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Governments are tempted to occasionally revise fiscal policy by increasing sales taxes or introducing a temporary high tax levy on capital income, because such sudden, unexpected policy revisions produce a wealth expropriation effect, whose efficiency is comparable to lump-sum taxation. However, expectations of wealth expropriation produce the opposite effect of a lump-sum subsidy and motivate the government to decrease the consumption tax and temporarily subsidize capital. When expected and realized expropriations coincide, the government faces exactly the same attainable resource allocation set and provides the same policy, as if there were neither expected nor realized expropriation at all. The Ramsey-optimal policy is then dynamically consistent, implies a near-zero capital income taxation from the beginning of the implementation of the optimal policy, and requires some endogenous adjustment of consumption and labor taxes at the beginning of the policy's implementation.

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Правительство склонно время от времени пересматривать фискальную политику, увеличивая каждый раз налог на потребление, либо вводя временно высокий налог на отдачу от капитала. Подобный пересмотр фискальной политики создает эффект экспроприации богатства, который по эффективности сравним с аккордным налогообложением. Однако ожидания эффекта экспроприации создают противоположный эффект аккордной субсидии и стимулируют правительство сокращать налог на потребление и временно субсидировать капитал. В ситуации, когда ожидаемый и фактический эффект экспроприации совпадают, правительство сталкивается в точности с таким же множеством достижимых распределений ресурсов и проводит такую же политику, как если бы не было ни фактической, ни ожидаемой экспроприации. В этой ситуации оптимальная политика Рамсея является динамически согласованной, предполагает близкий к нулю налог на отдачу от капитала с самого начала реализации оптимальной политики и требует эндогенной подстройки налогов на потребление и на труд в начале реализации оптимального плана.

Ключевые слова: согласованность, равновесная политика, оптимальное налогообложение
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1. Introduction

An unexpected policy revision in the form of an increased sales tax or a temporarily higher tax on capital income can produce a wealth redistribution effect, that is, a transfer of wealth from the private to the public sector without any associated deadweight loss. We call this effect an implicit household wealth *expropriation effect* due to policy revision.

Unexpected expropriation is thus comparable to lump-sum taxation to the extent that both decrease the excess tax burden needed to balance the government budget. Consequently, the government may be tempted to occasionally revise its fiscal plans to produce expropriation effects, creating the problem of dynamic inconsistency of optimal policy.

Modern literature on capital income taxation, which starts with the famous papers by Chamley (1986) and Judd (1985), implicitly accounts for an expropriation effect and reveals the related inconsistency problem. This paper not only takes into account the expropriation effect, but also explicitly models household expectations of such expropriation.

We consider the following situation. Assume that the economy is described by a continuous-time Ramsey-type growth model, and starts at some equilibrium trajectory with a given ex ante (may be suboptimal) fiscal plan. Households recognize that this plan may be revised and that an expropriation effect may arise. At some date, the fiscal policy is in fact revised, and from this date forward, the government implements the so-called Ramsey policy, which maximizes the households' utility in the decentralized economy with respect to fiscal policy instruments. The question of this paper is how expectations of expropriation due to possible policy revisions affect the attainable resource allocation set in the decentralized economy and the optimal policy.

We obtain a set of constraints for the optimal policy problem, which accounts for expropriation and expected expropriation effects. The variables associated with these effects enter into the constraints symmetrically with opposite signs, just like inflation and inflationary expectations enter into the Philips curve. Expropriation shares properties with lump-sum taxation and decreases future tax distortions, but the expectation of expropriation shares properties of lump-sum subsidy and increases them.

A special case of particular interest is that of *equilibrium policy*, where realized and expected expropriations coincide. In this case, the government faces the exact same budget constraint, as if there were no realized or expected expropriations. A natural question is what the properties are of a policy that simultaneously solves the Ramsey optimal policy problem and satisfies the equilibrium policy assumption.

Such an equilibrium Ramsey policy satisfies three basic properties. First, the policy is dynamically consistent. The reason for this is that under an equilibrium policy, any gain from expropriation is offset by a loss resulting from expectations. Therefore, an expropriation does not produce the effect of lump-sum taxation, and the government no longer has an incentive to revise its fiscal policy. Second, the Chamley (1986) and Judd (1985) result of a near-zero, medium-term capital income taxation holds from the very beginning of the implementation of an optimal plan. Third, any policy revision requires the endogenous adjustment of consumption and labor taxes. To clarify the latter point, assume that the government decides to tax labor instead of capital income. Under reasonable parameter values, such a policy revision would increase the real value of the household wealth, and this extra wealth would be implicitly expropriated by increasing the consumption tax. The increased consumption tax creates additional tax revenue, and to maintain the government budget satisfied as equality, the labor tax would be decreased.

