shock).

Secondly, we modify the Phillips curve under the assumption of asymmetric price rigidity, what allows us derive a kinked Phillips curve with a kink at the zero output gap. Such Phillips curve can be regarded as an approximation of a convex one, which was estimated empirically (Dolado at al., 2005). We study the implications of the kinked Phillips curve for the optimal monetary policy rule. Indeed, the monetary policy rule becomes asymmetric, but the optimal asymmetry may be different for different parameters of the economy. We show that while in case of strict inflation targeting or when prices are not sufficiently sticky and/or the weight of the output gap in the social loss function is small, inflationary supply shocks should be contracted more severely than deflationary ones of the same size should be accommodated. This case can describe the observed behavior of the monetary policy in western countries. But when the weight of the output gap is sufficiently high and/or the prices are very sticky, the asymmetry is reversed, so that deflationary shocks become more important. We also find that the degree of openness of an economy may affect the optimal direction of the monetary policy asymmetry.

We test the predictions of the model for a sample of developed countries. We analyze the response of the monetary policy to exchange rate shocks and find it to be in line with the model: the interest rate reacts to exchange rate shocks in order to smooth their effects on the domestic inflation, and the extent of this reaction depends positively on the degree of exchange rate pass-through. We also find some evidence of the asymmetry in monetary policies, which is in line with inflation targeting according to our model.

This paper proceeds as follows. In section 2 we lay out the theoretical model, derive the IS and Phillips curves and analyze the optimal monetary policy under symmetric and asymmetric price rigidity. In section 3 we perform the empirical tests of the predictions of our theoretical model. Section 4 is devoted to conclusions.

## 2. The theoretical model

In this section we build a New-Keynesian open economy model and analyze the optimal monetary policy in response to stochastic shocks.

### *2.1. Demand*

The demand side is represented by a New Keynesian IS curve. To derive it we follow the set-up of Gali and Monacelli (2005).

*Consumer choice*

We assume that the world consists of an infinite number of symmetric small open economies. Each economy i, $i \in [0,1]$, produces an infinite number of goods, indexed by *j*, $j \in [0,1]$. All goods are traded, and subscript *H* denotes the goods produced domestically while subscript *F* denotes imported goods. We analyze a typical economy which we call 'the domestic economy'.

In the domestic economy the representative consumer maximizes the following discounted expected utility function:

$$\max U_t = E_t \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi} \right) \quad (1)$$

where $\beta_t$ is the subjective discount factor, $N_t$ is the labor supply and $C_t$ is the following CES consumption index with the elasticity of substitution $\eta > 0$, $\eta \neq 1$:

$$C_t \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2)$$

where $\alpha$ is the share of imported goods, $C_{H,t}$ and $C_{F,t}$ are CES consumption indices of domestic and imported goods respectively with the corresponding elasticities of substitution $\varepsilon>0$, $\varepsilon \neq 1$ and $\gamma>0$, $\gamma \neq 1$:

$$C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{F,t} \equiv \left( \int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where $\gamma$ is the elasticity of substitution of goods imported from different countries and $C_{i,t}$ is the index of consumption of goods, imported from country *i*:

$$C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$



The utility function (1) is characterized by diminishing marginal utility of consumption ($0 < \sigma < 1$) and increasing marginal disutility of labor supply ($\phi > 0$). In this model we assume that consumers do not derive utility from holding money, and money serves as a mean of exchange only.

In every period the utility function (1) is maximized subject to the following period budget constraint:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t \left\{ \frac{1}{1+r_t} D_{t+1} \right\} = \\ = D_t + W_t N_t + R_t \quad (3)$$

where $P_{H,t}(j)$ is the price of the domestic good *j*, $P_{i,t}(j)$ is the domestic price of the good j, imported from country i, $D_t$ is the value of investment portfolio, $r_t$ is the nominal interest rate, $W_t$ is the wage rate and $R_t$ represents net transfers[2]. Thus, the left-hand side of the budget constraint describes the consumer's spending on consumption of domestic and foreign goods and investment, while the right-hand side describes the consumer's current wealth.

Budget constraint (3) may be written in a more concise way as:

$$P_t C_t + E_t \left\{ \frac{1}{1+r_t} D_{t+1} \right\} = D_t + W_t N_t$$

where $P_t$ is the aggregate price index in the economy and $C_t$ is the aggregate consumption index.

The first-order condition of the consumer's intratemporal maximization problem is

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}$$

which is log-linearized as follows:

$$\sigma c_t + \varphi n_t = w_t - p_t \quad (4)$$

and the one of her intertemporal maximization problem is the Euler equation:

$$\beta(1+r_t) E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\} = 1 \quad (5)$$

which is log-linearized as follows:

[2] We assume that the government issues bonds and redistributes the proceeds in the form of net transfers.

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(\tilde{r}_t - E_t \pi_{t+1} - \rho\right) \qquad (6)$$

where lower cases denote logarithms, $\tilde{r}_t \equiv \ln(1+r_t)$ , $\pi_{t+1} \equiv p_{t+1} - p_t = \ln\left(\frac{P_{t+1}}{P_t}\right)$ is the domestic CPI inflation and $\rho \equiv -\ln\beta = \ln(1+r)$ , where $r$ is the subjective discount rate.

Euler equation (6) is one of the key equations in our model. It shows that the optimal consumption depends positively on the future expected consumption (consumption smoothing effect) and negatively on the real interest rate (intertemporal substitution effect).

*Exchange rate, terms of trade and inflation*

We use the following notations: $S$ — terms of trade, $E$ — nominal exchange rate, $Q$ — real exchange rate.

We assume that the law of one price holds for all individual imported goods:

$$P_{i,t}(j) = \mathrm{E}_{i,t} P^i_{i,t}(j)$$

where $\mathrm{E}_{i,t}$ is the nominal exchange rate of country $i$, an increase of which means depreciation of the domestic currency against the currency of country $i$, and $P_{i,t}^{i}(j)$ is the price of good $j$ denominated in the currency of country $i$.

Then the price index of goods imported from country $i$ equals:

$$P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} = \mathrm{E}_{i,t} P^i_{i,t},$$

and the price index of all imported goods equals:

$$P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \mathrm{E}_t P_t^*$$

where $\mathrm{E}_t$ is the nominal effective exchange rate and $P_t^*$ is the world price index.

The above equation in log-linear form looks as follows:

$$p_{F,t} = e_t + p_t^* \qquad (7)$$

The terms of trade of the domestic economy and economy $i$ are defined as the ratio of prices of the national goods, expressed in the domestic currency (that is the currency of the economy under consideration):



$$S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$$

Then the effective terms of trade are:

$$S_t \equiv \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} = \frac{P_{F,t}}{P_{H,t}}$$

or in log-linear form:

$$s_t = \int_0^1 s_{i,t} di = p_{F,t} - p_{H,t} \qquad (8)$$

Substituting equation (7) into equation (8) we arrive at the following expression for the terms of trade:

$$s_t = e_t + p_t^* - p_{H,t} \qquad (9)$$

The real exchange rate of economy $i$ equals the ratio of consumer price indices, expressed in the domestic currency:

$$Q_{i,t} \equiv \frac{\mathrm{E}_{i,t} P^i_{i,t}}{P_t}$$

and in log-linear form:

$$q_{i,t} = e_{i,t} + p^i_{i,t} - p_t$$

Then the logarithm of the real effective exchange rate equals:

$$q_t \equiv \int_0^1 q_{i,t} di = \int_0^1 \left(e_{i,t} + p^i_{i,t} - p_t\right) di = e_t + p_t^* - p_t \qquad (10)$$

Substituting equation (9) into equation (10) and making use of the following expression for the consumer price index:

$$p_t \equiv (1-\alpha) p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t \qquad (11)$$

we arrive at the following expression of the logarithm of the real exchange rate:

$$q_t = (1-\alpha) s_t \qquad (12)$$

We assume that the uncovered interest parity condition between the domestic economy and economy $i$ holds:

$$E_t \mathrm{E}^i_{t+1} = \frac{1+r_t}{1+r^i_t} \mathrm{E}^i_t \qquad (13)$$

The nominal exchange rate itself is subject to an exogenous stochastic shock, which may be viewed, for example, as a financial markets shock, so that equation (13) in log-linear form looks as follows:

$$\Delta e^i_{t+1} = \tilde{r}_t - \tilde{r}^i_t + \psi^i_{t+1}$$

where $\psi^i_{t+1}$ is the country i's nominal exchange rate shock, $\psi^i_t \sim N(0,\sigma^2_{\psi^i})$.

Then the change in the nominal effective exchange rate of the domestic economy is:

$$\Delta e_{t+1} \equiv \int_o^1 \Delta e^i_{t+1} di = \tilde{r}_t - \tilde{r}^*_t + \psi_{t+1} \tag{14}$$

where $\tilde{r}^*_t \equiv \int_0^1 \tilde{r}^i_t di$ is the average foreign interest rate and $\psi_{t+1}$ is the nominal effective exchange rate shock, $\psi_t \sim N(0,\sigma^2_\psi)$.

Equation (10) for the real exchange rate may be re-written in first differences as:

$$\Delta q_{t+1} = \Delta e_{t+1} + \pi^*_{t+1} - \pi_{t+1} \tag{15}$$

Substituting equation (14) into equation (15) and taking expectations we arrive at the following expression for the expected real exchange rate depreciation:

$$E_t \Delta q_{t+1} = \tilde{r}_t - E_t \pi_{t+1} - (\tilde{r}^*_t - E_t \pi^*_{t+1}) \tag{16}$$

Equation (16) may be interpreted that the expected real exchange rate depreciation equals the difference between home and world real interest rates, what follows from the interest parity condition.

