Dynamically Optimal Executive Compensation when Reservation Utilities Are History-Dependent[1]

Stanimir Morfov[2]

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Abstract:

This paper computes the optimal executive compensation in an infinite-horizon moral hazard framework characterized by limited commitment and history-dependent reservation utilities. The model is given a recursive form and some properties of the state space are established. I derive a sufficient condition for the optimal contract to provide the CEO with insurance against fluctuations in the value of his/her outside options under short-term history dependence. In the numerical computation of the endogenous state space, I use an innovative algorithm which does not rely on the convexity of the underlying set. Exerting effort appears to be the predominant strategy for the principal, but shirking may still be optimal when the manager is rich enough. The optimal wage scheme and the future utility of the CEO tend to grow in both his/her current utility and in the firm's future profit. The manager's utility tends to increase weakly in the long run and appears to have a non-degenerate long-term distribution depending on the initial utility promise but not on the initial history.

Keywords: principal-agent problem, moral hazard, dynamic contracts, limited commitment, executive compensation

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[2] State University - Higher School of Economics, International College of Economics and Finance, Pokrovski bulvar 11, Moscow 109028, Russian Federation, smorfov@hse.ru

Динамически оптимальное вознаграждение топ-менеджеров, когда гарантированные полезности зависят от предыстории

(на английском языке)

Морфов Станимир

Работа принята к публикации в серии научных докладов ГУ-ВШЭ “Исследования по экономике и финансам” (WP9) в мае 2010 г.

Данное исследование вычисляет оптимальную заработную плату топ-менеджеров в модели морального риска, охарактеризованной ограничениями исполнения обязательств и зависимыми от предыстории гарантированными полезностями. Модель представлена в рекурсивной форме, и установлены некоторые свойства пространства состояний. Выведено достаточное условие, чтобы оптимальный контракт предоставлял топ-менеджеру страховку от колебаний в стоимости его/ее внешних вариантов при краткосрочной зависимости от предыстории. При вычислении эндогенного пространства состояний был использован инновационный алгоритм, который не использует свойство выпуклости базового множества. Проявление усилия является преобладающей стратегией для принципала, но уклонение может все еще быть оптимальным, когда менеджер достаточно богатый. Оптимальная схема заработной платы и будущая полезность топ-менеджера стремятся расти как в его/ее текущей полезности, так и в будущей прибыли фирмы. Полезность менеджера имеет тенденцию к росту и обладает невырожденным долгосрочным распределением, которое зависит от начальной полезности, но не зависит от начальной истории.

1 Introduction

Executive pay is a topic that has continuously interested media and academia alike. Since Jensen and Murphy (1990)'s seminal paper, there has been a big debate about the effectiveness of the observed compensation schemes in inducing the proper incentives while providing insurance to risk-averse managers. Empirical surveys and recipes abound.[3] The most important question, however, is how the optimal compensation scheme should actually look like. Estimating the top-management pay is not a trivial. task. It relates to the vast literature on dynamic contracts pioneered by Green (1987) and Spear and Srivastava (1987). In a dynamic model of adverse selection, Thomas and Worrall (1990) demonstrated that a legally enforceable contract would have the borrower's utility converging to minus infinity with probability one. Phelan (1995) showed that in a dynamic insurance setting characterized by one-sided commitment, there exists a non-degenerate long-run distribution of consumption. While the agency literature has mainly focused on deriving contracts inducing optimal effort, the participation constraints have largely been ignored. Some notable exceptions are Sleet and Yeltekin (2001) and Spear and Wang (2005) who concentrate on contract terminations and Cao and Wang (2008) who endogenize agent's reservation utility.

In the current paper, I compute the dynamically optimal executive compensation. Since I am interested in the long term dynamics of the contract and the resulting wealth distribution, I focus on long-term self-enforcing schemes that are incentive compatible. The setting is an infinite-horizon moral hazard problem characterized by limited commitment and history-dependent reservation utilities. Each period, the firm's shareholders (treated as a risk-neutral principal) and the CEO (a risk-averse agent) sign a contract which specifies a recommended level of effort to be exercised by the agent this period and the compensation the agent will receive in the end of the period. The effort exerted by the agent is not observed by the principal and influences the firm's (gross) profit in a non-deterministic fashion. Therefore, the compensation of the agent cannot be based on the specific level of effort exercised. However, it can be made contingent on firm's realized profit. More generally, since the firm's profit is publicly observable and no amnesia is introduced in the model, the contract

[3] See Murphy (1999) and Jensen and Murphy (2004) for a review.

can be based on the whole history of profit realizations and the compensation can additionally be made contingent on the profit to be realized in the end of the period. Moreover, the contract should provide the proper incentives to the manager in order for him/her to exert exactly the level of effort recommended by the firm's shareholders. Limited commitment is assumed on both parts in the sense that both the shareholders and the CEO can commit only to short-term (single-period) contracts. This assumption is intended to reflect legal issues on the enforcement of long-term contracts. However, at the initial period the shareholders can offer a long-term contract (a supercontract) that neither they, nor the manager would like to renege on, and that would provide the necessary incentives for the manager to exercise the sequence of effort levels suggested by the principal.

Wang (1997) and Aseff (2004) use a similar framework in order to analyze the optimal contract. The former, however, does not rigorously analyze the effects of limited commitment on both parts of the relationship while the latter restrictively pre-supposes the optimality of high effort and effectively estimates the optimal compensation scheme that induces it.

Furthermore, in my treatment of limited commitment, I allow for correlation between the reservation utilities of the agent and the principal and the (finitely truncated) history of profits. This extension directly affects the set of possible endogenous utilities, but also permits the analysis of some interesting dynamic effects. For example, if the outside offer for the manager is positively correlated with current profit (due to, say, a belief on part of the outside employers that the firm's performance reveals information about the quality/type of the manager), we may expect that he/she would be motivated to increase the probability of high profits in the future (by choosing a higher level of effort). At the same time, the risk-averse managers would like to smooth consumption across states, which may require that their participation constraint does not bind for lower profit realizations. Moreover, it may become increasingly more difficult to motivate richer CEOs, especially when the shareholders face some borrowing constraints, which may lead to the suboptimality of inducing high effort for such CEOs.

The current paper is the first to look at how shocks on the reservation utilities may affect the parties to a dynamic contractual relationship. In particular, we investigate whether the optimal contract insures the manager against variability in the value of his/her outside options. We build up the intuition behind the possible effect of such an insurance on the manager's utility in the short and the long run and relate it to the properties of the limiting distribution.

The framework falls into the scope of Morfov (2010); therefore, an optimal contract exists and the problem can be characterized recursively and addressed by dynamic programming techniques.

The estimation is conducted in three steps. First, the state space of an auxiliary problem that does not require the participation of the principal but binds the wage from above is recovered as the limit of a generalized Bellman equation. Second, the aforementioned auxiliary problem is solved by a standard recursive procedure. Third, the optimal recursive contract and its state space are recovered by severely punishing the principal for each violation of his/her participation constraint.

In order to estimate the model, I parameterize it following the calibration of Aseff (2004) and Aseff and Santos (2005) based on the results of Hall and Liebman (1998) and Margiotta and Miller (2000).

Regarding the numerical computation, one point deserves special attention. In computing the endogenous state space we are iterating on sets and therefore need to represent them efficiently. For the class of infinitely repeated games with perfect monitoring, Judd, Yeltekin and Conklin (2003) are able to construct inner and outer convex polytope approximations based on the convexification of the equilibrium value set through a public randomization device. The algorithm I use may be of independent interest since it does not rely on the convexity of the underlying set. The main idea is to discretize the guess for the equilibrium set elementwise, extract small open balls around the gridpoints unfeasible with respect to the (non-updated) guess and use the remaining set, i.e., the guess less the extracted intervals, as a new guess for the equilibrium set. The procedure stops if the structure of the representations of two successive guesses coincides[4] and the suitably defined difference between the representations is less than some prespecified tolerance level.

I derive the state space under constant reservation utilities. Then, I consider a single-period history dependence and show theoretically that if the manager's reservation utilities are sufficiently dispersed, his/her participation constraint does not bind under the worst case scenario, which is also observed when the manager can essentially commit when his/her outside option is at its lowest value. In other words, the minimum utility the CEO can be promised for initial histories characterized by lower reservation utility is generally boosted by higher reservation utilities for other states. Alternatively put, the optimal contract pro-

[4]Namely, if the representations have the same number of closed sets element by element.

vides the CEO with some insurance against fluctuations in the value of his/her outside options, which ultimately smooths his/her consumption across (initial history) states. In case of positive correlation between firm's profits and manager's reservation utilities, this translates into the participation constraint of the manager being non-binding in states characterized by low profits. Computing the model actually shows that utility promises close to the reservation level are possible only under the manager's best-case scenarios when his/her reservation utility is the highest (i.e., when the highest profit has been observed).

The numerical results suggest that with a loose upper bound on wages, the optimal contract can support extremely high values for the expected discounted utility of the CEO when the participation of the principal is not guaranteed. However, when solving for the self-enforcing contract, these values naturally disappear since they violate principal's participation constraint. Exerting effort appears to be the predominant strategy for the principal, but shirking may still be optimal when the agent is rich enough. The optimal wage scheme and the future utility of the manager tend to grow in both current utility and future profit. Intuitively, both current and future compensation are used to induce poor and mid-range managers to work hard, while rich managers prove too difficult to motivate. The latter shirk and while they may face some fluctuations in their current income stream in case of binding credit constraints on part of the firm, their lifetime utility remains relatively flat.

Simulations suggest that CEO's utility weakly increases in the long run. In particular, agents who start rich tend to keep their utility level while those who start poor get richer in time. The increase is most pronounced for managers with initial utilities below the highest reservation utility. These managers first have their utilities pushed well above their reservation level. Then, the principal motivates them to work hard by rewarding success through continuation utilities while providing insurance through flatter wages. In this way, the probability of success and, therefore, of a higher reservation utility tomorrow increases which rises the manager's expected continuation utility. The long term distribution of manager's utility is non-degenerate and depends on the initial utility promise but not directly on the relevant initial history at least as far as short initial histories are concerned.

The rest of the paper is structured as follows. Section 2 presents the model in a general and recursive form. Section 3 explains the numerical algorithm at a practical level and discusses the results. Section 4 concludes. Appendix 1 contains all the proofs. Appendix 2 presents the results.

2 Model

The setting describes a dynamic interaction between the shareholders of a corporation and its chief executive officer (CEO). The shareholders are exclusively interested in the profit realized by the corporation. They need the CEO to run the company but cannot observe the level of effort he/she exerts on the job. If offered a fixed compensation, the manager will naturally prefer to shirk rather than work hard, so such a scheme would have no incentive impact whatsoever. On the other hand, since the shareholders know the distribution of firm's (gross) profits conditional on executive's effort, they can offer a wage scheme contingent on the future profit realization in order to invoke the manager to adhere to a certain type of behavior. While the shareholders would prefer the manager to work hard every period, it may be costly to induce such a behavior. The CEO who is risk averse in the money he/she receives, would require a higher average remuneration in order to compensate him/her for the increased volatility of his/her current income. Since the manager is free to walk out of the relationship, incentive compatibility may go against individual rationality, namely, it may become difficult to induce the CEO to work hard and keep him/her in the company. The situation may further be complicated by the shareholders' own limited commitment. While it is very interesting to see how the optimal contractual agreement would look like in terms of incentives, insurance, compensation, induced behavior, and wealth distribution, characterizing it may prove quite involved given the parties' inability to commit and the realistic possibility that the manager's outside job offers/opportunities may vary with firm's realized profit (different types of agents whose ability may be considered related to firm's performance by outside potential employers; different economic environments: harder to find a job in a through than in a boom, etc.)[5]. Albeit the technical difficulties, analyzing this problem increases our understanding of the mechanics of incentive compatibility and self-enforcement in a dynamic setting. Would the manager require some form of insurance against fluctuations in the value of his/her outside options? How would this affect the CEO's utility in the short and the long run? Would shocks to reservation utilities have an impact on the long term distribution of executive's wealth? All these questions fall

[5] For example, in order to address the wide use of broad-based stock option plans, Oyer (2004) builds a simple two-period model where adjusting compensation is costly and employee's outside opportunities are correlated with the firm's performance.

into the scope of the current paper which brings more structure to the model presented in Morfov (2010), establishes some interesting properties of the state space, computes the model numerically and provides intuition for the results.

Let us formally introduce the environment. Time is discrete and the set of firm's possible profits, Y, is a time- and history-invariant set of $n > 1$ distinct real numbers. For concreteness, we will index the set of possible profits of length $\theta \geq 0$, Y^{θ}, by $L := \{1, ..., n^{\theta}\}$. Hereafter, we will refer to a particular element of Y^{θ} as an initial history and will frequently denote it by its index $l \in L$.[6] Moreover, all functions and correspondences with domain Y^{θ} will be considered as vectors or Cartesian products of sets indexed by L. At the beginning of period 0, the firm's shareholders and the manager sign an incentive-compatible, self-enforcing supercontract. The wage received by the CEO has a uniform lower bound $\underline{w}$ which can be considered a minimum wage level. The level of effort exerted by the manager belongs to the compact, time- and history-invariant set A. Additionally, we make the following assumptions.

Assumption 1 *The profit realization at any period of time depends only on the effort exerted by the CEO in the beginning of the same period and is characterized by the probability function* $\pi(., a) : Y \rightarrow (0, 1)$, $\forall a \in A$, *where* $\pi(y, .)$ *is continuous on* A *for any* $y \in Y$.

Assumption 2 *The shareholders of the corporation are proxied by a "principal" with period utility* $y - w$ *for any (gross) profit realization* y *and wage* w. *They discount the future by a factor* $\beta_P \in (0, 1)$.

Assumption 3 *The CEO's period-utility is specified as* $\upsilon(w) - a$ *for any wage* w *and level of effort* a, *where* $\upsilon(.)$ *is assumed continuous, strictly increasing and concave.*[7] *He/she discounts the future by a factor* $\beta_A \in (0, 1)$.

[6] Occasionally, we will treat l as a bijective function mapping Y^{θ} to L.

[7] This specification actually requires that the manager should consume his/her exact wage at each contingency thus preventing him/her from smoothing his/her consumption stream through borrowing and/or saving. Ceteris paribus, the principal will find it cheaper to motivate the CEO. Note, however, that in our framework problems with commitment are likely to have an adverse effect on the provision of incentives, so by ignoring possible readjustments in the

Assumption 4 *At any time t, the reservation utilities map Y^{θ} to $\mathbb{R}$. Given a particular initial history l observed at the beginning of period t, the reservation utilities of the principal and the CEO are respectively $\underline{U}_l$ and $\underline{V}_l$ (denoted as $\underline{U}$ and $\underline{V}$ if $\theta = 0$).*

Given a profit history $y^{t-1} \in l \times Y^t$ observed in the beginning of period $t \geq 0$, and an admissible supercontract $c = (a, w)$ signed at node l at the beginning of period 0,[8] define the expected discounted utility of the principal at node y^{t-1} as $U_t\left(c, y^{t-1}\right)$. Analogously, define $V_t\left(c, y^{t-1}\right)$ as the expected discounted utility of the manager at that node. The supercontract specifies a recommended level of effort and a contingent compensation scheme on all possible contingencies after signing. The admissibility of the contract refers to the effort belonging to A and the wage being greater or equal to its minimum level $\underline{w}$ at any contingency (after signing).

