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**“UNPLEASANT MONETARIST
ARITHMETIC” REVISITED:
THE ROLE OF TRANSITIONAL
MONETARY DYNAMICS**

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P38 **Pekarski S.** “Unpleasant Monetarist Arithmetic” Revisited: The Role of Transitional Monetary Dynamics”. Working Paper WP13/2007/13. — Moscow: State University — Higher School of Economics, 2007. — 20 p.

In the celebrated paper “Some unpleasant monetarist arithmetic,” Sargent and Wallace (1981) showed that tight monetary policy is not feasible unless it is not supported by appropriate fiscal adjustment. In this paper we explore a simple forward-looking monetary model to show that gradual or anticipated tightening of monetary policy is not necessarily characterized by “unpleasant arithmetic.” This is due to possible short-run gains in seigniorage revenues during the transition period. By applying the principle of sustainable macroeconomic policy that constrains the future choices of both fiscal and monetary authorities, we show that high interest rates on public debt work for the benefit of “pleasant arithmetic.”

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В широко известной работе Томаса Саржента и Нейла Уоллеса «Некоторая неприятная монетаристская арифметика», опубликованной в 1981 г., авторами было показано, что центральный банк не может проводить ограничительную монетарную политику в отсутствие соответствующей подстройки фискальной политики. В данной работе мы рассматриваем простую модель вперед-смотрящей динамики денежного рынка, на основе которой демонстрируем, что постепенное или ожидаемое ужесточение монетарной политики не обязательно приводит к «неприятной арифметике». Данный факт объясняется возможностью увеличения сеньоража во время переходной динамики. Применяя принцип устойчивости фискальной политики, который выступает как ограничение на фискальную и монетарную политику в будущем, мы показываем, что высокая ставка процента по обслуживанию государственного долга делает «приятную арифметику» более вероятным исходом.

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1. Introduction*

From the point of view of traditional monetarism, the growth rate of base money to a significant extent determines inflationary processes, since inflation as a phenomenon is a decrease in the purchasing power of existing money due to the emission of new money. However, can we consider only the central bank’s monetary policy as the sole determinant of inflation, without also considering the fiscal policy of the government?

This problem arises quite naturally, since one of the sources of financing the budget deficit of the government is seigniorage, the real income from the emission of the base money, which is collected and controlled by setting the growth rate of base money by the central bank. In this regard monetary policy should not be considered in isolation, but rather in the context of the interaction of fiscal and monetary policies. One of the first to note this were Thomas Sargent and Neil Wallace in 1981 in their famous paper: “Some unpleasant monetarist arithmetic” (referred to below as SW and UMA)¹. The main result can be stated as follows. For a given fiscal policy a tightening of monetary policy can bring about a decrease in the rate of inflation in the short run only at the expense of higher growth rates of base money and inflation in the future. Moreover, if the behavior of economic agents is rational and forward-looking, then a tight monetary policy today may bring about high inflation not only in the future, but also in the present.

SW consider the following theoretical experiment. Fiscal policy is assumed to be dominant. At the initial point in time the central bank decreases the growth rate of base money, and this brings about a decrease in the volume of seigniorage. For a given trajectory of the primary budget deficit, the less the volume of seigniorage, the higher the public debt at all points in time in the future. Indeed, by compensating for the decrease in the volume of seigniorage, the government is forced to borrow more in order to service the existing debt, and this brings about an increase in the volume of borrowings. However, there are many reasons why the public debt cannot increase infinitely². According to the argument by SW, for instance, the in-

* This paper was prepared when S. Pekarski visited CORE, UCL.

¹ Earlier the necessity to analyze fiscal and monetary policy interaction was proposed by Christ (1979).

² SW assume that the interest rate on public debt is greater than the growth rate of output in the economy. In this case the backward-looking dynamics of both the public debt and the ratio of public debt to output is unstable, which is an important reason for the existence of the problem of coordinating macroeconomic policies. If the interest rate were less than the growth rate of output, then the ratio of public debt to output would exhibit a stable trajectory, and this would remove many possible problems for fiscal policy.

crease of public debt is bounded from above by the volume of private savings in the economy, or, what is more likely, by some other, lesser quantity. In order to stabilize the increasing public debt in the future (to fix its volume at a certain point in the future) it will be necessary to have a higher volume of seigniorage, and therefore a higher growth rate of base money and rate of inflation. Modeling the demand for money according to the original quantity theory of money, SW showed that a lower growth rate of base money (rate of inflation) today will inexorably be replaced by a higher growth rate of base money (inflation) in the future. If, however, the demand for money decreases with an increase in expected rate of inflation (as, for example, for Cagan's money demand), and expectations are rational (forward-looking), then the unavoidable increase in the growth rate of base money at a certain point in time will already bring about an increase in inflation even today.

