Marginal Deadweight Loss when the Income Tax is Nonlinear

Sören Blomquist and Laurent Simula

Uppsala University and Uppsala Center for Fiscal Studies

SCW 2010, Moscow
1 Introduction

2 Setting

3 Simple Example (Smooth Budget Constraint)

4 General Case (Smooth Budget Constraint)

5 Marginal Deadweight Loss for Piecewise Linear Budget Constraint

6 Conclusions
Motivation

- Large interest in marginal deadweight loss of taxes. Goes back to Jules Dupuit: "De la mesure de l’utilité des travaux publics" (1844).

- Textbooks and almost all applications: computations for a linear tax linear budget constraint.

- Quite surprising since most income tax systems are nonlinear, generating nonlinear budget constraints.
Equivalent variation

Historically, much interest in labour supply. More recently, focus on taxable income.
Marginal DWL for linear income tax

- Utility maximisation programme:
  \[ \max_{A,C} U(C, A, \nu) \text{ s.t. } C \leq A - tA. \]

- Expenditure function:
  \[ E(t, \nu, \bar{u}) = \min_{A,C} \{ C - A + tA \} \text{ s.t. } U(C, A, \nu) \geq \bar{u}. \]

- Hicksian supply: \( A^h(t, \nu, \bar{u}). \)

Marginal deadweight loss for linear income tax

\[ DWL = \frac{dE(t, \nu, \bar{u})}{dt} - \frac{d[tA^h(t, \nu, \bar{u})]}{dt} = A^h() - A^h() - t \frac{dA^h}{dt} = -t \frac{dA^h}{dt} \]
Actual tax systems are:

- nonlinear $\rightarrow$ usually piecewise linear;
- defined by many parameters.

Many possible ways to vary the tax:

- changing break points;
- changing the intercept;
- changing the slope (in different ways).
Marginal DWL for nonlinear income tax (II.)

Here: focus on a change in the slope such that the marginal tax changes with the same number of percentage points at all income levels.

- Tilt of budget constraint.
- Compensation by an increase in the intercept.
- Closest to a variation of a linear budget constraint.
Clean experiment similar to a change in the slope of a linear budget constraint.
Interpretation

Deadweight loss of a marginal increase in:

- a payroll tax,
- or a proportional local (state) income tax, etc.
Questions

- How to correctly calculate the marginal deadweight loss when the income tax is nonlinear?
- Evaluate bias in results that obtains when the traditional linearization procedure is used.

Main results

- For tax systems where the marginal income tax increases with the taxable income, the usual linearization procedure often leads to a significant overestimate of the marginal deadweight loss.
- Difference concentrated at the kinked points for piecewise linear tax systems.
- Computation of the marginal deadweight loss of the US tax system.
Marginal DWL with smooth budget constraint (I.)

- $A$: taxable income.
- Tax on $A$:
  \[
  T(A) = g(A) + tA
  \]

  with $g'(A) > 0$, $g''(A) > 0$ and $t \geq 0$.
  
  - $g(A)$: nonlinear federal tax.
  - $tA$: a payroll tax or a proportional state income tax. Example: local community tax in Scandinavia.
  - Real tax systems are of the form $g(A) + tA$. 

S. Blomquist and L. Simula (Uppsala U.)

Deadweight Loss

SCW 2010 10 / 23
Marginal DWL with smooth budget constraint (II.)

- Utility maximization problem:
  \[ \max_{A,C} U(C, A, v) \text{ s.t. } C \leq A - g(A) - tA + B. \]
  \[ \rightarrow A(t, B, v), C(t, B, v). \]

- Expenditure function:
  \[ E(t, v, \bar{u}) = \min_{A,C} \{ C - A + g(A) + tA - B \} \text{ s.t. } U(C, A, v) \geq \bar{u}. \]
  \[ \rightarrow A^h(t, v, \bar{u}), C^h(t, v, \bar{u}). \text{ [h for Hicksian]} \]

- Compensated revenue function:
  \[ R(A^h(t, v, \bar{u})) = g(A^h(t, v, \bar{u})) + tA^h(t, v, \bar{u}). \]

Marginal deadweight loss for nonlinear tax

\[ DW = \frac{dE(t, v, \bar{u})}{dt} - \frac{dR(A^h(t, v, \bar{u}))}{dt} = - \left( g'(A^h) + t \right) \frac{dA^h}{dt}. \]
Choose $v^*$, $t^*$ and $B^*$. Budget constraint linearized around $A^* = A(t^*, v^*, B^*)$, $C^* = C(t^*, v^*, B^*)$:

$$C \leq A - p_A A + M$$

with $M = C^* - A^* + p_A A^*$ and $p_A = g'(A^*) + t^*$. 

- "Linearized" utility maximization problem:
  $$\max_{A, C} U(C, A, v^*) \text{ s.t. } C \leq A - p_A A + M$$
  $$\rightarrow A_L(p_A, v^*, M), C_L(p_A, v^*, M) \ [L \text{ for linear}]$$

- "Linearized" expenditure function:
  $$E_L(t, v, \bar{u}) = \min_{A, C} \{C - A + p_A A - M\} \text{ s.t. } U(C, A, v) \geq \bar{u}$$
  $$\rightarrow A^h_L(t, v, \bar{u}), C^h_L(t, v, \bar{u}) \ [L \text{ for linear, } h \text{ for Hicksian}]$$

- Compensated revenue function:
  $$R(A^h_L(t, v, \bar{u})) = g(A^h_L(t, v, \bar{u})) + tA^h_L(t, v, \bar{u})$$
Marginal deadweight loss with linearization

\[ DW_L = \frac{dE_L(t, v, u)}{dt} - \frac{dR_L(A^h_L(t, v, u))}{dt} = - \left( g' (A^h_L) + t \right) \frac{dA^h_L}{dt}. \]
Commonly used procedure (II.)

