

Micro Frictions, Asset Pricing, and Aggregate Implications*

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Abstract

There is a long standing debate in the macro-economics literature about the importance of non-convex micro level adjustment costs for aggregate quantities. In particular, (Thomas 2002), and (Khan and Thomas 2008) argue that non-convex micro frictions are unimportant for aggregate quantities; (Bachmann, Caballero, and Engel 2010) have argued that such frictions are crucial to match the heteroscedasticity and time-dependence of aggregate investment rate. However these models typically do not perform well when one considers asset pricing and the identification of micro-frictions is not clear. (This debate has to an extent spilled over to papers considering asset pricing, such as (Gomes, Kogan, and Zhang 2003) and (Garleanu, Panageas, and Yu 2009)). On the other hand, convex adjustment costs have been used to help match asset pricing moments in models by (Jermann 1998), (Zhang 2005), (Tuzel 2010), (Croce 2010), (Kaltenbrunner and Lochstoer 2010) however little attention has been paid to micro level implications. We explore micro level frictions in a model guided by asset pricing insights: in our model the relevant stochastic variable is a stationary growth rate (as opposed to a trend-stationary level) of productivity. We use asset pricing insights to show that non-convex micro frictions are not necessary to match the aggregate moments, including those emphasized by Bachmann et. al. suggesting that convex costs alone are an appropriate modeling choice when considering only the aggregate. However, non-convex frictions can affect aggregate quantities for certain parameter choices, confirming the findings of Bachmann et. al. Our best model, involving a combination of convex and non-convex costs, is able to match aggregate macroeconomic moments, micro-level investment moments, as well as the high Sharpe Ratio of equity returns. The model also suggests that partial equilibrium insights on the value premium may not carry over to general equilibrium.

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1 Introduction

There exists a long standing debate in the macro economic literature over whether modeling micro level frictions is important for matching aggregate quantities. (Khan and Thomas 2008) argue that in general equilibrium prices adjust making micro frictions irrelevant. However (Bachmann, Caballero, and Engel 2010) show that non-convex frictions can matter for aggregate quantities under certain conditions; in particular non-convex frictions help match the heteroscedasticity of aggregate investment, which standard frictionless models cannot do.

The heart of the debate lies in whether general equilibrium forces can cancel out aggregate investment demand implied by micro lumpy investment. Consider firms that face a non-convex (ie fixed) cost to invest; such firms will have a cut off rule in deciding whether to invest a large amount or none at all. Suppose many firms are just below the cutoff and not investing, then a small positive aggregate shock can drive a large number of firms over the hump, resulting in large swings of investment as everyone suddenly invests (extensive margin). Without fixed costs firms will only adjust the quantity of investment (intensive margin) and such large swings in response to small shocks would not occur. By the logic of Khan and Thomas, general equilibrium forces prevent a large number of firms from concentrating just below the cutoff because investment is valuable and prices would reflect this, imploring some of the firms to invest earlier. However, (Bachmann, Caballero, and Engel 2010) show that both adjustment costs and general equilibrium forces play a relevant role. In particular, when extensive margin is calibrated to have a more important role in shaping aggregate investment than general equilibrium constraints, non-convex frictions can have a consequential effect on aggregate quantities.

While the models above are solely calibrated to match multiple aggregate and firm level quantities, they fail to match the asset prices observed in the data; this is a problem endemic to most standard models, as observed by (Mehra and Prescott 1985). Moreover different calibrations of capital adjustment costs in (Khan and Thomas 2008) versus (Bachmann, Caballero, and Engel 2010) result in lack of restrictions in identifying the form and size of micro-rigidities making it harder to interpret the results. Asset pricing moments can provide an additional mechanism

to identify the appropriate way to model frictions and shed light on the above debate. To that end, we use insights from the finance literature to build a model with a non-trivial cross-section of heterogeneous firms and realistic asset prices. In particular household preferences are recursive and the growth rate (as opposed to the level) of shocks is stationary. (Tallarini 2000), (Bansal and Yaron 2004), (Croce 2010), (Kaltenbrunner and Lochstoer 2010) show that such shocks are important for asset prices. We then study the implications of frictions for aggregate quantities and for asset pricing in such a model.

Our key finding is that while non-convex frictions can matter for aggregate quantities under some calibrations (consistent with Bachmann et. al.), these types of frictions are not necessary to match the aggregate investment moments emphasized by Bachmann et. al. This is because shocks to the growth rate of productivity (exactly the types of shocks necessary to produce high Sharpe Ratios in a Long Run Risk world) naturally imply aggregate investment rates which are heteroscedastic and time dependent. Our preferred model combines convex costs (which help with aggregate investment dynamics) and non-convex costs (which help with firm level investment dynamics but are not necessary for the aggregate) and is able to sufficiently match aggregate real business cycle moments; additional aggregate investment dynamics emphasized by Bachmann et. al.; high Sharpe Ratios and interest rates which are low and smooth; as well as firm level investment rates and distributions. Since most of the work in our preferred model is done by convex costs, yet the model sufficiently matches both micro and macro observations, the results provide a justification for models such as (Jermann 1998), (Croce 2010), (Kaltenbrunner and Lochstoer 2010) who use a representative firm with convex costs only. As with most production models, our model fails to match the high volatility of equity returns observed in the data, although this volatility is twice that of standard models. Interestingly, we also find that partial equilibrium intuition about cross-sectional asset pricing may not carry over to general equilibrium.

When, as in standard models, shocks are to the level of productivity, firms have an optimal level of capital associated with each productivity level. When the productivity level increases

due to a positive shock, so does optimal capital and firms choose investment rate based on the distance to the optimum. Subsequent positive shocks are counterbalanced by mean reversion, resulting in little change to the currently optimal (high) productivity and capital levels. The result is an initial jump in aggregate investment rate, followed by a slow decline towards zero as more positive shocks come because firms are closer and closer to their optimal capital; this can be seen in the upper panel of Figure 3. On the other hand when the growth of productivity is trend stationary, a shock to productivity implies a permanent change in the level of productivity. Subsequent positive shocks are again counterbalanced by mean reversion, but this time it is the growth rate, rather than level of productivity that stays high. This results in further increases to productivity and optimal capital, requiring even more investment. This leads to time dependence in investment rate, with investment rate growing (falling) as the expansion (recession) gets longer.

These trend stationary shocks to the growth rate (as opposed to the level) of productivity imply changes to productivity not only today or tomorrow, but for many years to come. Through production, these shocks also affect consumption over many periods forward. While an agent with CRRA preferences only cares about changes to today's consumption growth rate, agents with recursive preferences also care about these long term changes, making this economy riskier from their perspective. Therefore, these shocks, combined with recursive preferences, produce high Sharpe Ratios as in (Bansal and Yaron 2004).

(Zhang 2005) is a partial equilibrium model in which the combination of shocks to the level of productivity and convex adjustment costs leads to a conditional CAPM where low market-to-book (value) firms are more risky and have high expected returns. This is because they are low productivity firms burdened with high capital. We find these results continue to hold in general equilibrium when we model shocks as level. Interestingly, when shocks are growth rate stationary and the elasticity of intertemporal substitution is as parameterized by (Bansal and Yaron 2004), this result reverses and value firms are actually less risky with lower expected returns. This result suggests an alternative mechanism for matching the value premium should be considered if long run risk is the underlying mechanism driving aggregate returns.

2 Review of Literature and Empirical Facts

2.1 Investment

We will first briefly review empirical properties of firm level and aggregate investment, as well as some of the models used to match these facts.