As it will be seen later, in the absence of shocks the real interest rate is a constant. Since in our model all economies are infinitely small and face uncorrelated shocks, we assume for simplicity that the average foreign real interest rate, represented by that last term in brackets in equation (16), is some constant as well.

*Equilibrium in the goods market*

In equilibrium, the domestic production of good *j* $Y_t(j)$ is equal to the world demand for it (i.e. the demand for good j of all consumers living in the domestic economy and all other economies) and can be expressed as follows (see Appendix 1 (a) for the derivation):



$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j) di = \tag{17}$$

$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[ (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathrm{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \right]$$

Then the aggregate output of the domestic economy equals (see Appendix 1 (b)):

$$Y_t \equiv \left[ \int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} =$$

$$= \left[ (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathrm{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \right] =$$

$$= S^{\alpha\eta}_t C_t \left[ (1-\alpha) + \alpha \int_0^1 \left(S_{i,t} S^i_t\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di \right] \tag{18}$$

where $S_t$ are effective terms of trade, $S_{i,t}$ are terms of trade with country i and $S^i_t$ are effective terms of trade of economy *i*.

Taking into account that $\int_0^1 s^i_t di = 0$, equation (18) is log-linearized as follows:

$$y_t = c_t + \alpha\gamma s_t + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t$$

Substituting the expression for $s_t$ from equation (12) into the above equation we get:

$$y_t = c_t + \alpha\gamma \frac{q_t}{1-\alpha} + \alpha\left(\eta - \frac{1}{\sigma}\right) q_t = c_t - \nu q_t \tag{19}$$

$$\text{where } \nu \equiv \alpha\left(\frac{1}{\sigma} - \frac{\gamma}{1-\alpha} - \eta\right).$$

Then we substitute the expression for $c_t$ from equation (19) into equation (6) and arrive at the following version of the IS curve for the economy:

$$y_t = \frac{\rho}{\sigma} + E_t y_{t+1} - \frac{1}{\sigma}\left(\tilde{r}_t - E_t \pi_{t+1}\right) + \nu E_t \Delta q_{t+1} \tag{20}$$

Finally, we substitute the expression for the expected real exchange rate depreciation (16) into (20) and simplify the notations:

$$y_t = a + E_t y_{t+1} - b\left(\tilde{r}_t - E_t \pi_{t+1}\right) \quad (21)$$

where $a \equiv \frac{\rho}{\sigma} - \nu(\tilde{r}_t^* - E_t \pi_{t+1}^*)$ and $b \equiv \frac{1}{\sigma} - \nu = (1-\alpha)\frac{1}{\sigma} + \alpha(\frac{\gamma}{1-\alpha} + \eta) > 0$.

We assume that the potential output of the economy, $\bar{y}$, is the output level in the absence of shocks and flexible prices. We also assume that the economy is subject to an exogenous stochastic demand shock $\xi_t$, $\xi_t \sim N(0, \sigma_\xi^2)$. Such a demand shock may be either a government spending shock, which we do not model here and, hence, assume fully exogenous, or a consumption shock caused, for example, by a change in the subjective discount factor $\beta$[3], or a rest-of-the-world real interest rate shock. Then the equation (21) may be re-written in terms of the output gap, as it is common in the literature:

$$\hat{y}_t = a + E_t \hat{y}_{t+1} - b\left(\tilde{r}_t - E_t \pi_{t+1}\right) + \xi_t \quad (22)$$

where $\hat{y}_t \equiv y_t - \bar{y}$ is the output gap.

Equation (22) represents the IS curve for an open economy. This New Keynesian IS curve shows that the output gap depends positively on the expected future output gap due to the consumption-smoothing effect and negatively on the real interest rate due to the intertemporal substitution effect. What distinguishes this open-economy IS curve from a closed-economy one are the coefficients $a$ and $b$[4]. In particular, the higher are the elasticities of substitution between the domestic and the foreign goods ($\eta$) and between the foreign goods ($\gamma$), the more responsive is the output gap to changes in the local real interest rate. This is due to the higher elasticity of net exports with respect to changes in the exchange rate (according to equation (19)) resulting from changes in the interest rate via the interest rate parity. Also the open-economy IS curve shifts in response to changes in the world real interest rate (and, hence, changes in parameter $a$). Thus, the domestic economy is vulnerable to foreign monetary policy, if it is significant enough to influence the world real interest rate.

The above derivation of the IS curve is similar to Gali and Monacelli (2005) in the beginning, but finally they model the output gap as a function of the expected inflation of the domestically produced goods only, while in our model

[3] Recall that $\rho \equiv -\ln\beta = \ln(1+r)$, where r is the subjective discount rate. If r is subject to stochastic shocks, then these shocks will be translated into the demand shock $\xi_t$.

[4] In a closed economy the share of import goods $\alpha = 0$, and therefore $a \equiv \frac{\rho}{\sigma}$ and $b \equiv \frac{1}{\sigma}$.



the output gap depends on the expected CPI inflation. Hence, in this paper we provide a microfounded derivation of the open-economy IS curve, which is so widely used in the literature (see Clarida, Gali and Gertler, 1999), and claim that the same functional form can be obtained for both closed and open economies with the only difference in the coefficients.

### *2.2. Supply*

The supply is represented by a New Keynesian Phillips curve. To derive it, we distinguish between domestic goods' pricing and foreign goods' pricing.

*Domestic goods' pricing*

We assume that the domestic producers set prices in a staggered fashion a la Calvo (1983). In every period a producer receives a signal with probability $(1 - \theta)$ that it should re-set its price. Therefore, its price will stay intact with probability $\theta$. The value of $\theta$ is assumed to be constant, so that the probability of changing the price in a given period does not depend on whether the price was changed in the previous period or not. The higher is the parameter $\theta$, the more sticky are domestic goods' prices in the economy.

Having received the signal to adjust the price, the producer of $j$-th good sets the new price $P_{H,t}(j)$, which minimizes his expected losses from log-deviations of this price from an optimal price $\tilde{P}_{H,t}(j)$:

$$\min \frac{1}{2}\sum_{k=0}^{\infty}\left(E_t p_{H,t+k}(j) - E_t \tilde{p}_{H,t+k}(j)\right)^2$$

where the lower cases denote logarithms.

Since the price will remain unchanged for k periods with probability $\theta^k$, the function of the expected discounted losses from deviation of the price $P_{H,t}(j)$ from the optimal price looks as follows:

$$\frac{1}{2}\sum_{k=0}^{\infty}\theta^k\left(p_{H,t}(j) - E_t \tilde{p}_{H,t+k}(j)\right)^2 \quad (23)$$

Minimizing function (23) with respect to $p_{H,t}(j)$ we get the following recursive first-order condition (see Appendix 2 (a)):

$$p_{H,t}(j) = (1-\theta)\tilde{p}_{H,t}(j) + \theta E_t p_{H,t+1}(j) \quad (24)$$

Equation (24) means that the current price equals the average of the desired price and the following period price, weighted with the probability of changing the price.

Since in every period the share of firms (1 — θ) adjust their prices while the share of firms θ keep their prices constant, the aggregate price index equals:

$$P_{H,t} \equiv \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta) P_{H,t}(j)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

or in log-linear form:

$$p_{H,t} = \theta p_{H,t-1} + (1-\theta) p_{H,t}(j) \tag{25}$$

Equation (25) is iterated forward and transformed (see Appendix 2 (b)), after which we arrive at the expression for the expected price in the future period:

$$E_t p_{H,t+1}(j) = \frac{1}{1-\theta} E_t \pi_{H,t+1} + p_{H,t} \tag{26}$$

which is then substituted into equation (24):

$$p_{H,t}(j) = (1-\theta)\tilde{p}_{H,t}(j) + \frac{\theta}{1-\theta} E_t \pi_{H,t+1} + \theta p_{H,} \tag{27}$$

The optimal price of the producer $\tilde{p}_{H,t}(j)$ is found from the profit maximization problem below. We assume a linear production function:

$$Y_t(j) = T_t N_t(j) \tag{28}$$

where $T_t$ is a technology parameter and $N_t(j)$ is the employment in the production of good *j*.

Each producer maximizes the following profit function:

$$\Pi_t(j) = Y_t(j)(P_{H,t}(j) - MC_t(j))$$

subject to the demand constraint (17). Then the optimal price of good *j* equals

$$\tilde{P}_{H,t}(j) = \frac{\varepsilon}{\varepsilon - 1} MC_t(j) \tag{29}$$

which can be re-written in log-linear form as follows:

$$\tilde{p}_{H,t}(j) = \ln\frac{\varepsilon}{\varepsilon-1} + mc_t(j) \tag{30}$$

It follows from the production function (28) that $MC_t(j) = W_t / T_t$ . Let $MC_t^r(j)$ denote the real marginal cost defined as follows:

$$MC_t^r(j) \equiv \frac{MC_t(j)}{P_{H,t}} = \frac{W_t}{P_{H,t} T_t} \tag{31}$$



which can be written in log-linear form as:

$$mc_t^r(j) = w_t - p_{H,t} - t_t \tag{32}$$

Using the consumption optimality condition (4) and equation (11) we obtain the following relationship for the real wage rate:

$$w_t - p_{H,t} = \sigma c_t + \varphi n_t + \alpha s_t \tag{33}$$

The production function for good j (28) can be log-linearized and then aggregated over all goods produced in the economy and approximated as follows[5]:

$$y_t = t_t + n_t \tag{34}$$

Finally, we substitute equation (33) into equation (32) and make use of equations (34), (19) and (12) to derive the following expression for the real marginal cost:

$$mc_t^r(j) = \chi y_t + (\sigma\nu(1-\alpha) + \alpha)s_t - (\varphi + 1)t_t \tag{35}$$

where $\chi \equiv \sigma + \varphi$ .

