Noise and aggregation of information in large markets

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The idea of the paper

- Since Grossman (1976) and Grossman and Stiglitz (1980), competitive REE equilibrium extensively studied.
- Add noise to solve GS paradox: Hellwig (1980), Diamond and Verrecchia (1981), Verrecchia (1982)
- We study how quantity of noise influences price formation in a large economy when info acquisition decisions are endogenous
- Explicitly study equilibria in a limit when the number of traders N goes to infinity

The main results

- Study a novel competitive REE in which per capita noise is negligible (DNE)
- Differs from the standard setting, (Hellwig (1980), Verrecchia (1982), where per-capita noise finite (SNE))
- Diamond and Verrecchia (1981) study an equilibrium with negligible per-capita noise but ignore information acquisition (assume all informed)
- We show that that is not an innocuous assumption
- DNE equilibrium leads to qualitatively different results for: informational content of price, risk premium, trading volume, serial correlation; in applications

Plan for the rest of the talk

- Formulation of the model for finite N
- Taking the limit when N goes to infinity and formulation of DNE and SNE
- Comparative statics. Compare and contrast the two limiting equilibria

Applications

- Info acquisition in multi-asset markets
- Sale of information by information monopolist

Extensions

- Imperfect competition and competitive limits
- Heterogeneous risk-aversion
- Different sources of noise

Finite-N model

- One trading period. All variables normal, uncorrelated and zero mean (unless otherwise stated)
- All agents have CARA preferences with risk aversion τ. Consume terminal wealth $W_i = \theta_i (X - P_N)$.
- A risk-less asset (perfectly elastic supply, zero return) and a risky asset with terminal payoff X (non-zero mean and unit variance)
- Risky asset has random supply Z_N.
- Each agent can acquire costly noisy signal (c>0):

$$Y_i = X + \varepsilon_i$$

Finite-N model

- Going backward: agents choose trading strategies to maximize their expected utility conditional on their info set (informed: signal and price; uninformed: just price)
- Markets need to clear, i.e.

$$\sum_{i=1}^{N} \theta_i = Z_N$$

- Ex-ante stage: information acquisition decision. Those who acquire info have ex-ante same utility as those who do not

Finite-N model

- Equilibrium consists of the number of informed m_N, trading strategies θ_i and price function P_N.
- Price function (confirmed in equilibrium, like in Hellwig (1980)) is

$$P_N = a_N + b_N \sum_{i=1}^{m_N} Y_i - d_N Z_N$$

Limiting equilibria

- An economy has large noise if there exists $\beta > 0$ and σ_z such that

$$\lim_{N \to \infty} \mathrm{var}(N^{-\beta} Z_N) = \sigma_z^2$$

- If $\beta = 1$, leads to systematic noise equil. (SNE)
- If $0 < \beta < 1$, diversifiable noise equil. (DNE)

Limiting equilibria

- Intuitively, if

$$Z_N = \sum_{i=1}^{N} Z_i$$

- DNE (β=0.5) obtains if individual Z_i are i.i.d., while the SNE obtains if they are systematically correlated (i.e. have a common risk factor)
- Useful to define per-capital average supply of the risky asset and normalized noise in the limit

$$\lim_{N\to\infty} \frac{E(Z_N)}{N} = \mu_z$$

$$Z_\beta \equiv N^{-\beta}(Z_N - E(Z_N))$$

Limiting equilibria

Limiting equilibrium price

$$P_\beta = \lim_{N\to\infty} P_N(\beta)$$

- Prices are partially revealing info if

$$0 < \mathrm{var}(X \mid P_\beta) < 1$$

- Under a technical condition ensuring that cost of info acquisition is not too high w.r.t. precision of the signal, a partially-revealing limiting equilibrium exists for both DNE and SNE. Prices have the same form

$$P_\beta = a_\beta + b_\beta X - d_\beta Z_\beta$$

Limiting equilibria

- The equilibrium number of informed traders in DNE and SNE model satisfy the the expression for intensity of information acquisition

$$\lim_{N \to \infty} N^{-\beta} m_N(\beta) = \lambda_\beta = \lambda$$

$$\lambda_1 = \min(1, \lambda)$$

- Number of informed grows with N at the same pace as noise. If more informed – prices fully revealing. If less, not revealing at all.
- In both equilibria the right mass of traders becomes informed