Then, at period 0 at node l, the principal will be solving the following problem:

[PPx]

$\sup_{c} U_0\left(c, l\right)$ s.t.:

$$c \text{ admissible} \tag{1}$$

$$V_t\left(c, y^{t-1}\right) \geq V_t\left(c', y^{t-1}\right), \forall c' = (a', w) \text{ admissible}, \forall y^{t-1}, \forall t \tag{2}$$

$$V_t\left(c, ., \widetilde{l}\right) \geq \underline{V}_{\widetilde{l}}, \forall t, \forall \widetilde{l} \in L \tag{3}$$

$$U_t\left(c, ., \widetilde{l}\right) \geq \underline{U}_{\widetilde{l}}, \forall t, \forall \widetilde{l} \in L \tag{4}$$

manager's consumption stream, we will be able to study the role of limited commitment in isolation. On a practical level, without imposing a very strong set of assumptions on the primitives of the model in order to justify the use of the first-order approach, allowing the agent to save will significantly complicate the numerical estimation of the model.

[8]Note that the history y^{t-1} consists of θ initial outcomes observed before period 0 and t profit realizations from time 0 to time $t-1$.

where (1) is an admissibility constraint, (2) requires that the recommended plan of effort levels is incentive compatible at every node, while (3) and (4) are participation constraints for the manager and respectively the principal which are required to hold at any node after (and including) l.[9]

Having defined the problem, we will assume that the set of constraints forms a non-empty set.[10]

Assumption 5 *$\forall l \in L$, $\{c : (1)–(4)$ hold after $l\} \neq \emptyset$.*

Proposition 1 *Let (1) and (4) hold after some l. Then at any node after (and including) l we have $w_t(.) \leq \overline{w}$, where $\overline{w} := \underline{w} + \frac{1}{\underline{\pi}}\left(\frac{\overline{y}-\underline{w}}{1-\beta_P} - \underline{\underline{U}}\right)$ with $\underline{\pi} := \min_{(y,a)\in Y\times A} \pi(y,a)$, $\overline{y} := \max Y$, and $\underline{\underline{U}} := \min_{\widetilde{l}\in L} \underline{U}_l$.*

The proposition says that an admissible contract that guarantees the commitment of the principal effectively binds the wage from above. Note that the upper bound $\overline{w}$ does not depend on the initial history l. Therefore, for any contract in the constrained set of the problem [PPx], we have that $w_t(.) \in W := [\underline{w}, \overline{w}]$ which is a compact subset of $\mathbb{R}$. Consequently, all the results of Morfov (2010) are valid for such a contract. In particular, there exists an equivalent recursive representation of [PPx] which is stationary upon a properly defined state space. A brief outline of the characterization follows.

Let AP denote an admissible, incentive-compatible supercontract that only guarantees the participation of the agent, while 2P stays for an admissible, incentive-compatible supercontract that guarantees the participation of both parties. Denote by $V^{AP}(l)$ the set of expected discounted utilities for the manager signing an AP contract at l with $\overline{w}$ imposed as a uniform upper bound

[9] In the current paper, the environment, the principal's problem and the recursive form are only presented schematically. For a more detailed and motivated exposition, refer to the more general framework of Morfov (2010).

[10] This assumption is the equivalent of Assumption 3 in Morfov (2010) [for details, see the comments in Footnote 14 in the aforementioned paper].

for the wage.[11] Let $V^{AP} := \left\{V^{AP}(l)\right\}$ be the Cartesian product of such sets indexed by L. Let V^{2P} be the corresponding product of sets of expected discounted utilities for the CEO signing a 2P contract. For any $\forall V = \{V_l\} \in V^{AP}$, define $U^{AP^*}(V)$ as a vector with a general element $U^{AP^*}(V_l, l)$ that stays for the maximum utility the principal can get by signing an AP contract offering V_l to the manager. Respectively, for any $V \in V^{2P}$, $U^*(V)$ is a vector with a general element $U^*(V_l, l)$ that denotes the maximum utility the principal can get by signing a 2P supercontract offering V_l to the manager. Let $\widehat{U}^*$ be the extension of U^* on V^{AP} s.t. for any $V \in V^{AP}$, $\widehat{U}^*(V)$ is a vector with a general element

$$\widehat{U}^*(V_l, l) = \begin{cases} U^*(V_l, l) \text{ if } V_l \in V^{2P}(l) \\ -\infty \text{ otherwise} \end{cases}$$

Let $l_+ : L \times Y \to L$ map today's initial histories and current profit realizations to tomorrow's initial histories. Finally, three important operators are defined.

For any $X = \{X_l\} \in \mathbb{R}^{n^\theta}$, $\widetilde{B}(X) = \left\{\widetilde{B}_l(X)\right\}$ with $\widetilde{B}_l(X) := \{V \in X_l : \exists$ a (single-round) contract $c_R(V, l) = \{a_-, w_+(y), V_+(y)\}_{y \in Y}$ s.t.:

$$a_- \in A \tag{5}$$

$$w_+(y) \in W, \forall y \in Y \tag{6}$$

$$\sum_{y \in Y} [\upsilon(w_+(y)) - a'_- + \beta_A V_+(y)] \pi\left(y, a'_-\right) \leq V, \forall a'_- \in A \tag{7}$$

$$\sum_{y \in Y} [\upsilon(w_+(y)) - a_- + \beta_A V_+(y)] \pi(y, a_-) = V \tag{8}$$

[11] Note that the "true" AP contract does not require that $w_t(.) \leq \overline{w}$. This condition comes from the participation constraints of the principal which only hold for the 2P contract. Therefore, we will actually characterize the AP contract that allows for wages not higher than $\overline{w}$. Imposing this additional condition to the AP contract, however, has no impact on the 2P contract since by Proposition 1, the original problem [PPx] is equivalent to one where wages are bounded from above by $\overline{w}$. Also note that working with explicit bounds for the wage will be an advantage in the forthcoming numerical computation.

$$V_+(y) \in X_{l_+(l,y)}, \forall y \in Y \tag{9}$$

hold}.

For any $U = \{U_l\}$ with $U_l : V^{AP}(l) \to \mathbb{R}$ upper semi-continuous (usc) and bounded with respect to the sup metric, and any $V \in \{V_l\} \in V^{AP}$, $T(U)_{(V)}$ is a vector with a general element defined as follows:

$$T_l(U)_{(V_l)} := \max_{c_R} \sum_{y \in Y} [y - w_+(y) + \beta_P U_{l_+(l,y)}(V_+(y))] \pi(y, a_-) \text{ s.t.:}$$

$$(5) - (8) \text{ hold, and}$$

$$V_+(y) \in V^{AP}(l_+(l,y)), \ \forall y \in Y \tag{10}$$

For any $l \in L$ and $V_l \in V^{AP}(l)$, let $\Gamma_R(V_l, U, l) := \{c_R : (5)-(8), (10)$ hold at (V_l, l) and $U_{l_+(l,y)}(V_+(y)) \geq \underline{U}_{l_+(l,y)}, \forall y \in Y\}$ for some function $U : V^{AP} \to (\mathbb{R} \cup \{-\infty\})^{n^\theta}$. Additionally, let

$$\Lambda_R(V_l, U, l) := \begin{cases} \Gamma_R(V_l, U, l) \text{ if } U_l(V_l) \geq \underline{U}_l \\ \emptyset \text{ otherwise} \end{cases}$$

For any $U = \{U_l\}$ with $U_l : V^{AP}(l) \to \mathbb{R} \cup \{-\infty\}$ usc and bounded from above, and any $V \in \{V_l\} \in V^{AP}$, $\underline{T}(U)_{(V)}$ is a vector with a general element:

$$\underline{T}_l(U)_{(V_l)} := \begin{cases} -\infty \text{ if } \Lambda_R(V_l, U, l) = \emptyset \\ \max\limits_{\substack{c_R \in \\ \Lambda_R(V_l, U, l)}} \sum\limits_{y \in Y} [y - w_+(y) + \beta_P U_{l_+(l,y)}(V_+(y))] \pi(y, a_-) \text{ otherwise} \end{cases}$$

Following the results of Morfov (2010), the optimal 2P contract is recursively characterized in three steps[12]:

[12] Step 1 generalizes on Abreu, Pearce and Stacchetti (1990). Step 2 is standard dynamic programming over upper semi-continuous, bounded functions. Step 3 is based on Rustichini (1998).

Step 1. Start with the set $\widetilde{X}_0 := \left\{\left[\underline{V}_l, \widehat{V}\right]\right\}$ where $\widehat{V} = \frac{v(\overline{w})-\underline{a}}{1-\beta_A}$ with $\underline{a} := \min\{A\}$ and iterate on the set operator $\widetilde{B}$ until convergence. The limit is V^{AP}.

Step 2. Take a function $U = \{U_l\}$ with $U_l : V^{AP}(l) \to \mathbb{R}$ usc and bounded with respect to the sup metric, $\forall l \in L$. Iterate on $T(.)$ until convergence. The limit is $U^{AP^*}(.)$.

Step 3. Take $U^{AP^*}(.)$ as an initial guess and iterate on $\underline{T}(.)$ until convergence. The limit is $\widehat{U}^*(.)$. Moreover, $V^{2P}(l) = \{V \in V^{AP}(l) : \widehat{U}^*(V,l) \geq \underline{U}_l\}$. Then, for any $V \in V^{2P}(l)$, we have $U^*(V,l) = \widehat{U}^*(V,l)$.

Although we cannot solve the model analytically, we have constructed an equivalent recursive representation that can be addressed by numerical techniques in a three-step procedure as outlined above. Now, we are ready to parameterize the model and compute the optimal solutions. Before that, I will provide some intuition for the results to follow.

Proposition 2 *If $\theta = 0$, we have* $V^{AP} = \left[\max\left\{\underline{V}, \frac{v(\underline{w})-\underline{a}}{1-\beta_A}\right\}, \widehat{V}\right]$.

This proposition derives the state space of the optimal AP contract when manager's reservation utility is constant across profit histories. V^{AP} is an interval and its lower limit is either the CEO's reservation utility or his/her discounted utility under a supercontract paying the minimum wage and inducing the lowest level of effort at every single node whichever is bigger. Indeed, when in the computation, I consider $\underline{V} = \frac{v(\underline{w})-\overline{a}}{1-\beta_A}$, where $\overline{a} := \max A$, the lowest possible utility supportable by an AP contract is exactly $\frac{v(\underline{w})-\underline{a}}{1-\beta_A}$ (cf. Table 2 in Appendix 2, LLL). The other possible values for $\underline{V}$ are chosen to be greater than $\frac{v(\underline{w})-\overline{a}}{1-\beta_A}$, so they immediately become the lower limit of the respective state spaces (cf. Table 2 in Appendix 2, MMM and HHH). Regarding the upper limit of the interval, it is given by $\frac{v(\overline{w})-\underline{a}}{1-\beta_A}$, i.e., the discounted utility of the manager under a contract that pays him/her the highest possible wage $\overline{w}$ and induces the lowest possible effort at every node. Note that $\overline{w}$ was obtained in Proposition 1 as a theoretical bound on wages under the AP contract that would not affect the subsequent derivation of the optimal 2P contract. In practice, we can improve on this bound using economic considerations (see next section). Proposition 2 will not be affected by any uniform bound on wages. We simply need to redefine $\overline{w}$. One economically interesting case, however, requires that wage does not exceed the future (gross) profit realization reflecting the inability of firm's shareholders in raising additional funds to support higher compensation values for the manager. In this case, the upper limit of the state space is, indeed, affected.

For the purposes of the next proposition, let $E_a(.)$ denote the mathematical expectation conditional on a current effort level a. For example, $E_{\underline{a}}(y) = \sum_{y \in Y} y\pi(y, \underline{a})$.

Proposition 3 *If* $\theta = 0$ *and* $w_t(., y) \leq y$, $\forall y \in Y$, *we have that* $\min V^{AP} = \max\left\{\underline{V}, \frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}\right\}$ *and* $\max V^{AP} = \frac{\max_{a\in A}\{E_a\upsilon(\min\{y,\overline{w}\})-a\}}{1-\beta_A}$. *Moreover, if* $\underline{a} \in \arg\max_{a\in A}\{E_a\upsilon(\min\{y, \overline{w}\}) - a\}$, *then* V^{AP} *is convex.*

This proposition establishes that when the shareholders are effectively prohibited from borrowing, the maximum of the state space of the AP contract is simply the expected discounted utility of the manager under a supercontract that maximizes his/her period utility across the set of admissible actions and wages. For example, if $A = \{\underline{a}, \overline{a}\}$, $\overline{y} \leq \overline{w}$ and $E_{\overline{a}}\upsilon(y) - E_{\underline{a}}\upsilon(y) < \overline{a} - \underline{a}$, then $V^{AP} = \left[\max\left\{\underline{V}, \frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}\right\}, \frac{E_{\underline{a}}\upsilon(y)-\underline{a}}{1-\beta_A}\right]$ (cf. Table 2 in Appendix 2, LLL, MMM, and HHH).

So far, we have established the limits of the state space V^{AP} for the case where the reservation utility of the manager remains constant across profit realizations. Since the focus of the paper is history dependent participation constraints, it would be interesting to see if we can say something about the case where the outside options vary with the history of observables. In what follows, I will concentrate on a one-period dependence.

Proposition 4 *Let* $\theta = 1$ *and* $\underline{V}_{\widehat{l}} = \min_{l\in L}\underline{V}_l$. *Then,* $\max V^{AP}(l) = \widehat{V}$, $\forall l \in L$. *Moreover, if* $\max_{a\in A}\{\beta_A E_a \underline{V} - a\} > \underline{V}_{\widehat{l}} - \upsilon(\underline{w})$, *then* $\min V^{AP}\left(\widehat{l}\right) > \underline{V}_{\widehat{l}}$; *otherwise,* $\min V^{AP}(l) = \underline{V}_l$, $\forall l \in L$.

Here, $E_a\underline{V}$ is the expected reservation utility of the agent tomorrow conditional on a current effort level a. Formally, $E_a\underline{V} = \sum_{y\in Y} \underline{V}_{l(y)}\pi(y,a)$.

Before commenting on the proposition, I will introduce some more structure. Let us order the elements of Y ascendingly and index them accordingly such that the lowest element corresponds to an index 1 and the highest to an index n. We also let the reservation utilities of the manager be positively correlated with the firm's realized profit. This last assumption is made solely for the purpose of illustration; it is not necessary for establishing the result of the proposition.

