# ГОСУДАРСТВЕНН ЫЙ УНИ ВЕРСИТЕТ 

 ВЫСШАЯ ШКОПА ЭКОНОМИКИN. Arefiev, T. Baron<br>CAPITAL TAXATION<br>AND RENT SEEKING

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Arefiev N.G., Baron T.Y. Capital Taxation and Rent Seeking. Working Paper A73 WP12/2007/07. - Moscow: State University - Higher School of Economics, 2007. 16 p .
We find optimal capital income tax rate in an imperfectly competitive economy, where some part of recourses is devoted to rent-seeking activity. Optimal tax offsets the difference between marginal social and marginal private return to capital, which is a result of rent seeking, and the difference between the ion. Optimal capital income tax rate depends neither on other tax rates nor on overall tax burden. Nuax rate depends neither on other tax rates nor on overall tax burden. Numerically it is close to zero.

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Мы определяем оптимальную ставку налога на капитал в несовершенно конкурентной экономике, в которой часть ресурсов тратится на поиск ренты. Оптимальное налогообложение компенсирует разницу между предельной частной и предельной социальной производительностью капитала, которая возникает в результате поиска ренты, и разницу между ставкой процента и предельной производительностью капитала, которая возникает в результате несовершенной конкуренции. Оптимальный налог на отдачу от капитала не зависит ни от других ставок налогов, ни от общего налогового бремени. Численно он близок к нулю

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At first glance, the nature of optimal fiscal policy in an imperfectly competitive economy or under decreased returns to scale is clear: pure profit should be intensively taxed, and subsidies should offset market distortions, which arise from imperfect competition; in all other respects the policy should be the same as under perfect competition and constant returns to scale. However, this recommendation is not applicable in practice, because fiscal authorities cannot distinguish pure profit from factor remuneration. Thus, it is not possible to tax pure profit without taxing wages or capital income.

To make the problem of taxation of imperfectly competitive economy more realistic, it is usually supposed that the pure profit tax is either zero or exogenously given (see Judd 1997, Guo \& Lansing 1999, Auerbach \& Hynes 2001). In this case, if higher stock of capital leads to higher economic profit, then the marginal productivity of capital should be higher than after-tax interest rate. It implicitly assumes that subsidies should offset distortions between marginal productivity of capital and before tax interest rate, which arise from market power. The optimal capital income tax rate in such an economy is determined by many factors (consumption and pure profit taxes, marginal excess tax burden, and others), and its numerical value considerably varies when we slightly change the model structure or its calibration. For example, Guo and Lansing found that for US economy the optimal capital income tax rate is somewhere between $-10 \%$ and $+22 \%$.

We take into consideration the fact that once pure profit has been produced, rent-seeking agents will spend their resources in order to seize it. This hypothesis makes the analysis more realistic: just as in the real world, we cannot distinguish pure profit from factor remuneration, and cannot tax them at different rates. Thus, we substitute the traditional hypothesis that pure profit enters into household budget constraint through a special channel by one that pure profit turns into private factor remuneration as a result of rent seeking.

We get general and intuitively clear results. Rent seeking distorts factor allocation. If an economy accumulates additional $\$ 1$ of capital, some part of it, say $\xi_{K}$, will be used for production of final goods, and the rest, $1-\xi_{K}$, for rent seeking. If the marginal productivity of capital used to produce final good is $F_{K}$, then the marginal productivity of capital, accumulated in the whole economy is $\xi_{K} F_{K}$. Therefore, the marginal social and marginal private returns to capital differ, and optimal policy offsets this distortion. Private and social returns to capital may differ also because capital accumulation may impact the division of labour between production and rent seeking. In addition, just as in previous researches, optimal policy offsets distortions, which arise from market power.