The natural level of output $\bar{y}_t$ is the one that would arise in the flexible-price steady state, when $mc_t^r = -\ln\frac{\varepsilon}{\varepsilon-1}$. Then, setting equation (35) to $-\ln\frac{\varepsilon}{\varepsilon-1}$ , we can find the expression for the natural level of output. Putting it back into equation (35), the real marginal cost equation (35) can be re-written in terms of the output gap as follows:

$$mc_t^r(j) = \chi\hat{y}_t - \ln\frac{\varepsilon}{\varepsilon-1}$$

Then, the equation for the nominal marginal cost is the following:

$$mc_t(j) = \chi\hat{y}_t - \ln\frac{\varepsilon}{\varepsilon-1} + p_{H,t} \tag{36}$$

Substituting equation (36) into equation (30) we find the optimal price of good j as a function of the output gap:

$$\tilde{p}_{H,t}(j) = \chi\hat{y}_t + p_{H,t} \tag{37}$$

Now, substituting this expression for the optimal price into the equation for the price of good j (27), and then into the expression for the general price in-

[5] See Gali and Monacelli (2005) Section 2.2.1.

dex (25), after some transformations, we arrive at the New-Keynesian Phillips curve for the inflation of the domestic goods:

$$\pi_{H,t} = E_t\pi_{H,t+1} + \frac{(1-\theta)^2\chi}{\theta}\hat{y}_t \tag{38}$$

With simplified notations and a stochastic cost shock $\omega_t$, $\omega_t \sim N(0,\sigma_\omega^2)$, which can be viewed as a technology shock, if the technology $T_t$ is a stochastic process, the Phillips curve for the domestic goods looks as follows:

$$\pi_{H,t} = E_t\pi_{H,t+1} + d\hat{y}_t + \omega_t \tag{39}$$

where $d \equiv \frac{(1-\theta)^2\chi}{\theta}$ is the slope of the Phillips curve.

The above domestic goods' Phillips curve has the same functional form, as the one that would be obtained for a closed economy. Its slope represents the degree of price stickiness in the economy.

*Import goods' pricing*

As it was already mentioned, we assume that the law of one price holds for all individual imported goods[6]. Hence, there is complete exchange rate pass-through onto import prices, as described by equation (7).

Then the inflation of the imported goods equals:

$$\pi_{F,t} = \Delta e_t + \pi_t^* \tag{40}$$

where $\Delta e_t \equiv e_t - e_{t-1}$ and $\pi_t^*$ is the world inflation.

*Aggregate supply*

Recall the following equation for the log-linearized consumer price index:

$$p_t \equiv (1-\alpha)p_{H,t} + \alpha p_{F,t}$$

Then the aggregate inflation is the following:

$$\pi_t \equiv (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{41}$$

Substituting the expressions for the domestic goods' inflation (39) and import goods' inflation (40) into equation (41), and making use of equation (14)

[6] We do not assume staggering in import pricing, as it would not affect the conclusions of the model significantly.



assuming the symmetric initial situation in all countries, we get the following expression for the overall inflation in the economy:

$$\pi_t = (1-\alpha)(E_t\pi_{H,t+1} + d\hat{y}_t + \omega_t) + \alpha(\psi_t + \pi_t^*) \tag{42}$$

Writing equation (41) for the period t+1, transforming it, using (40) and taking expectations we find the equation for the next period expected domestic goods' inflation:

$$E_t\pi_{H,t+1} = \frac{1}{1-\alpha}E_t\pi_{t+1} - \frac{\alpha}{1-\alpha}E_t\pi_{t+1}^* \tag{43}$$

Finally, we substitute equation (43) into equation (42) to arrive at the following expression of the New Keynesian Phillips curve for the economy:

$$\pi_t = E_t\pi_{t+1} + d(1-\alpha)\hat{y}_t + (1-\alpha)\omega_t + \alpha(\psi_t + \pi_t^* - E_t\pi_{t+1}^*) \tag{44}$$

This Phillips curve shows that the domestic inflation depends on the expected future inflation, current output gap and three types of shocks: cost shocks, nominal exchange rate shocks and foreign inflation shocks. The last term of equation (44) shows imported inflation with $\alpha$ measuring the exchange rate and foreign inflation pass-through onto CPI inflation: the higher is the share of the import goods $\alpha$ in the consumption basket, the higher is the pass-through effect in the economy. If $\alpha$ is zero, the Phillips curve simplifies to the closed-economy version (39).

### *2.3. Monetary policy*

*2.3.1. Symmetric price rigidity*

The equilibrium in the economy is described by the system of equations (22) and (44) holding simultaneously.

We assume that in every period the monetary authority chooses the value of its monetary policy instrument, the interest rate, discretionally by minimizing the following loss function:

$$\min L_t = E_t\sum_{k=0}^{\infty}\beta^k[(\pi_{t+k} - \pi^T)^2 + \lambda\hat{y}_{t+k}^2] \tag{45}$$

where $\pi^T$ is the target inflation rate.

Substituting the Phillips curve (44) into the loss function (45) and minimizing the losses with respect to $\pi_t$ we obtain the following reaction function of optimal inflation to expected inflation:

$$\pi_t = \frac{1}{d^2(1-\alpha)^2+\lambda}[d^2(1-\alpha)^2\pi^T + \lambda E_t\pi_{t+1} +$$

$$+\lambda(1-\alpha)\omega_t + \lambda\alpha(\psi_t + \pi_t^* - E_t\pi_{t+1}^*)] \quad (46)$$

Taking expectations of the both sides of (46), assuming that the average expected foreign inflation equals to its target ($E_t\pi_{t+1}^* = \pi^{T*}$), and solving for the expected inflation we get:

$$E_t\pi_{t+1} = \pi^T \quad (47)$$

Expression (47) says that the expected inflation always equals to the target since the expectations of all shocks are zero. Substituting it back into the reaction function (46), we obtain the expression for the equilibrium inflation, which minimizes the losses of society:

$$\pi_t = \pi^T + \frac{\lambda}{d^2(1-\alpha)^2+\lambda}[(1-\alpha)\omega_t + \alpha\psi_t + \alpha(\pi_t^* - \pi^{T*}) \quad (48)$$

We see that in order to minimize the social losses, domestic inflation should be adjusted to cost, exchange rate and foreign inflation shocks. It should be noticed that there is no dynamic inconsistency here since the monetary authority targets the zero output gap. Therefore, the expected inflation equals the target inflation and there is no incentive to deviate from this target unless there are some unexpected shocks.

Substituting equation (48) back into the Phillips curve (44) we find the equilibrium output gap:

$$\hat{y}_t = -\frac{d(1-\alpha)}{d^2(1-\alpha)^2+\lambda}[(1-\alpha)\omega_t + \alpha\psi_t + \alpha(\pi_t^* - \pi^{T*})] \quad (49)$$

From equation (49) it follows that in the absence of shocks the output gap is zero and positive shocks lead to the output being lower than the potential output.

The final step is to derive the optimal instrument rule where the real interest rate serves as the instrument for the monetary policy. To do this we substitute equation (49) into the IS curve (22) and solve for *r*:

$$\tilde{r}_t^{opt} = \frac{a}{b} + \pi^T + \frac{d(1-\alpha)}{b(d^2(1-\alpha)^2+\lambda)}[(1-\alpha)\omega_t + \alpha\psi_t + \alpha(\pi_t^* - \pi^{T*})] + \frac{1}{b}\xi_t \quad (50)$$

which can be re-written in a general form as:

$$\tilde{r}_t^{opt} = r_0 + A\omega_t + B(\psi_t + \pi_t^* - \pi^{T*}) + F\xi_t, \quad (51)$$



where $A \equiv \frac{d(1-\alpha)^2}{b(d^2(1-\alpha)^2+\lambda)}$, $B \equiv \frac{d(1-\alpha)\alpha}{b(d^2(1-\alpha)^2+\lambda)}$ and $F \equiv \frac{1}{b}$ are positive coefficients.

This rule (the Taylor rule) states that in order to minimize the social losses the real interest rate should respond to all types of shocks in the economy: cost shocks, nominal exchange rate shocks, foreign inflation shocks and demand shocks — with positive coefficients. This means, for example, that positive shocks, being inflationary, should be accompanied by a contractionary monetary policy, leading to negative output gap and lower equilibrium inflation, than what would be without intervention.

***Proposition 1****. The interest rate should be increased following an inflationary cost shock, exchange rate shock or foreign inflation shock and should smooth their effect on the domestic inflation. The optimal interest rate response should be more significant if:*

— *prices are more sticky (parameter d is lower, provided that* $d(1-\alpha) > \sqrt{\lambda}$ *);*

— *the share of the import goods and, hence, pass-through effect is lower (parameter* $\alpha$ *is lower) for the cost shock*

— *the share of the import goods and, hence, pass-through effect is higher (parameter* $\alpha$ *is higher) for the exchange rate and foreign inflation shocks;*

— *the elasticity of consumption with respect to the interest rate is lower (parameter b is lower);*

— *the government cares less about the output gap (parameter* $\lambda$ *is lower).*

**Proof.** We differentiate expressions for *A* and *B* in equation (51) with respect to each of the parameters and determine the signs of the corresponding derivatives. The expressions for the derivatives and their signs are presented in Appendix 3.