Firm level investment tends to be lumpy, with periods of inaction followed by periods of large investment (spikes); similar plant level findings have been documented by (Doms and Dunne 1998), (Davis and Haltiwanger 1992), (Caballero, Engel, and Haltiwanger 1995), (Cooper and Haltiwanger 2006). These papers have found that non-convex (fixed) costs of investment are necessary at the plant level to match these spikes and inactions. For example at the plant level, investment rate has an autocorrelation of 0.05. (Cooper and Haltiwanger 2006) show that such a low autocorrelation can happen if firms face fixed costs and invest nothing most of the time while spike up investment periodically. Convex costs instead lead to positively autocorrelated investment because firms with high investment demand invest step by step as huge investment spikes in a single period are extremely costly. We find that at the firm level autocorrelation is 0.36, making fixed costs less important.

A large fraction of variation in aggregate investment is due to this lumpiness or extensive margin. (Gourio and Kashyap 2007) show that plants with spikes ($I/K > 0.20$) account for half of total investment and 98% of the variation in investment. Their model, in which plants face fixed costs of investment, is able to match these facts.

Despite this rich structure for micro investment, it is not clear that it is relevant for thinking about aggregate investment. For example (Cooper and Haltiwanger 2006) solve a partial equilibrium model with a rich specification of adjustment costs. In their model, non-convex costs are crucial for matching firm level investment, however when they compute their model's implied aggregate investment, they find that a model with convex costs alone does reasonably well. Convex models do not track aggregate data well only around turning points in aggregate productivity. (Thomas 2002) argues that even during such turning points, non-convex costs are irrelevant once general

equilibrium forces are taken into account; essentially prices adjust in a way to prevent large swings in investment. Their general equilibrium model with firm level non-convex costs has aggregate results identical to a frictionless model. This is a sharp contradiction to partial equilibrium analysis of (Caballero, Engel, and Haltiwanger 1995), (Caballero, Engel, and Haltiwanger 1997), and (Caballero and Engel 1999). (Khan and Thomas 2003), (Khan and Thomas 2008) confirm that the above irrelevance results holds in a more general model. (Veracierto 2002) shows that investment irreversibility does not play a significant role in a standard real business cycle model.

Aggregate investment rate (I/K) is less volatile than output. (Bachmann, Caballero, and Engel 2010) highlight two additional features of aggregate investment: it is heteroscedastic and time dependent. Volatility in investment rate is high when past I/K is high. In Figure 2 (reproduced from (Bachmann, Caballero, and Engel 2010)) I/K is regressed against lagged I/K and the squared residual is plotted against lagged I/K ; the upward slope is evidence of heteroscedasticity. Additionally, investment rate tends to increase (fall) as the expansion (recession) gets longer. (Bachmann, Caballero, and Engel 2010) refer to this as time dependence and it implies that longer expansions (recessions) will be associated with larger increases (decreases) in investment. In the lower panel of Figure 1 we plot the average change in I/K against the length of NBER defined expansions and recessions between 1947 and 2008; coefficients from the associated regressions are positive and significant for expansions and negative and marginally significant for recessions.

[Figure 1 about here.]

These heteroscedasticity and time dependence are especially important during times of high stress (ie long recessions or expansions) but cannot be matched by standard frictionless models as will be discussed in our results. To improve the model performance along this dimension, (Bachmann, Caballero, and Engel 2010) provide a counter-example to the (Thomas 2002) claims of irrelevance of micro-frictions for aggregate dynamics. They argue that the finding of irrelevance is due to calibration: the calibration in (Thomas 2002) implies sectoral investment volatility that

is too high¹. With an alternative calibration (Bachmann, Caballero, and Engel 2010) show that adding non-convex frictions can help their model match the heteroscedasticity in investment rate. Our analysis contributes to this literature by investigating the asset pricing implications and investment quantities of a general equilibrium model with non-trivial micro-heterogeneity. Different from the existing analysis, we gauge the importance of TFP growth rate shocks instead of the more standard TFP level shocks. We show that the growth rate shocks have important implications for both aggregate investment and asset prices.

[Figure 2 about here.]

2.2 Asset Pricing

This paper is also related to papers that study asset pricing in models of production and focus on relationships between firms' capital investment and the time series and cross sectional risk premia². For example, (Gomes, Kogan, and Zhang 2003) study the cross section of returns in a general equilibrium model with production and growth options; however project sizes are fixed and options are take it or leave it. They show that size and book-to-market are correlated with model implied conditional beta and therefore predict stock returns. They also show that such a model is identical to an aggregate AK model with convex adjustment costs. (Garleanu, Panageas, and Yu 2009) relax the take it or leave it assumption and show that the result looks more like a convex aggregate production function. (Zhang 2005) finds that asymmetric adjustment costs can help quantitatively explain the value premium in a partial equilibrium model. Using a similar mechanism, (Tuzel 2010) shows that due to adjustment frictions, firms with slower depreciating capital (ie high real estate holdings) are riskier and have higher expected returns. Our work differs from (Gomes, Kogan, and Zhang 2003) in that we employ (Epstein and Zin. 1989) preferences together with TFP growth rate shocks which delivers a sizable equity premium; furthermore our

¹See other related work, e.g., (Gourio and Kashyap 2007), (House 2008), (Bloom, Floetotto, and Jaimovich 2009), etc.

²An incomplete list of contributions in this area include (Cochrane 1991), (Cochrane 1996)(Berk, Green, and Naik 1999), (Carlson, Fisher, and Giammarino 2004), (Zhang 2005), (Li, Livdan, and Zhang 2009), (Liu, Whited, and Zhang 2008), (Lin 2009), (Bazdresch, Belo, and Lin 2009), etc.

firms are free to adjust their capital stock. In (Gomes, Kogan, and Zhang 2003) equity risk is mainly from the volatility of the risk free rate. We differ from (Zhang 2005) in that we build up a full-fledged general equilibrium model and allow for a richer set of investment frictions and preferences.

Our work also relates to a growing literature on the relationship between asset prices and low frequency shocks. For instance, (Tallarini 2000) shows that modeling the growth rate instead of the level of shocks as trend stationary has important implications for welfare and asset pricing. (Bansal and Yaron 2004) demonstrate that a small long-run predictable component in the consumption growth process can explain key asset markets phenomena. (Alvarez and Jermann 2005) derive a lower bound for the volatility of the permanent component of asset pricing kernels and show that permanent innovations to consumption are key determinants of financial securities. We confirm in our model that the permanent shocks are important for asset prices but differ from (Bansal and Yaron 2004) and (Alvarez and Jermann 2005): i) instead of assuming shocks to an exogenous consumption growth rate process, consumption is endogenously determined in general equilibrium and the permanent shock in our model is to TFP; ii) the general equilibrium setup allows us to investigate the implications of permanent shocks to aggregate quantities as well as prices.

Finally, our work is related to several papers which study the effect of investment frictions on asset prices in production economies. (Jermann 1998) and (Boldrin, Christiano, and Fisher 2001) combine habit persistence preferences with investment frictions and show that this helps match both asset pricing and quantity moments. Our economy is closest to recent models by (Croce 2010) and (Kaltenbrunner and Lochstoer 2010) who show that a combination of low frequency shocks, recursive preferences, and adjustment costs can improve the asset pricing performance in a real business cycle model. However, the above papers consider a representative firm and a representative household; we allow for heterogeneity and study the effects of firm level frictions.