It remains to note that this paper proceeds examination of hypotheses which contradict the Chamley (1986) and Judd (1985) result of zero long run optimal capital tax. In addition to the case of imperfect competition, the optimal capital tax in the long run is not zero under uncertainty (Zhu (1992), Chari and Kehoe
(1994), Aiyagari (1995)), if some agents face with liquidity constraints (Hubbard and Judd (1987)), in the case of incomplete fiscal system (Correia (1896)), or under no-commitment (Benhabib and Rustichini (1997)).

The rest of the paper is organized as follows. In the first section we describe the economy with rent-seeking agents. Section 2 intuitively derives resource and implementability constraints, which we use to formulate the Ramsey policy problem; a formal proof may be found in the appendix. Section 3 poses the Ramsey problem of optimal policy, gives the first order conditions, and derives steadystate optimal capital income tax. Section 4 is devoted to numerical estimation of optimal capital tax rate, section 5 concludes.

## 1. Model description

A representative household solves the following problem:

$$
\begin{align*}
& \max _{C, L} \int_{0}^{\infty} e^{-\rho t} u(C, L) d t  \tag{1}\\
& \dot{A}=r A+w L-p_{c} C \tag{2}
\end{align*}
$$

where $C$ is consumption, $L$ - labour, $\rho$ - discount factor, $A$ - household's wealth, $r, w$ and $p_{c}$ - after-tax capital income, wage and commodity price. Household's wealth consists of capital $K$ and government bonds $B$. The number of households is normalized to unity and producer price of final good is the numeraire. The first-order conditions are

$$
\begin{gather*}
U_{C}=p_{c} \gamma  \tag{3}\\
U_{L}=-w \gamma  \tag{3b}\\
\dot{\gamma}=\gamma(\rho-r) \tag{3c}
\end{gather*}
$$

where $\gamma$ is the co-state variable.
There are two types of business activity: production and rent seeking. To produce final goods, firms use $K_{1}$ units of capital and $L_{1}$ units of labour:

$$
\begin{equation*}
Y=F\left(K_{1}, L_{1}\right) \tag{4}
\end{equation*}
$$

Profit maximization requires:

$$
\begin{gather*}
\hat{r}+\delta=(1-\sigma) F_{K}  \tag{5a}\\
\hat{w}=(1-\sigma) F_{L} \tag{5b}
\end{gather*}
$$

Where $\hat{w}$ and $\hat{r}$ are before tax wage and interest rate, and $\delta$ is the depreciation rate. Parameter $\sigma$ emerges as a result of imperfect competition on final goods market and may be measured by the inverse of demand elasticity for one firm's output. a may depend on resource allocation in the economy.

Profit is given by

$$
\begin{equation*}
\pi=F\left(K_{1}, L_{1}\right)-(1-\sigma)\left[F_{K} K_{1}+F_{L} L_{1}\right] \tag{6}
\end{equation*}
$$

Setting $\sigma$ equal to zero corresponds to the situation where all firms are pricetakers, and the only source of profit is decreasing returns to scale.

Rent-seekers compete with each other in order to seize the profit. Probability of success depends on amounts of capital and labour devoted to rent seeking. The seeker that achieves higher value of a function $G(K, L)$ has higher probability of success. Firms assume all the risks. Rent-seekers' optimization requires:

$$
\begin{equation*}
\frac{G_{K}}{G_{L}}=\frac{\hat{r}+\delta}{\hat{w}} \tag{7}
\end{equation*}
$$

For simplicity we assume that depreciation rates are equal for both types of activities.

Free-entry assumption leads to the following market clearing condition:

$$
\begin{equation*}
\hat{r} K_{2}+\hat{w} L_{2}=\pi \tag{8}
\end{equation*}
$$

where $K_{2}$ and $L_{2}$ are capital and labour used to seek the rent.
Other market clearing conditions are

$$
\begin{gather*}
Y=C+G+\dot{K}+\delta K  \tag{9}\\
K=K_{1}+K_{2}  \tag{10a}\\
L=L_{1}+L_{2} \tag{10b}
\end{gather*}
$$

The government collects taxes in order to finance an exogenously given amount of public goods $G$. Its budget constraint is

$$
\begin{equation*}
\dot{B}=r B+G-\left(p_{c}-1\right) C-[Y-r K-w L] \tag{11}
\end{equation*}
$$

The government solves the Ramsey problem. In other words, it chooses a tax system, which maximizes utility of a representative household in decentralized economy.