So, an exchange rate shock, a foreign inflation shock and a cost shock, all being supply shocks, affect the economy in a similar way. Therefore, the monetary policy should react to them similarly with the only difference that the degree of exchange rate pass-through influences the optimal response in the opposite way.

The finding that the higher is pass-through effect on the aggregate inflation the higher should be the optimal adjustment of the interest rate to an exchange rate shock is in line with Devereux and Engel (2000) who claim that although under low pass-through freely floating exchange rate (a monetary policy in which exchange rates are not taken into consideration) may be optimal in some circumstances, this is never true in case of producer currency pricing leading to full pass-through.

The analysis of a demand shock $\varepsilon_t$ is straightforward. Since such a shock does not create any trade-off between the targeted parameters the task of the optimal monetary policy is simply to adjust the interest rate along the IS curve in order to bring the economy back to the target. Therefore, the magnitude of the interest rate adjustment does not depend on the parameters of the model except for the elasticity of consumption to the interest rate b (the slope of the IS curve).

*2.3.2. Asymmetric price rigidity*

The above analysis assumed symmetrically rigid prices and, hence, a linear interest rate rule. Now, we assume that the optimal price of a domestic producer depends on the output gap asymmetrically, and equation (37) transforms into the following:

$$\tilde{p}_{H,t}(j) = \begin{cases} p_{H,t} + \chi_1 \hat{y}_t, \hat{y}_t > 0 \\ p_{H,t} + \chi_2 \hat{y}_t, \hat{y}_t < 0 \\ \chi_1 > \chi_2 \end{cases} \quad (52)$$

The above pricing rule states that a producer would increase her price by a larger amount in response to an increase in the output than she would cut her price in response to a fall in it. In this paper we assume such asymmetric price rigidity ad hoc. It may arise for several reasons. First, consumers may have asymmetric preferences. For example, consumers may have higher disutility of labor (higher $\phi$) if labor supply rises than if it falls. Or they may have lower marginal utility of consumption (higher $\sigma$) when consumption rises than when it falls. In the both cases the firm's marginal cost will be asymmetric for positive and negative output gap, which will be translated into the asymmetric pricing rule (52). Secondly, producers may be irrational and may increase their prices by more in response to an increase in the marginal cost than they would cut prices in response to a similar reduction in the marginal cost, what is an empirical observation (e.g. Alvarez et al., 2005). A behavioral explanation of the above 'frictions', based on reference-dependent preferences and loss aversion, and a formal derivation of the asymmetric pricing rule can be found in Dobrynskaya (2008c).

Using the pricing rule (52) instead of (37) and repeating the same steps, we derive the following region-linear Phillips curve:

$$\pi_t = \begin{cases} E_t\pi_{t+1} + d_1(1-\alpha)\hat{y}_t + (1-\alpha)\omega_t + \alpha(\psi_t + \pi_t^* - E_t\pi_{t+1}^*), \hat{y}_t > 0 \\ E_t\pi_{t+1} + d_2(1-\alpha)\hat{y}_t + (1-\alpha)\omega_t + \alpha(\psi_t + \pi_t^* - E_t\pi_{t+1}^*), \hat{y}_t < 0 \\ d_1 > d_2 \end{cases} \quad (53)$$



where $d_{1,2} \equiv \frac{(1-\theta)^2 \chi_{1,2}}{\theta}$.

The above Phillips curve is steeper for the positive output gap than for the negative one with a kink at the zero level, and it is illustrated on Figure 1. Such Philips curve is a region-linear approximation of a convex one, found in, for example, Alvarez Lois (2000) and Dolado et al. (2005). A similar Phillips was derived in Diana and Méon (2005) under the assumption of asymmetric wage indexation relative to expected inflation.

Now the equilibrium in the economy is described by the IS curve (22) and the Phillips curve (53) holding simultaneously. Minimizing the social loss function (45), after similar derivations as in the symmetric case, we obtain the following interest rate rule:

$$\tilde{r}_t^{opt} = \begin{cases} \frac{a}{b} + \pi^T + \frac{d_1(1-\alpha)}{b(d_1^2(1-\alpha)^2 + \lambda)}[(1-\alpha)\omega_t + \alpha\psi_t + \alpha(\pi_t^* - \pi^{T*})] + \frac{1}{b}\xi_{tt}, \hat{y}_t > 0 \\ \frac{a}{b} + \pi^T + \frac{d_2(1-\alpha)}{b(d_2^2(1-\alpha)^2 + \lambda)}[(1-\alpha)\omega_t + \alpha\psi_t + \alpha(\pi_t^* - \pi^{T*})] + \frac{1}{b}\xi_t, \hat{y}_t < 0 \end{cases} \quad (54)$$

Figure 1 analyses the optimal monetary policy rule in cases of a positive and a negative supply shocks of the same size under asymmetric price rigidity.

First, for illustrative purposes, consider a case with $\lambda = 1$. Then the social losses are represented by a circle around the target point. A deflationary supply shock shifts the kinked Phillips curve downwards by the same magnitude as an inflationary one of the same size shifts it upwards. If the prices were symmetrically rigid (represented by a hypothetical dashed *PC**), the optimal loss-minimizing points in cases of a positive and a negative shocks would be points *C* and *E* respectively. In order to reach these points, the interest rate should be adjusted by the same absolute value. But since prices are assumed to be stickier for the negative output gap than for the positive one, the optimal point in case of the inflationary supply shock is point *D*, which corresponds to the flatter part of the Phillips curve and, hence, lies to the right of point *C*. To reach point *D*, the interest rate should be increased by less than to reach point *C*. Therefore, the change in the interest rate should be more significant in response to a deflationary supply shock than in response to an inflationary one of the same size.

Now, consider another extreme case when $\lambda = 0$ (strict inflation targeting). Then the optimal points in cases of a positive and a negative supply shocks are points *G* and *F* respectively. Obviously, in such a case the interest rate should

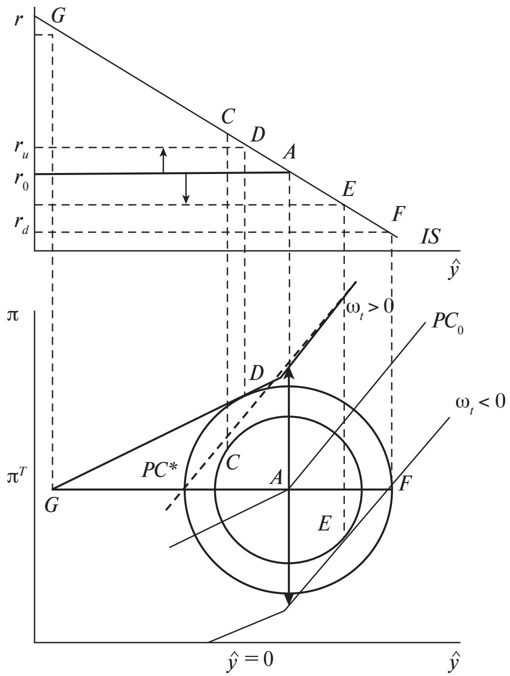


**Figure 1.** Monetary policy reaction to supply shocks under asymmetric price rigidity

adjust more in case of the inflationary shock than in case of the deflationary one of the same size.

Clearly, in general, the direction of the asymmetry in the monetary policy reaction to supply shocks depends on the relation between the parameters of the social loss function and the slope of the Phillips curve.

***Proposition 2.*** *The optimal monetary policy should react asymmetrically to positive and negative supply shocks (cost, nominal exchange rate and foreign inflation shocks) depending on the sign of the shock. If* $d(1-\alpha) > \sqrt{\lambda}$ *(prices are rather flexible and/or the weight of the output gap in the social loss function is low) then the monetary policy instrument should respond more to inflationary supply shocks than to deflationary ones of the same size due to higher downward price rigidity. The asymmetry is reversed in the opposite case.*

**Proof.** Recall from the proof of proposition 1 that the derivatives of $A$ and B with respect to parameter d are negative, provided that $d(1-\alpha) > \sqrt{\lambda}$, and positive in the other case. Consider the case when $d(1-\alpha) > \sqrt{\lambda}$. Since we assume that $d_1 > d_2$, the interest rate rule (54) has lower coefficients $A$ and $B$ in the top branch than in the bottom one. It follows from equation (49) for the equilibrium output gap that inflationary supply shocks lead to a negative output gap, while deflationary shocks lead to a positive output gap. Therefore, the monetary policy should follow the bottom branch with higher coefficients in case of inflationary shocks and the top branch with lower coefficients in case of deflationary shocks. In other words, the interest rate should be adjusted more in response to inflationary shocks than in response to deflationary ones of the same size. If $d(1-\alpha) < \sqrt{\lambda}$, then the interest rate rule (54) has higher coefficients in the top branch than in the bottom one, and the asymmetry is reversed.

