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MODELING A YIELD CURVE FOR THE RUSSIAN GOVERNMENT DEBT MARKET: A MACROECONOMIC APPROACH

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In this paper, we address the issue of modeling the term structure of interest rates on the Russian market of government bonds and discuss a general approach to incorporating macroeconomic factors into the model of term structure of interest rates. The analysis of the existing yield curve models and specific features of the Russian market allows us to determine the general methodology we would like to explore as applied to zero-coupon rates in the OFZ market. The outcome of our empirical exercise supports the existence of significant macro factor effects on the long-run level of yields and the slope of the yield curve. The approach should serve to a better understanding of the links between the evolution of the interest rates on the Russian government bonds, the economy and changes in the macroeconomic policies in recent years. It could be used as a starting point for analysis of the monetary policy transmission mechanism in Russia and forecasting of government bond yields for the purpose of public debt management.

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В работе рассматривается проблема моделирования кривой доходности российского рынка государственных облигаций. Приводится обзор моделей, используемых для описания динамики временной структуры процентных ставок, а также рассматриваются методики учета макроэкономических факторов при моделировании кривой доходности. На основе проведенного анализа формулируется подход к оценке воздействия макроэкономических условий на динамику бескупонных доходностей на рынке ОФЗ. Результаты проведенного статистического анализа свидетельствуют в пользу наличия значимого влияния макроэкономических переменных на динамику долгосрочного уровня процентных ставок и наклона кривой доходности. Предполагается, что рассмотренный подход к моделированию может быть использован в качестве отправной точки для исследования трансмиссионного механизма денежно-кредитной политики в России и прогнозирования кривой доходности государственных ценных бумаг в целях управления государственным долгом.

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Introduction

In this paper, we address the issue of modeling the term structure of interest rates in the Russian market of government bonds and discuss a general approach to incorporating macroeconomic factors into the model of term structure of interest rates. We focus on modeling of latent factors of zero-coupon yield curve on the OFZ market. Specifically, we explore the effects of inflation expectations, real economic activity and monetary condition on the level, slope and curvature factors in 2003–2007. This approach should lead to a better understanding of the links between the evolution of the interest rates on the Russian government bonds, the economy and changes in the macroeconomic policies in recent years.

The insight to the factors that drive movements in the term structure of interest rates is of potential interest to policymakers and market participants for a number of reasons. The first one is asset pricing. The yield curve has been a subject of extensive finance research as a natural starting point for pricing fixed-income securities and other financial assets.

The second reason is that a good understanding of the term structure dynamics is crucial for the implementation of monetary policy. On the one hand, the extent to which changes in the short-term policy rate affect (are transferred to) longer-term yields is important as a key part of the transmission mechanism of monetary policy. On the other hand, the term structure contains information about market participants' expectations of the future path of inflation. Thus, the indicators of market expectations might be used by the central banks in modeling and forecasting. In addition, in a regime of inflation targeting the difference between the expected inflation and the inflation target is an indicator of policy credibility.

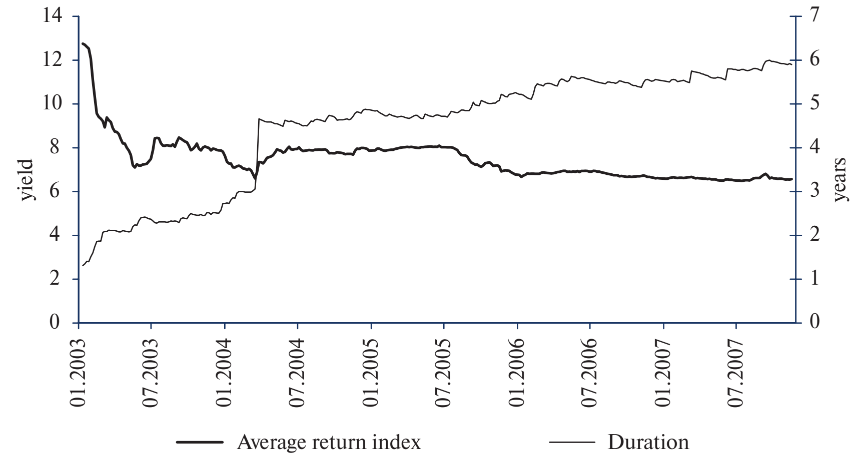
The yield curve modeling can also be useful for comparison of alternative strategies of the accumulation of public debt. When the government issues new debt, it has to decide on the maturity of the new bonds and to estimate the costs of various strategies of the public debt accumulation. A growing role of risk issues in management of sovereign assets and liabilities together with increasing complexity of tasks facing public debt managers call for development of advanced tools for evaluating public debt policy. In recent years simulation models have become popular among public debt managers. They allow stress-testing, ensuring consistency of debt management strategies and receiving quantitative estimates of their cost and risk characteristics. Yield curve models capturing both the influence of the macroeconomic development and the evolution of interest rates often represent integral parts of larger debt simulation models.

While some theories of term structure of interest rates explain the variation of yields across maturities through market expectations of future movements in short-term interest rate, the macroeconomics predicts that inflation expectations and future real economic activity are also important determinants of bond yields. It also concludes that effects of macroeconomic policy could be different for short-term and long-term interest rates. Thus, these effects could result in shifts and steepening/flattening of the yield curve. Empirical research supports these views. For example, Evans and Marshall (2001) explored macro-to-yield links and showed that macroeconomic factors have a substantial, persistent and statistically significant effect on the level of term structure. Ang and Piazzesi (2002) showed that models with macro factors forecast better than models with only unobservable factors. Diebold, Rudebusch, and Aruoba (2005) found strong evidence of the two-way effects of macro variables on movements in the yield curve and vice versa.