This conclusion can indeed be considered as a Fiscal Theory of Inflation (FTI). In a situation of inevitable policy interaction with the dependent role of monetary policy, the central bank is not able to permanently decrease the growth rate of base money, that is, to conduct long-term policies to bring inflation down. In this sense inflation becomes not only a monetary, but a fiscal phenomenon as well, since influencing it requires not only monetary policy actions, but also fiscal actions that have to do with correcting the budget deficit of the government. According to Sargent (1999) there indeed are "...limitations of what can be achieved by monetary policy...":

Proposition 3. *Monetary policy can influence the time path of price level (assuming that the authority's powers are augmented by sufficient powers to levy taxes).*

Sargent (1999, p. 1470)

Proposition 4. *Monetary policy cannot permanently prevent inflation (given a fiscal policy implying a stream of net-of-interest government deficits).*

Sargent (1999, p. 1472)

The main purpose of this paper is to show that while UMA (Propositions 3 and 4 stated above) is almost inevitable results if one looks at steady states, monetary arithmetic *may* be "pleasant" if one takes into account forward-looking transitional dynamics associated with the gradual or preannounced change in monetary policy. The simple intuition of this result is based on the fact that seigniorage is the product of *actual* growth rate of base money (tax rate) and the real money balances (tax base) that are demanded *regarding future* monetary policy. If information about future policy is made available ahead of the actual switch in the policy, then there is a time interval when real money balances and seigniorage change while the growth rate of money remains the same. Similar logic holds in the case when new policy is implemented gradually. We demonstrate that these transitional dynamics of seigniorage are of great importance, simply because its direction may be of the opposite sign with respect to the final change in the steady state level of seigniorage.

The possibility of a transitional gain in seigniorage allows us to re-emphasize the role of the interest rate on public debt. In the setup considered by SW a high interest rate may be seen as a strengthening factor for UMA: the higher the interest rate, the faster public debt grows following the initial cut in seigniorage revenue. However, if there is a transitional gain in seigniorage instead of loss, the role of a high interest rate is reversed. This is best seen if one applies the general principle of sustainable macroeconomic policy that constrains both fiscal and monetary authorities instead of original SW's setup, in which the role of this constraint is played by the upper limit of the public debt. For a given future path of budget deficits, the sustainability of public debt requires a certain *present discounted value* of future seigniorage revenues. A higher interest rate implies a higher discounting of future revenues. In this case a short-run gain in seigniorage becomes more important than its long-run values.

The rest of the paper is organized as follows. Section 2 presents a review of recent contribution to the literature on UMA. Section 3 contains a very simple forward-looking monetary model to analyze the interaction between fiscal and monetary policy. Special examples of the monetary policy tightening that may give rise to "pleasant monetarist arithmetic" are presented. The final section provides some concluding remarks.

2. Recent literature survey

The problem of the interaction of fiscal and monetary policy sparked an interesting discussion that still continues. While the theoretical results were undoubtedly interesting, some economists were skeptical about how realistic the basic assumptions were. Darby (1984) considers that SW's assumption that the interest rate is greater than the growth rate of output does not hold for the economy of the USA and other developed countries. This assumption is indeed critical for all analysis of macroeconomic policy. Answering Darby's criticism, Miller and Sargent (1984) note that the UMA can (and should) be considered in a wider context, and not just literally³. The growth of public debt as a result of a tightening of monetary policies can bring about an increase in the interest rate for a variety of reasons. If so, then Darby's methodology, which included the average interest rate for previous periods, could be erroneous⁴.

Agreeing that the assumption that the interest rate is greater than the growth rate of output is not incontestable, Bhattacharya, Guzman and Smith (1998) showed that this assumption is not necessary for the existence of UMA. The authors in-

³ Dwyer (1985) provides a similar line of reasoning on the validity of Darby's criticism.

⁴ In this sense, the analysis by Darby (1984) is subject to Lucas' critique.

clude an extra asset in SW's model that is available to the private sector and financial intermediaries. In doing so, savings, as an additional asset, conform to the requirement of partial reservation. It was shown that, taking these additions into account, UMA can exist if the economy contains at least one asset with a rate of return that is greater than the growth rate of output. In the real world such assets, obviously, almost always exist.