Inequality $d(1-\alpha) > \sqrt{\lambda}$ is crucial for the determination of the asymmetry. While for strict inflation targeting ($\lambda = 0$) the monetary policy should always react more to inflationary shocks than to deflationary ones, for flexible inflation targeting ($\lambda > 0$), which happens much more often in practice, the asymmetry may be reversed if prices are sufficiently sticky in response to changes in output. Since the share of import goods affects the level of the overall price stickiness in the economy, it affects the optimal asymmetry as well. This finding was neglected in the literature by assuming closed economies. Therefore, the prescription that the interest rate should always adjust more when inflation is above the target than when it is below the target is challenged here.

*2.3.3. Robustness: Pre-commitment to a rule*

Now assume that the monetary authority pre-commits to follow the interest rate rule of the following specification:

$$\tilde{r}_t = r' + \pi^T + G\Delta e_t + M(\pi_t - \pi^T) + J\hat{y}_t \tag{55}$$

where $G$, $M$ and $J$ are positive parameters. The above rule says that the interest rate is adjusted in response to an exchange rate shock, deviation of the inflation from the target or deviation of the output from its potential level. Although it is common in the literature to specify the monetary policy rule in terms of inflation and output only, in fact the monetary authorities in many countries manipulate exchange rates as well (e.g. Lubik and Schorfheide, 2007). However, as it will be seen below, the concrete specification of the rule is not important as it will only affect the optimal values of the coefficients $G$, $M$ and $J$, since exchange rate shocks affect inflation and output gap in any case.

Now the equilibrium in the economy is described by the system of the IS curve (22), the Phillips curve (44) and the interest rate rule (55). Since in the absence of shocks inflation equals to its target and the output gap is zero, $\tilde{r}_t = r' + \pi^T$, where $r' = \dfrac{a}{b}$ is the natural level of the real interest rate, as follows from the IS curve. Solving this system, we obtain the following expression for the equilibrium output gap:

$$\hat{y}_t = -\delta b\left(G + \alpha M\right)\psi_t - \delta b\alpha M(\pi_t^* - \pi^{T*}) - \delta b(1-\alpha)M\omega_t + \delta\xi_t \quad (56)$$

where $\delta \equiv \dfrac{1}{1 + bd(1-\alpha)M + bJ}$. We see that a depreciation of the domestic currency, a growth in the foreign inflation and a growth in costs lead to lower equilibrium output, while a growth in the demand leads to higher output. This dynamics is similar to the one obtained in the case of discretionary monetary policy, but the magnitude of the changes will depend on the parameters of the rule.

To find the optimal parameters of the rule (55) we substitute the Phillips curve (44) and the expression for the equilibrium output (56) into the social loss function (45) and minimize the resulting expression with respect to the three parameters.

Since all shocks influence both equilibrium inflation and output, it is difficult to isolate their effects. Therefore, in order to keep the analysis tractable, we restrict to the analysis of the exchange rate shock assuming all other shocks to be equal to zero. But a similar analysis would apply to all other shocks as well.

In the presence of exchange rate shocks, the equilibrium output equals:

$$\hat{y}_t = -\delta b\left(G + \alpha M\right)\psi_t \quad (57)$$

and the equilibrium inflation is

$$\pi_t = \pi^T + [\alpha - d(1-\alpha)\delta b(G + \alpha M)]\psi_t \quad (58)$$

From equations (57) and (58) follows, that if the interest rate does not react to exchange rate shocks and deviations of inflation from the target ($G = M = 0$), then the output gap is zero and inflation deviates from the target by $\alpha\psi_t$ due to the pass-through effect. But the higher are the parameters $G$ and $M$, the lower is the reaction of inflation to exchange rate shocks. This is achieved at the expense of lower output, as follows from equation (57). Therefore, the optimal values of $G$ and $M$ depend on the relative importance of inflation and output gap in the social loss function.

Substituting equations (57) and (58) into the social loss function (45) and minimizing it with respect to parameters $G$, $M$ and $J$, we obtain three first order conditions, which are linearly dependent and can be expressed, for example, as follows:

$$G = \frac{\alpha}{(d^2(1-\alpha)^2 + \lambda)}\left(\frac{d(1-\alpha)}{b} + d(1-\alpha)J - \lambda M\right) \quad (59)$$

The linear dependence of the first order conditions means that parameters $G$, $M$ and $J$ do not have single optimal values. Instead, there is infinite number of them with the only restriction that they must satisfy formula (59). Therefore, if the monetary authority follows the interest rate rule (55), only one parameter out of three is determined unambiguously, and the other two can be anything, and the social losses will be minimized. An example of such an interest rate rule is the following:

$$\tilde{r}_t^{opt} = \frac{a}{b} + \pi^T + \frac{\alpha}{(d^2(1-\alpha)^2 + \lambda)}\left(\frac{d(1-\alpha)}{b} + d(1-\alpha)J - \lambda M\right)\Delta e_t + M(\pi_t - \pi^T) + J\hat{y}_t \quad (60)$$

According to the rule (60), the more the interest rate adjusts to the inflation deviation, the less it should adjust to the exchange rate shock, and the more it adjusts to the output gap, the more it should adjust to the exchange rate shock. This happens because a positive exchange rate shock leads to positive inflation deviation (see equation (58)), so, coefficients $G$ and $M$ are complementary, while a positive exchange rate shock leads to a negative output gap (see equation (57)), so the interest rate should balance between the two.

It should be noted that since any two of the three parameters may be set exogenously at any level, we can set them equal to zero, so that the interest rate rule (60) explicitly targets only one variable. Then, there are three alternative specifications of the rule:

$$\tilde{r}_t^{opt} = \frac{a}{b} + \pi^T + \frac{d(1-\alpha)\alpha}{b(d^2(1-\alpha)^2 + \lambda)}\Delta e_t, \text{ if } M = J = 0 \quad (61)$$

$$\tilde{r}_t^{opt} = \frac{a}{b} + \pi^T + \frac{d(1-\alpha)}{b\lambda}(\pi_t - \pi^T), \text{ if } G = J = 0 \quad (62)$$

$$\tilde{r}_t^{opt} = \frac{a}{b} + \pi^T - \frac{1}{b}\hat{y}_t, \text{ if G = M = 0} \quad (63)$$

Interest rate rules (61)-(63) all respond to exchange rate shocks (although rules (62) and (63) do it implicitly, since they are specified either in terms of inflation deviation or output gap, which arise due to exchange rate shocks) and have exactly the same impact on the economy. Rule (61) responds to exchange rate shocks explicitly with the coefficient which is equal to the one obtained in the case of discretionary monetary policy. Therefore, no matter whether the monetary authority conducts its policy discretionary or with a pre-determined rule, it should react to exchange rate shocks with the same coefficient. Hence, our proposition 1 is valid for the case of monetary policy rules as well.

Now assume that the Phillips curve is characterized by the asymmetric price rigidity, as in equation (54). Then the interest rate rule will be modified: its parameters will depend on the sign of the exchange rate shock. A depreciation of the domestic currency leads to higher inflation and negative equilibrium output gap, which corresponds to the flatter part of the Phillips curve, than in case of a similar appreciation. Therefore, the interest rate should again adjust asymmetrically in response to positive and negative exchange rate shocks:

$$\tilde{r}_t^{opt} = \begin{cases} \frac{a}{b} + \pi^T + \frac{d_1(1-\alpha)\alpha}{b(d_1^2(1-\alpha)^2+\lambda)}\Delta e_t, \hat{y}_t > 0 \\ \frac{a}{b} + \pi^T + \frac{d_2(1-\alpha)\alpha}{b(d_2^2(1-\alpha)^2+\lambda)}\Delta e_t, \hat{y}_t < 0 \end{cases} \qquad (64)$$

According to rule (64), the interest rate should increase in response to a depreciation of the domestic currency and decrease in response to an appreciation. If $d(1-\alpha) > \sqrt{\lambda}$, then the coefficient in the bottom branch is higher due to higher downward price rigidity (lower value of parameter d), and the increase in the interest rate should be more significant. Thus, our proposition 2 about asymmetric monetary policy is valid for the case of a pre-determined monetary policy rule.

## 3. Empirical evidence

In this part we test the prescriptions of the model empirically for a set of countries. We concentrate our analysis on exchange rate shocks.

*Data*

Our sample is formed from developed countries with either clean or dirty floating exchange rate regimes according to Reinhart and Rogoff (2002) de facto classification: the USA, Canada, Australia, the United Kingdom, Euro area, Norway, Sweden, Czech Republic and Poland.

We study the following periods: 1990-2006 for the USA, Canada, Australia



and the UK, 1998-2006 for Euro area and 1999-2006 for Norway, Sweden, Czech Republic and Poland.

We use the following quarterly time series:

- Interest rate (r) — the federal funds rate (USA), money market rate (Canada, Australia, Czech Republic), interbank rate (UK, Euro area), discount rate (Norway, Poland), repurchase rate (Sweden). All interest rates are annual.
- Nominal effective exchange rate (neer) — exchange rate against a trade-weighted basket of currencies in logarithm. An increase in neer means appreciation of the domestic currency.
- Consume price index (p) - in logarithm.
- Real GDP (y or y_sa) — GDP volume (2000=100) in logarithm, seasonally adjusted for Norway, Sweden, Czech Republic and Poland. In other countries seasonality was not observed.

The source of all data is the IMF International Financial Statistics.

*Methodology*

To take into account the endogenous nature of the above variables we estimate the following VAR model:

$$\Delta\Omega_t = X\Delta\Omega_{t-1} + X_d dummy * \Delta neer_{t-1} + X_0 + Z_t \qquad (65)$$

where $\Omega_t$ is a vector of endogenous variables ($r_t$, neer$_t$, $p_t$, $y_t$), $X$ is a 4*4 coefficient matrix, the dummy variable equals 1 if neer goes up and 0 otherwise, $X_d$ is the dummy coefficient vector, $X_0$ is a vector of intercepts and $Z_t$ is a vector of residuals. Since the time series are non-stationary, we take their first differences, represented by Δ.