The major goal of this paper is to develop an approach to modeling the term structure of interest rates on the Russian market of government bonds. The structure of the paper is the following. We start with an overview of the OFZ market in 2003-2007. We indicate the major trends in the dynamic of the yield curve and postulate some hypotheses about the macro-factors that could have influenced this dynamics (section 1). Then we review the existing approaches to yield curve models that incorporate macroeconomic factors (section 2). Finally, we carry out initial empirical analysis of interaction between macro-factors and zero-coupon yields on Russian government bonds in recent years. In particular, we focus on the effects of inflation expectations, real economic activity and monetary condition on the yield curve latent factors – level, slope and curvature. The results of our empirical exercise indicate the existence of significant macro factor effects on the long-run level of yields and the slope of the yield curve. Section 4 gives general methodology and non-technical summary of our findings, while more detailed information on econometric estimates is provided in the Appendix.

1. Theoretical insight to the dynamics of interest rates in the Russian market for government debt securities

The dynamics of interest rates of ruble denominated government bonds in 2003-2007 was determined by the external and internal factors. The ongoing increase in world energy prices has resulted in foreign currency inflow to the Russian domestic market, growth in ruble liquidity and persisting expectations of ruble appreciation. These tendencies were reinforced by the significant improvements in the overall performance of the Russian economy and positive assessments of the recent internal macrodevelopments by international rating



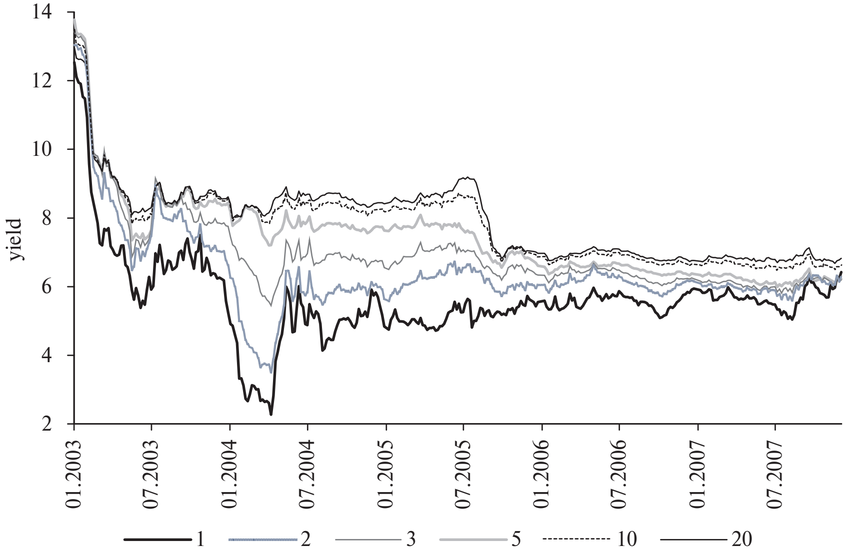
Source: Bank of Russia

Chart 1. Key characteristics of the Russian government bond market

agencies. Against this background, ruble denominated financial instruments overall and government bonds in particular have remained attractive for the investors, as evidenced by the consistent decrease in domestic interest rates. The situation on the international markets, characterized by the declining interest rates in light of ample liquidity, has also contributed to the reduction of domestic interest rates. As shown in Chart 1, most of period the downward trend in the OFZ average return index prevailed despite the increasing duration of the government internal debt portfolio[1].

Chart 2 demonstrates a flattening of the yield curve since 2004, meaning that most of the period the short and long-term interest rates have been converging. In 2003 the short-term interest rates fluctuated around a declining trend, which reversed in the first half of 2004. Developments in the international foreign exchange market induced temporary devaluation of ruble against dollar and brought to a sharp increase in demand for ruble liquidity, used for arbitrage operations with dollar. As a result, short-term interest rates soared. The further tendency of gradual rise in short-term rates can possibly be attributed to a number of structural factors, including the development of more effective liquidity absorption tools on the domestic market, in particular the development Russia's Stabilization fund and bonds issued by the central bank. A higher demand for monetary base due to robust economic growth might have

[1] A sharp rise in duration in 2004 corresponds to the issuance of 20-year securities.



Source: Bank of Russia

Chart 2. Estimates of zero-coupon rates of the Russian government bond market

been another factor that contributed to the rise of a short end of yield curve. The long-term rates were on the downward or horizontal trend since 2003. Periods of sharp drop in the long-term yields recorded in 2003 and mid-2005 reflect among other factors a decrease in the default risk associated with Russian government bonds[2].

Theories of term structure of interest rates offer numerous explanations for the variation of yields across the maturities. Market expectation hypothesis suggests that a shape of the yield curve depends on market participants' expectations of future interest rates and that long-term interest rates can be derived from the yields of short-term instruments.

According to the liquidity preference theory, long-term interest rates not only reflect investors' assumptions about future interest rates but also include a premium for holding long-term bonds.

[2] The long-term credit rating on Russia's government bonds was significantly upgraded in the end of 2002 and in the second half of 2005. In 2002 Russian obligations were rated 'BB' (less vulnerable to nonpayment than other speculative issues according to classification by Standard&Poors'), and in 2005 Russia got 'BBB' investment grade (according to classification by Standard&Poors').