DNE vs SNE

SNE	DNE
Finite fraction informed	Negligible fraction informed
Everyone can become informed and still prices would be partially revealing	It can never be the case that everyone becomes informed (comp. with D-V(1981))

DNE vs SNE

SNE	DNE
Both informed and uninformed participate in risk-sharing	Only uninformed participate in risk sharing. Informed affect only info revealed by prices.
Risk premium is decreasing in the amount of noise σ_z	Risk premium does not depend on the amount of noise σ_z

DNE vs SNE

- For DNE, unconditional volatility of prices has clear comparative statics: it is increasing in the precision of the signal, and decreasing in its cost and agent risk aversion
- For SNE, comparative statics depends on the value of these parameters
- For DNE, price changes are uncorrelated since the aggregate demand of the uninformed sector eliminates price reversals induced by noise traders. In equilibrium

$$E(X \mid P_\beta) = \eta_\beta + P_\beta$$

DNE vs SNE

- Thus, for DNE pricing is efficient since dollar movement in the asset price causes uninformed agent to update their expectation of asset payoff by one dollar
- On the other hand, for SNE, a dollar increase in asset price updates the expectation by less than a dollar (due to negative effect of noise trades)
- Expected trading volumes also differ: in DNE informed trade only for information purposes while in SNE, they trade for both information and risk-sharing purposes.

DNE vs RN market marker models

- An alternative class of competitive REE models is risk-neutral market maker model (Vives, 1995)
- There, a continuum of informed traders is assumed, prices set by a RN market maker
- If we set info acquisition intensity to 1, and average supply of the risky asset to zero, Vives (1995) price equation is recovered from DNE
- In that model prices are efficient as in DNE

DNE vs RN Market marker models

- Despite similarities, the two models are quite different
- Info acquisition – DNE endogenous vs Vives (1995) exogenous
- Different comparative statics
- DNE can have positive risk premia, while the model with RN market maker model cannot
- DNE model shows that starting with a continuum of informed traders is not innocuous.

Info acquisition in multi-asset markets

- Extend multi-asset setup of Admati (1985) to allow for endogenous info acquisition
- Same setup as before but now we consider two assets
- First asset has systematic noise. Then we consider two cases: when we add another systematic noise asset and when we add instead a diversifiable noise asset.
- How does that influence information acquisition by agents

Multi-asset markets

- In case of two systematic noise assets, a range of parameters exists such that all informed agents acquire information about both assets – i.e. all informed agents are generalists. In that equilibrium there are no specialists, i.e. those who acquire info only on one of the two risky assets
- This happens if there is an economy of scale from gathering info on both companies

Multi-asset markets

- For mixed-noise economy (first asset SN, second asset DN), a proper fraction of traders acquires info on asset 1, while only a negligible fraction of traders acquires info on asset 2.
- As a result, in this equilibrium it is possible to have specialists in asset 1 (not possible when both assets are SN). In addition, it is also possible to have generalists
- As the correlation between the two assets increases, info gathering about just asset 1 is decreasing and may cease completely if the correlation is sufficiently high.
- In that case, only generalists survive (and no specialists in asset 1).

Markets for information

- Admati and Pfleiderer (1986) use systematic noise model to study problem facing a monopolist seller of info about a risky asset.
- She sells info in form of a newsletter.
- In equilibrium has to be decided: who should receive info, what should be the precision of info and what is the equilibrium price.
- Equilibrium price has the same form as previously explained. Key to answer the first two questions.
- We start with a finite-N economy and take the limit as before

Market for information

- So, again we consider two limiting equilibria, SNE and DNE, but this time with respect to this particular problem
- In SNE model, monopoly seller sells newsletters of finite precision, and she sell them to everyone in the market.
- In DNE case, the solution is qualitatively different. Since the amount of noise is small, too much info would quickly reveal the true value rendering newsletter worthless.
- In that case, equilibrium is to provide as little information as possible to a small fraction of investors.
- Empirical studies Graham and Harvey (1996), Jaffe and Mahoney (1999), indicate that newsletters typically have very low informational contents – closer to DNE predictions

Market for information

- Cespa (2007) assumes that information seller can contact a continuum of agents that trade in a competitive market with a RN market maker
- He obtains, in one period model, that the monopoly seller would sell to the entire population a finite precision signal – just like SNE model
- DNE model leads to completely different predictions
- The order to taking large N limit is clearly very important and assumption in that regard need to be made with care.