3 Model

3.1 Households

In the model financial markets are complete, therefore we consider one representative household who receives labor income, chooses between consumption and saving, and maximizes utility as in (Epstein and Zin. 1989).

$$U_t = \max \left((1 - \beta)C_t^{1-1/\psi} + \beta E_t[U_{t+1}^{1-\theta}]^{\frac{1-1/\psi}{1-\theta}} \right)^{\frac{1}{1-1/\psi}}$$

$$W_{t+1} = (W_t + N_t * w_t - C_t)R_{t+1}$$

where R_{t+1} is the return to a portfolio over all possible financial securities. For simplicity, we assume labor supply is inelastic: $N_t = 1$.

3.2 Firms

The interesting frictions in the model are on the firm's side. Firms choose investment and labor to maximize the present value of future dividend payments where the dividend payments are equal to the firm's output net of investment, wages and various other costs. Output is produced from labor and capital. Firms hold beliefs about the discount factor M_{t+1} , which is determined in equilibrium. Firms are indexed by i , which is suppressed where the notation is clear.

$$V_t = \max_{I_t} E_t \left[\sum_{i=0, \infty} M_{t+i} D_{t+i} \right]$$

$$D_t = \Pi(K_t, N_t) - I_t - \Phi(I_t, K_t)$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$\Pi(K_t, N_t) = \max_{N_t} Z_t K_t^{\alpha_K} N_t^{1-\alpha_N} - w_t N_t - f$ is net income, given by output less labor and operating costs. It can be rewritten as $\Pi(K_t, N_t) = Z_t X_t K_t^\alpha - f$ where $\alpha = \alpha_K / \alpha_N$, X_t is an

aggregate variable that depends only on the aggregate wage and f is operational leverage. Note that when $\alpha_K = \alpha_N$, output is homogenous of degree one in capital and labor. We will consider decreasing returns to scale: $\alpha_K < \alpha_N$.

Z_t is the firm's productivity, it includes both aggregate and individual components. The aggregate component itself consists of transitory and permanent components. The calibration of Z_t is discussed below.

Total capital adjustment costs are given by $\Phi(I_t, K_t)$. We consider two types of costs; $\Phi(I_t, K_t)$ is the sum of the two. First, fixed costs given by 0 if $\frac{I_t}{K_t} \in (a, b)$ and $\phi * u_t$ otherwise where u_t is a firm specific, uniform random variable known to firms as of t^3 . Second, asymmetric convex costs given by $v_t \left(\frac{I_t}{K_t}\right)^2 K_t$ where $v_t = v^+$ if $\frac{I_t}{K_t} > 0$ and $v_t = v^-$ otherwise. Asymmetric costs have been shown to quantitatively help with the value premium by (Zhang 2005).

We define the firm's return on capital as $R_{t+1}^K = \frac{V_{t+1}}{V_t - D_t}$. However, real world firms are financed by both debt and equity, with equity being the riskier, residual claim. To compare the model's equity return to empirical equity returns we lever the return on capital using the 2nd proposition of (Modigliani and Miller 1958): $R_{t+1}^E = R_{t+1}^K + \lambda(R_{t+1}^K - R_t^f)$ where $\lambda = 5/3$ is the total firm value to equity value ratio⁴.

3.3 Equilibrium

We assume that there exists some underlying set of state variables S_t which is sufficient for this problem. Each firm's individual state variables are given by the vector S_t^i . Because the household is a representative agent, we are able to avoid solving the household's maximization problem and simply use the first order conditions to find M_{t+1} as an analytic function of consumption or

³Most papers on fixed costs choose this cost to be $\phi * u_t$ rather than just ϕ . The presence of u_t implies that some firms are low cost firms while others are high cost firms. If fixed costs are more concentrated, investment cycles will end up being coordinated among firms with the same cost and capital. This will lead to to predictable, oscillatory dynamics (aka "echo effects" or replacement cycles) in investment. It is unclear whether this type of oscillatory behavior is desirable, for example (Gourio and Kashyap 2007) argue that it is, however we do not want it to have first order effects in our model and therefore choose u_t to be uniform.

⁴In doing so we assume firms keep leverage constant. (Boldrin, Christiano, and Fisher 2001) lever up the return on capital in exactly the same way; they also choose $\lambda = 5/3$ which is consistent with the data.

expectations of future consumption. For instance, with CRRA utility, $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta}$.⁵

Equilibrium consists of:

- Beliefs about the transition function of the state variable and the shocks: $S_{t+1} = f(S_t, Z_{t+1})$
- Beliefs about the realized stochastic discount factor as a function of the state variable and realized shocks: $M(S_t, Z_{t+1})$
- Beliefs about aggregate wages as a function of the state variable: $w(S_t)$
- Policy functions (which depend on S_t and S_t^i) by the firms for labor demand N_t^i and investment I_t^i

It must also be the case that given the above policy functions all markets clear and the beliefs turn out to be rational:

- The firm's policy functions maximize the firm's problem given beliefs about the wages, the discount factor, and the state variable.
- Labor market clears: $\sum N_t^i = 1$
- Goods market clears: $C_t = \sum (\Pi_t^i - I_t^i - \Phi_t^i + w_t N_t^i) = \sum D_t^i + w_t N_t^i$
- The beliefs about M_{t+1} are consistent with goods market clearing through the household's Euler Equation⁶.
- Beliefs about the transition of the state variables are correct. For instance if aggregate capital is part of the aggregate state vector, then it must be that $K_{t+1} = (1 - \delta)K_t + \sum I_t^i$.

⁵More generally, given a process for C_t we can recursively solve for all the necessary expectations to calculate M_{t+1} .

⁶For example, with CRRA $M_{t+1} = \beta \left(\frac{\sum D_{t+1,i} + w_t N_{t+1,i}}{\sum D_{t,i} + w_t N_{t,i}} \right)^{-\theta}$

4 Results

4.1 Calibration

We solve the model at an annual frequency using a variation of the (Krusell and Smith 1998) algorithm. The model requires us to choose the preference parameters: β (time discount factor), θ (risk aversion), ψ (intertemporal elasticity of substitution); the technology parameters: α_N (one minus share of labor), α_K (share of capital), δ (depreciation), f (operational leverage); the adjustment cost parameters: η^+ (upward convex adjustment cost), η^- (downward convex adjustment cost), ϕ (fixed adjustment cost), (a,b) (interval within which no fixed cost is paid). The parameter choices for our preferred model (7), as well as several other models are in Table 1.

[Table 1 about here.]

We set $\beta = .975$ to match the level of the risk free rate. We set $\theta = 8$ to get a reasonably high Sharpe Ratio while keeping risk aversion within the range recommended by (Mehra and Prescott 1985). We set $\psi = 1.5$, the same number chosen by (Bansal and Yaron 2004).

The technology parameters are fairly standard and we use numbers consistent with prior literature. We set $1 - \alpha_N = .64$ to match the labor share in production and $\alpha_N + \alpha_K = .89$ to be consistent with estimated degrees of returns to scale⁷. We set $\delta = .1$ to match annual depreciation. Operational leverage allows us to match the average market-to-book ratio in the economy, we set it to $f = .05$.

We choose adjustment costs to jointly match volatility of aggregate investment and the distribution of firm level investment rate. Consistent with (Cooper and Haltiwager 2006), convex costs are more relevant for aggregate investment and fixed costs more relevant for firm level investment however each cost has some effect on both. (Zhang 2005) finds that $\frac{\eta^-}{\eta^+} = 10$ helps

⁷(Gomes 2001) uses .95 citing estimates of just under 1 by (Burnside 1996). (Burnside, Eichenbaum, and Rebelo 1995) estimate it to be between .8 and .9. (Khan and Thomas 2008) use .896, justifying it by matching the capital to output ratio. (Bachmann, Caballero, and Engel 2010) use .82, justifying it by matching the revenue elasticity of capital.

to quantitatively match the value premium and we choose the ratio to be the same. We also find that this asymmetry is helpful for matching the distribution of firm level investment rate. However, we find that the level of convex costs in (Zhang 2005) leads to aggregate investment that is far too smooth, therefore our level of costs is much smaller. The interval of no fixed cost is $(0, \infty)$ in our preferred model, implying no cost for positive investment; in some specifications the interval is $(-.005, .005)$ implying no cost only for near zero investment. The size of the fixed cost affects the autocorrelation of firm level investment, as well as the frequency of spikes and inactions.