We use the primal approach to the optimal fiscal policy problem developed by Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stokey (1983), Chari and Kehoe (1999), and many others. Chari, Christiano and Kehoe (1996) apply the primal approach to the optimal monetary policy problem; their method directly extends our results to the issue of monetary policy. Our solution to the equilibrium Ramsey fiscal policy problem is methodologically comparable to the solution to Woodford's (2003) problem of optimal monetary policy under commitment to a timeless perspective[1] (see Woodford (2003) for the definition of "timeless perspective" and his solution).

The Ramsey policy problem in the modern literature serves as a microfoundation for the Kydland and Prescott (1977) model, which introduces

[1] The solutions are similar but not equivalent. They would be equivalent under the assumption that there has not been any implicit expropriation of household wealth before time $t = 0$, which is where we start the analysis. Additionally, these solutions are derived from different initial problems.

the dynamic inconsistency issue into macroeconomic policy analysis. The assumption that households perfectly forecast inflation in the Kydland-Prescott framework implies that economy moves from the second-best optimum to the worse-than-second-best equilibrium. Their argumentation works well if we assume that policy takes into account the expropriation effect alone, without the expectations. However, the expectated expropriation motivates the Central Bank to decrease inflation below its second-best optimum, and this effect is omitted in the Kydland-Prescott framework. In our framework, in contrast to that of Kydland and Prescott, if households perfectly forecast the expropriation effect, the economy reverts to the second-best optimum.

Section 2 presents the model and explicitly introduces expropriation and expected expropriation into the analysis. In Section 3, we show that the attainable allocation set depends on the expropriation surprise alone and not on the expropriation and the expected expropriation taken separately. In Section 4, we compare the equilibrium policy with that of Chamley. Section 5 concludes.

2. Model

The objective of this section is to introduce explicitly the expropriation and expected expropriation effects into formal analysis of macroeconomic policy.

We consider a continuous-time version of the neoclassical growth model with endogenous labor, which is similar to the one used by Chamley (1986). There are three agents in the model: the representative household, the representative firm, and the government. The household sells labor, buys consumption goods and accumulates wealth. The firm hires labor and produces final goods. The government collects taxes to provide some exogenous amount of final goods. The economy starts at time $t = T_s < 0$ and, moving along some equilibrium trajectory, arrives at $t = 0$, from which we start the analysis.

The fiscal policy is defined by the dynamics of three taxes: consumption tax $\tau_C(t)$, labor tax $\tau_L(t)$, and capital income tax $\tau_K(t)$, which will be formally introduced later. At $t = T_s$, the government announces a policy path $\left[\tau_C(t), \tau_L(t), \tau_K(t)\right]_{t \in \left[T_s, \infty\right)}$. However, it can periodically revise the policy

and may have already done so several times before $t = 0$. Let $\left[\tilde{\tau}_C(t), \tilde{\tau}_L(t), \tilde{\tau}_K(t)\right]_{t \in [0,\infty)}$ be the last policy in $t \in [0,\infty)$ announced before $t = 0$, which we refer to as the *ex ante policy*.

The representative household maximizes expected utility, which depends on consumption C and labor L:

$$\max_{[C(t),L(t)]} E_0 \int_0^{\infty} e^{-\rho t} U\left(C(t), L(t)\right) dt. \tag{1}$$

We take the producer price of the final good to be equal to one. The consumer price of the final good equals $\left(1 + \tau_C(t)\right)$. The household's real wealth $A(t)$ consists of capital $K(t)$ and government debt holdings $B(t)$. The budget constraint is given by

$$\dot{A}(t) = r(t)A(t) + W(t)L(t) - \left(1 + \tau_C(t)\right)C(t), \tag{2a}$$

$$\lim_{t \to \infty} A(t) \; e^{-\int_0^t r(z)dz} \geq 0, \tag{2b}$$

$$A(0) = A_0, \tag{2c}$$

where $r(t)$ and $W(t)$ are the real after-tax equilibrium rate of return and the real wage.