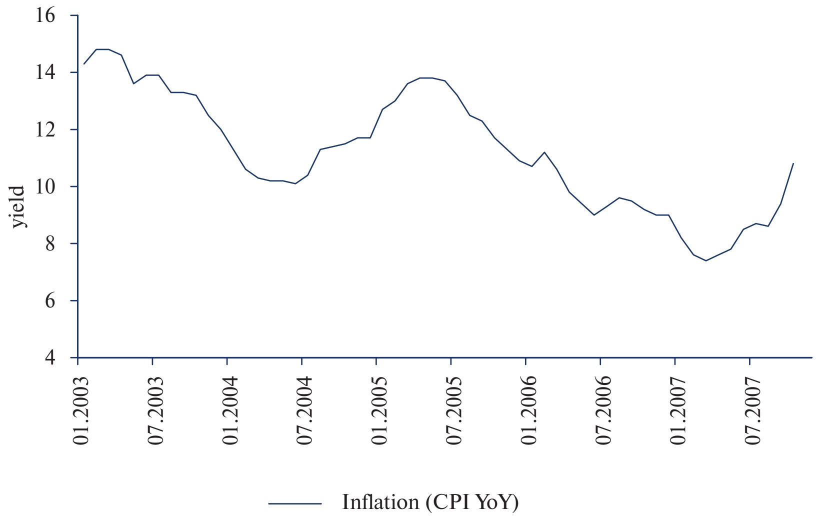
Market segmentation theory assumes that the supply and demand for short-term and long-term instruments are determined independently and that the bonds of different maturities are not substitutable. The market for short-term instruments tends to receive a higher (lower) demand due to the liquidity preferences of the investors, differential taxation, institutional structure of the economy or various legal constraints, which results in higher (lower) prices and lower (higher) yield.

The preferred habitat theory states that in addition to interest rate expectations, investors have distinct investment horizons and require a premium to buy bonds with maturities outside their "preferred" maturity, or habitat.

These theoretical approaches are mainly focused on a shape of the yield curve. When it comes to what stands behind its movements, these are the macro factors that advance to the forefront. Empirical studies indicate that the term structure conveys important message about the state of the economy. For this reason the term structure of interest rates has long been an object of a rapt attention of monetary policymakers. In particular, the information contained in the yield spread may predict future inflation. This statement builds on the Fisher decomposition of nominal yields. According to the Fisher hypothesis the movements in nominal interest rates can be decomposed into movements in real interest rates and changes in inflation expectations; hence, yields observed over time for a given maturity contains information about expected inflation measured at the time-horizon covered by that particular maturity. If we have a look at the inflation rates in Russia in 2003—2007 (Chart 3), we will see that two periods, when the downward trend in the long-term interest rates prevailed, coincide with the periods of steady disinflation.

The yield spread is also found to be a good predictor for the occurrence of recessions. In recessions the premium on long-term bonds tends to be high and the yield curves are upward sloping. At the same time, upward sloping yield curves might not only indicate bad times today, but better times tomorrow. The slope of the yield curve, measured as the difference between the long and the short yields, is usually used to predict the GDP growth. The higher the slope or term spread, the larger GDP growth is expected to be in the future. Thus, we expect to observe positive correlation between the expected rates of economic growth, the long-terms rates and term spread.

While the premiums on long bonds are countercyclical, yields on short bonds tend to be procyclical. It can be attributed to the fact that monetary authorities would lower the key interest rate in recessions in an effort to stimulate economic activity. Overall, macroeconomic policy is often cited to be a significant factor in term structure movements. Macroeconomic theory predicts that effects of macroeconomic policy could be different for short-term and long-term inter-



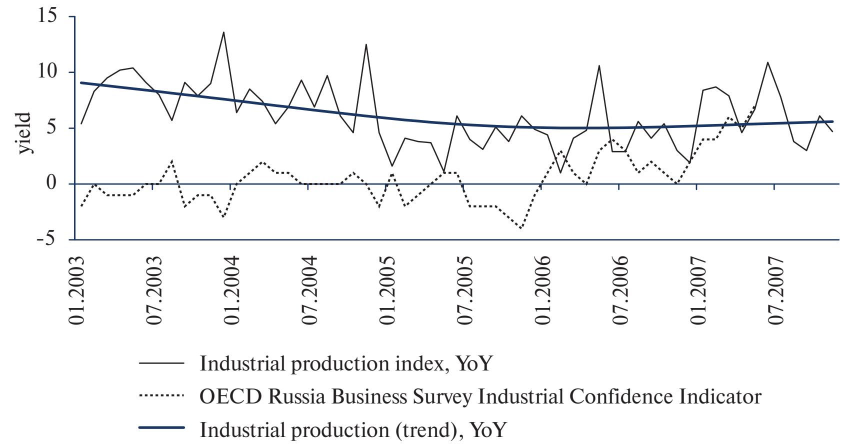
Source: Federal state statistic service

Chart 3. Inflation rates in Russia

est rates (Turnovsky, Miller, 1984; Blanchard, Fisher 1989; Turnovsky, 1989)[3]. Turnovsky (1989) put the term into stochastic macro economic model in order to find out the impact of fiscal and monetary policies on movements in the long-term and short-term yeilds. The results were understandable: unanticipated monetary expansions shocks, both permanent and temporary, lowed short term and long term nominal and real interest rates. Anticipated monetary changes didn't affect interest rates or operated in the same direction as unanticipated ones. Unanticipated fiscal disturbances were proved to push interest rates in the direction opposite to monetary ones.

In light of theoretical arguments that a narrowing of the spread foretells slower economic growth, one can question whether the recent changes in the term structure of interest rates of Russian government securities market predict an economic slowdown? It might be the case, as the chart below demonstrates that in the mid of 2005 a significant reduction in long term interest rates coincides with a gradually declining trend of industrial production growth and a

[3] It should be mentioned here that some macroeconomic models that do not abstract from the term rate structure assume that equilibrium in the goods and the money markets is driven by different interest rates. For example, the aggregate demand for goods depends on the long-term real interest rate, while the demand for money is defined by the nominal short-term rate.



Source: Federal state statistic service, Bloomberg, authors estimates

Chart 4. Indicators of economic growth in Russia

negative indicator of business industrial confidence, predicting a reduction in economic growth rates in 3–4 month.