Notice that the manager's reservation utility, the minimum wage, and the probability distribution of the firm's profit conditional on manager's effort are all exogenous to the model. Therefore, the proposition relates the slackness of the manager's participation constraint to the values of the exogenous parameters. Indeed, it is just a restatement of the fact that if the firm's profit is at its lowest level today and a temporary incentive-compatible contract providing the manager with the minimum wage and a continuation utility equal to the reservation value at each contingency tomorrow guarantees him/her today a utility strictly higher than his/her outside option, then the manager's participation constraint under the optimal contract will not bind at initial state y_1. Consider, for example, the case of two possible actions, $\underline{a} < \overline{a}$. If $E_{\overline{a}}\underline{V} - E_{\underline{a}}\underline{V} > \frac{\overline{a}-\underline{a}}{\beta_A}$, then $\{\overline{a}, \underline{w}, \underline{V}_l\}_{l=1}^n$ is the temporary incentive-compatible contract that minimizes the manager's current level of utility. The worst-case scenario (from the point of view of the manager) is when the firm's profit is lowest since then his/her reservation utility is at its minimum level. If, in such a case, inducing high effort by promising the minimum salary and the respective reservation utility on any node tomorrow guarantees utility of at most the reservation level today, then manager's participation constraint binds under the optimal contract irrespective of the history of profits. If, however, the manager can only be promised a current utility higher than his/her reservation one, then his/her participation constraint will not bind and the shrinking of the set of possible continuation utilities from below will eventually lead to increasing the lower limit of the state space for low enough profits. Note that the shrinking of $\widetilde{B}_1\left(\widetilde{X}_0\right)$ may lead to shrinking in $\widetilde{B}_l^i\left(\widetilde{X}_0\right)$, $l = 2, ..., n-1$, $i = 2, ...$ even if $\underline{V}_l > \underline{V}_1$ and $\max_{a\in A}\{\beta_A E_a\underline{V} - a\} > \underline{V}_l - v(\underline{w})$. The reason is that raising $\min \widetilde{B}_1^i\left(\widetilde{X}_0\right)$

increases $E_a \min \widetilde{B}^i \left(\widetilde{X}_0\right)$ relative to $\min \widetilde{B}^i_l \left(\widetilde{X}_0\right) = \underline{V}_l$ for any $a \in A$.[13] If $E_{\overline{a}}\underline{V} - E_{\underline{a}}\underline{V} < \frac{\overline{a}-\underline{a}}{\beta_A}$, then letting the manager shirk by paying him/her the minimum wage and promising him/her the reservation utility at any continuation node is temporary incentive-compatible and minimizes the agent's current level of utility. The same analysis as before applies.

Proposition 4 indicates that, ceteris paribus, decreasing (increasing) the variance of the manager's reservation utility, his/her patience, utility of effort, or utility of consuming the legally-established minimum wage level will increase (decrease) the number of scenarios under which the manager's participation constraint would actually be binding. In the extreme case where the manager's reservation utility, $\underline{V}$, is constant across the history of observables (i.e., $\theta = 0$) and is (reasonably assumed) higher or equal to the lowest utility level supportable by an admissible incentive-compatible contract ignoring the issue of manager's commitment, $\frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}$, then the result of Proposition 4 reduces to $\min V^{AP} = \underline{V}$ as also implied by Proposition 2, i.e., the poorest (in initial expected discounted utility terms) manager is guaranteed exactly his/her reservation utility level under the optimal contract. What would happen, however, if the manager's reservation utility actually varies across the observed profit histories?

Corollary 1 *If $\theta = 1$, $\underline{V}_{\widehat{l}} = \min_{l \in L} \underline{V}_l \leq \frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}$ and $\exists l \in L : \underline{V}_l > \underline{V}_{\widehat{l}}$, then $\min V^{AP}\left(\widehat{l}\right) > \frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}$.*

The corollary says that if we have a (non-reducible)[14] one-period dependence (i.e., reservation utilities at the beginning of each period do vary only with the profit realized at the end of the previous period) and we consider a manager who can essentially commit in the worst-case scenario (i.e., $\underline{V}_{\widehat{l}} \leq \frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}$), there will be cases under the optimal contract where the manager would receive utility strictly higher than the respective value of his/her outside option.

For higher values of $\underline{V}_{\widehat{l}}$, whether participation will bind or not depends on the specific parameter values. Nevertheless, if the manager's reservation utilities are

[13] Notice that $E_a \min \widetilde{B}^i \left(\widetilde{X}_0\right) = \sum_{y \in Y} \min \left(\widetilde{B}^i_{l(y)} \left(\widetilde{X}_0\right)\right) \pi(y, a)$.

[14] Note that assuming $\theta = 1$ and $\underline{V}_l = \underline{V}$, $\forall l \in L$, is equivalent (or, alternatively put, is reducible) to $\theta = 0$ with a manager's reservation utility of $\underline{V}$.

not bunched on a very tiny interval, we would expect some gain above reservation utility levels for the least wealthy of the managers with worse performance records.[15]

To summarize, if the manager's reservation utilities are sufficiently dispersed, his/her participation constraint will not bind under the worst case scenario, which is also observed if the manager can essentially commit when his/her outside option is at its lowest value. In other words, the minimum utility the CEO can be promised for initial histories characterized by lower reservation utility is generally boosted by higher reservation utilities for other states. Alternatively put, the optimal contract provides the CEO with some insurance against fluctuations in the value of his/her outside options. In case of positive correlation between firm's profit and manager's reservation utility, this translates into the participation constraint of the manager being non-binding in states characterized by low profits.

Another point that deserves attention is whether V^{AP} is convex. We have seen that when the reservation utility of the principal is constant across profit histories, the state space is indeed an interval. This result, however, cannot be easily generalized for the case of varying reservation utilities. Indeed, if for some $i = 1, ...$ the set $\widetilde{X}_i := \widetilde{B}^i\left(\widetilde{X}_0\right)$ exhibits a hole, then this hole can potentially persist into V^{AP}. Let us assume that $\theta = 1$, $A = \{\underline{a}, \overline{a}\}$, $\underline{a} < \overline{a}$, $\exists l \in L : \underline{V}_l > \min_{l \in L} \underline{V}_l$, i.e., we have a non-reducible one-period dependence and two possible levels of effort. Consider $\widetilde{B}\left(\widetilde{X}_0\right)$. Is it convex given that $\widetilde{X}_0$ is? Let $V_{\underline{a}}$ and $V_{\overline{a}}$ be the sets of initial utility values that are supportable by admissible incentive-compatible contracts guaranteeing continuation utilities in $\widetilde{X}_0$ and inducing low and, respectively, high effort. Note that both these sets are compact and convex. Then, $\widetilde{B}\left(\widetilde{X}_0\right)$ is convex if and only if $V_{\underline{a}} \cap V_{\overline{a}} \neq \emptyset$. From the proof of Proposition 4, we know that $\max V_{\underline{a}} > \max V_{\overline{a}}$, so the necessary and sufficient condition for the convexity of $\widetilde{B}\left(\widetilde{X}_0\right)$ is equivalent to

[15] In the next section, I consider positive correlation between yesterday's profit and manager's current reservation utility (i.e., a one-period positive dependence). The reservation utility values generally allow for the more interesting case of non-binding participation constraints. The state space V^{AP} is estimated numerically (for different combinations of reservation utility values and different borrowing arrangements) and the results are presented in Table 2 in Appendix 2. They indicate that if $\max_{a \in A} \{\beta_A E_a \underline{V} - a\} > \underline{V}_{\widehat{l}} - \upsilon(\underline{w})$, there is some utility gain on the lower limit of the state space for all but the best-record managers. Another observation is that the worse the record, the higher the gain.

$\max V_{\overline{a}} \geq \min V_{\underline{a}}$. It is not straight-forward, however, to derive this condition in terms of parameters. We can certainly derive sufficient conditions, but they need not be necessary. For example, let $\widetilde{y} \in \arg\max_{y \in Y} \{\pi(y, \overline{a}) - \pi(y, \underline{a})\}$ and $\underline{V} = \max_{l \in L} \underline{V}_l$. Take the contract $\{\underline{a}, \upsilon(\underline{w}), \underline{V}\}$ which is clearly incentive-compatible and guarantees the manager an initial utility of $\upsilon(\underline{w}) + \beta_A \underline{V} - \underline{a}$. Now, consider the contract recommending high effort while promising wage $\overline{w}$ and a continuation utility $\widehat{V}$ if the profit realization is $\widetilde{y}$ and, respectively, $\underline{w}$ and $\underline{V}$ otherwise. This contract would be incentive compatible and would guarantee the manager an initial utility of at least $\upsilon(\underline{w}) + \beta_A \underline{V} - \underline{a}$ if $\overline{a} - \underline{a} \leq \min\{(\pi(\widetilde{y}, \overline{a}) - \pi(\widetilde{y}, \underline{a}))\left(\upsilon(\overline{w}) + \beta_A \widehat{V} - \upsilon(\underline{w}) - \beta_A \underline{V}\right), \pi(\widetilde{y}, \overline{a})(\upsilon(\overline{w}) + \beta_A \widehat{V}) - \pi(\widetilde{y}, \underline{a})\left(\upsilon(\underline{w}) + \beta_A \underline{V}\right)\}$. This inequality is basically satisfied if $\underline{V}$ is not too high. Notice, however, that by Proposition 4 $\underline{V}$ will be higher the next iteration if $\max_{a \in A} \{\beta_A E_a \underline{V} - a\} > \underline{V}_{\widehat{l}} - \upsilon(\underline{w})$, i.e., if the guess for the state space shrinks. Also note that while we have constructed a sufficient condition for the convexity of $\widetilde{B}\left(\widetilde{X}_0\right)$, this condition is far from necessary.

Given the previous discussion, can we say anything more about the properties of the value function U^{AP^*} and its associated policies? We already know by Proposition 5 in Morfov (2010) that U^{AP^*} is upper semi-continuous (usc) and bounded. Is it continuous? For any $l \in L$ and $V \in V^{AP}(l)$, define $\Gamma_R^{AP}(V, l) := \{c_R : \text{(5)-(8), (10) hold at } (V, l)\}$ and $G_R^{AP}(V, l) := \{c_R \in \Gamma_R^{AP}(V, l) : U^{AP^*}(V, l) = E_a(y - w_+(y) + \beta_P U^{AP^*}_{l_+(l,y)}(V_+(y)))\}$. Namely, $\Gamma_R^{AP}(V, l)$ is the set of admissible, incentive-compatible, one-period contracts guaranteeing the participation of the manager and providing him/her with utility V at initial history l, while $G_R^{AP}(V, l)$ is the subset of optimal (from the point of view of the principal) contracts.

Proposition 5 *For any $l \in L$, $\Gamma_R^{AP}(., l)$ is upper hemi-continuous on $V^{AP}(l)$.*

To show that the value function U^{AP^*} is continuous on V^{AP}, we also need $\Gamma_R^{AP}(., l)$ to be lower hemi-continuous on V^{AP}. This is where the problem stems from. For example, consider two possible effort levels $\underline{a} < \overline{a}$ and let $V_{\underline{a}}^{AP}$ and $V_{\overline{a}}^{AP}$ be the sets of initial utility values that are supportable by admissible incentive-compatible contracts guaranteeing continuation utilities in V^{AP} and inducing low and, respectively, high effort. Fix $l \in L$. By Proposition 4,

$\max V_{\underline{a}}^{AP}(l) > \max V_{\overline{a}}^{AP}(l)$; therefore, if V^{AP} is convex, $\Gamma_R^{AP}(., l)$ may violate lower hemi-continuity at $\max V_{\overline{a}}^{AP}(l)$ and/or $\max\left\{\min V_{\underline{a}}^{AP}(l), \min V_{\overline{a}}^{AP}(l)\right\}$. Call these points V_1 and V_2, respectively. Then, by the theorem of the maximum[16] $U^{AP^*}(., l)$ will be continuous and $G_R^{AP}(., l)$ will be upper hemi-continuous on $V^{AP}(l) \setminus \{V_1, V_2\}$. If $\theta = 0$, we know that V^{AP} is convex, so the previous analysis applies.

Notice that the problems surrounding the potential discontinuities of Γ_R^{AP} may be related to the possible non-convexity of the set of effort levels, A.[17] However, in view of the numerical estimation, working with an interval of efforts is unfeasible. Moreover, multiple actions may require ranking conditions and the calibration of such a model may prove a difficult task. Therefore, in the next section, I will concentrate on the case of only two possible levels of managerial effort: high (working hard) and low (shirking).[18]

3 Computation and Results

The computation of the model starts with solving for V^{AP}, the set of manager's expected discounted utilities supportable by an AP contract. While Proposition 14 from Morfov (2010) gives the theoretical background for the estimation of V^{AP}, some caveats remain. In particular, $\widetilde{B}$ is a set operator and in order to apply the iterative procedure in practice we need an efficient representation of the sequence of sets $\left\{\widetilde{X}_i\right\}_{i \in Z_+}$. For the class of infinitely repeated games with perfect monitoring, Judd, Yeltekin and Conklin (2003) are able to construct inner and outer convex polytope approximations based on the convexification of the equilibrium value set through a public randomization device. Here, I

[16] See, for example, Stokey and Lucas (1989).

[17] Indeed, the problem may be attenuated if we assume A convex [cf. Phelan and Townsend (1991)].

[18] Note that if we presuppose the optimality of a certain level of effort, say high effort [see, for example, Aseff (2004)], we will have $V^{AP} = V_{\underline{a}}^{AP}$ convex, Γ_R^{AP} lower hemi-continuous and, therefore (given Proposition 5), continuous, so by the theorem of the maximum U^{AP^*} will be continuous and G_R^{AP} will be upper hemi-continuous. Such an assumption, however, is not as innocuous as it may seem since it appears that shirking (low effort) is optimal for a wide interval of initial utility values in the upper region of the state space (see Figure 10).

follow a more general approach which does not rely on assuming that V^{AP} is convex or convexifying it by introducing public randomization.[19] The main idea is to discretize the elements of the initial guess $\widetilde{X}_0$ and start extracting small open intervals, the midpoints of which are unfeasible with respect to $\widetilde{X}_0$. The extraction is done elementwise without updating the previous elements. In particular, I start from the discretization of the first[20] element of $\widetilde{X}_0$, find the points that cannot be supported by a one-period AP contract with a continuation utility profile contained in $\widetilde{X}_0$, i.e., the points of the discretization which are not in the first element of $\widetilde{B}\left(\widetilde{X}_0\right)$, and extract small open balls around these points. Next, I find the gridpoints in the second element of $\widetilde{X}_0$ which are unfeasible with respect to $\widetilde{X}_0$, extract their small open neighborhoods and proceed in a similar fashion until I cover all the elements of $\widetilde{X}_0$. The remaining set, i.e., $\widetilde{X}_0$ less the extracted intervals, becomes $\widetilde{X}_1$, our new guess for V^{AP}. Given that $\widetilde{X}_0$ is a vector of n^θ closed intervals in $\mathbb{R}$, each of the n^θ elements of $\widetilde{X}_1$ will be a finite union of closed intervals in $\mathbb{R}$. In order to increase efficiency, intervals with length less than some prespecified level are reduced to their midpoints. The procedure stops if for each element of $\widetilde{X}_i$ the number of closed intervals representing it equals the respective number for the same[21] element in $\widetilde{X}_{i-1}$ and, in addition, the representation of $\widetilde{X}_i$ differs from the representation of $\widetilde{X}_{i-1}$ by less than some prespecified tolerance level. In order to apply this stopping criterion, one still needs to construct a measure for the difference between representations. For this purpose, I find the difference in absolute terms between each endpoint (minimum or maximum point) of each interval of each element of $\widetilde{X}_i$ and $\widetilde{X}_{i-1}$ respectively and take the maximum one to be the difference between the representations of $\widetilde{X}_i$ and $\widetilde{X}_{i-1}$. This difference is well defined given that the two representations share the same structure, which is actually the first condition of the stopping criterion.