The tax rates are determined by ratios of consumer and producer prices.

## 2. Attainable allocation set

To derive the optimal policy we use primal approach to optimal taxation, developed by Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stockey (1983), Chari and Kehoe (1998) and many others. The first step of this approach is to describe the set of allocations that can be decentralized without lump-sum taxes. The second one is to maximize utility of a representative agent on this set. The last one is to find tax rates, which decentralize the optimal allocation.

Attainable allocation set may be described by two constraints: resource and implementability ones. The resource constraint ensures that the considered allocation is consistent with firms' optimization: if for a given allocation the resource constraint is satisfied then there exists a vector of producer prices such that this allocation satisfies firms' budget constraints and their first-order conditions. In the same sense the implementability constraint ensures that an allocation is consistent with the households optimization. If an allocation satisfies the both constraints then consumers and producers under some prices choose this allocation, and the government budget constraint will be satisfied by Walras law. The tax rates that decentralize the considered allocation are determined by ratios of consumer and producer prices.

In this section we intuitively derive both constraints from equilibrium conditions, and in the appendix we prove that these constraints exactly describe the attainable allocation set.

To get an implementability constraint, consider the value of household's wealth measured in units of utility:

$$
\begin{equation*}
a=\gamma A \tag{12}
\end{equation*}
$$

To get the implementability constraint, take a derivative of (12) with respect to time and substitute first-order conditions (3a, 3b, 3c) and household budget constraint (2) into obtained equation:

$$
\begin{equation*}
\dot{a}=\rho a-U_{C} C-U_{L} L \tag{13}
\end{equation*}
$$

We suppose that there are no implicit forms of expropriation or defaults, thus $a_{0}$ is given. This assumption is necessary to get a dynamically consistent solution, see Arefiev () for details.

To get the resource constraint we need to determine how $K_{1}$ and $L_{1}$ depend on $K$ and $L$. First, consider the Cobb-Douglass example: $Y=K_{1}^{\alpha} L_{1}^{\beta}, G=K_{2}^{\phi} L_{2}^{1-\phi}$. To get $K_{1} / K$ ratio, divide the share of $K_{1}$ income in $Y$ by the share of $K=K_{1}+K_{2}$ income in $Y$. The share of $K_{1}$ income in $Y$ is $(1-\sigma)$, and the share of $K_{2}$ income in $Y$ is equal to the share of $K_{2}$ income in profit, $\phi$, times the share of profit in Y , which is $[1-(1-\sigma)(\alpha+\beta)]$. We get:

$$
\begin{equation*}
\frac{K_{1}}{K}=\frac{\alpha(1-\sigma)}{\alpha(1-\sigma)+\phi 1-(1-\sigma)(\alpha+\beta)]} \tag{14}
\end{equation*}
$$

In a similar way:

$$
\begin{equation*}
\frac{L_{1}}{L}=\frac{\beta(1-\sigma)}{\beta(1-\sigma)+(1-\phi)[1-(1-\sigma)(\alpha+\beta)]} \tag{15}
\end{equation*}
$$

Thus, in the Cobb-Douglass case, the ratios $K_{1} / K$ and $L_{1} / L$ are constants. In a more general case, $\alpha, \beta$ and $\sigma$ may depend on $K_{1}$ and $L_{1}$, and $\phi$ may depend on $K_{2}$ and $L_{2}$. In this case we get a system of two equations, which implicitly gives $K_{1}$ and $L_{1}$ as functions of $K$ and $L$. Let's define

$$
\begin{align*}
& K_{1}=\xi(K, L)  \tag{16a}\\
& L_{1}=\eta(K, L) \tag{16b}
\end{align*}
$$

