Inter-temporal Poverty Measures: The Impact of Affluence

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Introduction

Our new class of inter-temporal poverty measures
  - Stylised examples
  - Axiomatic characterisation

A more general class of inter-temporal poverty measures
  - Axiomatic characterisation

Concluding remarks

Appendix: Review of literature on inter-temporal poverty measurement
Consider 2 individuals

Both are poor in some time periods but well off in others

They are each poor at different times, e.g.

- Individual 1 \( (p, 0, p) \)
- Individual 2 \( (p, p, 0) \)

Which one is poorer?

- Static literature can only tell us who is poorer at a given time
Our new class of inter-temporal poverty measures

- Focus on an individual’s inter-temporal poverty
- Weighted average of static poverty
  - using any static measure from the literature but normalised poverty gap perhaps most appropriate
- Impact of poor episode discounted according to number of periods of relative affluence directly preceding it
- Motivated by idea that the longer the period of relative affluence prior to becoming poor, the better equipped one is likely to be to deal with subsequent deprivation
  - Deprivation mitigated to some extent by resources accumulated during recent relatively affluent past
Our new class of inter-temporal poverty measures

Set up

- Poverty $p_t$ in period $t$, $t \in \{1, \ldots, T\}$
- Let $\mathbf{p} \in \mathbb{R}^T_+$ be an individual’s poverty profile
- Define $n_t :=$ number of consecutive non-poor periods immediately prior to period $t$.
- Individual poor at time $t$ if and only if $p_t > 0$
- Then $P_\beta(\mathbf{p}) = \frac{1}{T} \sum_{t=1}^{T} w_t p_t$ where $w_t = \left(\frac{1}{1+n_t}\right)^\beta$ and $\beta \geq 0$
- $\beta$ can be interpreted as social planner’s choice of how much to allow resources accumulated during preceding non-poor periods to mitigate impact of subsequent poor period
Our new class of inter-temporal poverty measures

\[ P_1\left(\frac{1}{2}, 0, \frac{1}{2}, 0\right) = \frac{1}{4} \left[ \left(\frac{1}{1}\right) \frac{1}{2} + 0 + \left(\frac{1}{1+1}\right) \frac{1}{2} + 0 \right] = 0.1875 > \]  

(1)

\[ P_1\left(\frac{1}{2}, 0, 0, \frac{1}{2}\right) = \frac{1}{4} \left[ \left(\frac{1}{1}\right) \frac{1}{2} + 0 + 0 + \left(\frac{1}{1+2}\right) \frac{1}{2} \right] \approx 0.17 \]  

(2)

- Our measures rank individual (1) poorer than (2)
- (1) has been out of poverty for only 1 period prior to suffering a second period of hardship whereas (2) is out of poverty for 2 periods before suffering a second poor period
Our new class of inter-temporal poverty measures

\[ P_1 \left( 0, \frac{3}{4}, \frac{1}{2}, \frac{3}{4} \right) = \frac{1}{4} \left[ 0 + \left( \frac{1}{1+1} \right) \frac{3}{4} + \left( \frac{1}{1} \right) \frac{1}{2} + \left( \frac{1}{1} \right) \frac{3}{4} \right] \approx 0.41 \] (3)

\[ P_1 \left( \frac{3}{4}, \frac{1}{2}, \frac{3}{4}, 0 \right) = \frac{1}{4} \left[ \left( \frac{1}{1} \right) \frac{3}{4} + \left( \frac{1}{1} \right) \frac{1}{2} + \left( \frac{1}{1} \right) \frac{3}{4} + 0 \right] = 0.5 \] (4)

- Rank (4) poorer than (3) as no affluent periods directly preceding poor periods
- Foster (2007) and Bossert, Chakravarty and D’Ambrosio (2008) rank both profiles equally
Focus on characterisation of measure for specific case $\beta = d$

\[ P_d(p) = \frac{1}{T} \sum_{t=1}^{T} w_t p_t \text{ where } w_t = \left( \frac{1}{1+n_t} \right)^d \]
Axiomatic characterisation of our new measures

- **Axiom 1** Single Period Equivalence

  For all $p \in \mathbb{R}_+$, $P(p) = p$

- **Axiom 2** Normalization

  For all $T \in \mathbb{N}$ and all $p \in \mathbb{R}^T_+$ such that $p_t = 0 \ \forall \ t \in \{1, \ldots, T\}$,
  $P(p) = 0$
Axiomatic characterisation of our new measures

- **Axiom 3** *Time Decomposability*

For all $T \in \mathbb{N} \setminus \{1\}$, for all $p \in \mathbb{R}^T_+$ and all $t \in \{1, \ldots, T - 1\}$, if $p_t \neq 0$, $P(p) = \frac{t}{T} P(p_1, \ldots, p_t) + \frac{T-t}{T} P(p_{t+1}, \ldots, p_T)$

**Definition**

Let $e^T_t$ be the $T$-vector in which $e_k = 0$ $\forall$ $k \in \{1, \ldots, T\} \setminus \{t\}$ and $e_t = 1$

- **Axiom 4** *Deprivation Mitigation*

For all $T \in \mathbb{N} \setminus \{1\}$ and all $b \in \mathbb{R}_+$, $P(b \cdot e^T_j) = \frac{P(b \cdot e^T_1)}{j^d}$
Axiomatic characterisation of our new measures

**Theorem**

An individual inter-temporal poverty measure $P(p)$ satisfies Single-period equivalence, Normalization, Time Decomposability and Deprivation Mitigation if and only if $P = P_d$. 
It must be stressed that in this particular axiomatic framework, every value of $\beta$ requires a slightly different version of Axiom 4

Justification for choosing any particular $\beta = d$ is unclear

A general axiomatisation of the entire class $P_\beta$ remains a topic for future research
Any variant of Axiom 4 is very strong and gives rise to a specific functional form.