Once V^{AP} is obtained, it is elementwise discretized and used as a state space in the dynamic program for obtaining U^{AP^*}. At each iteration, the guess for

[19] Such a general approach is particularly useful in addressing extensions as for example estimating the endogenous state space of agent's expected discounted utilities supportable by an AP stock option contract, because of the non-convexities inherent to the stock option contract.

[20] Note that $\widetilde{X}_0$ is a Cartesian product of n^θ sets.

[21] Here, 'same' refers to the index of the element, i.e. to the initial history to which it corresponds.

U^{AP^*} being defined only on the discretization needs to be interpolated over the state space. Interpolation is also required in the subsequent iterative procedure which uses U^{AP^*} as an initial guess for $\widehat{U}^*$, the extension of U^* on V^{AP}.

It should be noted that for computational purposes, I do not work with w directly, but use $\upsilon := \upsilon(w)$ instead. This simple change of variables makes the set of constraints linear in a, υ, and V_+, which significantly improves the numerical optimization. We can always recover the optimal wage by inverting the optimal υ.

Table 2 in Appendix 2 contains V^{AP}, the state space of the optimal AP contract. The results are obtained by parameterizing the model in line with the calibration of Aseff and Santos (2005) based on the results of Hall and Liebman (1998) and Margiotta and Miller (2000). Namely, the set of possible profit realizations which are interpreted as stock price returns $Y = \{y_{(1)}, y_{(2)}, y_{(3)}\} = \{0.55,\ 1.125,\ 1.7\}$, the space of effort levels $A = \{\underline{a}, \overline{a}\} = \{0.1253,\ 0.1469\}$, the conditional probabilities $\pi(y_{(1)}, \underline{a}) = 0.1508$, $\pi(y_{(2)}, \underline{a}) = 0.8121$, $\pi(y_{(3)}, \underline{a}) = 0.0371$, $\pi(y_{(1)}, \overline{a}) = 0.1268$, $\pi(y_{(2)}, \overline{a}) = 0.8082$, $\pi(y_{(3)}, \overline{a}) = 0.065$.[22] I fix $\underline{w} = 0$ and equalize the discount factors for the agent and the principal $\beta_A = \beta_P = 0.96$. The period utility with no effort, $\upsilon(.) = \sqrt{(.)}$, is as in Aseff (2004)[23]. The reservation utility of the principal is assumed constant across initial histories with a value $\underline{U} = 0$. As regards the upper bound of the manager's compensation, I consider three different cases. Case 1 uses Proposition 1 to derive the uniform upper bound for the wage $\overline{w}$ given the minimum reservation utility of the principal $\underline{U}$. Cases 2 and 3 still honor the upper bound $\overline{w}$, but impose further restrictions on the manager's period compensation[24] Case 2 bounds the wage by $\overline{y}$ at each contingency.[25] It implicitly allows the shareholders to borrow up to $\overline{y} - y$ every period given a realized profit y. Case 3 implicitly prevents the shareholders from borrowing. At each possible contingency, they can pay the CEO no more than the realized profit. For case 1, I take the upper bound for the initial guess $\widehat{V} = \frac{\upsilon(\overline{w}) - \underline{a}}{1-\beta_A}$, while for cases 2 and 3, I use $\widehat{V} = \frac{\upsilon(\min\{\overline{w}, \overline{y}\}) - \underline{a}}{1-\beta_A}$. I analyze the case of $\theta = 1$, which encompasses $\theta = 0$ as a subcase. Then, I

[22] Aseff and Santos (2005) actually consider two conditional distributions over an interval of possible stock price returns [0.55, 1.7]. In this numerical experiment, I concentrate the mass of each distribution on 3 points of this interval: the minimum, middle, and maximum point.

[23] Running the algorithm with $\upsilon(w) = log(1 + w)$ as in Aseff and Santos (2005) showed no qualitative changes in the results.

[24] Cf. Wang (1997).

[25] Remember that $\overline{y}$ is the highest possible profit realization, i.e. $y_{(3)}$ in our setting.

have to deal with $n^{\theta} = 3$ (initial history) states. I use the natural notation l for the state with initial history $y_{(l)}$, $l \in \{1, 2, 3\}$. I consider three possible values for the reservation utility of the CEO: L= $\frac{\upsilon(\underline{w})-\overline{a}}{1-\beta_A}$ = -3.6725, M = 0, H = -L. Then, I analyze the more interesting case of nonnegative correlation between initial histories and manager's reservation utilities. This limits the number of possible combinations of reservation utility values across initial histories to 10. For example, LMH, which stays for $\underline{V}_1$ = L, $\underline{V}_2$ = M, $\underline{V}_3$ = H, is allowed, while LHM is not. Note that KKK is equivalent to the case of $\theta = 0$ and $\underline{V}$ =K, where K$\in${L,M,H}. Each cell of Table 2, contains V^{AP} for a particular combination of reservation utility values (table rows) and a particular case (table columns). In each cell, the left subcolumn corresponds to the intervals' minimum points and the right - to the maximum points, while each subrow corresponds to a particular initial history. For example, for LMH, (case) 1, $V^{AP}(1)$ = [0.8275, 843.0178], $V^{AP}(2)$ = [0.8200, 843.0178], $V^{AP}(3)$ = [3.6725, 843.0178].

The results suggest that for any $l \in \{1, 2, 3\}$, $V^{AP}(l)$ is convex from where come the single intervals in Table 2. Note that at least for cases 1 and 2 the upper bound of $V^{AP}(.)$ remains constant across initial histories and reservation utility combinations. In fact, it equals the theoretical bound given the case: $\frac{\upsilon(\overline{w})-\underline{a}}{1-\beta_A}$ for case 1 and $\frac{\upsilon(\overline{y})-\underline{a}}{1-\beta_A}$ for case 2. This means that wages can be high enough to support high expected discounted utilities for the manager. Note, however, that $V^{2P} \subset V^{AP}$ and we lose high utility values in solving for U^* as Figure 1 in Appendix 2 indicates. The reason is that the value function is decreasing in the upper region of V^{AP}, which results in violations of the principal's participation constraint for high utility values of the manager.

Since the results are similar across cases, we concentrate on the economically motivated cases 2 and 3 with a special focus on case 3.[26] Figures 1 and 2 plot U^* and U^{AP^*} over V^{AP} for cases 2 and 3 respectively. In each graph, the left panel corresponds to an initial history 1, the middle - to 2, and the right - to 3. Note that although similar, the value functions are not the same across initial profit histories for both the auxiliary and the original problem. The main difference

[26] As regards the numerical computation, case 3 is the clearest case followed by case 2. Case 1 is the noisiest case since the state space of the auxiliary problem, V^{AP}, is the largest due to the higher upper bound of the manager's utility, $\widehat{V}$. This requires a coarser grid and also introduces numerical mistakes due to the high absolute values of the negative numbers the guess for (and the actual) U^{AP^*} takes in the upper regions of the state space, regions which we in fact lose when estimating U^* since they violate principal's participation.

comes from the substantial shrinking of the state space from the left when the initial history is the one characterized by the highest reservation utility (i.e., 3). Given an initial history 3, the maximum utility the principal can get by signing an AP or 2P contract with the CEO is less than what he/she can obtain under 1 or 2 since the contract should guarantee a higher initial utility to the manager. Note that U^* and U^{AP^*} are almost identical for case 3, while U^* does not cover the uppermost part of the domain of U^{AP^*} in case 2. The reason is that very high initial utility promises should be supported with sufficiently high wages, which would eventually decrease the expected discounted utility of the principal below its reservation value at some node. Therefore, in case 2, the minimum utility the principal can obtain by signing a 2P contract with the manager is higher than the minimum under an AP contract. This is not observed (or, in general, less pronounced) for case 3 since then the principal is essentially prevented from borrowing, so he/she cannot offer the manager wages that are sufficiently high to violate his/her own participation constraint under the 2P contract. The graph also suggests that the value functions are concave and monotonically decreasing, properties which, however, are not so easy to generalize.

Regarding the characteristics of the optimal contract, the recommended effort level is predominantly the high one. However, low effort appears to be optimal in some utility regions. Since the results are similar across cases, I only report the relationship for LMH, case 3. As Figure 3) indicates, shirking is optimal for sufficiently high initial utility values. Intuitively, the manager is so rich (in expected utility terms) that the firm cannot effectively reward or punish him/her and, therefore, finds motivating him/her to exert high effort suboptimal. The CEO's utility tomorrow increases in both the end-of-period profit and the initial utility promise as illustrated in Figures 4 and 5 respectively. Let us focus on Figure 4 which plots the relation for each possible initial utility. While the future utility promise is basically flat for high initial utilities, for low utility values the increase is driven by the participation constraint of the agent which is binding tomorrow at a profit realization $y_{(3)}$. This is also reflected on the left and the middle panel of Figure 5 as the kink of the graph of $V_+(., y_3)$. Note that this is not the case for initial history 3 which requires higher future utility promises. In general, the manager's wage increases in both the end-of-period profit and the initial utility promise. Notice that the compensation scheme is much flatter across profit realizations for case 2 than for case 3 (Figures 6 and 7 respectively). This is because current consumption smoothing (across profit realizations) which is achieved by a flat wage scheme for the initially poor (in terms of utility promises) managers, is no longer possible for richer CEOs because the credit constraint imposed in case 3 starts to bind. This is particularly relevant for the lowest profit realization $y_{(1)}$. The same point can be illustrated by Figures 8 and 9. Note that wage contingent on a low profit tomorrow is strictly increasing on the whole domain of initial utilities for case 2, while in case 3 it steadily increases until $y_{(1)}$ is reached and then with the credit constraint binding stays constant at that level.

The results suggests that both current and future compensation are used to induce poor and mid-range managers to work hard, while rich managers prove too difficult to motivate. The latter shirk and while they may face some fluctuations in their current income stream due to binding credit constraints on part of the firm, their lifetime utility remains relatively flat.

Since there is a sufficient dispersion in agent's reservation utility values,[27] the minimum utility supportable by an AP/2P contract for initial histories characterized by lower reservation utility is boosted by higher reservation utilities for other states. More specifically, in the presence of positive correlation between profits and reservation utilities, the participation constraint of the agent does not bind in states characterized by low profits. In other words, the AP/2P contract provides the manager with some insurance against fluctuations in the value of his/her outside options, which ultimately smooths his/her consumption across (initial history) states. Interestingly, while the theoretical result of Proposition 1 only refers to initial history 1, we observe a cascade effect which leads to a significant rise in the lower limits of the possible utility promises for both 1 and 2. Finally, note that if the reservation utility remains the same across some, but not all of the truncated initial histories, $V^{AP}(.)$ is identical for the initial histories with the same reservation utility. While this seems obvious for $\theta \leq 1$, longer history dependence will potentially break the relation since the set of possible tomorrow's histories will depend on the history today.

Table 1 in Appendix 2 shows the effect of changing the value of the minimum reservation utility of the principal for LLL, case 1. Theoretically, we have that increasing $\underline{U}$ decreases $\overline{w}$, which in turn causes $\widehat{V}$ to fall. Since the analysis so far suggests that the theoretical upper bounds for agent's utility can be supported by an AP contract, the only effect of changing $\underline{U}$ comes from the resulting change in the theoretical bound. Moreover, since the 2P contract cannot support

[27] The only exception observed is when the reservation utility remains flat across past outcomes, so in fact we are in the case of $\theta = 0$.

manager's utilities in the upper region of $V^{AP}(.)$, the optimal self-enforcing contract is not affected.

I also use Monte Carlo simulations to investigate the dynamic behavior of the optimal contract. Namely, I construct "typical" time paths of length T for the manager's effort, wage, and expected discounted utility, the firm's profits, and the principal's expected discounted utility. Each such path is taken to be the mean of I independently generated paths which are constructed following the transition and the policies (and if relevant, the value function) of the 2P contract. The "typical" path is well defined given an initial condition (V_0, l), where $V_0 \in V^{2P}(l)$ and $l \in \{1, 2, 3\}$. Figures 10-17 present the results for LMH, case 3 where I take $T = 50$ and $I = 450$.[28]

Figure 10 illustrates how the manager's effort optimally develops in time. Each curve on the l'th panel of the graph represents a time path conditional on a particular expected discounted utility being promised to the manager in the beginning of period 0 given an initial history $l \in \{1, 2, 3\}$. Relating each curve to its corresponding initial utility indicates that initial effort persists for sufficiently low or sufficiently high initial utilities (high effort for low initial utilities, and low effort for high initial utilities), there is some dynamics in the middle-utility range, mostly expressed in diminishing effort.

Figures 11 and 14 suggest that manager's compensation and, respectively, his/her expected discounted utility grow weakly in the long run where the increase is pronounced for sufficiently low initial utilities, while the mid-range and high initial utility paths tend to be relatively stable at their initial levels. In other words, CEOs who start rich (in expected utility terms) tend to keep their utility level while those who start poor get richer in time. Note that the increase is most pronounced for managers with initial utilities below the highest reservation utility, i.e., the poorest managers in 1 and 2.[29] These managers first have their utilities pushed well above their reservation level based on the insurance effect outlined in Corollary 1. Then, the principal motivates them to work hard by rewarding success through continuation utilities while providing them with insurance through flatter wages. In this way, the probability of a higher profit and, therefore, higher reservation utility tomorrow increases, which rises

[28] Longer paths were also simulated but the results did not show significant difference from the ones presented here while memory limitations progressively restricted the precision of the estimates.