To solve the optimization problem, the household needs to have some idea about how frequently the policy may be revised and what part of its wealth will be implicitly expropriated given that a revision takes place.

To understand the consequences of policy revisions and to introduce a variable associated with the expropriation effect, consider the following example. Assume that the government increases the consumption tax, decreases the labor tax, and gives a lump-sum transfer to the households in such a way that the consumer price $(1 + \tau_C(t))$ and the household initial wealth A_0 double but the real wage $W(t) / (1 + \tau_C(t))$ remains unchanged. It is easy to verify that the household would face exactly the same budget constraint in terms of $C(t)$ and $L(t)$, as before. Moreover, as this policy revision does not affect firms, by Walras' law, the government budget constraint would also not change. As a result, the resource allocation in the economy remains unchanged. Consequently, an unexpected policy revision that increases the consumption tax and leaves the real wage unchanged

produces the same effect as a direct expropriation of some part of the household wealth.

Policy revision affects the shadow price of the household wealth $\gamma(t)$ (the co-state variable for equation (2a)). In the example above, $\gamma(t)$ would decrease by a factor of 2 (this is clear from equation (8a) and from the fact that the policy revision would not affect the choice of $C(t)$ and $L(t)$ but $(1 + \tau_C(t))$ doubles). Moreover, in Section 3, we will see that a downward jump in $\gamma(t)$ modifies the attainable resource allocation set in the decentralized economy in exactly the same way that a direct expropriation of $A(t)$ does. Therefore, we introduce an auxiliary variable $a(t)$, defined as $a(t) = \gamma(t)A(t)$, which measures household wealth in units of the utility function and accounts for both direct expropriations and indirect expropriation effects produced by policy revisions.

Let $X(t)$ be a step function that accounts for the accumulated wealth expropriation effect at date t. We assume that $X(t)$ is constant during the periods in which the policy is not revised, and if a revision takes place, it discontinuously jumps to account for the new wealth expropriation effect:

$$dX(t) = \begin{cases} 0, \text{ if there is no policy revision at date } t \\ -\dfrac{a(t) - \tilde{a}(t)}{\tilde{a}(t)}, \text{ if there is a policy revision,} \end{cases} \tag{3}$$

where $a(t)$ is the actual value of a just after the announcement of a new policy, and $\tilde{a}(t)$ is the value of a that would be if there were no policy revision at t. The initial condition for X is $X(T_s) = 0$.

Let $x(t)$ be the derivative of $X(t)$ with respect to time:

$$x(t) = \dot{X}(t). \tag{4}$$

By definition, $x(t)$ is the so-called Dirac delta function, with $x(t) = 0$ in the intervals where the policy is not revised, and where, at the dates of policy revision, the value of $x(t)$ tends to infinity. However, the integral of $x(t)$ is bounded on any time interval.

The household takes into account the fact that the policy may be revised. It expects that during dt, there will be a revision of the policy with probability $p(t)dt$. If a revision takes place, there is an implicit expropriation of $\phi \times a(t)$ of the wealth, where ϕ is a random variable defined[2] in $\left(-\infty, 1\right)$

[2] For generality we assume that ϕ may be negative or positive. From $\gamma > 0$ we see that

with a distribution function $\xi(\phi,t)$. We do not impose any specific restriction on the functions $p(t)$ and $\xi(\phi,t)$; the only sensitive assumption is that each particular household considers $p(t)$ and $\xi(\phi,t)$ as given. Let $x^e(t)$ be the expected expropriation rate at any given instant:

$$x^e(t) = p(t)\int_{-\infty}^{1}\phi\xi(\phi,t)d\phi. \tag{5}$$

Similarly to $x(t)$, the value of $x^e(t)$ may tend to infinity at some particular points in time, but the integral of $x^e(t)$ on any time interval remains bounded. In contrast to $x(t)$, $x^e(t)$ may be positive in some time intervals.

