Relative Performance Evaluation and Managerial Outside Options

Stanimir Morfov

International College of Economics and Finance,

State University - Higher School of Economics

Manuel Santos

University of Miami

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INTRODUCTION

- Relative Performance Evaluation (RPE) for executives
    - Removing the market component of executive compensation
    - Performance relative to a benchmark (industry or market return)
- RPE puzzle (little or no evidence of RPE)

LITERATURE

- **Theory behind RPE**

  Holmstrom (1982)

- **Empirical evidence (CEOs)**

  Antle and Smith (1986), Lambert and Larcker (1987), Gibbons and Murphy (1990), Barro and Barro (1990), Janakiraman, Lambert and Larcker (1992), Garen (1994), Joh (1999), Aggarwal and Samwick (1999a, 1999b)

- **Towards explaining the RPE puzzle**:
  - Executives can adjust their total financial wealth:

    Feltham and Xie (1994), Maug (2000), Jin (2002), Garvey and Milbourn (2003)
  - Softening competition:

    Salas Fumas (1992)
  - Participation constraint:

    Oyer (2004)
  - Technology and participation are affected by aggregate shocks:

    Himmelberg and Hubbard (2000)
  - Marginal return of effort depends on the aggregate state:

    Celentani and Loveira (2006)

FRAMEWORK

- Principal-Agent problem (optimal contracting)
- Moral hazard with hidden action
- Limited commitment
- Aggregate shocks affect:
    - firm's technology
    - managerial outside options

BASICS

- Principal needs a manager to operate a stochastic technology mapping effort to outcomes

- The distribution of outcomes depends on managerial effort (unobservable) and an aggregate outcome (observable)

- Manager has outside options with value depending on the aggregate outcome

TIMING

1. Aggregate outcome y_A is realized according to a distribution described by $p(.)$. The outcome determines the manager's reservation utility $\underline{V}(y_A)$

2. The principal offers the manager a contract recommending an effort a and specifying compensation scheme w mapping outcomes to wages

3. If the manager rejects, both parties enjoy their reservation utilities. If the manager accepts, (s)he exerts some effort a' unobservable by the principal

4. An outcome y is realized according to a distribution conditional on y_A and a' described by $\pi(.,a',y_A)$

5. The principal pays $w(y)$ to the manager

MODEL

Assumptions:

- Two possible efforts of level: $\{\underline{a}, \overline{a}\}$
  - $\underline{a} < \overline{a}$
  - The outcome distribution conditional on $\overline{a}$ stochastically dominates the distribution conditional on $\underline{a}$, $\forall y_A$
- Given a contract (a, w) and an outcome y :
  - the manager's utility is $v(w) - a$, where v is twice continuously differentiable, strictly increasing and strictly concave.
  - the principal's utility is $y - w(y)$

Principal's Problem

$$\max_{a,w(.)} \sum_{y \in Y} (y - w(y))\pi(y, a, y_A) \text{ s.t.:}$$

$$a \in \{\underline{a}, \overline{a}\} \tag{1}$$

$$\sum_{y \in Y} (v(w(y)) - a)\pi(y, a, y_A) \geq \underline{V}(y_A) \tag{2}$$

$$a \in \arg\max_{a' \in \{\underline{a}, \overline{a}\}} \sum_{y \in Y} (v(w(y)) - a')\pi(y, a', y_A) \tag{3}$$

Note: (2) is individual rationality, (3) is incentive compatibility

If $a^*, w^*(.)$ solve the principal's problem, then:

$$\frac{1}{v'(w^*(y))} = \lambda + \mu\left(1 - \frac{\pi(y, a', y_A)}{\pi(y, a^*, y_A)}\right), \tag{4}$$

where the non-negative constants λ, μ are Lagrange multipliers for (2) and (3) respectively

Assumption: Strong monotonicity of the likelihood ratio (SMLR) holds for any y_A.

Proposition 1: Fix y_A.

(a) The principal implements low effort by a fixed wage $w_{\underline{a}} = v^{-1}(\underline{V}(y_A) + \underline{a})$, $\forall y$;

(b) The principal implements high effort by a strictly increasing compensation scheme such that $\lambda > 0$ and $\mu > 0$, i.e., both individual rationality and incentive compatibility bind.