A more general class arrives if we abandon Axiom 4 and replace it with two weaker axioms.

Consider the more general measure

\[ P_g(p) = \frac{1}{T} \sum_{t=1}^{T} w_t p_t \] where \( w_t = f(n_t) \) is such that

\[ f(n_t + 1) \leq f(n_t) \leq 1 \text{ and } f(0) = 1 \]
A more general class of inter-temporal poverty measures

Together the following 2 axioms capture much of the spirit of Axiom 4:

- **Axiom 5** Period Monotonicity

Let $1 < t \leq T$. Then $P(p \cdot e_{t-1}^T) > P(p \cdot e_t^T)$

- **Axiom 6** Independence

Let $1 < t \leq T$ and $\lambda > 0$. Suppose that $P(p \cdot e_t^T) = P(p' \cdot e_{t-1}^T)$. Then $P(\lambda p \cdot e_t^T) = P(\lambda p' \cdot e_{t-1}^T)$
A more general class of inter-temporal poverty measures

**Theorem**

An individual inter-temporal poverty measure $P(p)$ satisfies Single-period equivalence, Normalization, Time Decomposability, Period Monotonicity and Independence if and only if $P = P_g$. 
Concluding remarks

- Have presented 2 new classes of inter-temporal poverty measures and their characterisations
- Individual inter-temporal poverty is a weighted average of static poverty in each time period
- Weights are determined by number of affluent periods directly preceding each poor period
Concluding remarks

- Both classes similar in spirit to measure of Bossert et al. (2008)
- Have advantage that 2 poor spells which are close together but not consecutive are deemed more debilitating than 2 spells separated by longer period of relative affluence
- Motivated by intuitive notion that the longer the preceding period of affluence, the better equipped one is likely to be to deal with the subsequent poor spell
- Also have advantage that the mitigating impact of affluence is consistent with the direction of flow of time
Concluding remarks

Topics for further research include:

- Characterisation of the first class $P_\beta$ in its most general form
- Finding empirical evidence for what the functional form of $w_t = f(n_t)$ should be
- Finding a persuasive means of allowing the extent of affluence in the non-poor periods to have an impact
Appendix: Review of literature on inter-temporal poverty measurement

- Possible ways of dealing with time dimension include:
  - All periods equally important
    - e.g. Jalan and Ravallion (2000), Foster (2007), Foster and Santos (2009)
  - Deprivation amplified by consecutive periods of poverty
    - e.g. Bossert, Chakravarty & D’Ambrosio (2008)
  - Assigning greater importance to spells early in life
    - e.g. Hoy & Zheng (2008)
  - Deprivation increases as distance between spells decreases
    - e.g. Hoy & Zheng (2008)
Appendix: Review of literature on inter-temporal poverty measurement

- Bossert et al. (2008)
  First considers an individual’s inter-temporal poverty
  Weighted average of static poverty (using any static measure from the literature)
  Weights assigned to individual’s level of poverty in each time period to account for
  - Number of consecutive periods of poverty
  - Number of periods of relative affluence between poverty spells, e.g.

\[
P_B \left( \frac{1}{2}, \frac{1}{2}, 0 \right) = \frac{1}{3} \left( 2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 1 \cdot 0 \right) = \frac{2}{3} > \\
P_B \left( \frac{1}{2}, 0, \frac{1}{2} \right) = \frac{1}{3} \left( 1 \cdot \frac{1}{2} + 1 \cdot 0 + 1 \cdot \frac{1}{2} \right) = \frac{1}{3}
\]

- Societal poverty a simple average of individual inter-temporal poverty
An interesting contribution in terms of dealing with the amplified deprivation caused by longer periods of poverty

However focusing solely on consecutive periods of poverty is limiting

Doesn’t account for deprivation caused by poverty episodes which are close together but not quite consecutive, e.g.

\[
P_B\left(\frac{1}{2}, 0, \frac{1}{2}, 0\right) = P_B\left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)
\]
Appendix: Review of literature on inter-temporal poverty measurement

- Foster (2007) also evaluates $P\left(\frac{1}{2}, 0, \frac{1}{2}, 0\right) = P\left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$
- Hoy and Zheng (2008) and Foster and Santos (2009) may or may not determine $P\left(\frac{1}{2}, 0, \frac{1}{2}, 0\right) > P\left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$
  - Depends on extent of wealth during the non-poor periods
- This apparent gap in literature provides some of the motivation for our classes of measures
References


Thank-you!