Recall that a firm's productivity is given by Z_t^i which is a combination of firm specific and aggregate shocks. $Z_t^i = A_t^i Z_t^L Z_t^G$ where A_t^i is firm specific productivity, Z_t^L is the part of aggregate productivity whose level is trend-stationary, and Z_t^G is the part of aggregate productivity whose growth rate is stationary. We choose A_t^i to approximately match the second moment of firm level investment; it is a two-state Markov variable with annual volatility of 30% and autocorrelation of .6.⁸

While the setup allows for simultaneous shocks to growth and level of productivity, so far we have only solved the model for either only growth or only level productivity shocks. The process for level shocks is $\log(Z_{t+1}^L) = g * t + \rho^L \log(Z_t^L) + \sigma^L \epsilon_{t+1}^L$. The process for growth shocks is $\log(\frac{Z_{t+1}^G}{Z_t^G}) = g + \rho^G \log(\frac{Z_t^G}{Z_{t-1}^G}) + \sigma^G \epsilon_{t+1}^G$ where $g=2\%$ is the mean growth rate of the economy. Note that ϵ^L are transitory shocks to the level of productivity while ϵ^G are transitory shocks to the growth rate of productivity which permanently affect the level of productivity. We set ρ and σ to roughly match the autocorrelation and volatility of output.

4.2 Preferred Model

We will first present results from our preferred calibration and then show several alternative calibrations to understand what is driving the results.

⁸This is roughly consistent with other models. For example 15% and .62 in (Gomes 2001) and 35% and .69 in (Zhang 2005).

[Table 2 about here.]

Panel A of Table 2 presents the standard business cycle moments for the preferred model and the data. Overall, the model matches these aggregate moments quite well. In the data, depending on the time period output volatility ranges from 1.58% (1954-2008) to 3.94% (1930-2008) with higher volatility for the period including the Great Depression. We choose productivity parameters so that the model's output volatility is closer to the lower end of this range. The model also does well with the volatilities and autocorrelations of consumption, investment, and investment growth. The volatilities of investment rate and consumption growth are somewhat high, yet still within the range of the two time periods. The model produces autocorrelation of consumption growth that is too high.

Asset pricing moments are given in Panel B of Table 2. The risk free rate is relatively low and even smoother than the data; this is often a difficult feature to reproduce and occurs here because of the high elasticity of intertemporal substitution. On the other hand, the Sharpe Ratio is slightly higher than in the data despite having a risk aversion coefficient of only 8. This happens due to the long run risk channel of (Bansal and Yaron 2004): households with a high elasticity of intertemporal substitution are afraid of changes to expected long run consumption growth. These changes occur due to shocks to expected long run productivity growth, as in (Croce 2010) and (Kaltenbrunner and Lochstoer 2010).

Note that expected excess return on equity are low, however this is not due to low price of risk but rather low quantity of risk. $E[R_e - R_f] = SR * \sigma(R_e - R_f)$ thus both the price of risk (SR) and the quantity of risk must be high⁹. This model is able to match the price of risk, but fails on the quantity of risk. Matching the volatility of equity is a failure of most production models; in this model the volatility of equity is about twice that of a standard model (see Table 3) but still low relative to the data. The reason equity volatility improves relative to standard

⁹In a similar model (Croce 2010) chooses a risk aversion of 30 to get an expected return of 3.0. However he achieves this by overshooting on the Sharpe Ratio (1.9) without having a high enough equity volatility (1.6%). On the other hand, (Kaltenbrunner and Lochstoer 2010) keep risk aversion low, however choose a very high amount of financial leverage. While we believe the channel in (Croce 2010) and (Kaltenbrunner and Lochstoer 2010) is the right one for the price of risk, we believe future work should resolve both the price of risk and volatility puzzles rather than overshooting on one.

models is a combination of shocks to the growth rate rather than level of productivity and of decreasing returns to scale. One way to increase equity volatility is to increase the volatility of output shocks to the (1930-2008) level; doing this would result in equity volatility of around 7%.

Panel C of Table 2 has firm level investment moments, as well as information on heteroscedasticity and time dependence of aggregate investment rate. The model is close to the data for both volatility and autocorrelation of firm level investment. The mean is somewhat below the data because the distribution in the data is more skewed. Similarly, the model does a good job at matching the distribution of investment, which has a large number of spikes ($I/K > 20\%$) and a small number of firms disinvesting. Investment by firms with spikes is very important for aggregate investment in both the model and the data, however quantitatively this importance is not quite as high in the model as in the data: investment by firms with spikes accounts for 40% of all investment and for 69% of the volatility of aggregate investment in our preferred model compared to 50% and 98% in the data. Fixed costs are important to match these facts, as is discussed by (Gourio and Kashyap 2007) and will be seen in Table 3.

(Bachmann, Caballero, and Engel 2010) emphasize the importance of heteroscedasticity and time dependence of aggregate investment rate. Below we discuss how to summarize these two observations with summary statistics from the data and the model. Following (Bachmann, Caballero, and Engel 2010) we regress aggregate investment rate on its lag and then regress the squared residuals from this regression on lagged investment rate. We then define σ_x to be the fitted value of the x percentile of this regression. In the data, $\log(\frac{\sigma_{95}}{\sigma_5}) = 0.30$, indicating that investment rate is more volatile when it is high.

To summarize time dependence we use NBER quarterly recession dates and calculate the change in investment rate for each recession and expansion. In particular we calculate the quantity $\overline{\Delta I/K}_j = \frac{1}{T_j} \sum_{t=0_j, T_j} \frac{I_t}{K_t} - \frac{I_0}{K_0}$ where the expansion or recession j starts at 0_j and ends at T_j .¹⁰ This measure is plotted against the length of each recession and expansion in the lower

¹⁰We choose this as opposed to $\frac{I_T}{K_T} - \frac{I_0}{K_0}$ because NBER dates do not exactly match true changes in underlying shocks and the later measure is more sensitive to such errors, nevertheless, the two give qualitatively similar results. We plan to redo this using TFP growth rather than NBER dates in the future.

panel of Figure 1. We regress $\overline{\Delta I/K}$ on the length of each recession and separately expansion and report the slope coefficients γ^E and γ^R . We calculate $\log(\frac{\sigma_{95}}{\sigma_5})$, γ^E , and γ^R for simulated data from the model in the same way.

The preferred model is able to qualitatively capture these features of the data. Investment rate is heteroscedastic $\log(\frac{\sigma_{95}}{\sigma_5}) = 0.20$, although somewhat less so than 0.30 in the data. As in the data, investment rate rises through an expansion, and falls through a recession: $\gamma^E = 0.20$, and $\gamma^R = -0.34$ compared to 0.08 and -0.49 in the data.