Substitution of (16a and 16b) and (4) into (9) gives us the resource constraint:

$$
\begin{equation*}
\dot{K}=F(\xi(K, L), \eta(K, L))-C-G-\delta K \tag{17}
\end{equation*}
$$

## 3. Optimal capital income taxation

The government maximizes utility of a representative household on the set of allocations, attainable in a decentralized economy:

$$
\begin{gather*}
\max _{C, L}^{\infty} \int_{0}^{\infty} e^{-\rho t} u(C, L) d t  \tag{18a}\\
\dot{a}=\rho a-U_{C} C-U_{L} L  \tag{18b}\\
\dot{K}=F(\xi(K, L), \eta(K, L))-C-G-\delta K  \tag{18c}\\
a(0)=a_{0}  \tag{18d}\\
K(0)=K_{0} \tag{18e}
\end{gather*}
$$

Let $\lambda$ and $\mu$ be co-state variables for $a$ and $K$. First order conditions are

$$
\begin{gather*}
U_{C}\left[1-\lambda\left(1+H_{C}\right)\right]=\mu  \tag{19a}\\
U_{L}\left[1-\lambda\left(1+H_{L}\right)\right]=-\mu\left(F_{K} \xi_{L}+F_{L} \eta_{L}\right)  \tag{19b}\\
\dot{\lambda}=0 \tag{19c}
\end{gather*}
$$

$$
\begin{equation*}
\dot{\mu}=\mu \rho-\mu\left[F_{K} \xi_{K}+F_{L} \eta_{K}-\delta\right] \tag{19d}
\end{equation*}
$$

where

$$
\begin{gather*}
H_{i}=\frac{U_{C i} C+U_{L i} L}{U_{i}}  \tag{20a}\\
i=C, L \tag{20b}
\end{gather*}
$$

To get an optimal capital income tax, let's use Judd (1999) multiplier:

$$
\begin{equation*}
\Lambda=\frac{\gamma}{\mu} \tag{21}
\end{equation*}
$$

On the one hand, this multiplier is determined by first-order conditions of the household and Ramsey problems. Substitution of equations (3a) and (19a) into (21) gives us:

$$
\begin{equation*}
\Lambda^{-1}=p_{c}\left[1-\lambda\left(1+H_{C}\right)\right] \tag{22}
\end{equation*}
$$

On the other hand, logarithmic derivative of (21) with respect to time with conditions (3c) and (19d) gives the optimal capital income tax rate:

$$
\begin{equation*}
\frac{\dot{\Lambda}}{\Lambda}=\left[F_{K} \xi_{K}+F_{L} \eta_{K}-\delta\right]-r \tag{23}
\end{equation*}
$$

The Chamley - Judd result follows from the fact that $H_{C}$ is constant on a balanced growth path. In our framework, if $H_{C}$ is constant then the capital tax is implicitly given by the following equation:

$$
\begin{equation*}
F_{K} \xi_{K}+F_{L} \eta_{K}=r+\delta \tag{24}
\end{equation*}
$$

Thus, optimal Fcapital tax offsets the difference between private and social marginal productivity of capital, which is determined by $\xi_{K}$ and $\eta_{K}$, and the difference between before tax interest rate and marginal productivity of capital, which is given by $\sigma$. To be exact, if capital tax $\tau_{K}$ is defined by $(r+\delta)=\left(1-\tau_{K}\right)(\hat{r}+\delta)$, the optimal value of $\tau_{K}$ on a balanced growth path is given by

$$
\begin{equation*}
\tau_{K}=1-\frac{\left(\xi_{K}+\frac{F_{L}}{F_{K}} \eta_{K}\right)}{1-\sigma} \tag{25}
\end{equation*}
$$