To test whether the exchange rate is an important variable in a country's monetary policy we perform the variance decomposition test. This test shows us the percentage of the interest rate variance explained by real exchange rate shocks. Our theoretical model prescribes that this variance should be higher in countries with higher exchange rate pass-through effect on consumer prices (proposition 1).

*Hypothesis 1. The percentage of variance of the interest rate explained by exchange rate shocks is higher in countries with higher pass-through effect.*

To test the first hypothesis we need to know the degree of pass-through in the studied countries. Although there exist numerous empirical literature on the pass-through effect, their samples of countries are different from ours, and the estimates would be incomparable. Therefore, we estimate pass-through elasticities ourselves and compare them across our countries.

To estimate the pass-through elasticities we first estimate a similar VAR model, but without the dummy variable:

$$\Delta\Omega_t = \widehat{X}\Delta\Omega_{t-1} + \widehat{X}_0 + \widehat{Z}_t \qquad (66)$$

We build an impulse-response function to trace the effect of an exchange rate shock on consumer prices. We use the following Cholesky ordering:

$$\text{neer} \rightarrow \text{r} \rightarrow \text{p} \rightarrow \text{y} \qquad (67)$$

i.e. we assume that the monetary authority reacts to an exogenous exchange rate shock by adjusting the interest rate which affects prices and output[7].

To test how monetary policy in the studied countries reacts to exchange rate shocks we test the sign and significance of the coefficient of neer in the interest rate equation in (65). According to the theoretical model the domestic currency appreciation should be accompanied by a reduction of the interest rate while its depreciation — by an increase in the interest rate, ceteris paribus (proposition 1).

*Hypothesis 2. Coefficient of neer in the interest rate equation in model (65) is negative.*

This coefficient shows immediate reaction of the monetary policy to an exchange rate shock. But in fact the interest rate may adjust gradually. Indeed, as noted by Woodford (1999), gradual adjustment in the interest rate to changes in economic conditions is optimal as small but consistent interest rate changes have greater impact on long rates and, hence, on economic activity. To estimate such gradual adjustment we also build impulse-response functions for the interest rates for the model (65) using the ordering (67).

Finally, to test whether the monetary policy is indeed asymmetric in response to positive and negative exchange rate shocks we test the sign and significance of coefficient of dummy in the interest rate equation in model (65). According to our model, the domestic currency appreciation should be accompanied by a smaller by the absolute value reduction in the interest rate than the increase in the interest rate after the domestic currency depreciation of the same size, if the weight of the output gap in the social loss function is small (proposition 2).

*Hypothesis 3. Coefficient of dummy in the interest rate equation in model (65) is positive.*

*Results*

Table 1 presents selected results of the estimation of models (65) and (66). Columns 3 and 4 report the estimates of the coefficients which are most important for our analysis, columns 5 and 6 show characteristics of the model as a whole, columns 7—9 show the average values of the endogenous variables and columns 10—11 show the variance decomposition results. Sign "+" in column 11 means that the exchange rate explains the highest percentage of the interest rate variance in comparison with the other variables (inflation and output). Column 12 reports the estimated degree of exchange rate pass-through over 2 years[8].

[7] Actually changing the ordering does not alter the estimation results.



*Table 1.* Results of the estimation of models (65) and (66)

| **Country** | **Period** | $x_{12}$ | $x_{13}$ | $R^2$ | **F** | **Mean r pa, %** | **Mean infl pa** | **Var* for 2 years (%)** | **Max VAR?** | **PTE for 2 years** |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 |
| USA | 1990Q3 2006Q2 | **–4,35** | **0,04** | 0,96 | 223,21 | 4,20 | 0,02 | **62,28** | + | **–0,022** |
| | | [–5,38] | [ 1,74] | | | | | [5,35] | | |
| Canada | 1990Q3 2006Q2 | 1,45 | 0,00 | 0,92 | 116,22 | 4,85 | 0,02 | 0,125 | – | 0,006 |
| | | [ 0,77] | [ 0,06] | | | | | [0,02] | | |
| Australia | 1990Q3 2006Q2 | 1,00 | 0,01 | 0,97 | 311,96 | 6,11 | 0,03 | 8,04 | + | –0,018 |
| | | [ 1,22] | [ 0,51] | | | | | [0,84] | | |
| UK | 1990Q3 2006Q1 | –0,31 | **0,07** | 0,97 | 357,55 | 6,26 | 0,03 | 4,80 | – | 0,007 |
| | | [–0,33] | **[ 2,79]** | | | | | [0,75] | | |
| Euro Area | 1998Q3 2006Q1 | **–3,72** | –0,03 | 0,93 | 52,73 | 3,02 | 0,02 | **46,9** | + | –0,006 |
| | | **[–1,92]** | [–1,04] | | | | | [2,62] | | |
| Norway | 1999Q1 2006Q2 | **–6,89** | **0,06** | 0,97 | 126,60 | 6,60 | 0,02 | **28,56** | + | 0,002 |
| | | [–2,82] | [ 1,69] | | | | | [1,67] | | |
| Sweden | 1999Q1 2006Q2 | **–4,86** | 0,02 | 0,94 | 57,13 | 2,98 | 0,01 | 1,67 | – | –0,014 |
| | | **[–1,89]** | [ 1,10] | | | | | [0,15] | | |
| Czech Republic | 1999Q1 2006Q2 | **–7,59** | 0,00 | 0,98 | 247,51 | 3,75 | 0,02 | **52,44** | + | **–0,053** |
| | | **[–4,19]** | [–0,19] | | | | | [3,22] | | |
| Poland | 1999Q1 2006Q2 | **–8,04** | 0,11 | 0,97 | 147,94 | 11,08 | 0,04 | **39,39** | + | **–0,041** |
| | | **[–1,88]** | [ 1,06] | | | | | [2,16] | | |

* The percentage of variance of the interest rate explained by the exchange rate; t-statistics in parentheses; the significant variables at least at 10% significance level are in bold.

[8] The negative value of the pass-through elasticity is expected since the domestic currency depreciation should lead to an increase in prices and visa versa.

The variance decomposition test shows that the exchange rate is a significant variable in explaining the interest rate behavior in 5 countries out of 9: the USA, Euro Area, Norway, Czech Republic and Poland. Furthermore, in all these countries the exchange rate explains the highest percentage of the interest rate variance. And in most of these countries (the USA, Czech Republic and Poland) the estimated pass-through effect is the greatest among all countries in the sample, what can explain the high percentage (over 40%) of the interest rate variance explained by the exchange rate, according to our model.

Although pass-through effect in the Euro Area and Norway is close to zero, the exchange rate nevertheless plays a significant role in the monetary policy of these countries. We see the following explanations for this. Norway, being a resource exporter, has been experiencing the real appreciation of its currency after 1999 due to the rising prices of resources. And it is after 1999 when the real exchange rate started to play a role in its monetary policy: the interest rate is increased to smooth the currency appreciation. And the estimated pass-through effect during this period is zero (even positive) because the currency was mainly appreciating, but there is significant empirical evidence in favor of asymmetric pass-through effect[9]. Concerning the Euro Area, since Euro is one of the most influential currencies, it is important to take its fluctuations into account in designing the optimal monetary policy, while the pass-through effect may be small due to a low share of imported consumer goods.

The variance decomposition test shows that in Canada, Australia, the UK and Sweden the exchange rate does not play a significant role in the long run in determining the monetary policy. And if we look at the pass-through elasticities in these countries, they are either positive and almost zero (Canada and the UK) or have the correct sign (negative) but very low (less than 2% over two years in Australia and Sweden).

Figure A4.1 in Appendix 4 plots the percentage of the interest rate variance explained by the exchange rate fluctuations against pass-through elasticities. We can observe the following relationship: the higher is the pass-through by the absolute value, the higher is the variance, and the more significant role is played by the exchange rate in determining the interest rate fluctuations in the long run. This supports our first hypothesis.

Concerning the immediate reaction of the interest rate to a real exchange rate shock, it is significant for the same countries plus Sweden. In other words, coefficient of the neer in the interest rate equation turns out to be significantly negative for all countries except Canada, Australia and the UK. This means that in these countries the domestic currency appreciation is followed by a reduction in the interest rate, while its depreciation — by an increase. Therefore, we cannot reject our second hypothesis for the countries in which the exchange rate indeed plays a role.

Moreover, the reaction of the interest rate to an exchange rate shock is greater the higher is the pass-through effect, what supports our first hypothesis again. Figure A4.2 in Appendix 4 clearly demonstrates this relationship.

To analyze how the interest rates adjust to exchange rate shocks over time, we estimate the impulse-response functions, which are presented in Figure A4.3 in Appendix 4 together with the corresponding confidence intervals. We can observe a significantly negative long run reaction of the interest rates in the USA, Euro Area, Czech Republic and Poland. Indeed, the interest rates in these countries adjust gradually to exogenous exchange rate shocks, as proposed by Woodford (1999). And again, these are the countries with the highest pass-through effect.

