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MACROECONOMIC PROBLEMS

An Empirical Model of the Relationship between Value Added Elements and Final Product in the Russian Economy

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Abstract—This is a sequel to a paper published in the previous issue of this journal (No. 1, 2008), which dealt with a system of tables to provide a theoretical description of the transformation of primary incomes into final consumption and saving expenditures. On the basis of the proposed system of tables an empirical model was built, which can describe in quantitative terms the relationship between the elements of value added and end product in the Russian economy over the period 1995-2003. We have proposed and put into practice an algorithm for estimating primary income parts (flows) spent on financing various end-use items. The calculated flows are interpreted as elements of Quadrant IV of the interindustry balance (or tables of use of goods and services) in current consumer prices. We have estimated a system of dynamic equations for these shares, which can be used as the basis for predicting the end-use structure under given volumes of value added elements.

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Statement of the Problem

A mathematical model of the relationship between the elements of value added and end product is *empirical*, i.e., it describes the dynamics of flows of primary income and expenditures on final consumption, capital accumulation and net savings[1] of economic agents. Model-building is based on the developed in the work [1] system of tables providing a detailed theoretical description of the transformation of incomes into end expenditures and savings. Applying this system implies not only borrowing its *principal* structure but also giving a detailed representation of those units of the system for which there are real data in the freely accessible statistics. Therefore, the empirical model and theoretical system of tables should be regarded not as two different models but as closely interrelated constituents of a single structure designed to reflect the transformation of the primary incomes into expenditures and net savings. In other words, the theoretical model serves as a necessary basis for building a model on empirical data.

With the help of the proposed empirical model one can determine what parts (flows) of any given type of primary incomes are spent on an final-demand item aggregated in a certain manner as well as what sources are used to finance any given type of end expenditures. The specified flows are viewed as elements of the *modified* Quadrant IV of the interindustry balance[2] of production and distribution of products and services designed to reflect the interrelation between Quadrants II and III of the interindustry balance, or elements of the end product and gross value added (GVA). Though major items of economic agents' primary incomes and their end use, the GVA elements and end product, do not comprise, nevertheless, all the incomes and expenditures in the economy. Therefore, correct modeling of coordination between primary incomes and end expenditures requires accounting for some flows not reflected in GVA and end product, which implies expansion of Quadrants II and III of interindustry balance and, hence, considering the modified in this manner Quadrant IV.

Note that at the present stage it appears to be impossible to track or calculate the elements of the modified Quadrant IV by means of the simplest arithmetic operations on the basis of the available statistics, which speaks for the need in their modeling by means of mathematical methods. A doubtless advantage of this model is not only the opportunity to obtain estimates of these flows, but also a system of dynamic equations for them, which, in its turn allows one to prognosticate these indicators on the basis of the major income flows in economy as a whole.

Empirical Model Design Methodology

The empirical mathematical model is based on the yearly data for 1995–2003. Choice of the time interval is determined by the attempt to connect the model with the interindustry balance. Unfortunately, the systems of "input-output" tables [3], whose part is interindustry balance, are published with a three year delay; therefore, at the moment of building the model, the most recent information was available only for the year 2003.

Since the length of the time interval under study is nine years, and the source data are of annual nature, it would have been incorrect to apply methods of econometric analysis to modeling the relationship between the elements of GVA and end product. Moreover, the

[1] Under net savings we understand savings net of expenditures on capital accumulation.
[2] For details see [2, p. 214].

problem of model description of the indicated relationships understood as the task of filling-in Quadrant IV of the interindustry balance, implies estimating simultaneously both the elements of Quadrant IV for every year in the period, and the coefficients of the dynamic equation system for these elements. Therefore, statement of this problem is principally different from the one used in traditional econometric analysis based on some optimization principle of estimating *only* the model coefficients under the condition that there are observations of both the interpretable and explanatory variables. In this case we know only the sums by rows and columns of the Quadrant IV elements (let us call them *bounding sums*). Thus, the model under consideration includes partially latent, statistically unobservable variables; therefore it is a priori more complex than the traditional econometric problem statement. This engenders the need to estimate its unknown parameters in a somewhat different manner.

We propose modeling the relationship between the elements of GVA and end product by means of constructing an optimization problem of a special type, which would allow one to estimate the parts of income flows spent on certain types of end consumption and accumulation as well as the coefficients of their dynamic links on the basis of the information on the bounding sums and hypothesis of the share dynamics. Note that in the absence of observations on the considered shares (parts) of incomes in the expenditures it is rather difficult to test the hypothesis regarding the nature of their dynamic links. Moreover, there is practically no possibility to a priori set different functional forms for different shares. However, the assumption that the nature of the dynamic functions for all the sought-for parts (let us view them as elements of Quadrant IV of the interindustry balance) is the same, does not seem quite plausible. And as one can hardly differentiate a priori the functional types of the dynamic equations, in order to account for all the possible diversity in the nature of these equations, we shall apply the method of setting limitations on the calculated elements of Quadrant IV of interindustry balance. Let us comment on this. The theoretical scheme developed in [1] and official statistics do not give enough information to exactly determine the share of income flows in the end expenditures, however, one can set upper or lower limits on some of them. At the same time, the boundary values are different not only by elements but also in time. Thus, one sets for the number of model elements a certain *range* of their changes on the basis of real statistical information. The introduced limitations may exercise indirect influence on those elements on which one could not set limits beforehand. This is due to the fact that dynamic equations of the shares of income flows in the expenditures are considered not separately but as a *system* estimated within the framework of a single optimization problem statement. Note that these limitations allow one to correct in the necessary direction not only the values of the interindustry balance Quadrant IV elements but also dynamic equations coefficients.