Bhattacharya et al. (1997) stress the general role of reserve requirements as an instrument that allows inflation tax base, and thus seigniorage collection, to be regulated. Bhattacharya and Haslag (2003) go further and show that in the case when a monetary tightening is realized via a higher reserves ratio, monetary arithmetic is "more pleasant". This result is based on the relationship between reserves requirements, gross real return on deposits and capital accumulation. An important implication of this paper is that while fiscal dominance imposes constraints on monetary policy as a whole, it does not do so for a particular monetary policy instrument.

Espinosa and Russell (1998a,b) and Bhattacharya and Kudoh (2002) consider the plausibility of UMA when the initial interest rate is lower than growth rate of output under different monetary policy rules (fixed bonds-money ratio rule versus money-growth rule)⁵. This avenue of research goes far beyond the original subject of UMA, considering it in a more general framework of the real effects of fiscal and monetary policy. Nikitin and Russell (2006) contribute to this literature and provide a well-structured survey.

Dornbusch (1996) suggests additional considerations that strengthen UMA. First, tight monetary policy leads to higher interest rates, and thus higher debt service and more rapid accumulation of public debt. Second, tight monetary policy may worsen the fiscal position by lowering tax revenues and increasing unavoidable government spending. Third, higher interest rates can depress economic growth, thus leading to more rapid growth of the debt to GDP ratio⁶.

While most of the papers cited above propose substantial departures from the precise logic of UMA, the purpose of this paper is to show that UMA is not a necessary outcome using the very simple setup of the forward-looking monetary model. In this respect our paper is close to Buffie (2003a). Using the model developed by Liviatan (1984) and Drazen (1985)⁷ he found the equilibrium path of the real money balances and public debt "overlooked" by Drazen that gives "pleasant mone-

⁵ McCallum (1984) and Liviatan (1988) provide an earlier discussion of UMA under a specific money finance to bond finance ratio. This approach originated in the old macroeconomic literature on the stability of pure bond or pure money finance of the budget deficit.

⁶ These considerations are in line with those proposed by Miller and Sargent (1984).

⁷ The original contribution of Liviatan (1984) and Drazen (1985) consist in showing that UMA holds only if the demand for money is inelastic with respect to the nominal interest rate. In this case the economy operates on the increasing branch ("efficient" side) of the inflation tax Laffer curve. Velasco (1993) arrived at similar results after modifying the Drazen model for an open economy, floating exchange rate and perfect capital mobility.

tarist arithmetic" for tight monetary policy. This path followed by a decrease in the growth rate of base money is associated with the *transitory* gain in seigniorage revenue and thus does not require the extraction of extra revenues from money creation (or fiscal adjustment) to stabilize public debt later. Along this path the economy moves toward the new equilibrium with a lower growth rate of base money and inflation. In fact, in arriving at this result Buffie modifies the original Drazen model in one important respect: he allows the real money balances to be a jump variable, while Drazen considered a dynamic system of real money balances and public debt on a backward-looking basis. And this is indeed a crucial modification.

We go further and consider the forward-looking dynamics both for the real money balances (inflation rate) and public debt. For the latter, it is the principle of sustainable macroeconomic policy that future budget surpluses and seigniorage revenues must to provide appropriate backing for the accumulated public debt. This seems to be a prominent approach. The backward-looking system analyzed by Drazen has an unstable steady state (for a given constant level of the growth rate of base money and budget deficit)⁸. This result falls short of the Samuelson's Correspondence Principle and may be justified only if one applies the somewhat artificial assumption that fiscal and monetary authorities can keep the economy in equilibrium in the absence of any shocks. By contrast, the forward-looking dynamics of the system are stable.

Another important element of our analysis is that we allow for the preannouncement of future policy changes. This is important because it generates transitional dynamics in the system up to the time of the actual policy switch. We show that there is a gain in seigniorage during this transition. This is similar to Buffie's finding. However, while the existence of the special path associated with the transitory gain in seigniorage following a tightening in monetary policy in the Buffie model requires certain restrictions on preferences (money demand parameters) and the parameters of the policy switch (timing and magnitude of the change in the growth rate of base money), a preannouncement unambiguously provides a transitory gain in seigniorage. Nevertheless, even in the case where monetary tightening is associated with a short-run gain in seigniorage, it does not automatically follow that public debt is kept sustainable. This is because tight monetary policy unambiguously leads to lower steady state seigniorage on the increasing branch ("efficient" side) of the inflation tax Laffer curve. Thus the question is what is more important: short-term gain or long-run decrease in seigniorage revenue? In this respect the analysis in this paper helps us to stress the crucial role of the interest rate on public debt that is present, but rather undermined in Buffie's analysis. High interest rates lead to a heavy discounting of future seigniorage, making short-run gain more important than long-run loss.