Compare:

1. VRU (Varying Reservation Utilities) [indexed by y_A]

Manager's reservation utilities vary across aggregate outcomes. Let $V := E_p \underline{V}$

2. CRU (Constant Reservation Utilities) [denoted by c]

Manager's reservation utility equals V for any aggregate outcome

Note: The fixed wage inducing low effort, $w_{\underline{a}}$, is strictly increasing and strictly convex in $\underline{V}(y_A)$

Proposition 2: The cost of implementing low effort averaged over realizations of the aggregate outcome (under VRU) is higher than the cost of implementing low effort under CRU:

$$E_p w_{\underline{a}}^{y_A} > w_{\underline{a}}^c.$$

Proof: $E_p w_{\underline{a}} = E_p v^{-1}(\underline{V} + \underline{a}) > v^{-1}(E_p(\underline{V} + \underline{a})) = v^{-1}(V + \underline{a}) = w_{\underline{a}}^c$

Assumption: two possible outcomes: $\underline{y}$ (low) and $\bar{y}$ (high), $\underline{y} < \bar{y}$.

Notation:

$\underline{\pi}^{y_A} := \pi(\underline{y}, \underline{a}, y_A)$, i.e., the probability of low outcome conditional on low effort

$\bar{\pi}^{y_A} := \pi(\underline{y}, \bar{a}, y_A)$, i.e., the probability of low outcome conditional on high effort

Note: Stochastic dominance requires $\underline{\pi}^{y_A} > \bar{\pi}^{y_A}$, which also guarantees that SMLR holds

Results:

1. Low effort is implemented by the flat scheme $v^{-1}(\underline{V}(y_A) + \underline{a})$

2. High effort is implemented by the strictly increasing scheme paying $v^{-1}(\underline{v}_{\bar{a}}^{y_A})$ after $\underline{y}$ and $v^{-1}(\bar{v}_{\bar{a}}^{y_A})$ after $\bar{y}$, where:

$$\underline{v}_{\bar{a}}^{y_A} = \underline{V}(y_A) + \frac{(1-\bar{\pi}^{y_A})\underline{a}-(1-\underline{\pi}^{y_A})\bar{a}}{\underline{\pi}^{y_A}-\bar{\pi}^{y_A}}$$

$$\bar{v}_{\bar{a}}^{y_A} := \underline{V}(y_A) + \frac{\underline{\pi}^{y_A}\bar{a}-\bar{\pi}^{y_A}\underline{a}}{\underline{\pi}^{y_A}-\bar{\pi}^{y_A}}$$

CASE 1: The distribution of individual outcomes <u>does not</u> depend on the aggregate outcome:

$$\overline{\pi}^{y_A} = \overline{\pi} \text{ and } \underline{\pi}^{y_A} = \underline{\pi} \text{ for any } y_A$$

Proposition 3: Implementing high effort is more costly under VRU than under CRU:

$$E_p(E_{\overline{a}} w_{\overline{a}}) > E_{\overline{a}} w_{\overline{a}}^c.$$

Proof:

$$E_p w_{\overline{a}}(y) = E_p v^{-1}(v_{\overline{a}}(y)) > v^{-1}(E_p v_{\overline{a}}(y)) = v^{-1}(v_{\overline{a}}^c(y)) = w_{\overline{a}}^c(y) \Rightarrow$$

$$E_p(E_{\overline{a}} w_{\overline{a}}) = E_{\overline{a}}(E_p w_{\overline{a}}(y)) > E_{\overline{a}} w_{\overline{a}}^c.$$

Note: By Propositions 2 and 3, implementing either effort is more costly under VRU than under CRU. How does this affect the choice of optimal effort (VRU vs. CRU)? Difficult to establish for general utility functions.

Result under specific utility:

Under <u>logarithmic utility</u> lower reservation utilities reinforce the implementation of high effort, while higher reservation utilities reinforce the implementation of low effort. The result is formalized in Proposition 4 presented on next slide.