[29] Remember, that 3 is the manager's best initial history since it is associated with his/her highest reservation utility $\underline{U}_3 = \mathrm{H}$.

the manager's expected continuation utility. Since wage is increasing in initial utility, the resulting pattern is observed. Therefore, in the long run, both consumption (wage) and wealth (utility) are smoother across initial history states. The result can also be interpreted as a decreasing (wage- and utility-) inequality (as far as the poorest managers are concerned).

Figure 12 shows the profit fluctuations under the optimal contract. The average profit realization is substantially higher for lower than for higher initial utilities with some sudden drop at the mid range. This is understandable given that high effort is optimal for lower utility values while low effort is optimal for high utility values.

As Figure 13 indicates, the principal's expected discounted utility tends to decrease weakly in the long run where the decrease is more pronounced when lower initial utility is promised to the agent. For higher utility promises, the principal tends to keep his/her initial utility value. This is easily explained by the dynamics of the manager's utility given that the value function is decreasing.

In a setting of dynamic risk sharing, Green (1987) and Thomas and Worrall (1990) demonstrate that the agent becomes infinitely poor in the long run. Phelan (1995) shows that this result does not hold if limited commitment is introduced on part of the agent, namely that there exists a non-degenerate limiting distribution of agent's expected discounted utility and consumption. In a CARA setup with unobservable actions, Wang (1997) shows numerically that agent's wealth and consumption tend to fluctuates over time. Aseff (2004) numerically demonstrates that in a contract that optimally induces high effort, the agent's expected discounted utility increases in the long run and has a non-degenerate limiting distribution. In a more general setup characterized by limited commitment on both parts and history-dependent reservation utilities, I obtain a similar result as indicated in Figures 15-17. Each of these graphs considers an initial state l and plots the empirical distributions of manager's expected discounted utility after 50 periods conditional on 12 different initial utility promises. Since the lower bound of the set of possible initial utility promises for 3 is greater than those for the other two initial history states and I use an equidistant grid of 100 points ($V_{(1)}$,...,$V_{(100)}$), the i-th point of the grid for 3 will generally larger than the i-th point of the grids for 1 and 2 respectively. Having this in mind, we see that the limiting distribution does not vary considerably across initial history states, i.e., in the long run it would not matter what the initial profit was as far as the initial utility promise was the same (at least for a single-period history dependence). Note however that since the curves on each panel of Figure 14 generally do not cross, it still matters where (in terms of utility promise) you start - the poor get rich but it is still better to start richer.

4 Conclusion

This paper considers the dynamic principal-agent interaction between firm's shareholders and a CEO in a setting characterized by limited commitment and history-dependent reservation utilities. I analyze the state space of the recursive form of the problem under a short-term history dependence and derive conditions under which the optimal contract offers the manager a utility strictly higher than the reservation level. The model is parameterized and computed under different structural arrangements. I find evidence that the optimal contract provides the manager with insurance against (non-negligible) fluctuations in the value of his/her outside options, which ultimately smooths his/her consumption across (initial history) states. Exerting effort appears to be the predominant strategy for the principal, but shirking may still be optimal when the CEO is rich enough. The optimal wage scheme and the future utility of the manager tend to grow in both his/her current utility and in the future profit realization. In the long run, the CEO does not get poorer in utility terms. In particular, managers who start rich tend to keep their utility level while those who start poor get richer in time. The manager's utility tends to increase weakly in the long run and appears to have a non-degenerate long-term distribution depending on the initial utility promise.

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APPENDIX 1

Proof of Proposition 1. At any node $y^{\tau-1}$ after (and including) l, we have $\underline{\underline{U}} \leq U_\tau\left(., y^{\tau-1}\right) \leq \frac{\overline{y}-\underline{w}}{1-\beta_P}$, where the first inequality follows from (3) and Assumption 4, and the second from (1), Assumptions 1, 2, and the properties of A and Y. If we define $\underline{y} := \min Y$, it is straight-forward that $\sum_{t=\tau}^{\infty}\beta_P^{t-\tau}\sum_{y_t\in Y}...\sum_{y_\tau\in Y} y_t\prod_{i=\tau}^{t}\pi\left(y_i, a_i\left(y^{i-1}\right)\right) \in \left[\frac{\underline{y}}{1-\beta_P}, \frac{\overline{y}}{1-\beta_P}\right]$. Consequently, we have that $\widehat{W}\left(c, y^{\tau-1}\right) := \sum_{t=\tau}^{\infty}\beta_P^{t-\tau}\sum_{y_t\in Y}...\sum_{y_\tau\in Y} w_t\left(y^t\right)\prod_{i=\tau}^{t}\pi\left(y_i, a_i\left(y^{i-1}\right)\right) \in \left[\frac{\underline{w}}{1-\beta_P}, \frac{\overline{y}}{1-\beta_P} - \underline{\underline{U}}\right]$. Let us take some admissible a. Since $\widehat{W}\left(a, w, y^{\tau-1}\right) = \sum_{y_\tau\in Y} w_\tau\left(y^\tau\right)\pi\left(y_\tau, a\left(y^{\tau-1}\right)\right) + \beta\sum_{y_\tau\in Y}\widehat{W}\left(c, y^\tau\right)\pi\left(y_\tau, a\left(y^{\tau-1}\right)\right)$, we obtain $\sum_{y_\tau\in Y} w_\tau\left(y^\tau\right)\pi\left(y_\tau, a\left(y^{\tau-1}\right)\right) \in \left[\underline{w}, \frac{\overline{y}-\beta_P\underline{w}}{1-\beta_P} - \underline{\underline{U}}\right]$. Now, consider $w_\tau\left(y^{\tau-1}, y\right)$ for some $y \in Y$. Note that by Assumption 1 and the properties of A and Y, $\underline{\pi}$ is well defined and $\pi\left(y, a\left(y^{\tau-1}\right)\right) > \underline{\pi} > 0$. Then, we have:

$$
\begin{gathered}
w_\tau\left(y^{\tau-1}, y\right) \leq \\
\frac{1}{\pi\left(y, a\left(y^{\tau-1}\right)\right)}\left(\frac{\overline{y}-\beta_P\underline{w}}{1-\beta_P} - \underline{\underline{U}} - \sum_{y_\tau\in Y\setminus\{y\}} w_\tau\left(y^\tau\right)\pi\left(y_\tau, a\left(y^{\tau-1}\right)\right)\right) \leq \\
\frac{1}{\pi\left(y, a\left(y^{\tau-1}\right)\right)}\left(\frac{\overline{y}-\beta_P\underline{w}}{1-\beta_P} - \underline{\underline{U}} - \sum_{y_\tau\in Y\setminus\{y\}} \underline{w}\pi\left(y_\tau, a\left(y^{\tau-1}\right)\right)\right) = \\
\frac{1}{\pi\left(y, a\left(y^{\tau-1}\right)\right)}\left(\frac{\overline{y}-\underline{w}}{1-\beta_P} - \underline{\underline{U}}\right) + \underline{w} \leq \\
\frac{1}{\underline{\pi}}\left(\frac{\overline{y}-\underline{w}}{1-\beta_P} - \underline{\underline{U}}\right) + \underline{w},
\end{gathered}
$$

where the last inequality follows from $\left(\frac{\overline{y}-w}{1-\beta_P}-\underline{\underline{U}}\right)$ being nonnegative by Assumption 5. Since $\left(y^{\tau-1},y\right)$ was taken randomly, we are done. ■

Proof of Proposition 2. Given that $\theta=0$, the initial guess for V^{AP} in step 1 will be $\widetilde{X}_0:=\left[\underline{V},\widehat{V}\right]$. Then, $\max\widetilde{B}\left(\widetilde{X}_0\right)=\min\left\{\upsilon\left(\overline{w}\right)+\beta_A\widehat{V}-\underline{a},\widehat{V}\right\}$ $=\min\left\{\widehat{V},\widehat{V}\right\}=\widehat{V}$ since $\widehat{V}=\frac{\upsilon(\overline{w})-\underline{a}}{1-\beta_A}$. Consequently, by Proposition 14 from Morfov (2010), we have $\max V^{AP}=\widehat{V}$.

The stationary contract $\{\underline{a},\underline{w},\underline{V}\}$ promises the same wage and the same continuation utility for any profit realization. It is temporary incentive compatible and guarantees a current expected discounted utility of $\upsilon\left(\underline{w}\right)+\beta_A\underline{V}-\underline{a}$ to the manager. Can we find a contract that guarantees a current utility strictly lower than that? Assume such a contract exists, i.e., $\exists\left\{a_-,w_+\left(y\right),V_+\left(y\right)\right\}_{y\in Y}$ admissible, such that $E_a\left\{\upsilon\left(w_+\right)+\beta_AV_+\right\}-a<\upsilon\left(\underline{w}\right)+\beta_A\underline{V}-\underline{a}$, where E_a is the expectation over the profit realization y conditional on the current action being a. However, this contract will fail to satisfy temporary incentive compatibility. Indeed, (7) requires that $E_a\left\{\upsilon\left(w_+\right)+\beta_AV_+\right\}-a_-\geq E_{\underline{a}}\left\{\upsilon\left(w_+\right)+\beta_AV_+\right\}-\underline{a}\geq$ $\upsilon\left(\underline{w}\right)+\beta_A\underline{V}-\underline{a}$ which contradicts our assumption that $\left\{a_-,w_+\left(y\right),V_+\left(y\right)\right\}_{y\in Y}$ guarantees a strictly lower current utility than $\{\underline{a},\underline{w},\underline{V}\}$ does. Therefore, $\min\widetilde{B}\left(\widetilde{X}_0\right)=\max\left\{\upsilon\left(\underline{w}\right)+\beta_A\underline{V}-\underline{a},\underline{V}\right\}$. Note that $\upsilon\left(\underline{w}\right)+\beta_A\underline{V}-\underline{a}\geq\underline{V}$ is equivalent to $\underline{V}\leq\frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}$. Then, by Proposition 14 from Morfov (2010), it is trivial that $\min V^{AP}=\max\left\{\underline{V},\frac{\upsilon(\underline{w})-\underline{a}}{1-\beta_A}\right\}$.

Finally, we will show that V^{AP} is an interval. $\widetilde{X}_0$. Let $\upsilon_+\left(.\right):=\upsilon\left(w_+\left(.\right)\right)$. Given that $\upsilon\left(.\right)$ is strictly increasing by Assumption 3, the inverse function of $\upsilon\left(.\right)$ is well defined and we have $w_+=\upsilon^{-1}\left(\upsilon\left(w_+\right)\right)$. Then, we can effectively work with υ_+ instead of w_+. Indeed, (6) becomes $\upsilon_+\in\left[\upsilon\left(\underline{w}\right),\upsilon\left(\overline{w}\right)\right]$. Now, let us concentrate on stationary contracts of the form $\left\{a_-,\upsilon_+\left(y\right),V_+\left(y\right)\right\}_{y\in Y}$. Note that $\widetilde{B}\left(\widetilde{X}_0\right)$ is a compact set and its lower and upper limits are utilities supportable by stationary contracts inducing the lowest possible level of effort. Then, any utility between $\min\left\{\widetilde{B}\left(\widetilde{X}_0\right)\right\}$ and $\max\left\{\widetilde{B}\left(\widetilde{X}_0\right)\right\}$ can be obtained as a linear combination of the respective stationary contracts that support them. The linear combination will satisfy (5)-(8). (9) will also hold since $\widetilde{X}_0$ is an interval. In that way, we can show that $\widetilde{X}_i$ is a convex set for any $i=0,1,...$ Since $\widetilde{X}_i$ is a sequence of decreasing (nested), compact, convex sets, we have that their limit is also convex. ■

Proof of Proposition 3. The minimum of the state space is obtained as in the proof of Proposition 2. Note that $\max_{a\in A}\{E_a v(\min\{y,\overline{w}\})-a\}$ is well defined given that A is compact and $\pi(y,.)$ is continuous on A for any $y\in Y$ by Assumption 1. Let $\widetilde{X}_0 := \left[\underline{V},\widetilde{V}\right]$, where $\widetilde{V} = \frac{v(\min\{\overline{y},\overline{w}\})-\underline{a}}{1-\beta_A}$. Here, $\widetilde{V}$ is chosen so that $V^{AP}\subset\widetilde{X}_0$. Let $\widehat{A} = \arg\max_{a\in A}\{E_a v(\min\{y,\overline{w}\})-a\}$. Choose $\widehat{a}\in\widehat{A}$. Then, the stationary contract $\left\{\widehat{a},\min\{y,\overline{w}\},\widetilde{V}\right\}_{y\in Y}$ satisfies (5)-(7), (9) and guarantees a current utility of $E_{\widehat{a}}v(\min\{y,\overline{w}\})+\beta_A\widetilde{V}-\widehat{a}\le\widetilde{V}$. Assume a contract $\{a_-,w_+(y),V_+(y)\}_{y\in Y}$ that has $w_+(y)\le y$, $\forall y\in Y$ and satisfies (5)-(7), (9) can guarantee a strictly higher current utility to the manager, i.e., $E_{a_-}\{v(w_+)+\beta_A V_+\}-a_- > E_{\widehat{a}}v(\min\{y,\overline{w}\})+\beta_A\widetilde{V}-\widehat{a}$. However, we have that $E_{\widehat{a}}v(\min\{y,\overline{w}\})+\beta_A\widetilde{V}-\widehat{a}\ge E_{a_-}v(\min\{y,\overline{w}\})+\beta_A\widetilde{V}-a_-\ge E_{a_-}\{v(w_+)+\beta_A V_+\}-a_-$, so a contradiction is reached. By Proposition 14 from Morfov (2010), we obtain $\max V^{AP} = \frac{\max_{a\in A}\{E_a v(\min\{y,\overline{w}\})-a\}}{1-\beta_A}$. In case $\underline{a}\in\widehat{A}$, the convexity of V^{AP} is established as in the proof of Proposition 2. ■

