Arbitrage Inefficiency and Regulation in Multi-Stage Market Games

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The aim is to investigate the efficiency of asset market regulatory schemes modeled as strategies in repeated market games.

In this framework, both “Arbitrage-free" (with risk aversion) and “bubble price" (with risk neutrality) equilibria can emerge as Nash equilibria of the stage game.

The question that arises is: Can we implement Pareto improving outcomes by designing regulations (strategies for the repeated game) that do not result a Nash equilibrium in the stage game?
Motivation

- Important role in this task plays the incorporation of an extended peer-monitoring mechanism that is self-enforcing for traders.
- Traders remain self-regulated and each operates as a "policeman" for the implementation of the designated regulatory scheme.
- Thus, equal attention is paid to the design of supervision mechanisms that accompany the optimal regulatory framework.
An Overview of the Literature

- The market mechanisms for our economy are met in Shapley & Shubik [JPE1977] and Dubey & Shubik [JET1978]
- Money is introduced as in Shubik & Tsomocos [J.Econ 92]
- Relevant to this work are Allen & Gale [JPE97], Freixas & Tsomocos [WP04], Peck & Shell [RES92] that emphasize on the financial intermediation rather than regulation while Giraud & Weyers [ET04] show how in a dynamic market game with incomplete markets asset mispricing emerge.
- Analytical tools for our results are the folk theorems in finitely repeated and OLG games given by Benoit & Krishna [Econometrica85], Kandori [RES92], Salant [GEB91], Smith [GEB92]
The economy

- A dynamic monetary economy $t = 1, 2, \ldots$
- A finite number of traders $I \geq 3$
- $L$ commodities with the $L$th commodity to be the numeraire
- A Bank converts the numeraire good into IOU notes. For all trades we use IOU money.
- Financial markets are composed of two securities
  1. (deposit certificate) A long-lived security that converts one unit of money today into one unit at any future time (storage technology), and has infinite supply.
  2. (a standard fixed income security) A short-lived security that converts one unit of money today into $c > 1$ units tomorrow (investment technology) and has fixed supply.
Security prices $\pi = (\pi_{\lambda}, \pi_s) \in \mathbb{R}^2_+$ are determined by the formation rule,

$$\pi^j(t) = \begin{cases} \frac{\sum_{i \in N} \sigma^i_j}{s}, & \text{if } j = s \\ 1, & \text{if } j = \lambda \end{cases}$$

and the allocations are,

$$\theta^i_j(t) = \begin{cases} \frac{\sigma^i_j}{\sum_{i \in N} \sigma^i_j}, & \text{if } j = s \\ \sigma^i_j, & \text{if } j = \lambda, \end{cases}$$

Actions in security markets are,

$$\Sigma^i(t) = \{ (\sigma^i_{\lambda}, \sigma^i_s) \in \mathbb{R}^2_+ | \sum_{j=\lambda,s} \sigma^i_j \leq m^i(t) \}.$$
The commodity markets

We use the "offer-for-sale" model for commodity markets. Prices are formed by the rule,

\[ p(t) = \begin{cases} \frac{\sum_{i \in N} b_i}{\sum_{i \in N} q_i}, & \text{if } l \neq L \\ 1, & \text{if } l = L \end{cases} \]

and the allocation will be

\[ x^i(t) = \begin{cases} w^i - q^i + \frac{b_i}{p_i} & \text{if } p_i \neq 0 \\ w^i & \text{if } \text{else} \end{cases} \]

Actions in commodity markets are,

\[ A^i(t) = \{(b^i, q^i) \in (R_+^L)^2 | \forall l \in L, i \in N, \sum b^i \leq m^i(t) + c\theta^i_s(t - 1) + \theta^i_\lambda(t - 1), q^i_l(t) \leq w^i_l(t)\} \]
Action sets for \( i \), \( B^i(t) = \Sigma^i(t) \times A^i(t) \)

- The standard security has non-negative return only if actions taken from \( F^i(t) = \{ \beta^i(t) \in B^i(t) | \sum_{s \in N} S \sigma^i_s \leq c \} \)

- Actions where a trader buys deposits only, are called *regulatory dominant*. When he buys both securities, these actions are called *prudential*.

- When a trader set his sight to the standard security and buys no deposits, we say that plays a *payoff dominant* action
The stage game

Proposition

At stage level, interior Nash equilibria satisfy $c = \pi^S$.