The accumulated expected wealth expropriation effect is $X^E(t)$ and defined as

$$X^E(t) = \int_{T_s}^{t} x^e(\tau)d\tau. \tag{6}$$

The expropriation surprise x^s and accumulated expropriation surprise X^s are introduced as follows:

$$x^s(t) = x(t) - x^e(t), \tag{7a}$$

$$X^s(t) = X(t) - X^e(t). \tag{7b}$$

The first-order conditions of the household problem are (see Appendix A for details)

$$U_C\big(C(t),L(t)\big) = \big(1+\tau_C(t)\big)\gamma(t), \tag{8a}$$

$$U_L\big(C(t),L(t)\big) = -W(t)\gamma(t), \tag{8b}$$

$$\dot\gamma(t) = \big(\rho - r(t) + x^s(t)\big)\gamma(t). \tag{8c}$$

Note that $\gamma(t)$ jumps at the dates where $X^s(t)$ jumps.

We assume perfectly competitive markets and constant returns to scale, which implies that there is no profit. The production function depends on labor $L(t)$ and capital $K(t)$ and is given by

$$Y(t) = F\big(K(t),L(t)\big). \tag{9}$$

The rate of depreciation is δ.

$\phi < 1$. Consequently, in the most general case ϕ may be defined on $(-\infty,1)$.

The capital income and labor taxes are $\tau_K(t)$ and $\tau_L(t)$. The before-tax interest rate and wage are $\hat r(t)$ and $\hat W(t)$: $r(t) = \big(1-\tau_K(t)\big)\hat r(t)$ and $W(t) = \big(1-\tau_L(t)\big)\hat W(t)$. The firms' first-order conditions are given by

$$\hat r(t) = F_K\big(K(t),L(t)\big) - \delta, \tag{10a}$$

$$\hat W(t) = F_L\big(K(t),L(t)\big). \tag{10b) (14) (15)}$$

The government collects taxes to supply an exogenous amount of public goods $G(t)$. Its budget constraint can be written as

$$\dot B(t) = r(t)B(t) + G(t) - \tau_C(t)C(t) -$$

$$-\tau_K(t)\hat r(t)K(t) - \tau_L(t)\hat W(t)L(t) \tag{11a}$$

$$\lim_{t\to\infty} B(t)\, e^{-\int_0^t r(z)dz} \le 0 \tag{11b}$$

$$B(0) = B_0. \tag{11c}$$

Market clearing requires

$$\dot K(t) = Y(t) - C(t) - G(t) - \delta K(t), \tag{12a}$$

$$K(t) \ge 0 \ \forall t, \tag{12b}$$

$$K(0) = K_0. \tag{12c}$$

In Section 3, we define the attainable allocation set in the decentralized economy, assuming that the government periodically revises the policy without any particular objective, and we allow X^s to change over time. In Section 4, we assume that the government solves the equilibrium Ramsey (1927) policy problem: it maximizes household utility in the decentralized economy with respect to the fiscal policy $\big[\tau_C(t),\tau_K(t),\tau_L(t)\big]_{t\in[0,\infty)}$ under the constraint that the resulting policy must satisfy the equilibrium policy condition $x^s(t) = 0\ \forall t \ge 0$.

3. Expropriation and the attainable allocation set

This section reveals why our conclusions differ from those of Chamley and Judd. Chamley and Judd implicitly assume that a positive value of x is possible only at $t = 0$ and $x^e = 0 \; \forall \geq 0$. Consequently, they arrive at the result that the greater degree to which the government expropriates at the beginning of the optimal policy's implementation leads to a better policy outcome.

However, the expected expropriation affects the attainable resource allocation set, and there is no reason to believe that x^e is always zero. In this section we demonstrate that under an equilibrium policy (defined as $x = x^e \; \forall t$) the expropriation does not affect the attainable resource allocation set.

The set of allocations that are attainable by the social planner (who finds the first-best allocation) is given by the resource constraint. This constraint may be found by the substitution of the production function (9) into the market clearing condition (12):

$$\dot{K}(t) = F\left(K(t), L(t)\right) - C(t) - G(t) - \delta K, \tag{13a}$$

$$K(t) \geq 0 \; \forall t, \tag{13b}$$

$$K(0) = K_0. \tag{13c}$$

The implementability constraint ensures that a particular allocation can be decentralized without lump-sum taxes. This constraint requires that for a considered allocation $\left[C\left(t\right), L\left(t\right)\right]_{t \in [0,\infty)}$, there exists a vector of consumer prices that simultaneously satisfies the household budget constraint and its first-order conditions[3]. To get the implementability constraint, differentiate $a(t) = \gamma(t)A(t)$ with respect to time and substitute (2) and (8) into the obtained equation. The expropriation-augmented implementability constraint is given by

$$\dot{a}(t) = \rho a(t) - U_C\left(C(t), L(t)\right) \cdot C(t) - U_L\left(C(t), L(t)\right) \cdot L(t) - x^s(t)a(t), \tag{14a}$$

$$\lim_{t \to \infty} a(t)e^{-\rho t} = 0. \tag{14b}$$

[3] In an economy with two goods, the implementability constraint coincides with the price-consumption curve.