## 4. Value of optimal capital income tax

Let's suppose that the shares of $K_{1}, K_{2}, L_{1}$ and $L_{2}$ income in $Y$, and also the share of profit in $Y$ are constants. From equation (14) we see that in this case

$$
\begin{gather*}
\xi_{K}=\frac{\alpha(1-\sigma)}{\alpha(1-\sigma)+\phi 1-(1-\sigma)(\alpha+\beta)]}  \tag{26}\\
\eta_{K}=0 \tag{27}
\end{gather*}
$$

and the optimal capital income tax is given by

$$
\begin{equation*}
\tau_{K}=1-\left[\left(1-\frac{\phi}{\alpha}(\alpha+\beta)\right)(1-\sigma)+\frac{\phi}{\alpha}\right]^{-1} \tag{28}
\end{equation*}
$$

If an economy exhibits constant returns to scale $(\alpha+\beta=1)$, and the share of $K_{1}$ income in $Y$ equals the share of $K_{2}$ income in profit $(\alpha=\phi)$, then the optimal capital income tax is zero. To get more general results we need an estimation of returns to scale and the share of profit in GDP. Guo and Lansing (1999) used the estimations as in Basu and Fernald (1997) and got that the optimal capital income tax rate is somewhere between $-10 \%$ and $+22 \%$. We take an estimation of returns to scale in the typical US industry from the same source (Basu, Fernald, 1997), and hence assume the degree of homogeneity of the production function to be equal to 1,01 , and the profit ratio of the typical US industry of about $3 \%$. Let the gross share of capital income in GDP (the denominator in 26) be equal to $35 \%$. Consequently, in our framework the optimal capital income tax is somewere between $-4,1 \%(\phi=0)$ and $4,6 \%(\phi=1)$. When no capital is involved in rent-seeking, the capital is subsidized in order to offset distortions arising from imperfect competition (represented by $\sigma$ ). When all capital is involved in rent seking, the effect of discouraging unproductive activity dominates and the tax rate is positive.

## 5. Conclusion

The central hypothesis of our research is that pure profit doesn't enter directly into households' budget constraints but turns into factor remuneration. This approach to remuneration creates additional incentives to invest and to work, this is why private and social returns differ. In section 3 we show that this is a sufficient assumption to get equations (14) and (15), which imply that optimal capital income tax is given by (24). Hence, the central results of this paper hold under more general assumptions than it is assumed in section 1.

Taking account of unproductive use of resources in rent-seeking has allowed us to compactly pose the Ramsey problem, and to get intuitively clear and interpretable results. In particular, we found that the optimal capital tax offsets the difference between private and social marginal productivity of capital (given by $\xi_{K}$ and $\eta_{K}$ ) and the difference between before tax return on capital and its marginal productivity (given by $\sigma$ ). The sign of the optimal tax in the long run is ambiguous. On one extreme, when all capital is tied up in rent seeking, the tax is positive, so that it distimulates capital accumulation. On the other extreme, when all capital is used in production, there arises a subsidy, which eliminates the distortions of imperfect competition. The bounds within which the tax variates are narrower than in previous works.

## Appendix

## A. Theorem

(i) The implementability (18b) and the resource (18c)constraints together with the initial conditions (18d), (18e) and transversality condition $\lim e^{-\rho t} a(t)=0$, with the latter traditionally presumed to be satisfied, are satisfied for any equilibrium allocation $\{C(t), L(t), t \in(0, \infty)\}$.
(ii) If the implementability (18b) and the resource (18c) constraints together with the initial conditions (18d), (18e) and transversality condition lime $e^{-\rho t} a(t)=0$ are satisfied for a given allocation $\{C(t), L(t), t \in(0, \infty)\}$, then for given dynamics of any tax $\left(\tau_{K}, \tau_{L}, \tau_{C}\right)$ there exists the dynamics of the other two taxes such that this allocation will be implemented in decentralized economy.