Proof of Proposition 4. The maximum is obtained as in the proof of Proposition 2. Let $\widetilde{X}_0 := \left\{\left[\underline{V}_l,\widehat{V}\right]\right\}$. Take $\widehat{a}\in\arg\max_{a\in A}\{E_a\underline{V}-a\}$. Then, the stationary contract $\{\widehat{a},\underline{w},\underline{V}_l\}$ satisfies (5)-(7), (9) and guarantees the manager a current utility of $v(\underline{w})+\beta_A E_{\widehat{a}}\underline{V}-\widehat{a}$. Assume that there exists another contract that satisfies (5)-(7), (9) and guarantees a strictly lower level of current utility to the agent. Let $\{a_-,w_+(y),V_+(y)\}$ be such a contract, i.e., $E_{a_-}\{v(w_+)+\beta_A V_+\}-a_- < v(\underline{w})+\beta_A E_{\widehat{a}}\underline{V}-\widehat{a}$. Then, $E_{a_-}\{v(w_+)+\beta_A V_+\}-a_-\ge E_{\widehat{a}}\{v(w_+)+\beta_A V_+\}-\widehat{a}\ge v(\underline{w})+\beta_A E_{\widehat{a}}\underline{V}-\widehat{a}$. where the first inequality follows from incentive compatibility and the second from (6) and (9). A contradiction is reached, so $\{\widehat{a},\underline{w},\underline{V}_l\}$ must bring minimum utility to the manager today. If $v(\underline{w})+\beta_A E_{\widehat{a}}\underline{V}-\widehat{a}\le\underline{V}_{\widehat{l}}$, we have that $\min\widetilde{B}_l\left(\widetilde{X}_0\right)=\underline{V}_l$, $\forall l\in L$ since $\underline{V}_{\widehat{l}}=\min_{l\in L}\{\underline{V}_l\}$. If $v(\underline{w})+\beta_A E_{\widehat{a}}\underline{V}-\widehat{a}>\underline{V}_{\widehat{l}}$, $\min\widetilde{B}_{\widehat{l}}\left(\widetilde{X}_0\right)=v(\underline{w})+\beta_A E_{\widehat{a}}\underline{V}-\widehat{a}$. Since applying $\widetilde{B}$ successively on $\widetilde{X}_0$ leads to a sequence of decreasing (nested) compact sets that converges to V^{AP}, we obtain that $\min V^{AP}\left(\widehat{l}\right)\ge\min\widetilde{B}_{\widehat{l}}\left(\widetilde{X}_0\right)>\underline{V}_{\widehat{l}}$. ■

Proof of Corollary 1. From Proposition 4, it is enough to show that $\max_{a\in A}\{\beta_A E_a \underline{V} - a\} > \underline{V}_{\widehat{l}} - \upsilon(\underline{w})$. We have that $\max_{a\in A}\{\beta_A E_a \underline{V} - a\} > \max_{a\in A}\left\{\beta_A \underline{V}_{\widehat{l}} - a\right\} = \beta_A \underline{V}_{\widehat{l}} - \underline{a} \geq \underline{V}_{\widehat{l}} - \upsilon(\underline{w})$, where the first inequality follows from the definition of $\underline{V}_{\widehat{l}}$, the assumption that for at least one $l \in L$, $\underline{V}_{\widehat{l}} = \min_{l\in L}\{\underline{V}_l\} < \underline{V}_l$, and $\pi(y, a) > 0$ from Assumption 1, the equality is trivial, and the last inequality results directly from $\underline{V}_{\widehat{l}} \leq \frac{\upsilon(\underline{w}) - \underline{a}}{1-\beta_A}$. ■

Proof of Proposition 5. Analogous to the proof of Lemma 2 in the Appendix of Morfov (2010). ■

APPENDIX 2

Table 1
Effects of Changing the Minimum Reservation Utility of the Principal
(LLL, case 1)

$\underline{U}$	0	5	10
$\overline{w}$	1145.5526	1010.7817	876.0108
$\widehat{V}$	843.0178	791.6873	736.8045
V^{AP}	[-3.1325, 843.0178]	[-3.1325, 791.6873]	[-3.1325, 736.8045]

Table 2
State Space of the Optimal AP Contract

Case		1		2		3	
		[	]	[	]	[	]
LLL		-3.1325	843.0178	-3.1325	29.4635	-3.1325	22.4035
LLM	$y_{(1)}$	-1.4325	843.0178	-1.4325	29.4635	-1.4325	22.4035
	$y_{(2)}$	-1.4325	843.0178	-1.4325	29.4635	-1.4325	22.4035
	$y_{(3)}$	0.0000	843.0178	0.0000	29.4635	0.0000	22.4035
LLH	$y_{(1)}$	0.8075	843.0178	0.8056	29.4635	0.8050	22.4035
	$y_{(2)}$	0.8075	843.0178	0.8056	29.4635	0.8050	22.4035
	$y_{(3)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.4030
LMM	$y_{(1)}$	-0.1425	843.0178	-0.1460	29.4635	-0.1461	22.4035
	$y_{(2)}$	0.0000	843.0178	0.0000	29.4635	0.0000	22.4035
	$y_{(3)}$	0.0000	843.0178	0.0000	29.4635	0.0000	22.4035
LMH	$y_{(1)}$	0.8275	843.0178	0.8182	29.4635	0.8280	22.4035
	$y_{(2)}$	0.8200	843.0178	0.8200	29.4635	0.8200	22.4035
	$y_{(3)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.4030
LHH	$y_{(1)}$	3.3575	843.0178	3.3635	29.4635	3.3632	22.3724
	$y_{(2)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.3783
	$y_{(3)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.3783
MMM		0.0000	843.0178	0.0000	29.4635	0.0000	22.4035
MMH	$y_{(1)}$	0.8100	843.0178	0.8100	29.4635	0.8100	22.4035
	$y_{(2)}$	0.8100	843.0178	0.8100	29.4635	0.8100	22.4035
	$y_{(3)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.4030
MHH	$y_{(1)}$	3.3600	843.0178	3.3594	29.4635	3.3592	22.3623
	$y_{(2)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.3703
	$y_{(3)}$	3.6725	843.0178	3.6725	29.4635	3.6725	22.3703
HHH		3.6725	843.0178	3.6725	29.4635	3.6725	22.4001

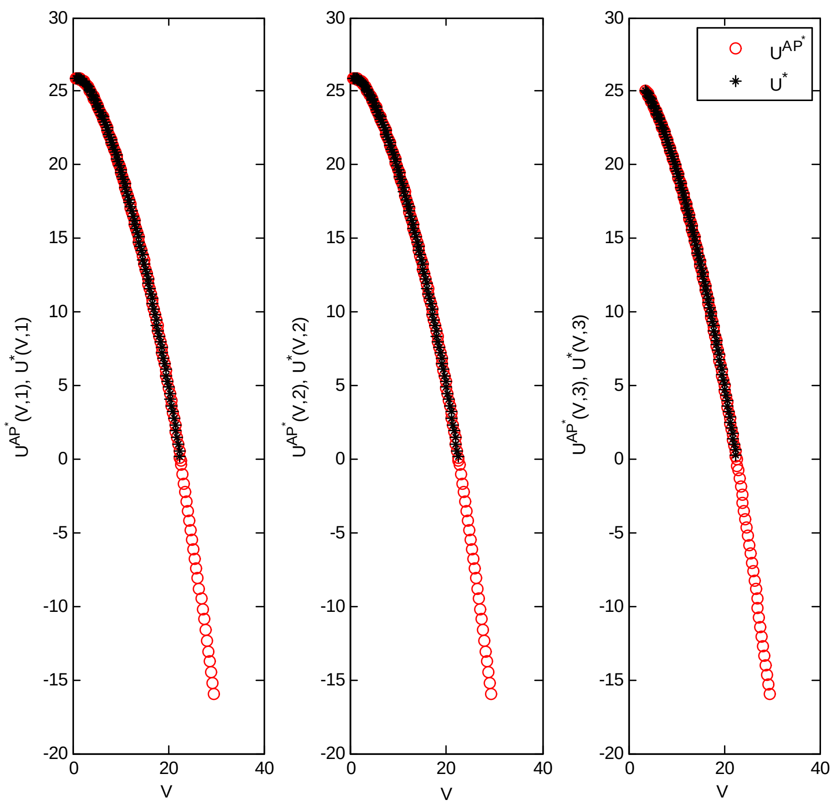

Figure 1: Value functions for the AP and 2P contracts ordered by initial history: $U^{AP^*}(.,l)$, $U^*(.,l)$, $l \in \{1,2,3\}$ (LMH, case 2)

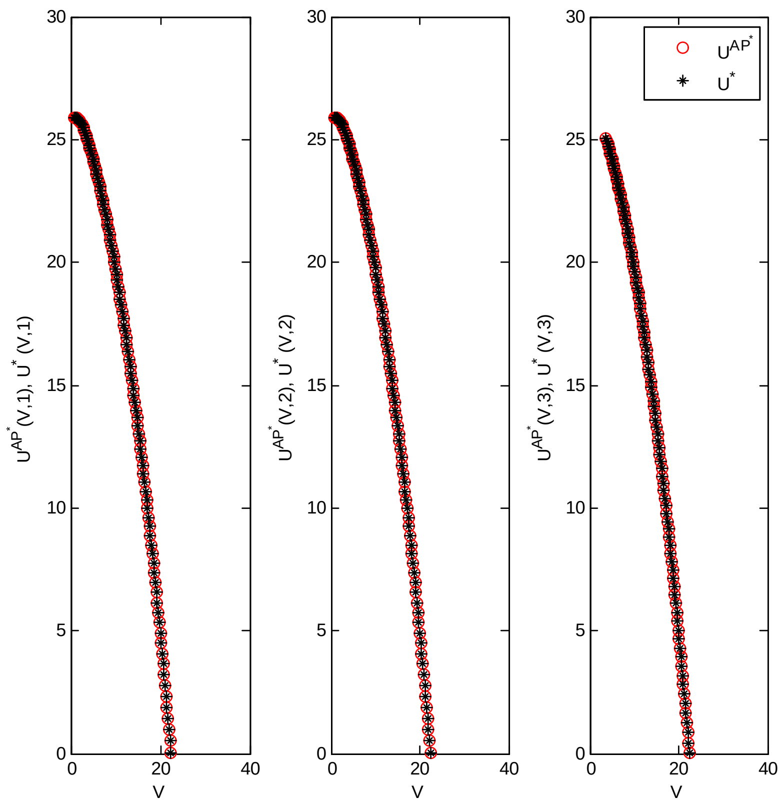

Figure 2: Value functions for the AP and 2P contracts ordered by initial history: $U^{AP^*}(.,l)$, $U^*(.,l)$, $l \in \{1,2,3\}$ (LMH, case 3)

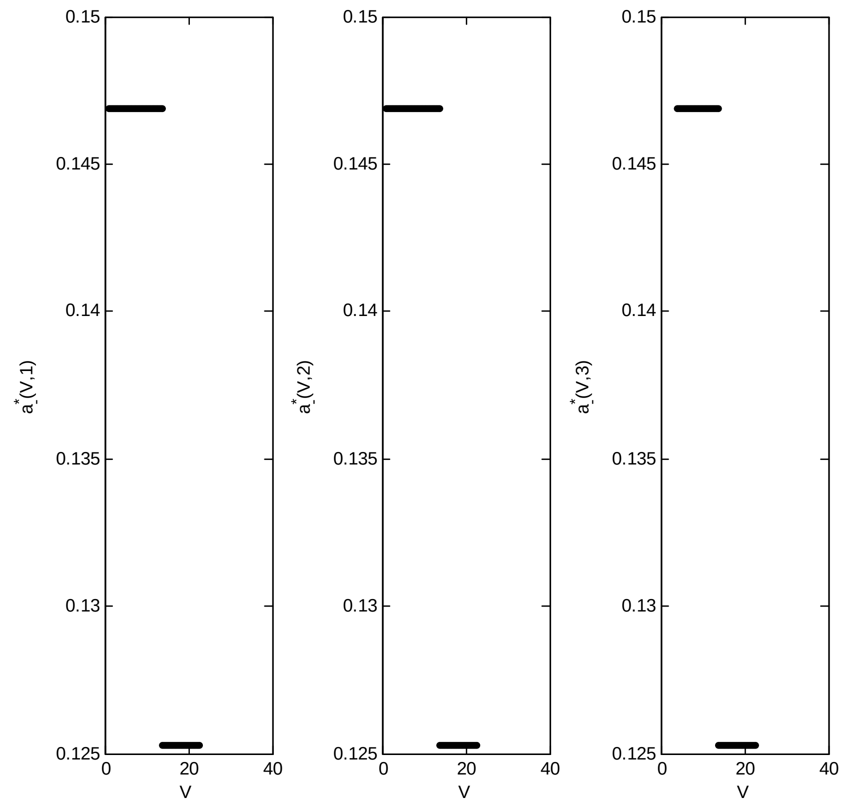

Figure 3: Optimal effort as a function of initial utility promise: $a^*_{-}(., l)$, $l \in \{1, 2, 3\}$ (LMH, case 3)

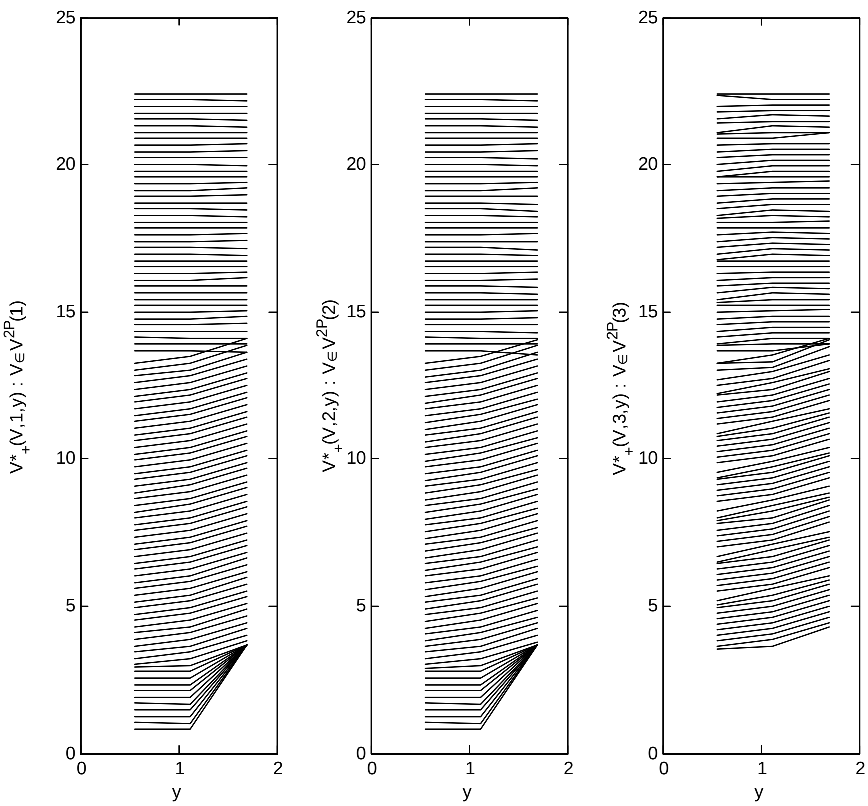

Figure 4: Optimal future utility promise as a function of future profit: $V_{+}^{*}(V, l, .)$: $V \in V^{2P}(l)$, $l \in \{1, 2, 3\}$ (LMH, case 3)