Marginal deadweight loss with linearization

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Marginal deadweight loss for nonlinear tax

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**Marginal deadweight loss with linearization**

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**Marginal deadweight loss for nonlinear tax**

\[ DW = \frac{dE(t, v, \bar{u})}{dt} - \frac{dR(A^h(t, v, \bar{u}))}{dt} = - \left( g' \left( A^h \right) + t \right) \frac{dA^h}{dt}. \]

- By construction, \( A^h \equiv A^h_L \).
- But, in general, \( \frac{dA^h}{dt} \neq \frac{dA^h_L}{dt} \).
For the given function $A - g(A) - tA + B$, we have the following conditions:

- $A(\cdot) = A_L(\cdot) = A^h(\cdot) = A^h_L(\cdot)$
**Simple example**

- \( U = C - \alpha A - \beta A^2 \). [no income effect on \( A \)]
- \( T(A) = \text{state tax} + \text{federal tax} = (tA) + (pA + \pi A^2) \).
- \( \frac{dA^h}{dt} = -\frac{1}{2(\pi + \beta)} \) and \( \frac{dA^h}{dt} = -\frac{1}{2\beta} \)

**Implication**

The true marginal deadweight loss is lower than the one obtained using the linearization procedure.
Simple example

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Example (Magnitude of the overestimation?)

\( \pi = \beta = 0.1 \). Then \( \frac{dA^h}{dt} = -2.5 \) and \( \frac{dA^h}{dt} = -5. \) \( \implies \) The linearization procedure overestimates the deadweight loss with a factor 2.
Figure: Deadweight loss when the budget constraint is nonlinear (left panel) and linearized (right panel)
E \left( t, v, \bar{u} \right) = \min_{A,C} \left\{ C - A + g\left( A \right) + tA - B \right\} \text{ s.t. } U\left( C, A, v \right) \geq \bar{u}.

• \( U\left( C, A, v \right) = \bar{u} \) in optimum \iff \( C = f\left( A, v, \bar{u} \right) \) \( \text{where } f \text{ defined by } U\left( f\left( A, v, \bar{u} \right), A, v \right) = \bar{u} \).

• \( E \left( t, v, \bar{u} \right) = \min_{A} f\left( A, v, \bar{u} \right) - A + tA + g\left( A \right) - B \).

• FOC: \( f'\left( A, v, \bar{u} \right) - 1 + t + g'\left( A \right) = 0 \rightarrow A^h\left( t, v, \bar{u} \right). \) \( \left[ f'\left( \right) \equiv \frac{\partial f}{\partial A} \right] \)

• Differentiating implicitly:

\[
\frac{dA^h}{dt} = -\frac{1}{g'' + f''}.
\]
General Utility Function (II.)

Result

The linearization procedure overestimates the "true" deadweight loss.

- $f'(A, v, \bar{u})$: slope of the indifference curve.
- $f''(A, v, \bar{u})$: curvature of the indifference curve.

The curvature of the budget constraint is as important for the size of the marginal deadweight loss as is the curvature of the indifference curve.

What matters is the curvature of the indifference curve in relation to the budget constraint.
Piecewise linear budget constraint (I.)

Tax system as above, but piecewise linear federal tax.

Sufficient to consider tax system with two linear segments and one kink point. Results easily generalize to tax system with many kinks.

\( \tau_1 \) = federal tax, first bracket;
\( \tau_2 \) = federal tax, second bracket;
\( t \) = state income tax.

<table>
<thead>
<tr>
<th>First segment:</th>
<th>Second segment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- the intercept ( R_1 ) is lump-sum income;</td>
<td>- virtual income ( R_2 = R_1 + (\tau_2 - \tau_1) A_1 ) independent of ( t );</td>
</tr>
<tr>
<td>- slope ( \theta_1 = 1 - \tau_1 - t ).</td>
<td>- slope ( \theta_2 = 1 - \tau_2 - t ).</td>
</tr>
</tbody>
</table>
Piecewise linear budget constraint (II.)
Changes in the budget constraint when $t$ increases?

- $R_1$ and $R_2$ do not change.

- Kink point still at $A_1$ but its $C$-coordinate decreases by the amount of the extra tax paid $dt \times A_1$. 
Changes in the budget constraint when $t$ increases?

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**Person located at kink point before and after the change**

- $dA^h/dt = 0$
- No marginal deadweight loss for him/her.

**Person with a tangency on one of the linear segments**

- Variation just like variation in linear budget constraint.
- Apply function generated by linear budget constraint.

[People moving in/out kink point: set of measure 0.]
Piecewise linear budget constraint (IV.)

- Aggregate marginal deadweight loss:

\[
DW_{TRUE} = - \sum_{i=1}^{2} \int_{S_i} (\tau_i + t) \frac{dA^h}{dt} \phi(v) \, dv + 0 \times \int_{K_1} \phi(v) \, dv.
\]

\(S_i\): People on a linear segment

\(K_1\): People at kink point

Results

- To compute the aggregate marginal deadweight loss, we only integrate along the segments of the budget sets; the contribution from individuals bunched at the kink point is zero.

- In the piecewise linear case, the difference in the two measures is concentrated to the kink.
Conclusion

- Derive correct way to calculate marginal deadweight loss when budget constraint is smooth and convex. Show that the curvature of the budget constraint is equally important for the size of the marginal deadweight loss.

- Show how to calculate the marginal deadweight loss when the tax system generates a piecewise linear budget constraint. Impact of the curvature of the budget constraint to diminish the deadweight loss now concentrated to the kink points.

- Numerical calculations and comparison with computations obtained by linearizing the budget constraint.