Proposition 4. Assume $v(w) = \log(w)$. Then:

(a) if low effort is optimal under a particular reservation utility of the manager, then it is also optimal for all higher reservation utilities. In particular, if low effort is optimal under CRU, then it is also optimal under VRU for reservation utilities above the mean, V;

(b) if high effort is optimal for a particular reservation utility of the agent, then it is also optimal for all lower reservation utilities. In particular, if high effort is optimal under CRU, then it is also optimal under VRU for reservation utilities below the mean, V.

RPE?

1. Under CRU

- Individual firm's outcome is independent from the aggregate ⇒ managerial compensation is not related to aggregate performance

2. Under VRU.

- Due to binding individual rationality, the contract depends on the agent's reservation utility which is in turn a function of the aggregate outcome.
- Both the fixed wage implementing low effort and the monotonic scheme implementing high effort are increasing in agent's reservation utility.

- If the demand for managerial services, and so the agent’s reservation utility, increase with the aggregate outcome, the agent’s compensation designed to implement a particular effort level will also increase in y_A.

Note: If the increase in the aggregate outcome does not change the optimal level of effort induced by the contract, the agent will enjoy a pay rise for any individual outcome, i.e., $\frac{\partial w^*_{y_A}}{\partial y_A} > 0, \forall y$ and since the distribution of individual outcomes is unaffected by the increase in y_A, (s)he should also have his/her average wage increase, i.e., $\frac{\partial E_{a^*(y_A),y_A} w^*_{y_A}}{\partial y_A} > 0$

CASE 2. The distribution of individual outcomes depends on the aggregate outcome (both $\overline{\pi}^{y_A}$ and $\underline{\pi}^{y_A}$ vary with y_A)

RPE?

1. Under VRU:

1.1. Pay implementing low effort rises with y_A (if managerial services are more demanded in a boom than in a trough)

1.2. Compensation scheme implementing high effort?

$$\frac{\partial \underline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \underline{V}}{\partial y_A} + \frac{\overline{a}-\underline{a}}{(\underline{\pi}-\overline{\pi})^2}\left((1-\overline{\pi})\frac{\partial \underline{\pi}}{\partial y_A} - (1-\underline{\pi})\frac{\partial \overline{\pi}}{\partial y_A}\right)$$

$$\frac{\partial \overline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \underline{V}}{\partial y_A} + \frac{\overline{a}-\underline{a}}{(\underline{\pi}-\overline{\pi})^2}\left(\underline{\pi}\frac{\partial \overline{\pi}}{\partial y_A} - \overline{\pi}\frac{\partial \underline{\pi}}{\partial y_A}\right)$$

Assumption: Aggregate effort has the same marginal effect on both distributions (the distribution conditional on low and the distribution conditional on high effort), i.e., $\frac{\partial \overline{\pi}}{\partial y_A} = \frac{\partial \underline{\pi}}{\partial y_A}$.

$$\frac{\partial \underline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \overline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \underline{V}}{\partial y_A} + \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} \frac{\partial \overline{\pi}}{\partial y_A}$$

$\frac{\partial \underline{V}}{\partial y_A} > 0$ (rising demand for managerial services in a boom)

$\frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} > 0$

$\frac{\partial \overline{\pi}}{\partial y_A} < 0$ for a pro-cyclical firm

$\frac{\partial \overline{\pi}}{\partial y_A} > 0$ for a counter-cyclical firm.

Results:

A marginal increase in the aggregate outcome:

- increases the wage compensation implementing high effort for a counter-cyclical firm
- increases/does not affect/decreases it for a pro-cyclical firm depending on whether the increase in reservation utilities dominates/cancels/is dominated by the decrease in the probability of failure (low outcome) weighted by the additional disutility of high over low effort divided by its contribution to the probability of success (high outcome).

Note: Managerial pay increases in individual outcome and decreases in aggregate outcome only for pro-cyclical firms where the reservation utility effect of aggregate effort is weaker than its direct effect on the probability of success!

2. Under CRU:

2.1. Pay implementing low effort does not change with y_A

2.2. Compensation scheme implementing high effort?

If $\frac{\partial \overline{\pi}}{\partial y_A} = \frac{\partial \underline{\pi}}{\partial y_A}$,

$$\frac{\partial \underline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \overline{v}_{\overline{a}}}{\partial y_A} = \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} \frac{\partial \overline{\pi}}{\partial y_A}$$

Results:

A marginal increase in the aggregate outcome:

- increases the wage compensation implementing high effort for a counter-cyclical firm
- decreases it for a pro-cyclical firm

Note: Managerial pay increases in individual outcome and decreases in aggregate outcome for all pro-cyclical firms!