⁸ This problem does not arise if one considers monetary policy in terms of money finance to bond finance ratio rather than setting the growth rate of base money. See Liviatan (1986).

Apart from the discussion of the validity of UMA and its extensions on a theoretical basis, there is a wide branch of literature on the empirical implications of UMA⁹. Indeed, the main prediction of FTI is that budget deficits are inflationary either in the short-run or in the long-run¹⁰. The evidence is not solid, but in general it does not support this view at least for developed countries. One possible explanation of this apparent failure is that one important assumption of FTI, namely that the economy is in a regime of fiscal dominance, may not hold in practice (at least all the time). That is, it may be the case that the government (not the central bank) adjusts its policy at times when public debt becomes high. Our analysis has important implications for this discussion. If there is indeed a possibility for tight money to sometimes have “unpleasant arithmetic” and “pleasant arithmetic” at other times, then FTI does not unambiguously predict the inflationary consequences of budget deficits under the assumption of fiscal dominance. In the final section of the paper we confirm that in some cases an increase in the budget deficit can be accompanied by some sort of tight (and low inflation) monetary policy. Thus, an appropriate case study is a good alternative to the time series analysis that may help clarify whether and when budget deficits are inflationary¹¹.

3. The model

To simplify the exposition we employ the forward-looking modification of the Cagan (1956) model proposed by Sargent and Wallace (1973). The demand for real money balances, $m^d = (M/P)^d$, is log-linear in the expected inflation rate, π^e ¹²:

$$m^d = Ae^{-\alpha\pi^e}. \quad (1)$$

In the following analysis we will assume that the money market is in equilibrium, $m^d = M^s/P = m$. Without loss of generality, we can assume that the scale parameter is normalized to unity, $A = 1$. The parameter $\alpha = -\left(dm^d/m^d\right)/d\pi^e > 0$ characterizes the semi-elasticity of demand for real money balances with respect to expected rate of inflation. Denote $x = \ln m^d$, and then (1) can be rewritten as

$$x = -\alpha\pi^e. \quad (2)$$

The dynamics of real money balances can be determined by simple arithmetic: their rate of growth is equal to the difference between the rate of growth of the base money, $\mu(t) = \dot{M}^s(t)/M^s(t)$, and the actual inflation rate, π ¹³:

$$\dot{x}(t) = \frac{\dot{m}(t)}{m(t)} = \mu(t) - \pi(t). \quad (3)$$

Following Sargent and Wallace (1973) we will analyze the dynamics of the money market under the assumption of perfect foresight in forming the expected rate of inflation:

$$\pi^e(t) = \pi(t). \quad (4)$$

Equation (3) along with equations (2) and (4) gives us:

$$\dot{\pi}(t) = -\frac{1}{\alpha}(\mu(t) - \pi(t)) \quad (5)$$

Imposing the additional condition for the absence of a hyperinflationary bubble,

$$\lim_{t \rightarrow \infty} \pi(t)e^{-\frac{1}{\alpha}t} = 0, \quad (6)$$

we arrive at the fundamental forward-looking solution to (5):

$$\pi(t) = \frac{1}{\alpha} \int_t^\infty \mu(\tau) e^{-\frac{1}{\alpha}(\tau-t)} d\tau. \quad (7)$$

The forward-looking solution for the dynamics of the logarithm of real money balances may be written in the same way:

$$x(t) = -\int_t^\infty \mu(\tau) e^{-\frac{1}{\alpha}(\tau-t)} d\tau. \quad (8)$$

Using the definition of seigniorage, $S = \mu m$, and the fact that $m = e^x$, we find the dynamics of seigniorage that is based on the forward-looking dynamics of inflation:

$$S(t) = \mu(t) e^{-\int_t^\infty \mu(\tau) e^{-\frac{1}{\alpha}(\tau-t)} d\tau}. \quad (9)$$

⁹ These are King and Plosser (1985), Viera (2000), Fischer, Sahay and Vegh (2002), Giannitsarou and Scott (2006), among others.

¹⁰ Drazen and Helpman (1990) stress that the theory does not predict the strong correlation between budget deficits and inflation that is due to uncertainty about the type and timing of future policy shifts. This, however, does not mean that a reduction in budget deficit is unnecessarily to stop high inflation.