Condition (14b) is derived from inequalities (2b), (11b), and (12b) and from the knowledge that $\lim_{t \to \infty} K\left(t\right) e^{-\int_0^t r(z)dz} \geq 0$, which complete each other and, together with (8c), ensure that (14b) is satisfied with equality.

There are two differences between the conventional implementability constraint (see, for example, Chari and Kehoe (1999)) and the expropriation-augmented constraint (14). First, there is a new term $x^s(t)a(t)$ in (14a). Second, for a given value of X_0, the value of a_0 is also given, while in the conventional constraint, a_0 is not determined.

The government finds the equilibrium under the ex ante policy $\left[\tilde{\tau}_C(t), \tilde{\tau}_L(t), \tilde{\tau}_K(t)\right]_{t \in [0,\infty)}$ and arrives at $\tilde{a}(0)$. The value of X_0 is historically given. Consequently, the initial conditions for are given by

$$a(0) = \tilde{a}(0), \tag{14c}$$

$$X(0) = X_0. \tag{14d}$$

Note that a policy revision that produces a wealth expropriation effect at date 0 will change not only a_0 but also X_0. Consequently, the pair of variables $\left\{a_0, X_0\right\}$ is predetermined, though each variable taken separately from the other is not.

The resource and implementability constraints with the initial and transversality conditions exactly describe the set of allocations that may be implemented in a decentralized economy without lump-sum taxes. Proof of this fact is well known in the literature; see Appendix B for details.

Proposition 1. *The attainable resource allocation set depends on the expropriation surprise x^s but not on x and x^e separately.*

Proof. The attainable resource allocation set is given by the resource constraint (13) and the expropriation-augmented implementability constraint (14). We see that only x^s enters into these constraints.

4. Equilibrium Ramsey Policy

In this section we derive the Ramsey policy under assumption of an equilibrium policy. In fact, the assumption of an equilibrium policy is not realistic; however, just as in the case of the rational inflationary expecta-

tions hypothesis, any alternative assumption would be even less realistic. The discussion of weak and strong features of the rational expectation hypothesis is applicable to the hypothesis of equilibrium policy. Additionally, the condition of equilibrium policy is a necessary (but not sufficient) condition for a Nash equilibrium in pure strategies. As a result, a natural question concerns the properties of the equilibrium Ramsey policy.

4.1. The modified Ramsey problem

Under equilibrium policy, we have $x^s(t) = 0 \ \forall t$, and the optimal policy problem takes the form

$$\max_{[C(t),L(t)]} \int_0^\infty e^{-\rho t} U\left(C(t), L(t)\right) dt \tag{15a}$$

$$\dot{K}(t) = F\left(K(t), L(t)\right) - C(t) - G(t) - \delta K(t), \tag{15b}$$

$$\dot{a}(t) = \rho a(t) - U_C\left(C(t), L(t)\right) \cdot C(t) - U_L\left(C(t), L(t)\right) \cdot L(t), \tag{15c}$$

$$\lim_{t\to\infty} a(t) e^{-\rho t} = 0, \tag{15d}$$

$$a(0) = \tilde{a}(0), \tag{15e}$$

$$K(0) = K_0. \tag{15f}$$

The co-state variable is $\lambda(t)$ (negative) for the implementability constraint and $\mu(t)$ (positive) for the resource constraint. The first-order conditions are

$$U_C\left(C(t), L(t)\right) \cdot \left(1 - \lambda(t)\left(1 + H_C\left(C(t), L(t)\right)\right)\right) = \mu(t), \tag{16a}$$

$$\frac{U_L\left(C(t), L(t)\right)}{F_L\left(K(t), L(t)\right)} \cdot \left(1 - \lambda(t)\left(1 + H_L\left(C(t), L(t)\right)\right)\right) = -\mu(t), \tag{16b}$$

$$0 = \dot{\lambda}(t), \tag{16c}$$

$$\left(\rho - \left(F_K\left(K(t), L(t)\right) - \delta\right)\right)\mu(t) = \dot{\mu}(t), \tag{16d}$$