Caveats (**RPE analysis**): Impact of dy_A on the choice of optimal effort

MODEL WITH CONTINUOUS-VALUED SHOCKS

<u>**Timing**</u>:

1. Aggregate shock η observed

2. Manager's reservation utility $\underline{V}(\eta)$, $\underline{V}'(\eta) \geq 0$, observed

3. Optimal contracting: $a, w(.)$

4. Manager exerts effort a, unobserved by the principal

5. Idiosyncratic shock ε correlated to η realized, unobserved by the principal

6. Firm's outcome y is realized and $w(y)$ is paid to the manager

Assumptions:

$$y = g(a) + \eta + \varepsilon,$$

$$g'(a) > 0$$

$$\begin{bmatrix} \eta \\ \varepsilon \end{bmatrix} \backsim N\left(\begin{bmatrix} \mu_\eta \\ \mu_\varepsilon \end{bmatrix}, \begin{bmatrix} \sigma_\eta^2 & \rho\sigma_\eta\sigma_\varepsilon \\ \rho\sigma_\eta\sigma_\varepsilon & \sigma_\varepsilon^2 \end{bmatrix} \right)$$

Results:

- Conditional distribution of firm's outcome:

$$y|\eta \backsim N(\mu(a), \sigma^2(a)),$$

$$\mu(a) := g(a) + \eta + \mu_\varepsilon + \rho\frac{\sigma_\varepsilon}{\sigma_\eta}(\eta - \mu_\eta)$$

$$\sigma^2(a) := (1 - \rho^2)\sigma_\varepsilon^2$$

- Optimal managerial compensation:

$$w^*(y) = \frac{1}{r}\log r +$$

$$\frac{1}{r}\log\left(\lambda_1(\eta) + \lambda_2(\eta)\left(1 - e^{-\frac{\left(g(a)-g(a')\right)\left(2y-g(a)-g(a')-2\left(\eta+\mu\varepsilon+\rho\frac{\sigma\varepsilon}{\sigma\eta}(\eta-\mu\eta)\right)\right)}{2\left(1-\rho^2\right)\sigma_\varepsilon^2}}\right)\right)$$

Best Linear Wage:

- Implementing low effort:

$$w_l(y) = \frac{1}{r}\log(-\underline{V}(\eta) - \underline{a})$$

- Implementing high effort:

$$w_l(y) = \alpha + \beta y$$

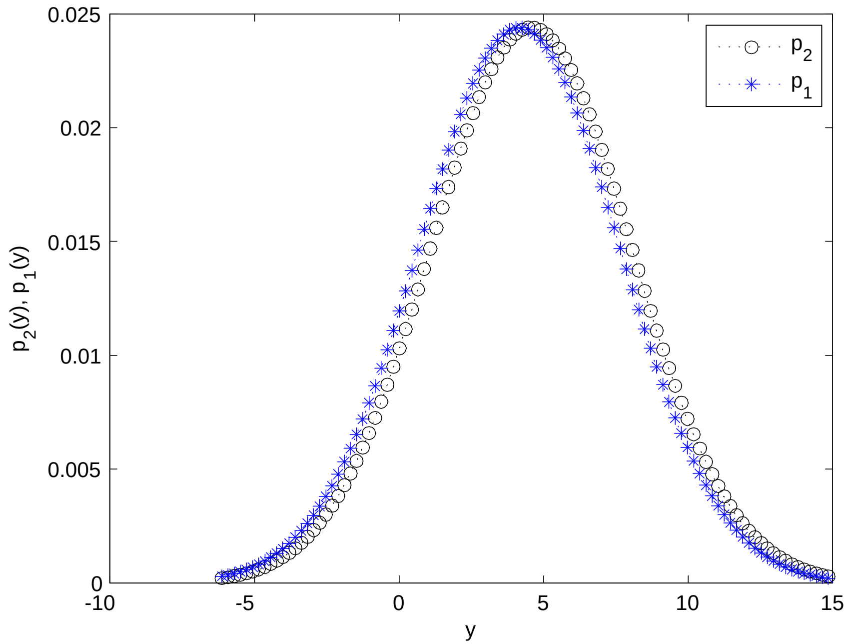
$$\beta = \frac{\log\left(\frac{\underline{V}(\eta)+\underline{\alpha}}{\underline{V}(\eta)+\overline{a}}\right)}{r(g(\overline{a})-g(\underline{\alpha}))} > 0,$$