¹¹ Buffie (2003b) provides such a case study of inflationary episodes in Sub-Saharan African countries and describes quite different consequences of tight monetary policies. Salamon (2001) finds empirical support for UMA using a non-linear time series model to study inflation in Brazil.

¹² The necessarily microeconomic foundations for the aggregate relationships may be easily found in the literature (see, for example, Drazen, 1985, and Calvo and Leiderman, 1992), and so we do not replicate them here for brevity.

¹³ In general all variables are functions of time. A dot over a variable denotes the derivative with respect to time.

In accordance with the logic of FTI, we must consider monetary policy in the interaction with fiscal policy. Let us assume, following SW's original assumption, that the fiscal policy is dominant. That is, when monetary authorities set the trajectory of the growth rate of base money they consider future budget deficits as given. Moreover, they must consider the government budget constraint as a *joint* constraint for future fiscal and monetary policy. SW and subsequent authors (Drazen, 1985; Buffie, 2003a) assume that monetary policy should be endogenous to the dynamics of public debt: when debt reaches some upper limit and can not be stabilized by means of fiscal adjustment, it should be monetized. While this assumption is reasonable, it is not general. It is more convenient to consider the principle of public debt sustainability that implies constraints on both fiscal and monetary policy as well. In Sargent's terminology, public debt must be backed by net revenues of the government and the central bank, that is, by future budget surpluses and seigniorage revenues¹⁴. Formally, macroeconomic policy is sustainable, if at each point in time t and for every given accumulated volume of public debt, $b(t)$ future policies are characterized by the choice of trajectories $d(t)$ and $S(t)$ which satisfy:

$$b(t) \leq \int_t^{\infty} (S(\tau) - d(\tau)) e^{-r(\tau-t)} d\tau, \quad (10)$$

where d is the real primary budget deficit, b is the real (indexed) public debt, and r is the real interest rate on public debt, taken to be constant for simplicity. This framework for the analysis of the interaction between fiscal and monetary policy is more general than that applied by SW. Given exogenous (dominant) fiscal policy, (10) determines the constraint on the *present discounted value* of future seigniorage, while its transitory dynamics may in fact be arbitrary. This is the crucial point in our analysis. In what follows we may characterize future monetary policy as credible in the sense that it remains compatible with the sustainability of public debt, if it does not lead to a decrease in the present discounted value of seigniorage.

To examine whether tight monetary policy is credible under a regime of fiscal dominance, we explore several “textbook” theoretical experiments on the forward-looking dynamics.

Permanent unexpected one-step decrease in the growth rate of base money

Consider first the case when monetary authorities conduct once and for all a decrease in μ . Prior to the date t_1 , the growth rate was kept constant at $\mu = \mu_0$. Starting from date t_1 , the growth rate is unexpectedly decreased and held constant at $\mu_1 < \mu_0$. It follows from equations (7)–(9) that the inflation rate, log of real money balances and seigniorage jump (without gradual transition dynamics) at time

t_1 from their initial steady levels, $\pi_0 = \mu_0$, $x_0 = -\alpha\mu_0$, and $S_0 = \mu_0 e^{-\alpha\mu_0}$, to their new steady levels, $\pi_1 = \mu_1$, $x_1 = -\alpha\mu_1$, and $S_1 = \mu_1 e^{-\alpha\mu_1}$, respectively. The inflation rate (log of real money balances) becomes permanently higher (lower). From now on we assume for convenience that economy operates on the efficient side of the inflation tax Laffer curve, where money demand is inelastic. In this case seigniorage becomes permanently smaller. Its present discounted value also decreases. If (10) holds with equality prior to time t_1 , then a tightening of monetary policy leads to a violation of (10). Thus, such monetary policy is not credible under fiscal dominance. This resembles the essence of UMA.

Permanent unexpected two-step decrease in the growth rate of base money

Surprisingly, the previous result does not hold *in general* if the growth rate of base money decreases gradually. Consider, for example, the simplest case of a two-step decrease in μ . Assume that initially the growth rate of base money was constant at $\mu = \mu_0$. At time t_1 agents learn that for the time interval $[t_1, t_2]$ μ will be set at $\mu_1 < \mu_0$, while later on (for $t \geq t_2$) it will be decreased further to $\mu_2 < \mu_1$. Equations (7)–(9) determine the transitory dynamics of the variables:

$$\pi(t) = \begin{cases} \mu_0, & t < t_1, \\ \mu_1 - (\mu_1 - \mu_2) e^{-\frac{1}{\alpha}(t_2-t)}, & t_1 \leq t < t_2, \\ \mu_2, & t \geq t_2. \end{cases} \quad (11)$$