Against such a background, we focus our analysis on the following strands.

- What macroeconomic factors determine a shape of a yield curve and the level of yields at various maturities? We will test whether any shifts in the term structure of interest rates in Russia can be attributed to the dynamics and the developments in the real sector and changes in inflation expectations.
- What monetary factors determine a shape of a yield curve and the level of yields at various maturities? We believe that the level of liquidity in the banking sector is the main determinant for money market conditions given the regime of managed floating. Thus, we will test how the level of liquidity available to the banking sector and growth in monetary aggregates influence the term structure of the interest rates.
- Do international financial markets determine a shape of a yield curve and the level of yields at various maturities? In light of this hypothesis, we will test whether the term structure of the interest rates in Russia is responsive to changes in yields of the Russian Eurobonds, that presumably capture policies and the conditions in the international markets, capital inflows and other balance of payments characteristics.

2. The variety of yield curve models

There is a vast variety of yield curve models and techniques used to describe and forecast movements in term structure of interest rates. Different modeling approaches and estimation methods partially reflect the particular modeling demands of various researchers and their different motives for modeling the yield curve. Financial models built for bond and option pricing typically describe movements in bond prices as a stochastic process. They assume that interest rate changes are driven by unobserved financial factors, rather than observable macroeconomic factors. The postulated modeling objective is to construct a stochastic process that would best imitate movements in the actual yields. The yield curve models developed by macroeconomists typically focus on the influence of inflationary expectations and future real economic activity on interest rates. The minimum set of fundamentals designed to capture basic macroeconomic dynamics usually includes the indicator for the real economic activity relative to its potential level, the monetary policy instrument, and the inflation rate.

From the perspective of our analysis, the approaches to modeling the influence of macroeconomic factors on the medium-term dynamics of interest rates can broadly be categorized into the models of stochastic process, parametric models and models of interest rate spreads.

Models of interest rate spreads imply separate modeling of short-term and long-term interest rates. Under this approach a researcher focuses on explaining movements in particular segments of the yield curve and spreads between the segments. To construct the whole yield curve, a linear interpolation between these rates (Swedish National Debt Office) or more sophisticated methods (French AFT uses OLS) are used to regress yield of intermediate maturities. Within this approach the influence of macroeconomic conditions and policies is considered when forecasting the spread between the short and long interest rates and the level of these rates. Being quite simple and straightforward, models of interest rate spreads could hardly be used to test specific linkages along the yield curve or to explain non-normal forms of the curve. However, this method could be implemented to provide empirical tests to theoretical macroeconomic models that distinguish reaction of short-term and long-term interest rates to changes in macroeconomic policy. Moreover, models of interest rate spreads give adequate estimates when being a part of large and complex models.

The class of ***stochastic process models*** represents evolution of interest rates as a *stochastic differential equation*, usually within the "factor model" framework. Such models compress all the information that influences yields of zero-coupon bonds (ZCB) into the behavior of one or several factors. Thus, the term structure of interest rates in *single-factor models* is traditionally driven by a short-term interest rate. This rate is considered to be a state variable, i.e. a variable that captures the state of the economy and contains all the relevant for ZCB pricing information about the term structure. The state variable is postulated to follow a Markov process, described by the stochastic differential equation[4]. As all that is known about future interest rates is compressed in the current short-term rate, the value of ZCB of any maturity may be written as a function of this short-term rate and time. Among the most famous one-factor models, these are the Vasicek (1977) and the Cox-Ingersol-Ross models (1985). Vasicek described the dynamics of the short rate by the stochastic differential equation with a constant across term-structure volatility parameter and a drift term constructed in a way to capture mean reversion property of interest rates. Being rather simple and analytically tractable, it is still popular with practitioners and academics. Its main disadvantage is the theoretical possibility for the interest rate to become negative. This shortcoming was addressed in the Cox-Ingersol-Ross model, which led to its economically more appealing properties and better performance in explaining the empirical data.

The advent of *multi-factor models* of the term structure marked the next step to a more realistic approach to yield curve modeling. The empirical study proves that they provide a better description of the shape and the movements of the term structure of interest rates than the one-factor models. In this class of models the short rate is determined by several state variables, depending not only on the current short rate level that follows stochastic process, but also on other factors like long-term mean and volatility.

During the last two decades the approach to modeling zero-coupon curves has evolved to the concept of no-arbitrage opportunities, which permits the derivation of a deterministic relationship between the term structure of interest rates and the state variables. Generally, these relationships (or functions) can take many forms, which means that in some cases the solution to the differential equations may not exist. However, one can ensure the existence of the solution by placing certain conditions on the coefficients for drift and diffusion terms. In particular, the coefficients may have an affine form (linear plus a constant). This assumption is a theoretical footing for the most common subclass of no-arbitrage models – *affine models*. When using these models, one comes to tractable pricing formulas, which favors such an approach. There is, none-

[4] Short rate dynamics is typically described in a following way $dr(t) = \underbrace{A_0 dt}_{Drift} + \underbrace{A_1 dW(t)}_{Diffusion}$

This stochastic differential equation says that the differential change in the state variable – dr(t) – is composed of a drift or trend term, which is non-random, and a diffusion or variance term. Here W(t) is a Wiener process modeling the random market risk factor. The coefficient A_1 determines the volatility of the interest rate. Thus, the short-term interest rate bounces along over time according to some kind of general trend.

theless, some criticism of this class of models. Affine models fail to adequately describe the current term structure of interest rates. They are good in describing the time-series properties of the term structure, but not its cross-sectional properties. The detailed overviews of this class of models and the implementation techniques can be found in Bold (2001) and Munk (2005).