$$\alpha = -\beta\left(\eta + \mu_\varepsilon + \rho\frac{\sigma_\varepsilon}{\sigma_\eta}(\eta - \mu_\eta)\right) + \frac{1}{2}r\beta^2(1-\rho^2)\sigma_\varepsilon^2 + \frac{g(\underline{\alpha})\log(-\underline{V}(\eta)-\overline{a})-g(\overline{a})\log(-\underline{V}(\eta)-\underline{\alpha})}{r(g(\overline{a})-g(\underline{\alpha}))}.$$

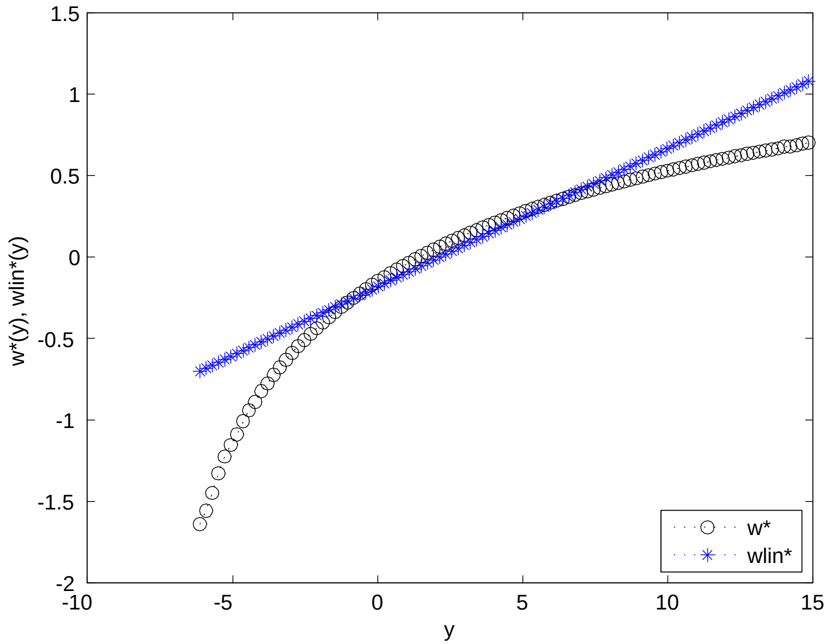
- Under CRU ($\underline{V}'(\eta) = 0$), η affects α, but not β. The effect on α is proportional to β.
  - negative effect for a pro-cyclical firm.
  - ambiguous effect for a counter-cyclical firm (it may become positive if the individual shock is sufficiently noisy and sufficiently correlated to the aggregate shock).
- Under VRU ($\underline{V}'(\eta) > 0$), η also affects β and positively so.
  - an ambiguous effect on α for a pro-cyclical firm
  - any positive response under CRU is reinforced under VRU for a counter-cyclical firm.

PARAMETERIZATION:

$$\mu_\eta = 0.2,\ \mu_\varepsilon = 0.5,\ \sigma_\varepsilon = 2,\ \sigma_\varepsilon = 4,\ \rho = -0.5,\ \eta = 0.5,\ r = 1,\ A = \{0.12, 0.15\},\ \underline{V} = -1,\ g(a) = 10\sqrt{a}$$



Outcome distribution conditional on effort



Optimal wage and best linear wage

APPROXIMATION ERROR IN TERMS OF COST TO THE PRINCIPAL:

$U^* - Ulin^* = 0.0042564;\qquad \frac{U^* - Ulin^*}{U^*} = 0.097681\%$

EXTENSIONS

1. Empirical

2. Theoretical: shocks, dynamics, insurance