Imperfect competition and competitive limits

- Imperfect competition as in Kyle (1989)
- Agents now behave strategically, internalizing the impact of their trades.
- Prices again take linear form
- Each agent conjectures that she faces a residual supply curve of the form

$$P_{N_i}(\theta_i) = P_{N_i} + d_{N_i}\theta_i$$

Imperfect competition and competitive limits

- Solving the game backwards, each trader maximizes her expected utility subject to information set (assumed fixed at that stage)
- Players take into account price impact of their trades
- Equilibrium prices are set for the fixed number of informed traders m, and total number of traders N so that the market clear

Imperfect competition and competitive limits

- Going backwards, the number of informed is determined from the condition that ex-ante expected utility of informed and uninformed is the same
- Thus, equilibrium price, strategies and the number of informed is determined in equilibrium (for fixed N)
- As in the base case, next we consider two limiting equililbria parametrizing noise as DN or SN

Imperfect competition and competitive limits

- The economy with imperfect competition with endogenous info acquisition exhibits the same limiting prices, limiting optimal trading strategies and limiting measures of informed trading coincide with competitive models SNE and DNE.
- The existence of large noise is a necessary and sufficient condition for the existence of a competitive limiting equilibrium

Imperfect competition and competitive limits

- In large economies with growing noise (either DNE or SNE), strategic decisions of agents are irrelevant in price determination
- Limiting equilibria converge to perfect competition
- The result is not a special case of the limits in Kyle (1989) since we endogenize info acquisition as we increase the number of agents and noise
- Thus, large noise is necessary and sufficient for having limiting competitive equilibria

Imperfect competition and competitive limits

- This result is related to Kovalenkov and Vives (2008), but we take the limit of N goes to infinity without assuming that there is a "free entry of informed speculators"
- The auction literature, when studying information aggregation, typically focused on perfect info revelation in markets with large numbers of risk-neutral bidders (Wilson (1977), Milgrom (1981), etc)
- As discussed in Jackson (2003) this leads to GS-like paradox

Imperfect competition and competitive limits

- Introduction of noise prevents auction prices from being perfectly revealing
- Our model with imperfect competition is a particular auction type where info acquisition is endogenous and price is partially revealing
- Analogy between multi-unit auctions with large number of players and the size of aggregate supply in our model (Swinkels (2001), Jackson (2003), etc)
- DNE analogous to auction in which a fraction of players getting the good goes to zero as the number of players goes to infinity
- SNE –a positive fraction of players gets the good

Other extensions

- For SNE, Verrecchia (1982) shows that agents with higher risk tolerance are more likely to purchase info
- We show that for DNE similar, but more extreme, result holds: only the least risk averse agents get informed
- Also, we demonstrate that the DNE is, in the limit of large N, equivalent to the case when there is no aggregate supply of noise but, rather, agents face idiosyncratic endowment shocks

Conclusion

- This paper contributes to the literature on endogenous info acquisition in asset markets
- We show that large noise is necessary and sufficient to yield economies where:

- Prices are partially revealing
- Trading is competitive (agents are price-takers)

Conclusion

- New equilibrium we discuss, DNE, shares some important features of the standard SNE and equilibrium with RN market maker
- Yet, it differs from them in many important ways
- The key difference lies in the role of informed traders who, in DNE, trade only for information reasons and not for risk sharing purposes
- This leads to very different comparative statics and results in applications

Possible extensions

- Study how applications differ in DNE vs other settings.
- Empirically differentiate between DNE and SNE predictions
- Extending the model to a dynamic case may be a promising venue for obtaining testable results