4.3 Inspecting the Mechanism

To inspect what works and what does not in allowing the preferred model to match the data, we will study six alternative parameterizations, these are described in Table 1. Models 1-3 have only shocks which are trend stationary in the level of productivity while models 5-6 have only shocks which are stationary in the growth rate of productivity. Models 1 and 4 are frictionless, models 2 and 5 have only convex adjustment costs, models 3 and 6 have only non-convex adjustment costs. Recall that model 7 (our preferred model) has a combination of convex and non-convex adjustment costs which we have found to work well. This is a calibration, rather than estimation, exercise and there are likely other parameter combinations that work as well, however we have experimented with a large range of parameters (not reported) and believe the best combination is not far from our preferred calibration¹¹.

Selected quantities from the data, the six alternative parameterizations, as well as the preferred model are in Table 3. Results are separated into asset pricing moments, firm level investment moments, and aggregate investment moments.

[Table 3 about here.]

Starting with the asset pricing moments, it is clear that growth rate stationary, rather than level stationary shocks to productivity are necessary to match high Sharpe Ratios of the data

¹¹Due to the length of time required to solve the model, and the large number of parameters, we have not so far estimated the model. We are considering estimating the model by Simulated Method of Moments as a future direction.

without resorting to very high risk aversion. This happens through the long run risk channel of (Bansal and Yaron 2004): households care about expectations of future consumption growth and not just this period's consumption growth. While we only report results in which $\psi = 1.5$, such a high level of the intertemporal elasticity of substitution is an additional necessary component for this channel; this has also been documented by (Croce 2010) and (Kaltenbrunner and Lochstoer 2010). We use these findings as a restriction on technology imposed by asset pricing¹². Models with growth rate stationary shocks also have equity return volatility that is twice that of models with level stationary shocks, though this volatility is still too low relative to the data.

Moving on to the firm level investment, a frictionless model (1 and 4) has investment rate that is far too volatile; too many firms have negative investment and those with spikes often have investment rates in excess 100%; in short investment rate is far wilder than in the data. A convex adjustment cost (2 and 5) fixes most of these problems by preventing firms from taking very large positive or negative investments. Autocorrelation also increases because for firms with high investment demand it is cheaper to invest over consecutive years rather than all at once. A fixed cost alone (3 and 6) does not do as well as a convex cost along most dimensions. It reduces volatility, but not by enough. The only dimension on which it provides a clearcut improvement is on share of investment by firms with spikes. This is because with fixed costs alone, firms follow an (s,S) rule. Most firms do not invest at all; once they choose to invest they invest a lot so as to avoid paying future adjustment costs. This result is similar to (Gourio and Kashyap 2007). It is possible that a richer structure of fixed costs could do a better job. Interestingly, asset pricing restrictions on technology do not make much of a difference along this dimension: the frictionless, convex, and fixed pairs look quite similar independent of the type of shock. In our preferred model, we rely mostly on convex costs as they are able to do most of the job; we add only a small fixed cost for negative investment to match the small number of firms with $I/K < 0$.

¹²An alternative promising channel to deliver a high price of risk in a general equilibrium, production economy is habit preferences, as in (Campbell and Cochrane 1999), (Jermann 1998), (Boldrin, Christiano, and Fisher 2001) among others. Both habit and long run risk have certain desirable as well as undesirable features, which have been debated at length in the literature.

All calibrations with shocks to the growth rate of productivity have a similar I/K pattern to the data: I/K grows as an expansion gets longer ($\gamma^E > 0$) and I/K falls as a recession gets longer ($\gamma^R < 0$). Models with shocks to the level of productivity have exactly the opposite pattern. This can be further seen in the lower panel of Figure 3. Here we plot impulse responses of I/K to output (1-3) or output growth (4-6) rising from x at time 0 to \bar{x} for either 1 (short expansion, solid line) or 4 (long expansion, dashed line) consecutive years and falling back to normal after¹³.

[Figure 3 about here.]

In models 1-3, as the expansion gets longer, I/K decreases. This is because each level of productivity is associated with an optimal level of capital. A temporary rise in productivity causes firms to increase their target capital level and increase investment immediately. As the expansion gets longer, the target capital level stays at the same level and I/K falls because the firms are closer to optimal capital due to past high investment. Adding capital adjustment costs (models 2 and 3) mitigates this feature of standard model by making it too costly to raise investment too much immediately. The result is a flatter decline in investment rate for the parameters we consider however never a reversal of the decline. Choosing larger adjustment costs may reverse the relationship between investment rate and length of a recession, but this would make both aggregate and firm level investment far too smooth.

In models 4-6, as the expansion gets longer, I/K increases. The reason this happens is straight forward: if the growth rate of productivity is high for a longer number of periods, the level of productivity, and therefore the optimal level of capital continues to grow. Therefore an unexpected lengthening of an expansion leads firms to ramp up investment rather than slow it down. Note that this happens even in a model without frictions.

In a standard model with shocks to the level of productivity investment rate is homoscedastic, for example in model 1, $\log(\frac{\sigma_{95}}{\sigma_5}) = -.01$. (Bachmann, Caballero, and Engel 2010) find that in models similar to 1 and 3, an increase in fixed costs increases $\log(\frac{\sigma_{95}}{\sigma_5})$. They list this as a reason

¹³ x is the mean level of the shock and $\bar{x} - x = \sigma$. Note that because shocks follow an AR(1), this amounts to either one $\epsilon > 0$ or four consecutive $\epsilon > 0$.

for importance of fixed costs. We confirm their finding, $\log(\frac{\sigma_{95}}{\sigma_5}) = .05$ when we add a fixed cost in model 3.¹⁴ However, models with shocks to the growth rate of productivity (including the frictionless model) all exhibit $\log(\frac{\sigma_{95}}{\sigma_5})$ moderately close to the data. Thus asset pricing restrictions suggest that non-convex frictions are unimportant for the aggregate quantities (at least those aggregate quantities that have been considered). Of course non-convex frictions are still important for jointly matching micro and macro findings as in model 7; this is consistent with partial equilibrium findings of (Cooper and Haltiwager 2006).

The above results suggest that it may not matter whether (Khan and Thomas 2008) or (Bachmann, Caballero, and Engel 2010) are fundamentally correct about the relevance of non-convex micro frictions for aggregates. Quantitatively, a model calibrated at the firm level to match asset pricing moments is able to deliver the desired investment moments with or without non-convex costs. Since model 5 (convex costs only) performs nearly as well as our preferred model (convex costs and small non-convex costs) even at matching firm level micro data, we provide a micro foundation for convex cost asset pricing models such as (Jermann 1998), (Zhang 2005), and (Croce 2010).

4.4 The Cross-Section

In this section we will explore the cross sectional asset pricing implications of the fixed cost and productivity issues considered above. Recall that (Zhang 2005) is a partial equilibrium model in which low market-to-book (value) firms are riskier, have higher betas, and higher expected returns. Low market-to-book firms are riskier because they are low productivity firms who, in bad times, are burdened with excess capital which is costly to adjust down. While the model is partial equilibrium, if aggregate consumption were to be perfectly correlated with aggregate productivity shocks, the model would be analogous to a general equilibrium model with CRRA preferences. Indeed, we find that the above intuition continues to hold in general

¹⁴Their increase is quantitatively bigger than here because their model is not exactly identical to this one. For example their firms are forced to replace some amount of depreciated capital, this likely increases the importance of partial equilibrium effects.

equilibrium with CRRA preferences irrespective of the type of shocks we use. However the value premium operational leverage relationship reverses when we raise the intertemporal elasticity of substitution and change productivity shocks to being growth stationary.

Table 4 presents investment and asset pricing statistics for firms sorted on either productivity or market-to-book from our preferred model. For simplicity we have only two portfolios, high and low, with half of all firms falling in each. High productivity firms have more capital, invest more, have higher valuations relative to capital, and are less risky. High (low) productivity firms have the characteristics we typically associate with growth (value) firms and indeed they have higher (lower) market-to-book ratios.