## A.1. Proof

(i) The full set of constraints that describe the equilibrium allocation for Ramsey problem looks as follows:

$$
\begin{gather*}
A_{0}=\int_{0}^{\infty} e^{-\int_{0}^{t} r(\tau) d \tau}\left(p_{C} C-w L\right) d t  \tag{29}\\
U_{C}=p_{c} \gamma  \tag{30a}\\
U_{L}=-w \gamma  \tag{30b}\\
\dot{\gamma}=\gamma(\rho-r) \tag{30c}
\end{gather*}
$$

$$
\begin{gather*}
Y=F\left(K_{1}, L_{1}\right)  \tag{31}\\
\pi=F\left(K_{1}, L_{1}\right)-(\hat{r}+\delta) K_{1}-\hat{w} L_{1}  \tag{32}\\
\hat{r}+\delta=(1-\sigma) F_{K}  \tag{33a}\\
\hat{w}=(1-\sigma) F_{L}  \tag{33b}\\
\frac{G_{K}}{G_{L}}=\frac{\hat{r}+\delta}{\hat{w}}  \tag{34}\\
\hat{r} K_{2}+\hat{w} L_{2}=\pi  \tag{35}\\
Y=C+G+\dot{K}+\delta K  \tag{36}\\
K=K_{1}+K_{2}  \tag{37a}\\
L=L_{1}+L_{2}  \tag{37b}\\
\dot{B}=r B+G-\left(p_{c}-1\right) c-[Y-(r+\delta) K-w L]  \tag{38}\\
K(0)=K_{0} \tag{39}
\end{gather*}
$$

This set of constraints ensures that an allocation is compatible with the household budget constraint and optimization (equations 29-30c), the firms' optimization (equations 31-34), free-entry condition (35), market clearing conditions (equations $36-37 \mathrm{~b}$ ) and the government budget constraint (38)

The first step is to prove that the initial household budget constraint (29) is equivalent to the equation of motion (2).To see it we solve the equation of motion (2) with respect to $A$ and take the limit of it when $t$ tends to infinity. Hence we get that the transversality condition

$$
\begin{equation*}
\lim _{t \rightarrow \infty} e^{-\int_{0}^{r(\tau) d \tau}} A(t)=0 \tag{40}
\end{equation*}
$$

is equivalent to the household's initial budget constraint (29) and this is true for any trajectory of $A$ subject to the equation of motion (2). Thereafter, if the equation of motion (2) and the transversality condition (39) are satisfied then the household budget constraint holds.

To prove that vice versa is true, we first write the budget constraint similar to (29) for some moment $t$ :

$$
\begin{equation*}
A_{(t)} e^{-\int_{0}^{t} r(\tau) d \tau}=\int_{t}^{\infty} e^{-\int_{0}^{t} r(\tau) d \tau}\left(p_{C} C-w L\right) d t \tag{41}
\end{equation*}
$$

which implies that the assets accumulated by the time $t$ should exactly offset the discounted flow of all future deficits, arising from the excess of consumptoin over labour income. Substracting from (41) the initial budget constraint (29) we get an expression that is exactly the solution of the equation of motion (2) with respect to $A$. Hence if the initial budget constraint holds, then the equation of motion holds (2) and so does the transversality condition (40).

The second step is to prove that the implementability constraint (13) is equivalent to the household's budget constraint given that the first-order conditions for the household problem are satisfied. To do this, we first need to find the dynamics of the co-state variable $\gamma$ from the first-order condition (3c):

$$
\begin{equation*}
\gamma=\gamma_{0} e^{\rho t} e^{-\int_{0}^{t} r(\tau) d \tau} \tag{42}
\end{equation*}
$$

Then using (42), the fact that $a(t)=A(t) \gamma(t)$ by definition and substitituting in (2) and transversality condition (40), we get exactly the implementability constraint (13) and the corresponding transversality condition $\lim _{t \rightarrow \infty} e^{-\rho t} a(t)=0$. We could equally have substituted in implementability constraint (13) to get the equation of motion (2). Thus provided corresponding transversality conditions, (13) and (2) are both equivalent to the initial budget constraint (29).