Recently a number of researches have been made to incorporate macroeconomic factors into the models of stochastic process in order to study fundamental determinants of interest rates. These macro-financial models differ from each other in two aspects: 1) the type of the basic model used to describe term structure and 2) the way the economy is modeled and incorporated into the basic model. The term structure is fitted to macroeconomic factors either by combining them within unobserved factors (see, Ang and Piazzesi (2003); Ang, Dong and Piazzesi (2005); Bernanke, Reinhart and Sack (2004); and Dai and Philippon (2005)), or by incorporating a no-arbitrage model of the term structure within a fully specified macroeconomic model that exhibits both rational expectations and nominal rigidities (see Hordahl, Tristani and Vestin (2006); Bekaert, Cho and Moreno (2005); Dewachter and Lyrio (2006); and Rudebusch and Wu (2004)).

The role of macroeconomic variables in a no-arbitrage affine model is explored by several papers. In Piazzesi (2005), the key observable factor is the Federal Reserve's interest rate target. The target follows a step function or pure jump process, with jump probabilities that depend on the schedule of policy meetings and three latent factors, which also affect risk premiums. The short rate is modeled as the sum of the target and short-lived deviations from target. The model is estimated with high-frequency data and provides a new identification scheme for monetary policy. The empirical results show that relative to standard latent factor models using macroeconomic information can substantially lower pricing errors. In particular, including the Fed's target as one of four factors allows the model to match both the short and the long end of the yield curve.

Rudebusch and Tao Wu (2004a) provide an example of a macro-finance specification that employs more macroeconomic structure and includes both rational expectations and inertial elements. They obtain a good fit to the data with a model that combines an affine no-arbitrage dynamic specification for yields and a small fairly standard macro model, which consists of a monetary policy reaction function, an output Euler equation, and an inflation equation.

The underlining principle of ***parametric models*** is the specification of a function that is defined over the entire maturity domain. The model parameters are determined through minimization of the squared deviations of theoretical prices from observed prices. In contrast to financial (or stochastic) models considered above this approach does not explicitly incorporate no-arbitrage condition. However, as pointed by Diebold, Piazzesi and Rudebusch (2005), parametric

models provide a convenient way of summarizing the voluminous yield information contained in the large number of bonds that are traded at any point in time point. As such models allow for compression of information, they are consistent with so called "parsimony principle" which implies that constraining models can be useful for producing good forecasting models.

According to the overview published by the Bank of International Settlement (2005), these are fitted Nelson-Siegel curve and the Svensson's extension of this model that are quite popular among market and central bank practitioners approaches. The Nelson-Siegel yield curve has the following functional form[5]:

$$r_t(m) = L_t + S_t\left(\frac{1-\exp(-m/\tau)}{m/\tau}\right) + C_t\left(\frac{1-\exp(-m/\tau)}{m/\tau} - \exp(-m/\tau)\right) + \varepsilon_t(m),$$

$$\varepsilon_t(m) - N(0, \sigma(m)^2)$$

where $r_t(m)$ denotes yield to maturity m at time t and l_t, s_t and c_t and τ are parameters that determine the shape of the yield curve and $\varepsilon_t(m)$ is an error term.

The Nelson-Siegel yield curve could be considered as a linear combination of the three functions or factor loadings – 1, $\left(\frac{1-\exp(-m/\tau)}{m/\tau}\right)$ and $\left(\frac{1-\exp(-m/\tau)}{m/\tau} - \exp(-m/\tau)\right)$ – with their corresponding latent or unobserved dynamic parameters or factors L_t, S_t and C_t. The three latent dynamic factors are respectively considered as level, slope and curvature factors. For long maturities, spot and forward rates approach asymptotically the value L_t which must be positive. The sum of L_t and S_t determines the starting value of the curve at maturity zero, and S_t thus represents the deviation from the asymptote L_t. In addition, $(L_t + S_t)$ must also be positive. The remaining two parameters C_t and τ are responsible for the "hump". The hump's magnitude and direction are given by the absolute size and the sign of C_t (a negative sign indicates U-shape, while a positive sign indicates a hump). Parameter τ, which again must be positive, determines the position of the hump.

Macroeconomic factors can be introduced into such models within the vector autoregression framework. In Andrew Ang and Piazzesi (2003) and Ang, Sen Dong, and Piazzesi (2004) the joint dynamics of inflation and real activity and additional latent factors are captured by VARs. In Ang and Piazzesi

[5] To improve the flexibility of the curves and the fit, Svensson extended Nelson and Siegel's function by adding a further term that allows for a second "hump". The extra precision is achieved at the cost of adding two more parameters, which have to be estimated.

(2003), the indicators of real activity and inflation are constructed as the first principal component of a large set of candidate macroeconomic series, to avoid relying on specific macro series. Both papers explore various methods to identify structural shocks. They differ in the dynamic linkages between the macro factors and the yields.

Diebold, Rudebusch, and Aruoba (2005) provide a macroeconomic interpretation of the Nelson-Siegel representation by combining it with VAR dynamics for the macroeconomy. Their maximum likelihood estimation approach extracts three latent factors (level, slope, and curvature) from a set of 17 yields on U.S. Treasury securities and simultaneously relates these factors to three observable macroeconomic variables (real activity, inflation, and a monetary policy instrument). The authors found strong evidence of the effects of macro variables on future movements in the yield curve and evidence for a reverse influence as well.

A different estimation approach is implied by Pick and Anthony (2006). The authors build a yield curve model for conventional gilts that is based on the three factor Nelson-Siegel function. Their model is developed to simulate interest rates at different maturities over time. It is specified so as to provide an economic explanation for the behavior of the yield curve over time. They link the evolution of the three dynamic latent factors directly to the evolution of the short interest rate, the CPI inflation and the output gap. The parameters are chosen by the combination of theoretical considerations and empirical evidence. At first, the authors impose restrictions on the level and the slope factors. Then using a state space model with conjunction of Kalman filter they get an estimate for the curvature factor. Finally, the estimated curvature factor is regressed on an intercept, the output gap, the short interest rate and CPI inflation.