However results reverse once we sort on market-to-book: high market-to-book firms have less capital, invest less, and are more risky. That is, high (low) market-to-book firms have characteristics we typically associate with value (growth) firms. Figure 4 plots V/K against K for all firms in a particular time period. If returns to scale were constant, the relationship of V/K against K would be flat for a given level of productivity; because of decreasing returns to scale V/K is decreasing in K . For any level of capital, high productivity firms indeed have higher valuations than low productivity firms. However high productivity firms typically hold more capital than low productivity firms. Because of decreasing returns to scale, many high productivity firms have high capital and low valuations while many low productivity firms have low capital and high valuations. This is the reason for the reversal.

[Table 4 about here.]

[Figure 4 about here.]

(Zhang 2005) finds that operational leverage is important for matching the value premium. In particular he finds that as operational leverage increases (through increasing f), value firms become more risky. In Figure 5 we solve four different versions of the model: level shocks and CRRA, level shocks and (Bansal and Yaron 2004) preference calibration, growth shocks and CRRA, growth shocks and (Bansal and Yaron 2004) preference calibration. For each of these

four versions we solve the model for different levels of operational leverage and then plot the average operational leverage in a calibration against the average difference in betas between value and growth stocks. Since conditional CAPM holds, this is the same as looking at the value premium directly¹⁵. For example, the solid black line represents the value premiums in a series of models with CRRA preferences, trend stationary productivity level, with different levels of operational leverage. Note that the average V/K is around 1.5 in the data.

The pattern found by (Zhang 2005) continues to hold in the general equilibrium version of his partial equilibrium model, which corresponds to the solid black line. When we keep CRRA preferences but change the productivity shocks to be stationary in the growth rate the pattern is still the same. When we increase intertemporal elasticity of substitution but keep level shocks, the pattern gets weaker. However when we combine a high intertemporal elasticity of substitution with shocks to the growth rate of productivity, the pattern reverses.

Recall that a combination of high intertemporal elasticity of substitution and shocks to the growth rate of productivity are both necessary (within our model) to deliver a high price of risk and match several aggregate investment moments. Our interpretation is that the adjustment cost channel alone is not likely to deliver a value premium within the long run risk framework and other channels should be found. Such channels may involve additional aggregate shocks with different loadings for value and growth firms as in (Campbell and Vuolteenaho 2004) and (Bansal, Kiku, and Yaron 2008), real options as in (Berk, Green, and Naik 1999) and (Gomes, Kogan, and Zhang 2003), differences in financing costs, or other types of heterogeneity.

[Figure 5 about here.]

¹⁵We look at the difference in betas rather than expected returns because models with CRRA preferences versus (Bansal and Yaron 2004) calibrated preferences have very different prices of risk and therefore very different scales for expected returns. On the other hand, despite differences in the price of risk, betas have similar scales and their magnitudes can be compared.

5 Conclusion

We solve a general equilibrium production economy with a firm specific productivity shocks resulting in a non-trivial distribution of firms. Within this model, we use insights from asset pricing to shed light on the irrelevance of non-convex frictions debate in the macro literature; we also use insights from the macro literature to guide our modeling choices for matching asset pricing moments.

We use a combination of preferences and technology shocks as in (Bansal and Yaron 2004), (Croce 2010), (Kaltenbrunner and Lochstoer 2010) which are known to help match aggregate asset pricing moments. We view asset prices as additional restriction and find that within such a model, non-convex frictions are unnecessary to match important features of aggregate investment. Previous literature, e.g. (Bachmann, Caballero, and Engel 2010), has been able to match some of these features by using non-convex frictions but not simultaneously with asset prices.

We also find that a model with convex costs alone does nearly as good of a job at matching firm level micro data as our preferred model with both convex and non-convex costs. This provides justification for papers such as (Jermann 1998), (Zhang 2005) and (Croce 2010) which use convex costs alone to explain asset prices.

Finally, we find that in our preferred model, the relationship between the value premium and operational leverage explored by (Zhang 2005) reverses. This suggests alternative channels are necessary to explain the value premium in a long run risk framework.

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A Data

Monthly stock returns are from the Center for Research in Security Prices (CRSP) and accounting information is from the CRSP/Compustat Merged Annual Industrial Files. Our sample is from 1975 to 2009. We exclude from the sample any firm-year observation with missing data or for which total assets or the gross capital stock are either zero or negative. In addition, as standard, we omit firms whose primary SIC classification is between 4900 and 4999 (regulated firms) or between 6000 and 6999 (financial firms). Following Cooper and Haltiwanger (2006), We define the capital stock, K , as gross property, plant and equipment, and investment (PPEGT in Compustat), I , as capital expenditures (CAPX) minus sales of property, plant and equipment (SPPE). Incorporating sales of capital is especially important in our analysis as the role of partial irreversibility is quite difficult to study with the use of capital expenditures data alone. Both capital and investment are deflated by the domestic fixed investment price deflator. Investment

rate IK is defined as investment scaled by lagged capital stock, which can be either positive or negative. The gross domestic fixed investment price deflator is from NIPA table 1.1.9.

B Numerical Solution

Table 1: Calibration

	β	θ	ψ	Z	$1 - \alpha_N$	$\frac{\alpha_K}{\alpha_N}$	δ	f	v^+	v^-	(a,b)	ϕ
1	0.975	8	1.5	L	0.64	0.69	0.1	0.05	0	0	-	0
2	0.975	8	1.5	L	0.64	0.69	0.1	0.05	0.05	0.5	-	0
3	0.975	8	1.5	L	0.64	0.69	0.1	0.05	0	0	(-.005,.005)	0.15
4	0.975	8	1.5	G	0.64	0.69	0.1	0.05	0	0	-	0
5	0.975	8	1.5	G	0.64	0.69	0.1	0.05	0.05	0.5	-	0
6	0.975	8	1.5	G	0.64	0.69	0.1	0.05	0	0	(-.005,.005)	0.15
7	0.975	8	1.5	G	0.64	0.69	0.1	0.05	0.05	0.5	(0,1)	0.05

Table 2: Baseline Model

Panel A: RBC moments

	Data 1930-2008			Data 1954-2008			Preferred Model		
	$\sigma(x)$	$\rho(x, y)$	AC(x)	$\sigma(x)$	$\rho(x, y)$	AC(x)	$\sigma(x)$	$\rho(x, y)$	AC(x)
y	3.94	1.00	0.54	1.58	1.00	0.32	2.05	1.00	0.55
c	1.99	0.37	0.39	1.13	.84	0.35	1.32		0.50
i	11.16	0.11	0.45	5.13	.80	0.42	5.95		0.51
Δc	2.63	0.13	0.31	1.50	0.42	0.22	2.40		0.66
Δi	13.49	0.15	0.43	6.40	0.62	0.27	7.82		0.37
i-k	2.07	0.03	0.94	0.82	0.50	0.78	2.01		0.92
p-d	47.46	0.05	0.89	40.52	0.05	0.90	26.48		0.93

Panel B: Asset Pricing

	Data 1930-2008				Data 1954-2008				Preferred Model			
	E[R]	$\sigma(R)$	AC(R)	SR	E[R]	$\sigma(R)$	AC(R)	SR	E[R]	$\sigma(R)$	AC(R)	SR
R_f	0.68	3.93	0.75		1.59	1.92	0.71		1.57	0.80	0.83	
$R_e - R_f$	7.34	20.79	.02	0.35	6.76	18.85	-0.06	0.36	1.52	3.60	0.02	0.42

Panel C: Investment

	E[I/K]	$\sigma(I/K)$	AC(I/K)	<0	-5<IK<15	>20	$\frac{E[I20/K]}{E[I/K]}$	$\frac{Cov[\frac{I20}{K}, \frac{I}{K}]}{Var[I/K]}$	$\log(\frac{\sigma_{95}}{\sigma_5})$	γ^E	γ^R
Data	13.2	11.2	0.36	4.1	67.4	18.8	0.50	0.98	0.30	0.08	-0.49
Model	10.5	10.0	0.25	3.5	72.3	16.9	0.40	0.69	0.20	0.20	-0.34

Table 3: Inspecting the Mechanism

The rows are separated into asset pricing moments (1-2), firm level investment (3-8), and aggregate investment (9-11).