$$x(t) = \begin{cases} -\alpha\mu_0, & t < t_1, \\ -\alpha\mu_1 + \alpha(\mu_1 - \mu_2) e^{-\frac{1}{\alpha}(t_2-t)}, & t_1 \leq t < t_2, \\ -\alpha\mu_2, & t \geq t_2. \end{cases} \quad (12)$$

$$S(t) = \begin{cases} \mu_0 e^{-\alpha\mu_0}, & t < t_1, \\ \mu_1 e^{-\alpha\mu_1 + \alpha(\mu_1 - \mu_2) e^{-\frac{1}{\alpha}(t_2-t)}}, & t_1 \leq t < t_2, \\ \mu_2 e^{-\alpha\mu_2}, & t \geq t_2. \end{cases} \quad (13)$$

Fig. 1 illustrates the dynamics¹⁵. The inflation rate (the log of real money balances) has one announcement jump at the time t_1 by $\Delta\pi(t=t_1) = \mu_1 - (\mu_1 -$

¹⁵ For the sake of space Fig. 1 and Fig. 2 present only the dynamics of μ , x and S . The time path of π is just a regular reflection of the time path of x .

¹⁴ Sargent (1985).

$-\mu_2)e^{-(1/\alpha)(t_2-t_1)} - \mu_0 < 0$ ($\Delta x(t=t_1) = -\alpha\mu_1 + \alpha(\mu_1 - \mu_2)e^{-(1/\alpha)(t_2-t_1)} + \alpha\mu_0 > 0$) and then it gradually declines (rises) towards its new steady state level. It follows from (13) that seigniorage is an increasing function of time for $t_1 \leq t < t_2$. It jumps twice at t_1 and t_2 . The last jump is unambiguously downward: $\Delta S(t=t_2) = (\mu_2 - \mu_1)e^{-\alpha\mu_2} < 0$. The first jump at time t_1 may be either downward or upward. The ratio of seigniorage at time t_1 to the initial steady state level of seigniorage is $\delta_1 = (\mu_1/\mu_0)e^{-\alpha(\mu_1-\mu_0)}e^{-\alpha(\mu_1-\mu_2)e^{-(1/\alpha)(t_2-t_1)}}$. The value of δ_1 may be either smaller or greater than unity. But even if the seigniorage experiences a downward jump at time t_1 , during the transitional period $[t_1, t_2]$ it *may* become higher than its initial level. Possible scenarios are depicted by different lines in Fig. 1.

The value of δ_1 is a function of underlying parameters: $\delta_1 = \delta_1\left(\alpha, t_2, \mu_1, \mu_2\right)$.

The intuition is rather simple. (i) The higher the semi-elasticity of money demand is (which represents agent's attitude to inflation), the higher the gain in seigniorage revenue during the time interval when agents expect a further tightening of monetary policy. (ii) Given the date t_1 , date t_2 determines the length of the transitional period. A prolonged transition results in a smaller initial adjustment to the news and more gradual transitional dynamics. (iii) Seigniorage is determined as a product of the growth rate of base money and real money balances that are driven by expectations about future monetary policy. Thus, given initial and final rates of growth, μ_0 and μ_2 , the closer μ_1 is to μ_0 , the higher the initial gain in seigniorage. (iv) Given μ_0 and μ_1 , the higher μ_2 is (the smaller the scope of tightening of monetary policy is), the smaller the gain in transitional seigniorage.

The fact that the direction of transitory dynamics of seigniorage differs from the direction of change in its steady state level has important implication. As long as the *present discounted value* of future seigniorage is important for the sustainability of public debt, a temporal increase in seigniorage above its initial steady state during the transitional period *may* lead to a higher (or at least the same) present discounted value. This is more likely if the interest rate is relatively high, and thus a future decrease in steady state seigniorage is heavily discounted. Putting all said together, this result implies that a gradual (two-step) decrease in the growth rate of base money *in some cases* may be consistent with the sustainability of public debt, and thus tight monetary policy conducted in this way *may* be credible.