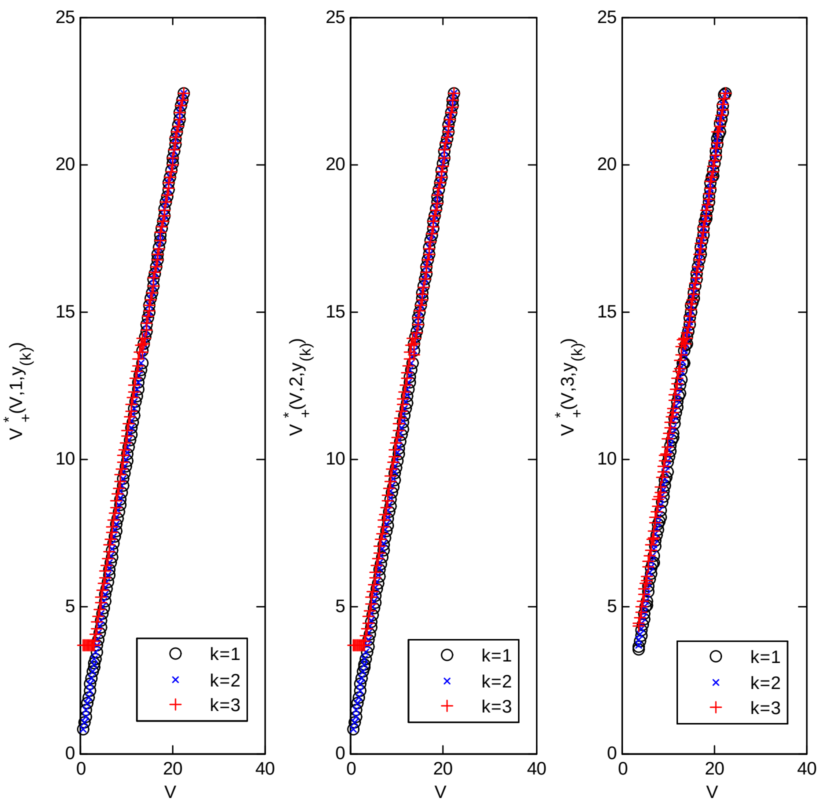

Figure 5: Optimal future utility promise as a function of initial utility promise: $V_{+}^{*}\left(.,l,y_{(k)}\right)$, $l,k \in \{1,2,3\}$ (LMH, case 3)

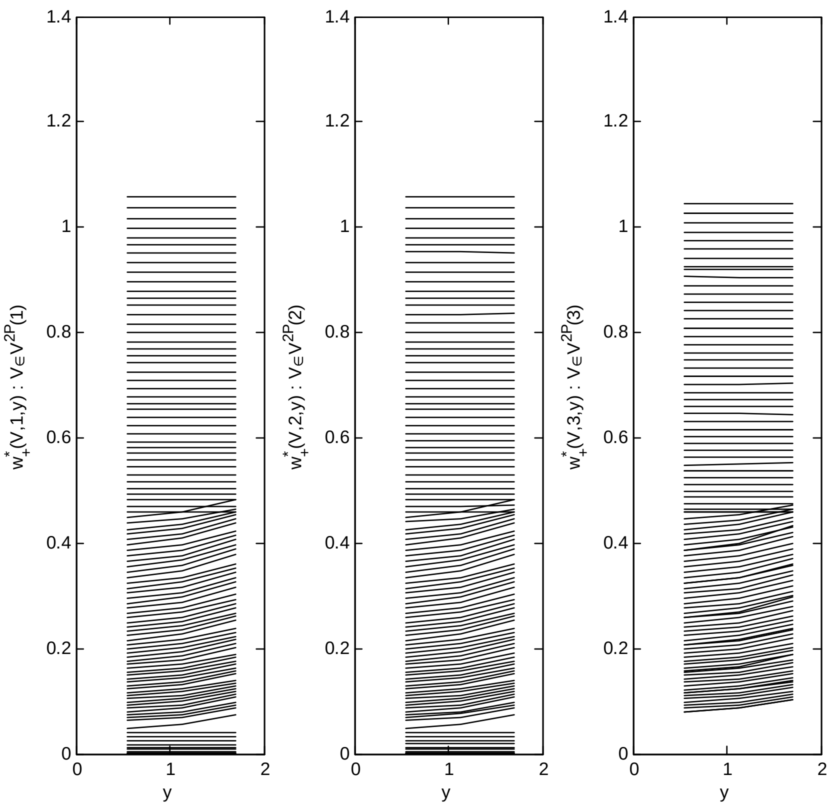

Figure 6: Optimal wage as a function of future profit: $w_{+}^{*}(V, l, .)$: $V \in V^{2P}(l)$, $l \in \{1, 2, 3\}$ (LMH, case 2)

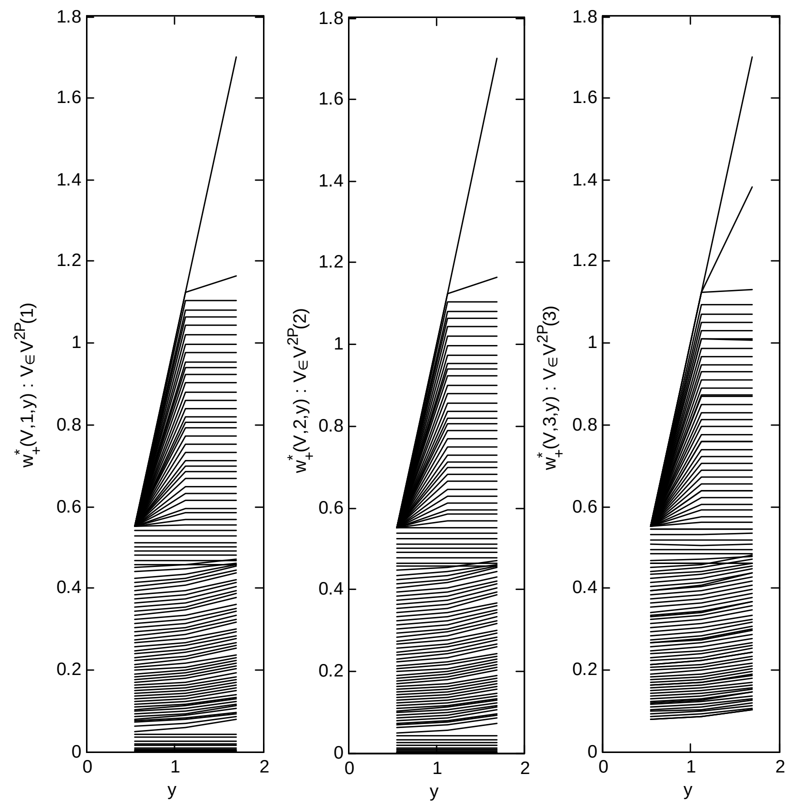

Figure 7: Optimal wage as a function of future profit: $w_+^*(V, l, .)$: $V \in V^{2P}(l)$, $l \in \{1, 2, 3\}$ (LMH, case 3)

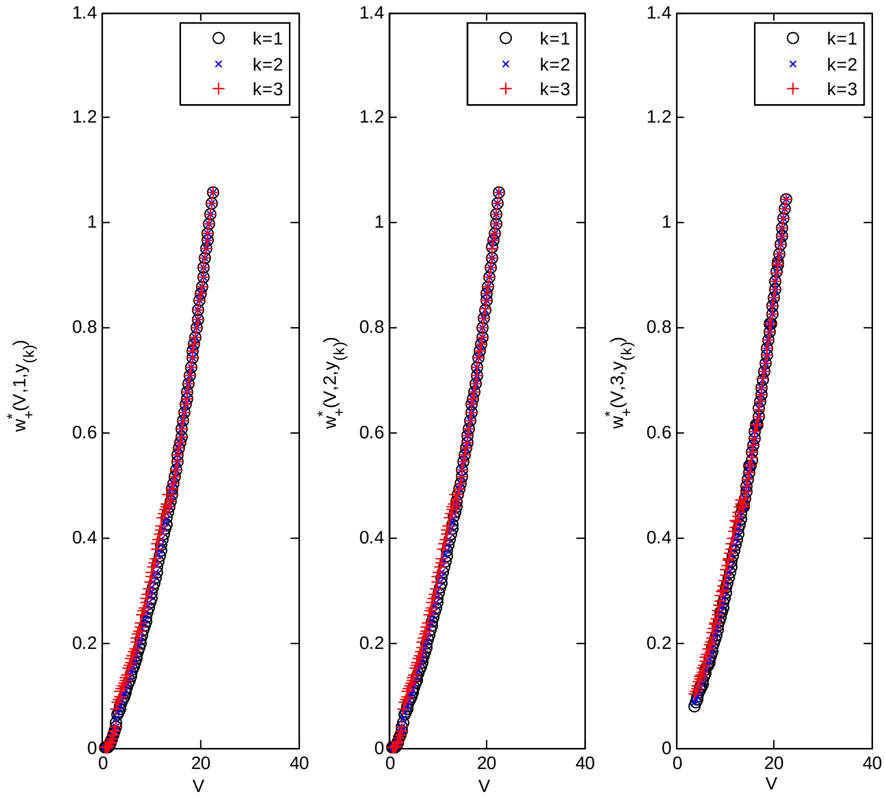

Figure 8: Optimal wage as a function of initial utility promise: $w^*_+\left(.,l,y_{(k)}\right)$, $l,k \in \{1,2,3\}$ (LMH, case 2)

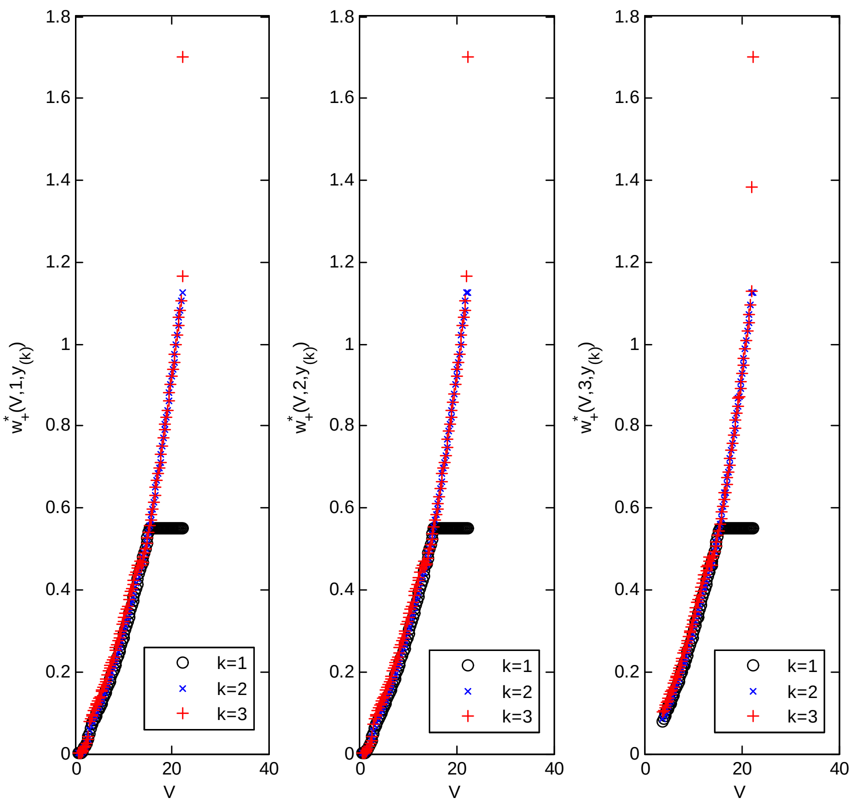

Figure 9: Optimal wage as a function of initial utility promise: $w_+^*\left(.,l,y_{(k)}\right)$, $l,k \in \{1,2,3\}$ (LMH, case 3)

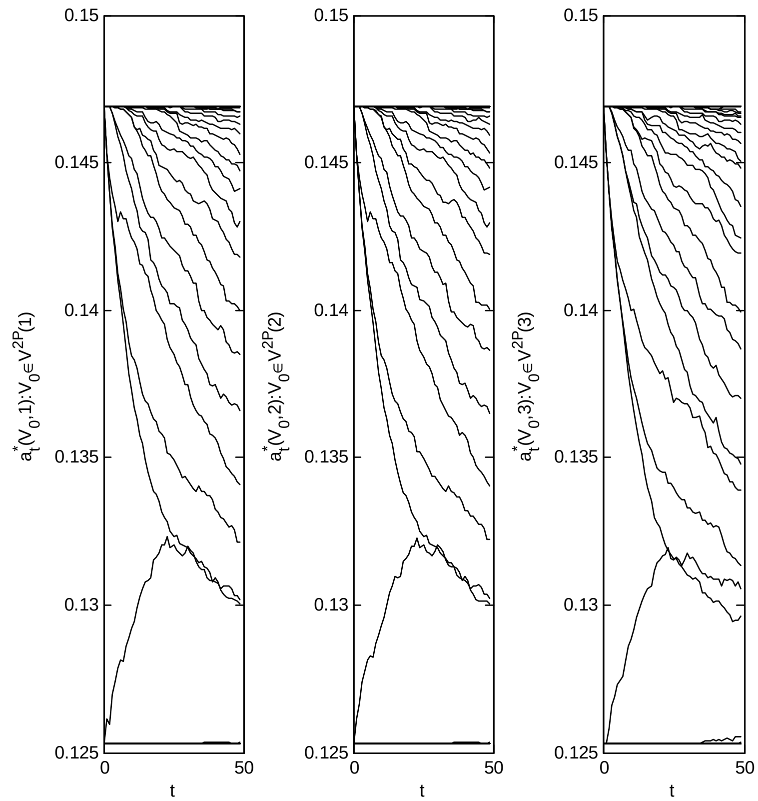

Figure 10: Optimal effort in time: $a_t\,(V_0, l)$: $V_0 \in V^{2P}\,(l)$, $l \in \{1, 2, 3\}$, LMH, case 3

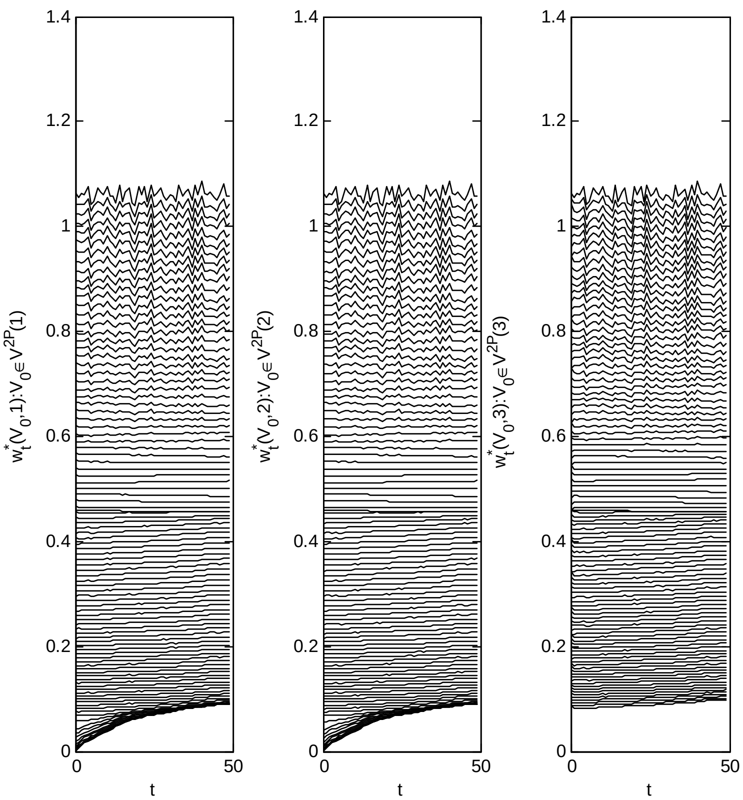

Figure 11: Optimal wage in time: $w_t\left(V_0, l\right)$: $V_0 \in V^{2P}\left(l\right)$, $l \in \{1, 2, 3\}$, LMH, case 3