Shock	Data	Model						
		1 L None	2 L Convex	3 L Fixed	4 G None	5 G Convex	6 G Fixed	7 G Both
Sharpe Ratio	0.36	0.06	0.06	0.06	0.42	0.42	0.42	0.42
$\sigma(R^e - R^f)$	20.0	1.80	1.86	1.86	3.52	3.58	3.53	3.60
I/K < 0	4.2	12.5	24.3	19.5	12.5	23.3	19.8	3.5
-5 < I/K < 15	67.4	74.9	75.0	82.4	74.8	74.6	82.3	72.3
I/K > 20	19.8	12.5	16.3	14.2	12.5	16.3	14.4	16.9
$\sigma(I/K)$	11.2	62.4	10.6	46.0	62.5	10.5	46.5	10.0
$AC(I/K)$	0.36	-0.22	0.23	-0.09	-0.22	0.25	-0.09	0.25
$\frac{E[I_{20}/K]}{E[I/K]}$	0.50	1.21	0.40	0.96	1.19	0.40	0.96	0.40
$\frac{Cov[I_{20}/K, I/K]}{Var[I/K]}$	0.98	0.19	0.64	0.99	0.19	0.67	0.96	0.69
$\log\left(\frac{\sigma_{95}}{\sigma_5}\right)$	0.30	-0.01	0.01	0.05	0.19	0.14	0.22	0.20
γ^E	0.08	-0.06	-0.05	-0.05	0.21	0.20	0.20	0.20
γ^R	-0.49	0.10	0.09	0.09	-0.41	-0.39	-0.36	-0.34

Table 4

	K	V/K	E[I/K]	I/K<0	I/K>0.2	β	$E[R^e - R^f]$
All	2.28	1.92	10.4	3.5	16.9	1.00	1.52
Low Z^i	2.13	1.86	3.3	7.0	0.0	1.05	1.60
High Z^i	2.44	1.97	17.6	0.0	33.8	0.96	1.46
Low $\frac{V}{K}$	2.54	1.77				0.97	1.47
High $\frac{V}{K}$	1.93	2.16				1.04	1.60

Figure 1: Investment Rate and the Business Cycle

The top panel plots investment rate over time, with NBER contractions dashed. The bottom panel plots $\overline{\Delta I/K}$ against the length of the associated expansion (contraction) where $\overline{\Delta I/K}$ is the difference between the average I/K during the expansion (contraction) and I/K at the start.

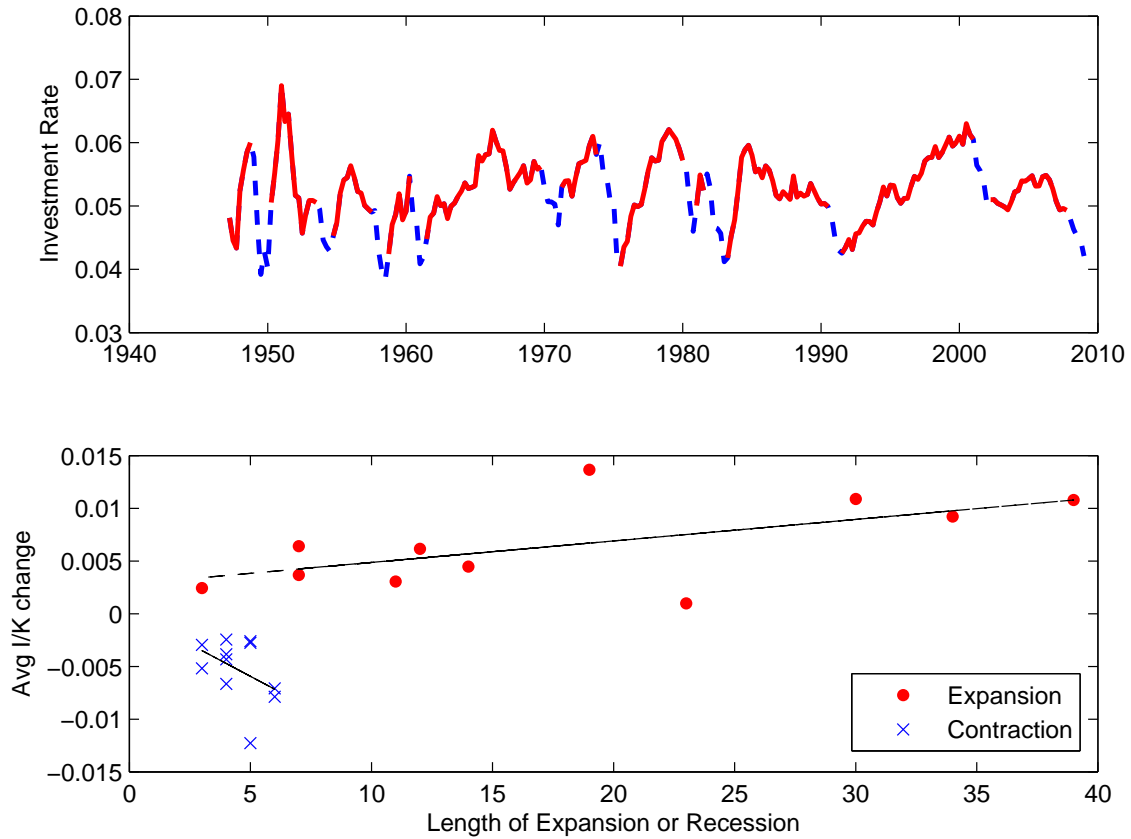


Figure 2: Heteros

Heteroscedasticity range from (Bachmann, Caballero, and Engel 2010). First I/K is regressed on its own lag, then residuals from this regression are squared. The mean of square residuals for each level of I/K lagged is plotted against I/K lagged.