The long run reaction of the interest rate to an exchange rate shock in Norway is insignificant, although negative, while coefficient of neer is significantly negative and the percentage of the interest rate variance explained by the exchange rate is significant and equals to 30%. Therefore, it can be concluded that the exchange rate plays a role in determining the monetary policy in Norway only in the short run, and in general this role is not so significant as in the USA, Euro Area, Czech Republic and Poland[10].

In Sweden, the exchange rate plays even smaller role, and only in the short run, as the long run impulse-response function and the variance decomposition tests give statistically insignificant results.

We do not find any evidence that the monetary policies in Canada, Australia and the UK take into account their exchange rate fluctuations. Having also estimated impulse-response functions for the interest rate in response to inflation and output shocks, we find that the only significant variable in Canada is the output, in the UK — inflation, and in Australia nothing is significant in explaining the interest rate behavior. The same conclusion follows from the variance decomposition test: in Canada the highest percentage of the interest rate variance is explained by the output (20%), in the UK — by inflation (10%), while in Australia each of the variables explains not more than 8% of the variance.

Our findings go in line with the conclusions of Nogueira Junior (2006), who studies monetary policies in Canada, Sweden, the UK, Czech Republic, Brasil, Mexico, South Africa and South Korea and does not find any evidence of foreign exchange interventions (including adjustments in the interest rate in response to exchange rate shocks — so-called "interest rate defense of exchange

[9] It was observed empirically that the domestic prices rise more as a result of the domestic currency depreciation, than they fall as a result of the domestic currency appreciation (e.g. Goldberg, 1995; Pollard and Coughlin, 2004, for the USA; Webber, 2000, for a set of Asian countries; Ohno, 1990, for Japan; Dobrynskaya and Levando, 2005, for Russia).

[10] The same can also be concluded from the variance decomposition test since the percentage of variance in Norway is the smallest among these five countries.

rate") in Canada, Sweden and the UK, while confirming significant interventions in Czech Republic and other countries.

What concerns the monetary policy asymmetry, coefficient of the dummy in the interest rate equation turns out to be significant for only three countries: the USA, the UK and Norway. This coefficient is estimated to be positive and much less by the absolute value, than the corresponding coefficient of neer. This means that while, for example, in the USA the interest rate rises by 4.35% in response to 1% depreciation of the dollar, it falls by 4.31% in response to a 1% appreciation of the dollar. A similar conclusion is valid for Norway. Such interest rate behavior corresponds to the prescriptions of our model and supports our hypothesis 3.

Dolado, Pedrero and Ruge-Murcia (2004) also analyze the US monetary policy and find non-linearity in its interest rate rule for the period 1983-2000. They conclude that when inflation is above the target the interest rate is adjusted more than when inflation is below the target, and they explain this by asymmetric central bank preferences. But the authors study the US as a closed economy, and do not analyze the exchange rate impact, which is significant according to our findings.

The situation in the UK is intriguing. Since the coefficient of neer turns out to be insignificant while coefficient of the dummy is significant, we can conclude that the monetary authorities behave asymmetrically and react mostly to positive exchange rate shocks. Taylor and Davradakis (2006) also find asymmetries in the UK monetary policy, but further research is needed here.

For the other countries in the sample we do not find significant asymmetries in the reaction of the monetary policy to exchange rate shocks. This can be explained by the fact that Central Banks normally follow a symmetric rule since the idea that the monetary policy rule should be asymmetric is new in the literature, and the Central Banks might not practice this yet. Nevertheless, the estimates of the dummy coefficient are positive in most cases, what corresponds to the prescriptions of our model if the countries follow inflation targeting rule.

## 4. Conclusion

In this paper we build a microfounded general equilibrium sticky price model of a small open economy, and obtain a closed form solution. Using a quadratic loss function, we find analytically that the optimal monetary policy rule is to adjust the interest rate in response to exogenous exchange rate, cost, foreign inflation and demand shocks in order to smooth their effect on the domestic inflation. We claim that the optimal degree of such adjustment depends on the value of pass-through effect and price stickiness in an economy.



Since the numerous empirical evidence speaks in favour of higher downward price rigidity, we modify our theoretical model to capture this asymmetry and derive a kinked Phillips curve. Under this setting the optimal monetary policy becomes asymmetric for positive and negative supply shocks. We claim that when prices are rather flexible and/or the weight of the output gap in the social loss function is low, the monetary policy instrument should respond more to inflationary supply shocks than to deflationary ones of the same size due to higher downward price rigidity. But the asymmetry is reversed in the case of the high weight of the output gap and/or high price stickiness. This can explain why in some countries which care about the output a lot deflationary exchange rate shocks are paid more attention than inflationary ones (e.g. Russia — see Dobrynskaya, 2008b), while in inflation targeting countries the opposite asymmetry is observed.

This analysis in the framework of a micro-founded New Keynesian model is novel, it is interesting from the theoretical point of view and has important practical implications for the conduct of monetary policy. It predicts that in order to minimise the social losses, the monetary authority should determine not only the direction of the required policy instrument change, but also its magnitude depending on the sign of a shock. If the monetary policy rule is specified so that it does not take into account such asymmetries, then following this rule may result in the equilibrium inflation and output gap, which are far from optimal. For example, if there is a significant asymmetry in price rigidity in an economy, while it may be optimal to increase the interest rate significantly as a result of a sharp depreciation of the domestic currency, it may also be optimal to respond to the domestic currency appreciation only slightly. Then, following a symmetric rule of a significant adjustment in the interest rate due to an exchange rate shock will lead to higher social losses in case of an appreciation of the domestic currency than would be without monetary policy reaction at all.

We test the predictions of our model for a set of developed countries. We find that the exchange rate plays a significant role in determining the monetary policy in most countries, and that the domestic currency appreciation is generally accompanied by a reduction of the interest rate while the domestic currency depreciation leads to an increase in the interest rate. We also find some evidence of the asymmetry in the monetary policy reaction to positive and negative exchange rate shocks, which is in line with inflation targeting, although this asymmetry is quite weak in most countries. This provides evidence that simple linear Taylor rules are still popular in practice, although they may not be optimal.

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## Appendix 1. Derivation of the IS curve

(a) Derivation of (17):

The following demand functions correspond to consumer price index (2):

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{A1}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{A2}$$

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{A3}$$

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \tag{A4}$$

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t} \tag{A5}$$

Substituting (A1) into (A4), we obtain the domestic demand for good $j$:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{A6}$$

Since all economies are characterized by the same preferences, we make use of equations (A1)-(A5) to derive the demand of economy $i$ for good $j$, keeping in mind that good $j$ is an import good for them:

$$C^i_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}/\mathrm{E}_{i,t}}{P^i_{F,t}}\right)^{-\gamma} \alpha \left(\frac{P^i_{F,t}}{P^i_{i,t}}\right)^{-\eta} C^i_t \tag{A7}$$

Substituting (A6) and (A7) into the aggregate demand function for good $j$, we get:

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t +$$

$$+\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{\mathrm{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \alpha \left(\frac{P^i_{F,t}}{P^i_{i,t}}\right)^{-\eta} C^i_t di = \tag{A8}$$

$$= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{\mathrm{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_{i,t}}\right)^{-\eta} C^i_t di\right]$$

(b) Derivation of (18):

From (A8) follows:

$$Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\int_0^1\left(\frac{P_{H,t}}{\mathrm{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_{i,t}}\right)^{-\eta} C^i_t di\right] =$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathrm{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma}\left(\frac{P^i_{F,t}}{P^i_{i,t}}\right)^{-\eta}\left(\frac{P_t}{P_{H,t}}\right)^{-\eta} C^i_t di\right] =$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathrm{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma}\left(\frac{\mathrm{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{-\eta}\left(\frac{P_t}{\mathrm{E}_{i,t}P^i_{i,t}}\right)^{-\eta} C^i_t di\right] = \tag{A9}$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathrm{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta}\left(\frac{P_t}{\mathrm{E}_{i,t}P^i_{i,t}}\right)^{-\eta} C^i_t di\right] =$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1\left(\frac{\mathrm{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} Q^{\eta}_{i,t} C^i_t di\right] =$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha\int_0^1\left(\frac{\mathrm{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} Q^{\eta}_{i,t}\frac{C^i_t}{C_t} di\right]$$

Since all economies are symmetric, and also the uncovered interest parity condition (13) holds, the optimal consumption in economy i is characterized by the Euler equation similar to (5):

$$\beta(1+r_t)E_t\left\{\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma}\left(\frac{P^i_t}{P^i_{t+1}}\right)\left(\frac{\mathrm{E}^i_t}{\mathrm{E}^i_{t+1}}\right)\right\} = 1 \tag{A10}$$

From equations (5) and (A10) follows:

$$C_t = C_t^i Q_{i,y}^{\frac{1}{\sigma}} \tag{A11}$$

We substitute (A11) into (A9):

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(\frac{\mathrm{E}_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} Q_{i,t}^{\eta} \frac{C_t^i}{C_t^i Q_{i,t}^{\frac{1}{\sigma}}} di\right] =$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(\frac{P_{i,t}^i \mathrm{E}_{i,t}}{P_{H,t}} \frac{P_{F,t}^i}{P_{i,t}^i}\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di\right] =$$

$$= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(S_{i,t} S_t^i\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di\right] =$$

$$= \left(\frac{P_{H,t}}{P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(S_{i,t} S_t^i\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di\right] =$$

$$= \left(\frac{P_{H,t}^{\alpha}}{P_{F,t}^{\alpha}}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(S_{i,t} S_t^i\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di\right] =$$

$$= S_t^{\alpha\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(S_{i,t} S_t^i\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di\right]$$