To finish the proof of the first part of the theorem it remains to demonstrate that the resource constraint (17) is equivalent to the constraints ( $31-37 \mathrm{~b}$ ).

The system of equations ( $32-35,37 \mathrm{a}-37 \mathrm{~b}, 39$ ) is a system with respect to $K_{1}$ and $L_{1}$ for given $K$ and $L$. The solution to this system can be easily expressed through the shares of capital and labour incomes in the product and profit and the degree of market power $\sigma\left(K_{1}, L_{1}\right)$. If:

$$
\begin{align*}
\alpha\left(K_{1}, L_{1}\right) & =\frac{F_{K} K_{1}}{F\left(K_{1}, L_{1}\right)}  \tag{43}\\
\beta\left(K_{1}, L_{1}\right) & =\frac{F_{L} L_{1}}{F\left(K_{1}, L_{1}\right)}  \tag{4}\\
\phi\left(K_{2}, L_{2}\right) & =\frac{G_{K} K_{2}}{G\left(K_{2}, L_{2}\right)}  \tag{45}\\
1-\phi\left(K_{2}, L_{2}\right) & =\frac{G_{L} L_{2}}{G\left(K_{2}, L_{2}\right)} \tag{46}
\end{align*}
$$

Then:

$$
\begin{equation*}
K_{1}=\frac{\alpha\left(K_{1}, L_{1}\right)}{\alpha\left(K_{1}, L_{1}\right)+\phi\left(K_{2}, L_{2}\right)\left[\left(1-\sigma\left(K_{1}, L_{1}\right)\right)^{-1}-\alpha\left(K_{1}, L_{1}\right)-\beta\left(K_{1}, L_{1}\right)\right]} K \tag{47}
\end{equation*}
$$

$$
\begin{equation*}
L_{1}=\frac{\beta\left(K_{1}, L_{1}\right)}{\beta\left(K_{1}, L_{1}\right)+(1-\phi(\cdot))\left[\left(1-\sigma\left(K_{1}, L_{1}\right)\right)^{-1}-\alpha\left(K_{1}, L_{1}\right)-\beta\left(K_{1}, L_{1}\right)\right]} L \tag{48}
\end{equation*}
$$

In Cobb-Douglass case all the income shares do not depend on the distribution of resources in the economy. In a general case equations (47 and 48) implicitly define $K_{1}$ and $L_{1}$ as functions of $K$ and $L$ :

$$
\begin{align*}
& K_{1}=\xi(K, L)  \tag{49a}\\
& L_{1}=\eta(K, L) \tag{49b}
\end{align*}
$$

On the last step we use (31, 49a and 49b) to substitute in (36) and get the resource constraint:

$$
\begin{equation*}
\dot{K}=F(\xi(K, L), \eta(K, L))-C-G-\delta K \tag{50}
\end{equation*}
$$

The government budget constraint is satisfied by Walras law.
(ii) For a given allocation $\{C(t), L(t), t \in(o, \infty)\}$ and initial value of $K(0)=K_{0}$, equation (50) is a first-order differential equation with respect to $K(t)$. For given trajectories of $K$ and $L$, the functions $\xi(K, L)$ and $\eta(K, L)$ determine $K_{1}(t)$ and $L_{1}(t)$ for each point of time. Productive firm's first order conditions (33a and 33b) define the corresponding values of $\hat{w}$ and $\hat{r}$. Suppose that the trajectory of one of three taxes is given. Then, the system of first-order conditions (30a-30c) uniquely determines all consumer prices and the value of $\gamma$. Thereafter all taxes are found through the ratios of consumer and producer prices.
Q.E.D.

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