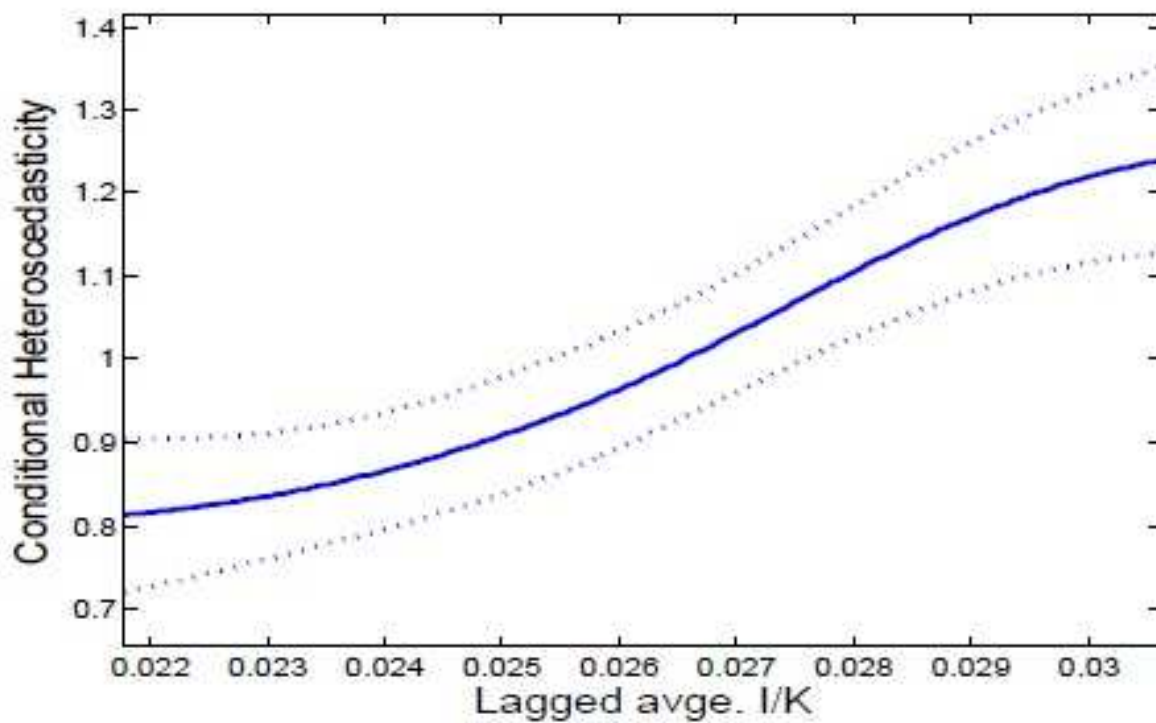


Figure 3: Impulse Response of Investment Rate

These are impulse responses of investment rate to 1 positive shock or 5 consecutive positive shocks (dashed). In models 1-3 shocks are to TFP level, in models 4-6 shocks are to TFP growth.

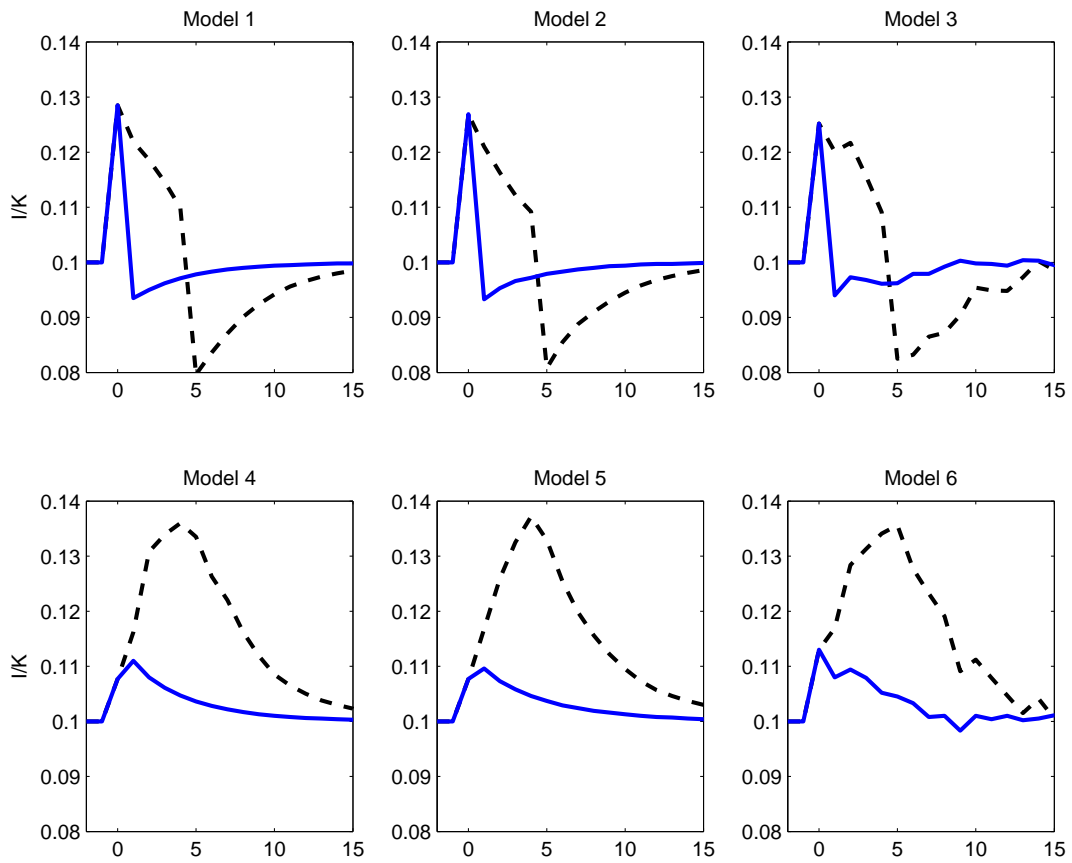


Figure 4:

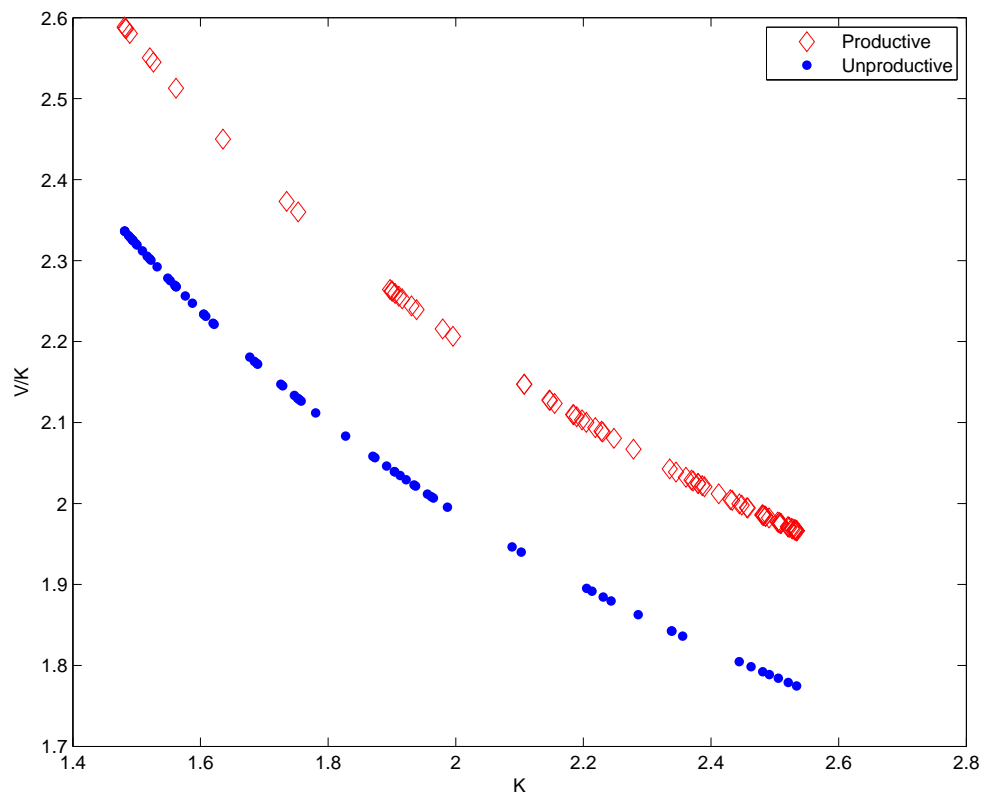


Figure 5: Value Premium and Volatility for models with different operational leverage