- Traders buy the standard security as long as the return remains positive.
- In equilibrium, traders become indifferent between the two securities.
- Hence, $\beta^* \in \bar{F}^i(t) = \{\beta^i(t) \in B^i(t) | \sum_{i \in N} \sigma^i_s = c\}$. 
The T-repeated game

Proposition

There is no “profitable" strategy that sustains an interior Nash equilibrium for the T-repeated game

- Traders play in equilibrium either “regulatory dominant" or they play “prudential".
- The next question is “can we figure out strategies that ensure a positive return to traders at least for some periods of their lifetime?"
Regulating the security markets

Important in the implementation of efficient regulation schemes in our model is supervision. We assume two different games where fundamental role plays a peer monitoring process. We solve two games:

1. A T-repeated game where the regulation of the market is enforced by the Bank
2. An infinitely horizon game with an overlapping term structure of traders where the market is self-regulated by using richer strategies
We assign the Bank the role of the regulatory authority.

The Bank enforces a designated strategy that regulates the actions taken by traders in the standard security market. I.e. you ought to play some prudential action when young and as a reward you will be eligible to play a payoff dominant action for some periods in your old age.

If someone deviates, the Bank penalizes him by holding a part of his numeraire good when redeemed.
Some drawbacks

The above mechanism standing alone may be problematic. Because:

- Market supervision is costly.
- Traders could deviate massively.

For we strengthen the mechanism by incentivizing peer monitoring.

- Peers can flag a trader as a deviator and in return they receive a bonus when redeem their gold from the Bank. Of course, the deviator is penalized by the Bank.
The Bank implements a **regulatory scheme** and bans deviator from the standard security market. A regulatory scheme is a triplet \((\theta^0, (\theta^1, \bar{\theta}^1))\)

- \(\theta^0\) is the allocation of fractional reserves when traders respect the designated strategy
- \(\theta^1\) the bonus the "policemen" receive and,
- \(\bar{\theta}^1\) the fractional reserves the deviant traders receive when there are no adequate periods to offset the gains from deviation.

Peers ban the deviator from real commodity markets

- If a "policeman" deviates from punishing the deviator, then he becomes the "deviator" and the penalty applies to him.
The Regulated Economy

The suggested strategy

The equilibrium strategy $\beta'$: $\beta'_i = (\beta'(1), \beta'(2), \ldots, \beta'(T); \theta^0_i)$. In case of deviation we shift to:

$$\beta_{ij} = \begin{cases} (\beta_{ij}(1), \ldots, \beta_{ij}(Q), \beta'(Q + 1), \ldots, \beta'(T); \theta^0_i) & \text{if } Q < T, \\ (\beta_{ij}(1), \ldots, \beta_{ij}(T); \bar{\theta}_1^i) & \text{if } Q \geq T. \end{cases}$$

with $Q$ the efficient punishment period.

**Proposition**

There is a reallocation rule of fractional reserves $\theta$, that can sustain $\beta'$ as a Subgame Perfect Equilibrium.
When Bank cannot enforce a regulatory scheme, we can still implement a regulation strategy. In this new setup,

- An overlapping term structure of traders is necessary
- We span trader’s life in three periods, "young" age, "middle" age and "old" age.
- Traders in "old" age are permitted to play the payoff dominant action for some periods as long as they committed to the designated equilibrium path along their previous lifetime.
Self-regulation

- If traders defect in "young" or "middle" age, they are minimaxed by others for several periods, enough to offset their instantaneous gains. The "losses" of policemen during the punishment period will be compensated in the "old age".

- Still, someone may deviate in the old age if the gains are significant and no sufficient time remains to punish them.

- For we introduce a buffer zone in the “old age”, called the payoff adjustment phase where we can reward or punish an old trader. The compensations and/or rewards for the young and the middle age will be received in the last phase of their lifetime, the reward phase.
The equilibrium strategy $\beta'$:

$$\beta' = (\bar{\beta}, \ldots, \bar{\beta}, \bar{\beta}', \ldots \beta', \beta', \ldots, \beta', \hat{\beta}, \ldots, \hat{\beta}).$$

- $\bar{\beta}$ is a regulatory dominant action
- $\hat{\beta}$ is a payoff dominant action

**Proposition**

*There is a self-regulating strategy $\beta'$ for the OLG market game such that for long enough lifespan $T$, can be sustained as a Subgame Perfect Equilibrium.*
We suggest a bonus-malus regulation scheme for the Bank aiming at regulating asset markets,

The regulation is supplemented by a peer monitoring scheme. Bank has not always the capacity to supervise efficiently the markets. Peer-monitoring fulfills this role as a supplementary surveillance mechanism.

Alternatively, a decentralized mechanism is possible and an optimal punishment scheme is suggested.

Future research could be directed to the role of central Bank in interbank lending markets as well as to the contemplation of self regulated organizations (bourses or brokers’ associations etc..)