A permanent anticipated decrease in the growth rate of base money

Tight monetary policy is credible in the setup of the two-stage reduction in μ , if the interest rate is high enough and additional constraints on semi-elasticity and time intervals are met. Now we will show that when a permanent decrease in μ is

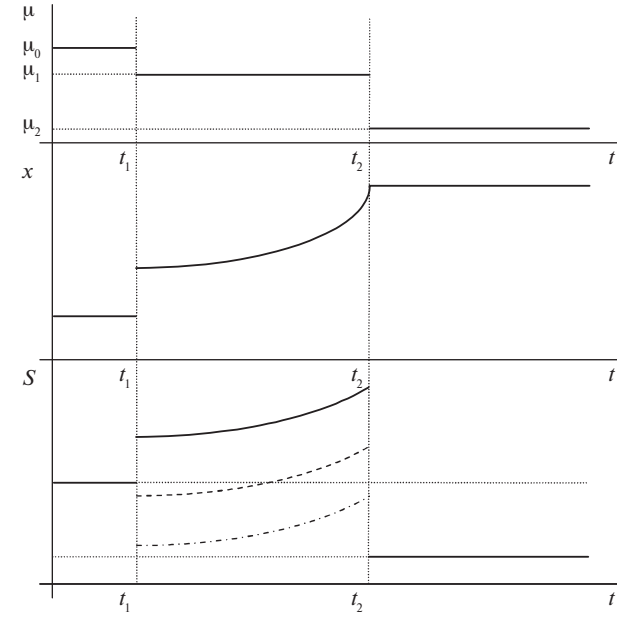


Fig. 1. Time path of the real money balances and seigniorage in the case of an unexpected two-step decrease in the growth rate of base money

preannounced, seigniorage is always higher than its initial steady state level and rises during the transition dynamics (though its present discounted value depends on various parameters).

Let us consider the following simple example. Starting with a constant growth rate of base money, $\mu(t) = \mu_0$, at time t_A the central bank announces that in the future, starting from $t_S > t_A$, the growth rate of base money will be increased to $\mu(t) = \mu_1 > \mu_0$. Using (7)–(9), we can describe the dynamics of inflation, the log of real money balances, and seigniorage:

$$\pi(t) = \begin{cases} \mu_0, & t < t_A, \\ \mu_0 - (\mu_0 - \mu_1)e^{-\frac{1}{\alpha}(t_S-t)}, & t_A \leq t < t_S, \\ \mu_1, & t \geq t_S. \end{cases} \quad (14)$$

$$x(t) = \begin{cases} -\alpha\mu_0, & t < t_A, \\ -\alpha\mu_0 + \alpha(\mu_0 - \mu_1)e^{-\frac{1}{\alpha}(t_S - t)}, & t_A \leq t < t_S, \\ -\alpha\mu_1, & t \geq t_S. \end{cases} \quad (15)$$

$$S(t) = \begin{cases} \mu_0 e^{-\alpha\mu_0}, & t < t_A, \\ \mu_0 e^{-\alpha\mu_0 + \alpha(\mu_0 - \mu_1)e^{-\frac{1}{\alpha}(t_S - t)}}, & t_A \leq t < t_S, \\ \mu_1 e^{-\alpha\mu_1}, & t \geq t_S. \end{cases} \quad (16)$$

Fig. 2 shows the time paths of these variables. Prior to the announcement, the money market is in a steady state. The announcement at time $t = t_A$ leads to discrete jumps in the log of real money balances, $\Delta x(t = t_A) = -\alpha(\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)} > 0$, and in inflation, $\Delta\pi(t = t_A) = (\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)} < 0$. Also, the decrease in the growth rate of base money initially results in a discrete increase in seigniorage: $\Delta S(t = t_A) = -\mu_0 e^{-\alpha\mu_0} \left[1 - e^{-\alpha(\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)}} \right] > 0$. The ratio of seigniorage at time t_A to seigniorage prior to the announcement of a policy switch is $\delta_2 = e^{-\alpha(\mu_1 - \mu_0)e^{-(1/\alpha)(t_S - t_A)}} = \delta_2 \left(\alpha, t_S, \mu_1 \right) > 1$. The intuition behind the role of the underlying parameters is the same as in the previous case¹⁶.

Up to time t_S , when monetary policy switches, the inflation rate and the log of real money balances gradually adjust to their new steady levels (decreasing and increasing, respectively). Seigniorage gradually increases on the interval $[t_A, t_S)$ and undergoes another discrete jump, $\Delta S(t = t_S) = (\mu_1 - \mu_0)e^{-\alpha\mu_1} < 0$, at time t_S . Assuming that the economy is on the efficient side of the inflation tax Laffer curve, the new steady state value of seigniorage is lower than it was initially. However, this does not unambiguously lead to a decrease in its present discounted value at time t_A . If the time interval $[t_A, t_S)$ is long enough, if the increase in real money balances is large, and (most importantly) if the interest rate is high, then the present value of future seigniorage revenues *may* increase. Thus, as in the previous case, tight monetary policy *may* be credible.