where the terms $H_C\left(C(t), L(t)\right)$ and $H_L\left(C(t), L(t)\right)$ are given by

$$H_C\left(C(t), L(t)\right) = \frac{U_{CC}\left(C(t), L(t)\right)}{U_C\left(C(t), L(t)\right)} C(t) + \frac{U_{CL}\left(C(t), L(t)\right)}{U_C\left(C(t), L(t)\right)} L(t) \tag{17a}$$

$$H_L\left(C(t), L(t)\right) = \frac{U_{CL}\left(C(t), L(t)\right)}{U_L\left(C(t), L(t)\right)} C(t) + \frac{U_{LL}\left(C(t), L(t)\right)}{U_L\left(C(t), L(t)\right)} L(t) \tag{17b}$$

The term $H_i\left(C(t), L(t)\right)$ is a measure of the excess tax burden related to a particular form of taxation. It plays the same role as the inverse elasticity of demand in the microeconomic analysis of the deadweight loss of taxation; see Atkinson and Stiglitz (1980). A possible interpretation of $(-\lambda(t))$ is the marginal excess burden of taxation measured in terms of utility.

Proposition 2. *The solution to the optimal policy problem is dynamically consistent.*

Proof. The optimal policy is implicitly defined by constraints (15b)-(15f) and first-order conditions (16). These equations do not depend on the time perspective from which the optimal policy problem is solved. Consequently, solutions found from different time perspectives imply the same optimal dynamics, and the solution is dynamically consistent.

4.2. Optimal policy

Equations (16) and the constraints to the Ramsey problem (15) give the resource allocation under the optimal policy. To determine the policy itself, it is necessary to combine the first-order conditions of the household's problem (8) with the first-order conditions of the optimal policy problem (16), taking into consideration the initial condition (15t).

There are an infinite number of policies that implement (16) and decentralize the optimal allocation. To obtain a unique policy, we exogenously define the dynamics of one of the tax rates. Suppose that the consumption tax is constant and that its value is chosen to satisfy (18a). The optimal policy is then given by

$$1 + \tau_C = (1 + \tilde{\tau}_C(0)) \frac{U_C(C(0), L(0))}{U_C(\tilde{C}(0), \tilde{L}(0))}, \tag{18a}$$

$$1 - \tau_L(t) = \left(1 + \tau_C\right) \frac{\Phi_L\left(C(t), L(t)\right)}{\Phi_C\left(C(t), L(t)\right)}, \tag{18b}$$

$$\tau_K(t)\hat{r}(t) = \frac{\dot{\Phi}_{C,t}\left(C(t),L(t)\right)}{\Phi_{C,t}\left(C(t),L(t)\right)}, \quad (18c)$$

where

$$\Phi_C\left(C(t),L(t)\right) = \left(1-\lambda\left(1+H_C\left(C(t),L(t)\right)\right)\right)^{-1}, \quad (19a)$$

$$\Phi_L\left(C(t),L(t)\right) = \left(1-\lambda\left(1+H_L\left(C(t),L(t)\right)\right)\right)^{-1}. \quad (19b)$$

Equation (18a) was derived from the definition $a(t)=\gamma(t)A(t)$, constraint (15t), and equation (8a). Equation (18b) was derived from (8a), (8b), (16a), and (16b). Equation (18c) was derived from (8a), (8c), (16a), and (16d).

4.3. Properties of the equilibrium Ramsey policy

There are three basic properties of the equilibrium Ramsey policy. First, by Proposition , it is dynamically consistent. Second, capital income taxation is close to zero even at the beginning of the optimal policy implementation, given that the economy close to the balanced-growth path. If $H_C\left(C(t),L(t)\right)$ is constant over time, the optimal capital income tax is zero. This is possible either if H_C does not depend on C and L (for example, if $U(C,L)=\frac{C^{1-\theta}-1}{1-\theta}+V(L)$), or if the economy moves along a balanced growth path, which implies that H_C is constant.

Third, the optimal consumption and labor taxes are adjusted at the beginning of the implementation of the optimal plan to avoid any change in a_0. If we neglect certain second-order effects that we discuss in the next paragraph, then the required changes in consumption and labor taxes may be approximately calculated in the following manner. Suppose that a decrease in the capital income tax increases the after-tax interest rate by 10%. Then, the capitalists become 10% richer, and to compensate for this effect, the consumer price of the final good $(1+\tau_C)$ should be increased by 10%. The new value of the labor tax should ensure that the intratemporal government budget constraint is satisfied.