3. Empirical analysis of the Russian OFZ market: methodology and discussion

The previous research on the Russian market of government securities was predominantly devoted to the analysis of the GKO market in the 1990s. There were several attempts to test market efficiency, the expectation hypothesis and to estimate the influence of macroeconomic variables on the term structure of the GKO market. For example, Entov, Radygin, Sinelnikov (1998) performed analysis of the predictive properties of forward rates. Gurvich and Dvorkovich (2000) and Drobyshevsky (2000) touched upon the influence of inflationary expectations, the interest rates for the Russian currency securities, the dynamics of monetary indicators and exchange rate on the GKO interest rates.

In our paper we focus on modeling the term-structure of zero-coupon yield curve on the Russian OFZ market. Specifically, we decompose the effect of

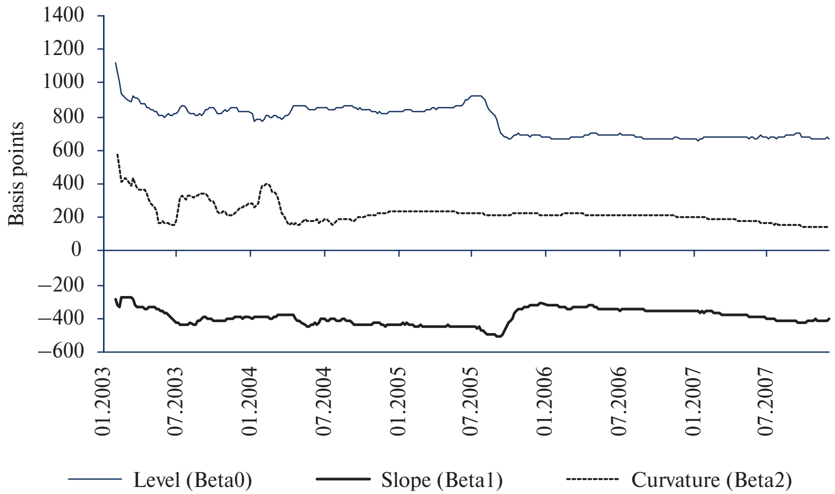
macroeconomic factors on yield curve movements as the effects of inflation expectations, real economic activity and monetary conditions on the yield curve latent factors, derived on the basis of parametric approach.

Several features should be taken into account when developing a yield curve model for the Russian market. First, the structure of the government bond portfolio is rather complex. The payment structure of bonds is typically characterized by amortization of debt or/and floating coupon rate. Thus, observable yields to maturities reflect not only the term structure of interest rates, but also premiums related to the structure of payments. Second, the level of liquidity on particular market segments is very low because of comparatively small market size of the internal government debt allocated over a significant number of issues. This means that no-arbitrage condition will not always hold and financial models could be too restrictive for developing a yield curve model for the Russian market. Moreover, the researcher should exclude illiquid government bonds from the estimation procedure, as their prices are determined by non-market forces. Third, there is no single short-term interest rate that plays the role of a base rate determining the whole term structure. There is rather a bundle of key interest rates. Thus, it is difficult to identify a basic factor for building a yield curve model based on stochastic process. Finally, due to the domestic foreign exchange policy, the domestic yields are highly sensitive to the dynamics of interest rates on international markets. Therefore, the minimum set of variables that determines the yield curve should also include indicators that reflect the situation on the international financial markets.

For the reasons mentioned above, we have adopted the following approach to analyze the effect of macroeconomic variables on the dynamics of government bond yields in recent years. We focus on zero-coupon yields rather than on actually observed yields of specific bonds. In particular, we examine latent factors behind the yields that are estimated within the parametric model of zero-coupon yield curve for the Russian market of Gambarov, Shevchuk and Balabushkin (2004). According to empirical analysis conducted by Gambarov, Shevchuk and Balabushkin, parametric models prove to give more robust and better fitted results for the Russian market as compared to stochastic approach.

The authors use the following extended version of the four-factor Nelson-Siegel yield curve:

$$R(m) = \beta_0 + (\beta_1 + \beta_2)\frac{\tau}{m}\left[1 - \exp\left(-\frac{m}{\tau}\right)\right] - \beta_2 \exp\left(-\frac{m}{\tau}\right) +$$
$$+ g_1 \exp\left(-\frac{m^2}{2}\right) + g_2 \exp\left(-\frac{(m-1)^2}{2}\right) + g_3 \exp\left(-\frac{(m-2)^2}{2}\right),$$



Source: Bank of Russia

Chart 5. Estimates of the level, slope and curvature factors of the Russian zero-coupon yield curve

where R(m) is an instantaneous yield (in basis points) of a zero-coupon bond with maturity of m years, and $\beta_0, \beta_1, \beta_2, \tau, g_1, g_2, g_3$ are the parameters that need to be estimated. Similar to the notation used above, β_0, β_1 and β_2 stand for the "level", "slope" (defined as the short yield minus the long one) and "curvature" factors respectively. A set of variables g_1, g_2, g_3 is required to improve the fit of the model to the short end of the curve. The estimation algorithm is similar to Kalman filter. It accounts only for those bonds that have sufficient liquidity level. The evolution of the estimated level, slope and curvature parameters is shown on the chart 5.

We take the estimates of three principal latent factors of zero-coupon yields derived from the Gambarov, Shevchuk and Balabushkin model (β_0, β_1 and β_2) and carry out statistical tests in order to find which macroeconomic impulses could have affected movements in the yield curve[6]. The empirical analysis is based on the weekly data for the period from January 2003 to August 2007. Data sets of lower frequencies were converted to weekly data by means of cubic-spline.