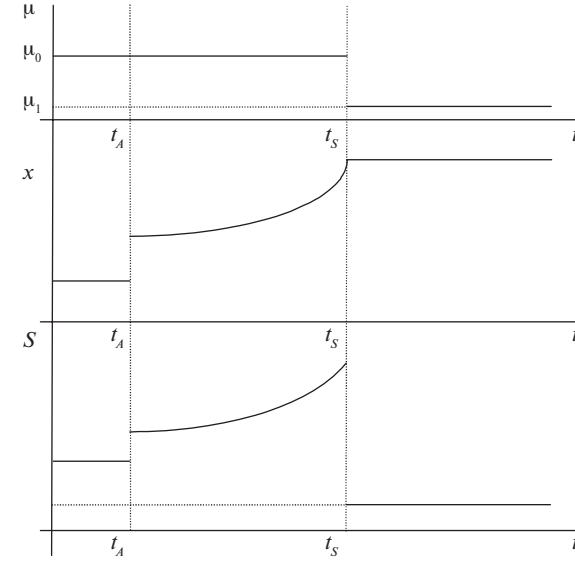


Fig. 2. Time path of the real money balances and seigniorage in the case of permanent anticipated decrease in the growth rate of base money

4. Concluding remarks

In the celebrated paper “Unpleasant monetarist arithmetic” Sargent and Wallace (1981) showed that tight monetary policy is not feasible if it is not supported by appropriate fiscal adjustment. In this paper we explore a simple forward-looking monetary model that does not assume any substantial departures from the SW’s original framework. Steady state analysis (that is sufficient in the case of an unexpected, permanent decrease in the growth rate of base money) unambiguously shows that UMA is the only possible outcome. However, when a tightening of monetary policy is conducted gradually or with a preannouncement, it generates transitional dynamics and “pleasant arithmetic” is possible. This is due to the short-run gain in seigniorage revenues during the transitional dynamics. The possibility of this gain and its value depend crucially on the semi-elasticity of money demand and the parameters of the theoretical experiment. By applying the principle of sustainable macroeconomic policy that constrains the future choices of both the fiscal and monetary authorities, we show that high interest rates on public debt work to

¹⁶ Actually, the permanent preannounced change in μ can be viewed as a special case of the two-step decrease in μ when $t_1 = t_A$ and $\mu_1 = \mu_0$.

the benefit of “pleasant arithmetic” simply because it makes a long-run decrease in the seigniorage revenue less important than the associated short-run gain.

There are two remarks on the implications of this result for the interaction of fiscal and monetary policies and on the simplifying assumptions of the model that was employed. First, under the assumption of fiscal dominance, we may consider what monetary authorities should do to accommodate a permanent increase in fiscal deficit¹⁷. For the public debt to remain sustainable, the fiscal expansion must be accompanied by an increase in the present discounted value of seigniorage that may be associated with different paths of transitional dynamics. Assume for concreteness the setup of anticipated permanent shifts in monetary policy. It follows from the analysis in the previous section that if the interest rate is high enough, then fiscal expansion can be accompanied by a tight monetary policy that benefits from a transitional short-run gain in seigniorage. However, when the interest rate is relatively low, the short-run gain in seigniorage may be outweighed by the long-run loss in seigniorage resulting from the lower growth rate of base money. In this case monetary policy needs to be loose. Indeed it is easy to show that when there is a preannounced permanent increase in the growth rate of base money seigniorage experiences an initial decline, while its new steady state value is higher. A low interest rate makes the short-run loss in seigniorage less important than its long-run increase and public debt remains sustainable. It is possible to apply the same logic to the case of monetary dominance. For example, a preannounced permanent tightening of monetary policy should be supplemented by a reduction in future budget deficits if the interest rate is relatively low. However, it gives fiscal authorities an opportunity to increase its spending or decrease taxes if interest rate is high enough. In sum, low and high interest rates determine different regimes of interaction between fiscal and monetary policies.

Second, a constant real interest rate is not always a realistic assumption to study the effects of monetary policy. Assume that tight monetary policy leads to higher interest rates through conventional transmission mechanisms or through an increase in the risk premium associated with the growing public debt as in the Drazen and Helpman (1990) model. As we have seen, high interest rates actually make “pleasant arithmetic” of the tight monetary policy the more plausible outcome. This is in a sharp contrast with Miller and Sargent’s (1984) arguments in the defense of UMA.

¹⁷ The ability of monetary policy to accommodate a large fiscal expansion seems to be limited. However, this problem lies out of the scope of this paper.

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