This arithmetic works well in "$Y=AK$"-type models, where the decrease of the capital income tax creates a permanent effect on the real interest rate and when the excess tax burden of distortionary taxation is not too high. However, in exogenous growth models, the effect of a decrease in τ_K on a is temporary; consequently, it requires a smaller increase in the consumption tax. In addition, the reduction in the capital income tax produces another effect: this tax will be substituted by the consumption or labor taxes. The increased consumption or labor tax will increase the ratio $\frac{1+\tau_C}{1-\tau_L}$, decrease the labor supply, decrease the before-tax interest rate, and reduce a_0. These effects require a decrease in the consumption tax. In the general case, it is not clear whether the consumption tax, or the labor tax, will be increased or decreased.

Finally, note that on the balanced-growth path, all taxes are constant. Consequently, the optimal debt-to-GDP ratio is also constant.

5. Conclusions

This paper introduces expectations of household wealth expropriation due to policy revisions into the analysis of fiscal policy. We show that actual and expected expropriations enter into the implementability constraint symmetrically with opposite signs. If expected and realized expropriation coincide, they offset each other, and the attainable allocation set in the decentralized economy is the same as if there were neither expected nor realized expropriation at all.

Under equilibrium policy, intensive capital income taxation at the beginning of the optimal policy does not imply a lump-sum taxation of households' initial wealth and creates only an unnecessary consumption distortion. As a result, in contrast to the Chamley result, we show that intensive capital income taxation at the beginning of the optimal policy is suboptimal.

The only reason for the inconsistency of the Chamley policy is the desire to produce a positive expropriation surprise. Under the equilibrium policy, an expropriation surprise is not possible, and the policy is dynamically consistent.

The Chari, Christiano, and Kehoe (1996) approach allows to apply the consistency result to the optimal monetary policy problem.

Appendix

Appendix A. First-order conditions for the expropriation-augmented household problem

Let $V(A(t),X(t),t)$ be the value function

$$V(A(t),X(t),t)=\max_{[C,L]}E_t\int_t^{\infty}e^{-\rho\tau}U\left(C(t),L(t)\right)d\tau. \tag{20}$$

Taking into account that

$$\begin{aligned}&E_tV(A(t+dt),X(t+dt),t+dt)=\\&=\left(1-p(t)dt\right)V(A(t+dt),X(t),t+dt)+\\&+p(t)dt\int_{-\infty}^{1}V(A(t+dt),X(t)+\phi,t+dt)\xi(\phi,t)d\phi,\end{aligned} \tag{21}$$

the Bellman equation can be written as

$$\begin{aligned}0=\max_{[C,L]}\Bigg(&e^{-\rho t}U\left(C,L\right)+\frac{V(A(t+dt),X(t),t+dt)-V(A(t),X(t),t)}{dt}+\\&+p(t)\int_{-\infty}^{1}\left(V(A(t+dt),X(t)+\phi,t+dt)-V(A(t+dt),X(t),t+dt)\right)\xi(\phi,t)d\phi\Bigg).\end{aligned} \tag{22}$$

We will use a Taylor decomposition for the second term and substitute $\dot{A}$ from (2). Taking the limit as $dt\to 0+$, this gives his gives:

$$\begin{aligned}0=\max_{[C,L]}\Bigg(&e^{-\rho t}U\left(C(t),L(t)\right)+V_A(A(t),X(t),t)(rA+WL-(1+\tau_C)C)+\\&+V_t(A(t),X(t),t)+p(t)\int_{-\infty}^{1}\left(V(A(t),X(t)+\phi,t)-V(A(t),X(t),t)\right)\xi(\phi,t)d\phi\Bigg).\end{aligned} \tag{23}$$

Equation (23) is the Bellman equation for the problem. The first-order conditions are

$$e^{-\rho t}U_C\left(C(t),L(t)\right) = (1+\tau_C(t))V_A\left(A(t),s(t),t\right), \tag{24a}$$

$$e^{-\rho t}U_L\left(C(t),L(t)\right) = -W(t)V_A\left(A(t),s(t),t\right). \tag{24b}$$

Let $\gamma(t)$ be the shadow price of the household's wealth:

$$\gamma(t)=V_A\left(A(t),s(t),t\right)e^{\rho t}, \tag{25}$$

then, equations (24) give (8a) and (8b).