## Appendix 2. Derivation of the Phillips curve

(a) Derivation of (24):

$$p_{H,t}(j) \sum_{k=0}^{\infty} \theta^k - E_t \sum_{k=0}^{\infty} \theta^k \tilde{p}_{H,t+k}(j) = 0$$

$$p_{H,t}(j) \frac{1}{1-\theta} = E_t \sum_{k=0}^{\infty} \theta^k \tilde{p}_{H,t+k}(j)$$

$$p_{H,t}(j) = (1-\theta)\tilde{p}_{H,t}(j) + (1-\theta) E_t \sum_{k=1}^{\infty} \theta^k \tilde{p}_{H,t+k}(j)$$



$$p_{H,t}(j) = (1-\theta)\tilde{p}_{H,t}(j) + \theta E_t p_{H,t+1}(j)$$

(b) Derivation of (26):

We iterate forward (25):

$$E_t p_{H,t+1} = \theta p_{H,t} + (1-\theta) E_t p_{H,t+1}(j)$$

We add and subtract $p_{H,t}$:

$$(1-\theta) E_t p_{H,t+1}(j) = E_t p_{H,t+1} - \theta p_{H,t} + p_{H,t} - p_{H,t} \tag{A12}$$

Since $E_t \pi_{H,t+1} \equiv E_t p_{H,t+1} - p_{H,t}$, equation (A12) turns into:

$$(1-\theta) E_t p_{H,t+1}(j) = E_t \pi_{H,t+1} + (1-\theta) p_{H,t}$$

## Appendix 3. Proof of propositions 1 and 2

*Table A3.1.* Derivatives of coefficient A in equation (51)

| Parameter | d | α | b | Λ |
|---|---|---|---|---|
| $\frac{\partial A}{\partial(Parameter)}$ | $\frac{(1-\alpha)^2(\lambda - d^2(1-\alpha)^2)}{b(d^2(1-\alpha)^2+\lambda)^2}$ | $\frac{-2d\lambda(1-\alpha)}{b(d^2(1-\alpha)^2+\lambda)}$ | $\frac{-d(1-\alpha)^2}{b^2(d^2(1-\alpha)^2+\lambda)}$ | $\frac{-d(1-\alpha)^2}{b(d^2(1-\alpha)^2+\lambda)^2}$ |
| Sign | - if $d(1-\alpha) > \sqrt{\lambda}$ | - | - | - |

*Table A3.2.* Derivatives of coefficient B in equation (51)

| Parameter | d | α | b | Λ |
|---|---|---|---|---|
| $\frac{\partial B}{\partial(Parameter)}$ | $\frac{\alpha(1-\alpha)(\lambda - d^2(1-\alpha)^2)}{b(d^2(1-\alpha)^2+\lambda)^2}$ | $\frac{d((1-\alpha)^2+\lambda)}{b(d^2(1-\alpha)^2+\lambda)^2}$ | $\frac{-d\alpha(1-\alpha)}{b^2(d^2(1-\alpha)^2+\lambda)}$ | $\frac{-d\alpha(1-\alpha)}{b(d^2(1-\alpha)^2+\lambda)^2}$ |
| Sign | - if $d(1-\alpha) > \sqrt{\lambda}$ | + | - | - |

# Appendix 4

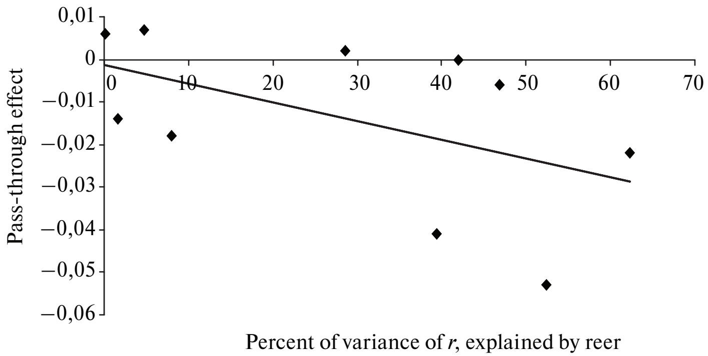


**Figure A4.1.** The relation between the percentage of the interest rate variance explained by the exchange rate and the degree of pass-through

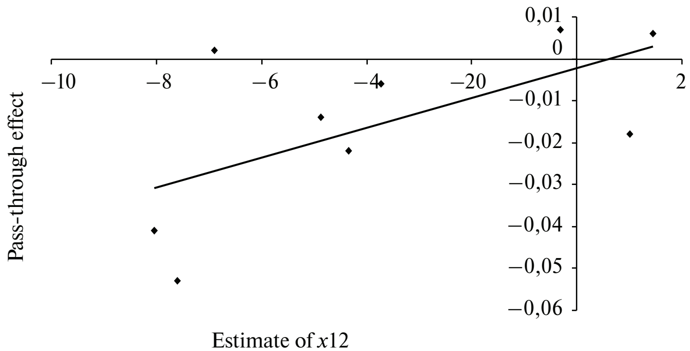


**Figure A4.2.** The relation between the degree of reaction of the interest rate to exchange rate and the degree of pass-through



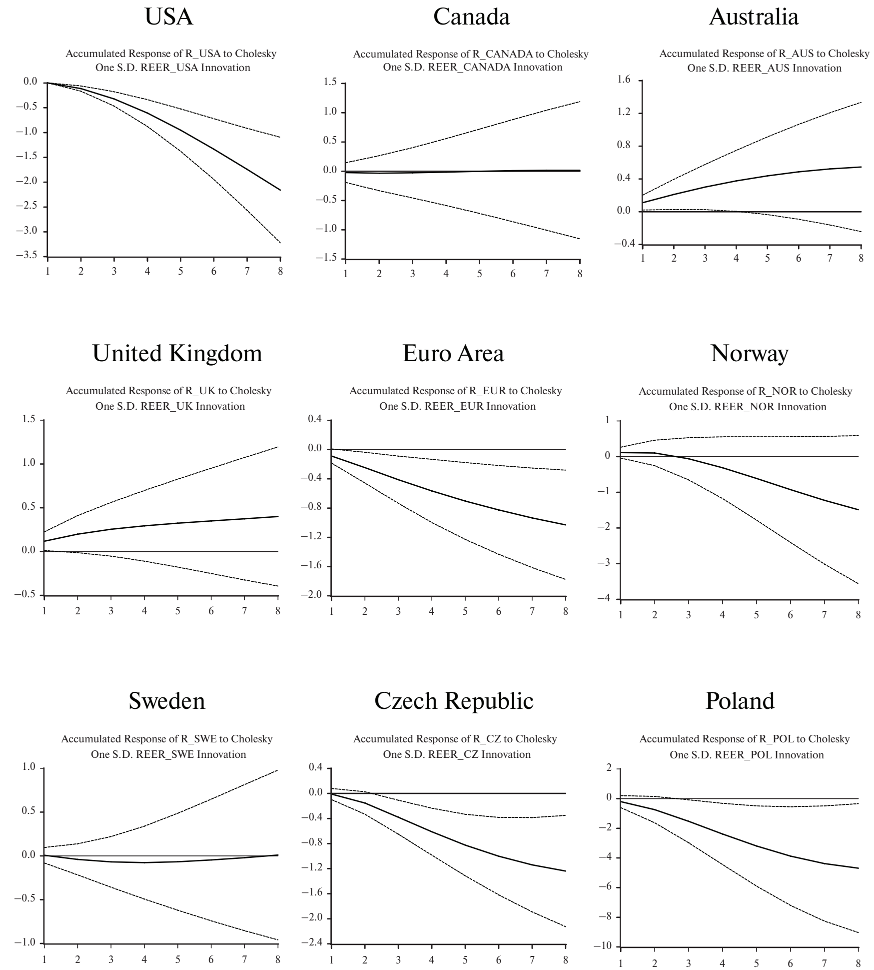


**Figure A4.3.** Response of the interest rate to an exchange rate impulse over 2 years

*Препринт WP9/2008/02*
*Серия WP9*
*Исследования по экономике и финансам*

**Добрынская В.В. Влияние асимметричной жесткости цен на денежно-кредитную политику в открытой экономике и эмпирический анализ для ряда стран**: Препринт WP9/2008/02. — М.: ГУ ВШЭ, 2008. — 44 с.

В работе предложена новокейнсианская модель малой открытой экономики, в рамках которой проанализирована оптимальная денежно-кредитная политика (ДКП) при симметричной и асимметричной жесткости цен. Показано, что при асимметричной жесткости цен ДКП должна реагировать на инфляционные и дефляционные шоки асимметрично и оптимальное направление асимметрии зависит от жесткости цен и социальных предпочтений. В частности, если цены достаточно гибкие и/или разрыв выпуска имеет малый вес в функции потерь (например, жесткое инфляционное таргетирование), то ДКП должна реагировать на инфляционные шоки сильнее, чем на дефляционные аналогичной величины. Но в противоположном случае асимметрия меняется.

В работе также проведен эмпирический анализ выводов модели на примере валютной политики развитых стран. Выявлено, что валютный курс играет немаловажную роль в ДКП большинства стран, и найдены свидетельства асимметрии валютной политики, соответствующей предписаниям модели для случая инфляционного таргетирования.

В.В. Добрынская

**Влияние асимметричной жесткости цен на денежно-кредитную политику в открытой экономике и эмпирический анализ для ряда стран**

*(на английском языке)*

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# Notes