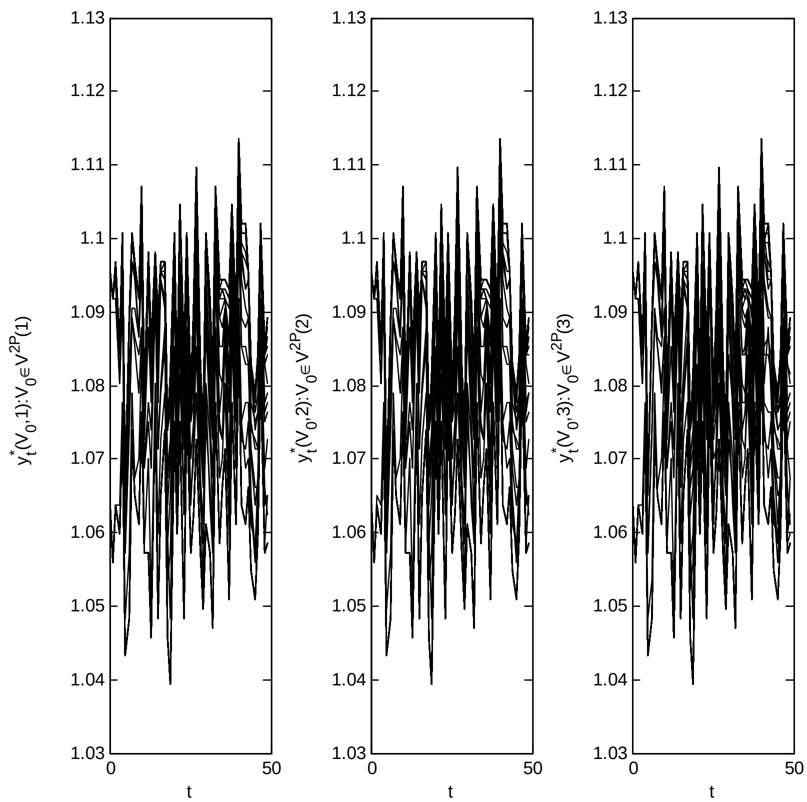

Figure 12: Firm's profit in time: $y_t\,(V_0, l)$: $V_0 \in V^{2P}\,(l)$, $l \in \{1, 2, 3\}$, LMH, case 3

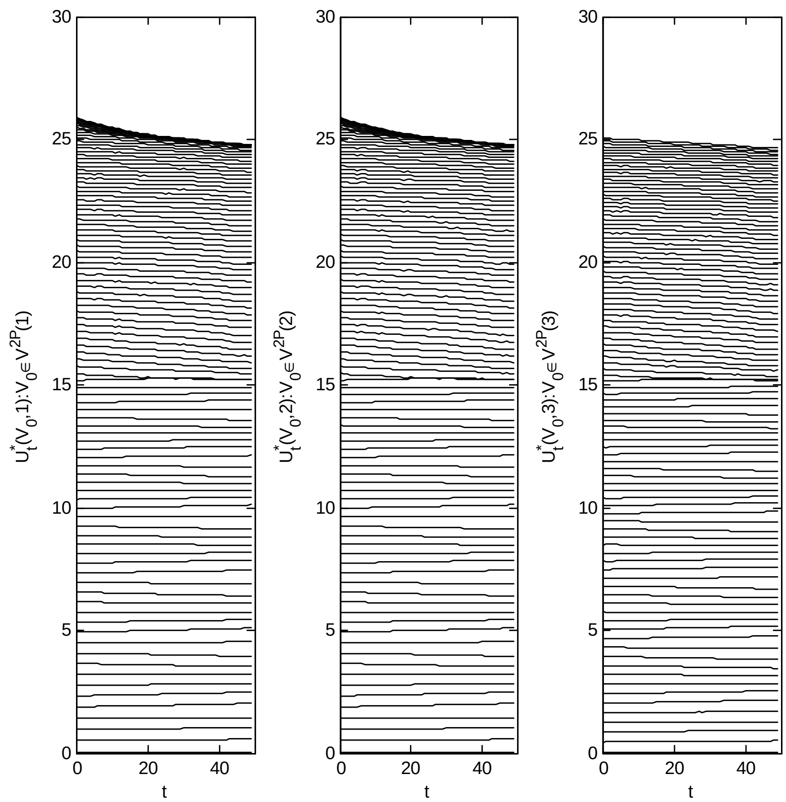

Figure 13: Principal's utility in time: $U_t\,(V_0, l)$: $V_0 \in V^{2P}\,(l)$, $l \in \{1, 2, 3\}$, LMH, case 3

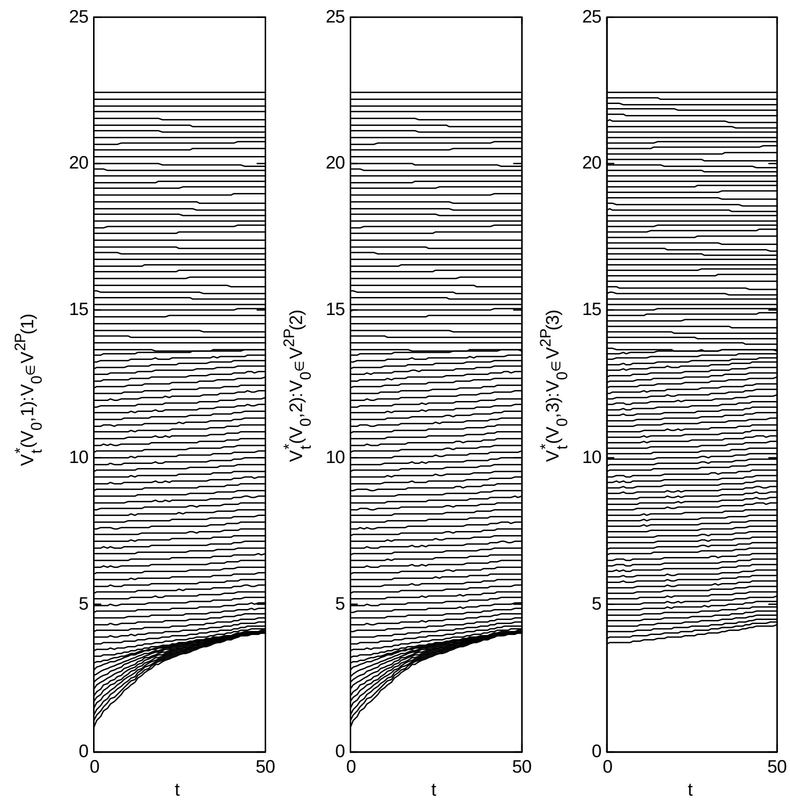

Figure 14: Manager's utility in time: $V_t\left(V_0, l\right)$: $V_0 \in V^{2P}\left(l\right)$, $l \in \{1, 2, 3\}$, LMH, case 3

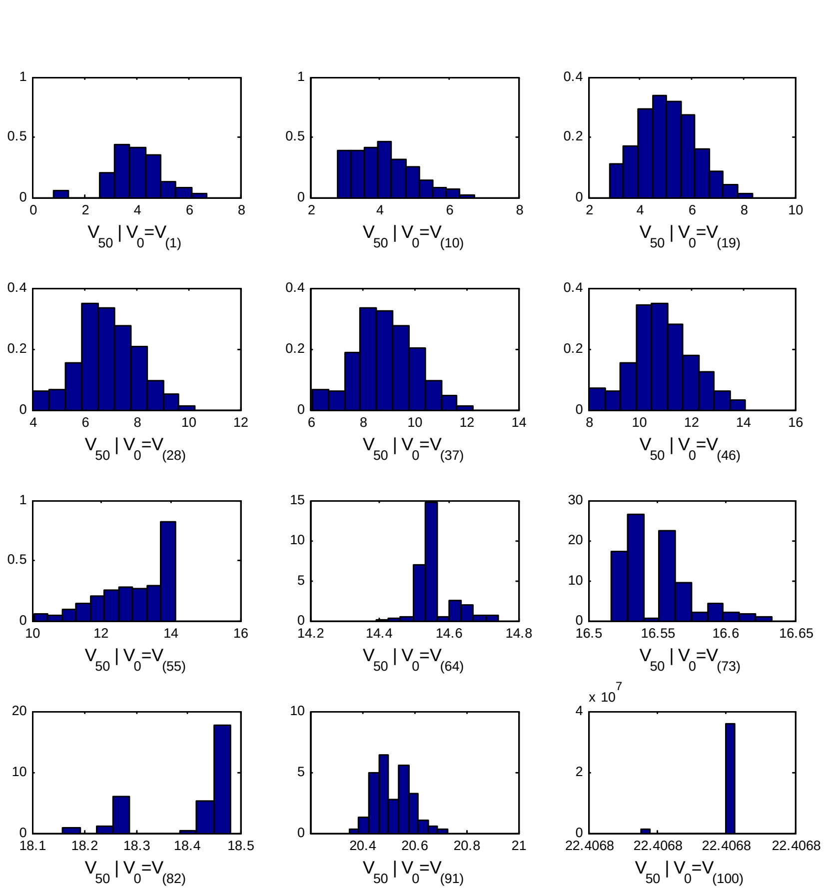

Figure 15: Empirical distribution of manager's utility after 50 periods, V_{50}, conditional on initial history $y_0 = y_{(1)}$ and initial utility promise $V_0 \in V^{2P}(y_0)$, LMH, case 3

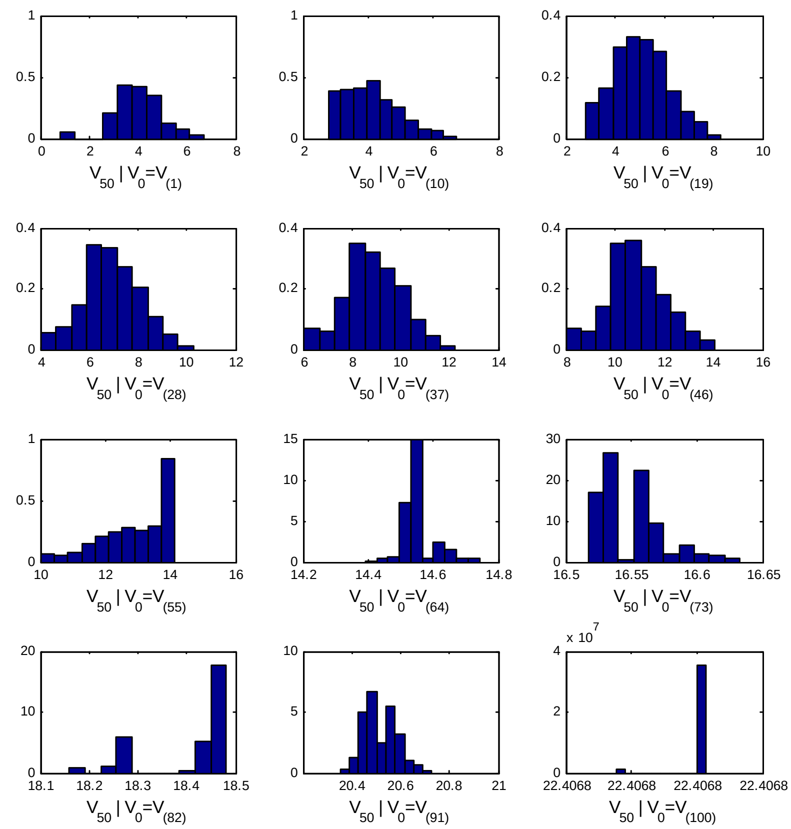

Figure 16: Empirical distribution of manager's utility after 50 periods, V_{50}, conditional on initial history $y_0 = y_{(2)}$ and initial utility promise $V_0 \in V^{2P}(y_0)$, LMH, case 3

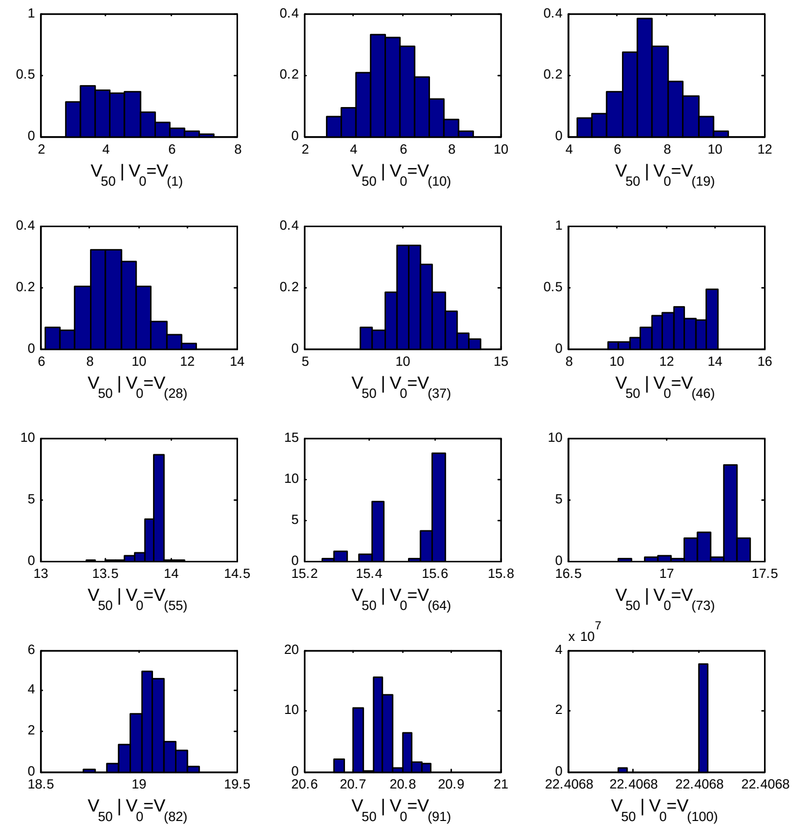

Figure 17: Empirical distribution of manager's utility after 50 periods, V_{50}, conditional on initial history $y_0 = y_{(3)}$ and initial utility promise $V_0 \in V^{2P}(y_0)$, LMH, case 3