To explore the data properties we assume that the movements in latent factors do not feed back to the macro variables and estimate alternative models of the dynamics of the level, slope and curvature factors, where macroeconomic factors are treated as exogenous variables.

Following theoretical considerations, we include four groups of indicators into the set of macroeconomic variables: the indicators of inflation expectation, the level of real economic activity related to potential level, the stance of the monetary policy, and the level of yield on the Russian foreign government debt. Within the each group of macroeconomic factors the indicators that are tested vary. The group of variables that capture real activity includes industrial confidence indicator from OECD Russia business survey and the cyclical components of year-on-year growth rates of industrial production, retail sales, and index of production of basic industries. The stance of monetary policy is modeled as a combination of a short-term lending interest rate on the Russian interbank market, a year-on-year growth in monetary aggregates, a level of liquidity available to the banking sector and a rate of appreciation of the ruble against the dollar.

The major findings of our empirical analysis are the following. First of all, we can not reject the hypothesis of a structural break in the middle of 2005. The period before August 2005 is characterized by significantly higher volatility of the yields (and consequently of all the latent factors), a higher level of long-term yields and a steeper yield curve (lower slope factor in our definition). To all appearance this structural break not only reflects the improvement of the Russian credit rating (up to investment grade) and consequently a decrease in the default risk associated with it's obligations, but it also can be attributed to the turning point in the trend of and overall deterioration in business conditions. In order to capture this change in regime we introduced a specific dummy variable to model the mean and the variance of our latent factors.

The next set of results corresponds to independent ARMA and GARCH models for each of the latent factors. The outcomes of our empirical analysis indicate the presence of significant macro factor effects on the long-run level of yields and the slope of the yield curve, while the curvature factor appears to be unrelated to the macroeconomic variables we explored.

The estimation results for selected models of the level factor (Beta0 series) are provided in Table A1, and the related forecasts for the period from October 2006 to September 2007 are depicted on Chart A1. While simple ARMA/GARCH model provides a good fit and captures the general trend in the mean of the level factor it fails to explain fluctuations around the trend. Introduction of macroeconomic factors allows to improve the forecasting power of the level factor models and to capture changes in the direction of movements of the yields. In particular, we found that the level factor is positively correlated with the yields of the Russian Eurobond (maturing in 2030) and the rate of currency depreciation. At the same time, an increase in inflation expectations (measured as a geometric average of inflation rates over the last four periods)

[6] For the purpose of this research, we ignore the set of g_1, g_2, g_3 parameters responsible for good fit in the short end.

also leads to a higher level of long-term yields. Therefore, our finding supports the hypothesis that the two periods of significant reduction in the level factor can be attributed to comparatively rapid appreciation of ruble and a steady decrease in inflation rates.

The hypotheses of any significant effects of money growth on the long-term interest rates on the interbank market were rejected.

We failed to find a robust model of level reaction to changes in the activity of the real sector. In particular, we did not find any statistical evidence that the growth rates of GDP or industrial production are significant for the movements in the long-term level of interest rates. However, some specifications support the idea that the indicators of economic growth perceptions (e.g. the growth rate of retail sales, which can be considered as a proxy for the expected growth rates of consumption) have an explanatory power. Thus, it is possible that a more general indicator of the stance of the real sector (for example, a set of principal component derived from a number of standard indicators of business activity) should be tested.

The dynamics of the slope factor (Beta1) for the most part can be explained by the values of the slope and the level realized in the previous period (Table A2). At the same time, we could not reject the hypotheses that changes in the level of liquidity and short term interest rate (one month interest rate on the interbank market) are important determinants for the slope factor. An increase in liquidity (or decrease in the short term rate) has a negative effect on the slope factor, meaning that the curve becomes less negatively sloped. Moreover, we found that the narrow monetary base (M1) proves to have the most significant effect on the yield curve compared to other monetary aggregates. That supports the hypothesis that short-term interest rates are more sensitive to changes in monetary policy, than the long end of the curve.

The curvature factor appears to follow ARCH process unrelated to the macroeconomic variables we explored. As in the cases of other latent factors we cannot reject the hypothesis of a structural break in the mean and variance of the curvature factor.

Conclusion

We have discussed the question of modeling the yield curve of government bonds in Russia. We focused on the parametric approach and used a decomposition of zero-coupon yields into three major unobservable factors — the level, slope and curvature — to incorporate macroeconomic variables into the yield curve model. We provided the initial evidence on the relationship between the term structure factors and macroeconomic variables. In particular, we found

that inflation, the level of yields on the international market, as well as narrow monetary aggregates and short term interest rate are significant determinants of the long-run level of yields and the slope of the yield curve.

Our analysis helps to get some economic explanation about the movements in term structure of interest rates in Russia in recent years. However, it should be extended in a number of ways in order to be used in analysis of monetary policy transmission mechanism or yield curve forecasting for the purpose of public debt management.

First, a more thorough analysis and modeling of the monetary policy reaction to changes in the macroeconomic conditions are required. In this paper, we assumed that the stance of monetary policy is adequately reflected by the rate of currency appreciation and the dynamics of the monetary base and the changes in the short-term interest rate on the interbank market. However, the changes in these indicators can be caused by the factors other than actions of monetary authorities in response to macro shocks. In particular, it is required to examine the influence of various instruments of the Bank of Russia on the market conditions (including REPO operations, operations with the Bank of Russia's bonds, and numerous deposit facilities) in order to reveal the bundle of effective instruments that, on the one hand, presumably control the very short end of the yield curve and the level of liquidity in the banking system, and, on the other hand, are subject to discrete changes in response to fundamental macroeconomic shocks.