Application of the envelope theorem gives

$$\begin{aligned}0=&V_{AA}(A(t),X(t),t)\dot{A}+rV_A(A(t),X(t),t)+V_{At}(A(t),X(t),t)+\\&+p(t)\int_{-\infty}^{1}\left(V_A(A(t),X(t)+\phi,t)-V_A(A(t),X(t),t)\right)\xi(\phi,t)d\phi\Bigg).\end{aligned} \tag{26}$$

Differentiating (25) with respect to time yields

$$\dot{\gamma}(t)=\left(V_{AA}\left(A(t),s(t),t\right)\dot{A}(t)+V_{At}\left(A(t),s(t),t\right)\right)e^{\rho t}+\rho\gamma(t). \tag{27}$$

From equations (25), (26), and (27), taking into account(3), (4), and (5), we arrive at the last first-order condition for the expropriation-augmented household problem (8c).

Appendix B. The attainable allocation set (comments to Section 3)

The derivation of the attainable allocation set that we use in Section 3 is well known in the literature; see, for example, Lucas and Stockey (1983).

We obtain the resource (13) and implementability (14) constraints from conditions that are satisfied for any equilibrium allocation; consequently, they are also satisfied for any equilibrium allocation.

If an allocation $\left[C(t),L(t)\right]_{t\in[0,\infty)}$ satisfies equation (14), then for any given strictly positive dynamics of one of the consumer prices $\left[r(t),\tau_c(t),W(t)\right]_{t\in[0,\infty)}$, there exist dynamics of the other prices such that the household will choose the given allocation. Indeed, the first-order conditions (8) and definitions $a(t)=\gamma(t)A(t)$ and (3) give prices such that these conditions are satisfied, and the substitution of these prices into the

implementability constraint gives the household's budget constraint. Consequently, the household's budget constraint is also satisfied.

If an allocation $\left[C(t), L(t)\right]_{t \in [0,\infty)}$ satisfies the resource constraint (13), then we can find the dynamics of the producer prices $\left(\hat{r}, \hat{W}\right)$ under which firms will choose an input-output vector such that the equilibrium market condition is satisfied. Indeed, from equation (13) and the initial conditions, we can calculate the dynamics $K(t)$ that give the dynamics of the output $Y(t) = C(t) + G(t) + \dot{K}(t) + \delta K(t)$. Knowing the dynamics of $Y(t)$, $K(t)$, and $L(t)$, we can use the firms' first-order conditions to find the prices $\left(\hat{r}(t), \hat{W}(t)\right)$ under which the firms choose the considered allocation, $\hat{r}(t) = F_K\left(K(t), L(t)\right)$ and $\hat{W}(t) = F_L\left(K(t), L(t)\right)$.

If both constraints are satisfied, the government budget constraint is also satisfied by Walras' law. Therefore, these constraints guarantee that there exist vectors of consumer and producer prices such that all equilibrium conditions are satisfied. The tax rates that decentralize the considered allocation may be found from the ratios of consumer to producer prices.

References

[1] Atkinson, A.B. and J.E. Stiglitz (1980). *Lectures on Public Economics*. McGrow-Hill, Maidenhead, UK.

[2] Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica* 54 (3): 607–622.

[3] Chari, V.V., P. J. Kehoe (1999). Optimal fiscal and monetary policy. In: J. B. Taylor, M. Woodford (ed.), *Handbook of Macroeconomics*, volume 1, chapter 26, pages 1671–1745 Elsevier.

[4]Chari, V. V., L.J. Christiano, P.J. Kehoe (1996) Optimality of the Friedman rule in economies with distorting taxes. *Journal of Monetary Economics* 37: 203–223.

[5] Judd, K. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics* 28: 59–83.

[6] Kydland, F. and E. Prescott (1977). Rules Rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* 85: 473–491.

[7] Lucas, R.E. and N.L. Stokey (1983). Optimal Fiscal and Monetary Policy in an Economy Without Capital. *Journal of Monetary Economics*, 12: 55–93.

[8] Ramsey, F. (1927). A contribution to the theory of taxation. *Economic Journal* 37: 47–61.

[9]Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

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