Second, in practice the yield curve may flatten for specific reasons not related to the economic slowdown or lower inflation expectations. For example, the investors might have become more willing to invest long term in recent years, encouraged by recent stability of the Russia's economic conditions and financial markets. As a result, the increased demand for long-term securities has caused yields on these securities to fall. Moreover in light of further liberalization of Russia's financial markets, new sources of demand could have supported the price of medium and long-term securities as the most liquid ones. At the same time, the limited supply of long and medium term government securities might have kept the lid over the long end of the yield curve Besides, the increased stability in global financial markets itself might have resulted in a low term premium.

The model specifications we tested did not take into account any consequences from the changes in a relative supply/demand of bonds on different segments for the yield curve. Thus, it might be interesting to examine how the structure of market players and the government securities' auction strategy contribute to the movements in the term structure of interest rates. Overall, such "market structure" effects should be analyzed in conjunction with other macroeconomic data to avoid misreading.

Appendix

Table A1. Selected models of the level factor (Beta0)

Mean regressors	Model			
	Level_1	Level_2	Level_3	Level_4
CON STANT	122.04 (47.6)	102.21 (35.62)	131,48 (15,29)	195,7 (19,69)
DUMMY_2005	–24.16 (9.14)	–24.05 (7.12)	–25,55 (3,11)	–36,95 (4,1)
BETA0(-1)	0.86 (0.06)	0.86 (0.04)	0,84 (0,02)	0,72 (0,02)
EBOND30			5,82 (1,57)	6,2 (1,22)
CPIY_GM	–	1.77 (1.07)	–	1,12 (0,67)
USD_Y	–		0,44 (0,16)	0,49 (0,15)
RSALES_Y	–	55.22 (30.91)	–	–
AR(1)	0.41 (0.11)	0.33 (0.1)	0,35 (0,07)	0,35 (0,05)
AR(2)	–	–0.16 (0.07)	–	–
Variance regressors				
DUMMY_2005	–	+	+	+
Sample	ARMA(1,0) 26/05/2003 26/11/2007	ARMA(1,0) 3/03/2003 30/07/2007	GARCH(1,1) 27/01/2003 26/11/2007	GARCH(1,1) 10/02/2003 27/08/2007
Number of observations	236	231	253	238
AIC	7.091	7.256	6,869	6.904887
SIC	7.150	7.361	7,009	7.079959
Forecast error	8.71	13.83	7.55	11.56

Estimation method: Least squares – Newey-West HAC Standard Errors;
ML – ARCH (Marquardt)

Table A2. Selected models of the slope factor (Beta1)

Mean regressors	Model		
	Slope_1	Slope_2	Slope_3
DUMMY_2005	–1,86 (0,74)	0.95 (0.01)	–
BETA1(-1)	0,95 (0,01)	–0.03 (0.01)	0,96 (0,01)
BETA0(-1)	–0,04 (0,01)	0.04 (0.01)	–0,05 (0,01)
BETA2(-1)	0,06 (0,01)	–	0,08 (0,05)
MIACR_30D	–	–	1,51 (0,39)
M1_Y(-12)	–	–0.09 (0.04)	–
AR(1)	0,13 (0,08)	0.16 (0.09)	0,22 (0,09)–
Variance regressors			
DUMMY_2005	+	+	+
Sample	GARCH(1,1) 27/01/2003 26/11/2007	GARCH(1,1) 20/01/2003 26/11/2007	GARCH(1,1) 20/01/2003 24/09/2007
Number of observations	253	254	245
AIC	6.541	6.665	6.955
SIC	6.681	6.762	7.041

Estimation method: ML – ARCH (Marquardt)

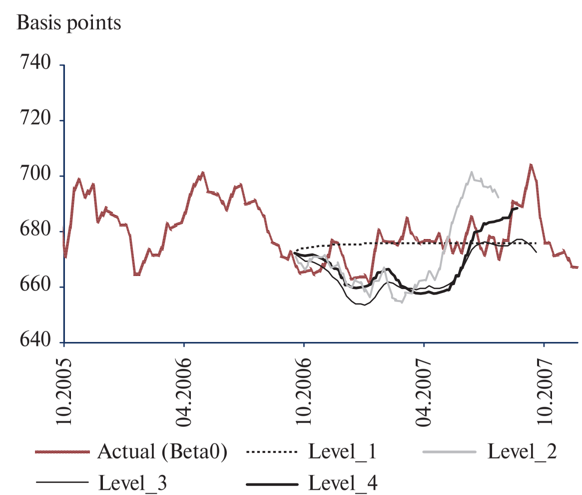


Chart A1. Forecasts of the level factor (Beta0)

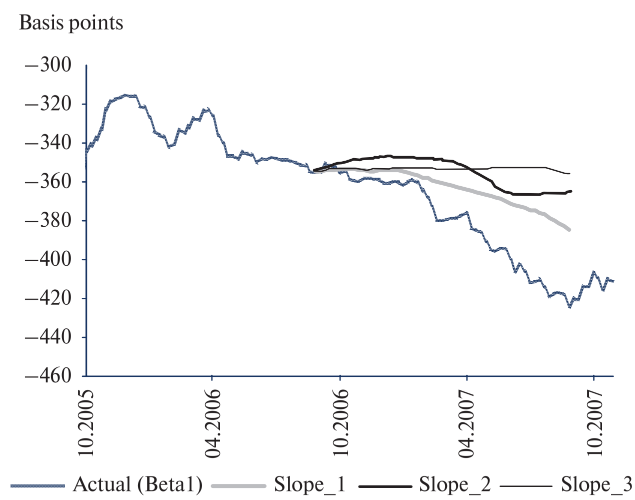


Chart A2. Forecasts of the slope factor (Beta1)

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Н.П.Новикова, Ю.Б. Устюгова

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на российском рынке государственного долга:
макроэкономический подход**

(на английском языке)

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