Let us assume that the dynamics of the investigated elements can be described by means of certain functional dependency with stable coefficients accurate to the errors, which should not exceed a certain threshold value. At this stage, we propose to reject a stochastic problem statement in favor of a determined one, which means treating errors as non-random values. Using a probabilistic toolkit in models of this kind may, generally speaking, be useful, for example, for estimating the coefficients' significance. However, in case of applying the probabilistic methods of regression analysis, the investigated time series should contain enough observations for the conclusions to be regarded as reliable. In our case, the time series used in the model are short, which largely levels the benefits of applying the stochastic problem statement.

The hypothesis on the existence of errors in the equations of the dynamic system suggests, in fact, that one should fit the unknown parameters of the model by means of minimizing these errors. Thus, the task of searching for the model solution can be reduced to finding the minimal value of some functional convolution of the errors given the set of limitations using the a priori established form of the dynamic links of each interindustry balance Quadrant IV element; *content-based* limitations setting the range of changes in the variables; balance identities providing for the conformity of the solution to the bounding sums, etc.

The analyzed mathematical model may be referred to the category of high-dimension conditional optimization problems since its solution requires filling in the matrices of the modified Quadrant IV for the nine years of the investigated time interval, calculating the dynamic equation coefficients for *each* element (we assume that the coefficients do not change with time), and calculating error matrices for each time period except for the first one. Apart from the high dimensionality, additional difficulties arise due to the need to introduce a big number of limitations. To relieve the acuteness of the problem caused by a big number of variables and limitations, we suggest aggregating in certain way the items of primary incomes and end use representing the bounding sums of the modified Quadrant IV of the interindustry balance, retaining at the same time the homogeneity and content sense of the consolidated flows. These flows aggregation is also caused by the need to set a limitation on the number of dimensions of the matrix representing the shares of income flows in the expenditures in order to prevent overcomplication of the analyzed dynamic system and generation of too many errors inevitable when one tries to estimate the values of some indicators by a model method, not statistical one.

Table 1. Schematic diagram of the matrix X^t

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1						
	Pr^t	2						
	MI^t	3						
	NOT^t	4			X_{ij}^t			
	NPT^t	5						
	PI_f^t	6						
	Bor^t	7						
	Inv_{res}^t	8						

Mathematical Statement of the Empirical Model

Modeling the modified Quadrant IV reduces to filling in the matrices X^t for all the years t of the time interval under study (let us refer to the elements of these matrices as X_{ij}^t) whose bounding sums are specifically aggregated flows of primary incomes and expenditures on end consumption and accumulation, and net savings in economy as a whole. The general structure of these 8×6 matrices bounded by the column with elements A_j^t ($i = \overline{1, 8}$) and row with elements B_j^t ($j = \overline{1, 6}$), is given in Table 1.

The sums of elements X_{ij}^t by row represent a column vector (A_j^t) of the flows of domestic economy incomes in the reporting period t, among which one should single out compensation of employees W^t, gross profit Pr^t, gross mixed income (GMI) MI^t, net other taxes on production NOT^t, net taxes on products NPT^t, incomes from property received from ROW, PI_f^t (aggregated with current transfers from abroad), and borrowing Bor^t. It is easy to see that the introduced classification of the income flows implies aggregating some GVA elements in market prices, in particular, taxes and subsidies on products, other taxes and subsidies on production, and financial intermediation services indirectly measured. Taking into account the content-based interpretation of the FISIM indicator [1], it was deducted from gross profits. Additionally, the model consolidates the flows from abroad which incomes from property and current transfers are referred to. We should point out that gross profit and wage indicators used in this model are more closely consistent with the national accounts methodology than the interindustry balance, because here concealed wages are included into the compensation of employees. In other words, on the basis of the national accounts data on concealed wages, this indicator was deducted from the gross profit item and added to the employee compensation item of Quadrant III of the interindustry balance. This reshaping of the flows structure serves the purpose of achieving their maximal content-wise homogeneity.

The sums of elements X_{ij}^t by column of the respective matrices X^t represented as row vectors (B_j^t) reflect the end expenditures and net saving in economy as a whole. Among these we distinguish expenditures on: end consumption of households C_{hh}^t; of public institutions, including nonprofit institutions serving households C_g^t; fixed capital accumulation K^t; increase in inventories $\mathrm{Inv}_{\exp}^t$; and incomes from property paid by the residents to ROW (taking into account the current transfers to foreign recipients) PC_f^t and lending (net savings) Cr^t. In this case, with respect to the interindustry balance classification, we have aggregated all the expenditures on end consumption of public institutions and nonprofit institutions serving households, inventories of producers, consumers and in trade, including net value acquisition. Being the resource of economic agents-residents, export costs are not shown on the expenditure side; import costs are included into the items of expenditures on end consumption and accumulation. Incomes from property transferred to ROW are aggregated with the respective current transfers.

Let us comment on the item "net lending". The value of the indicator "net lending" is calculated by the balance method in order to provide for the equality of the sums of the X_{ij}^t elements by row and column, which, in its turn, implicitly suggests that this indicator includes the statistical divergence. Content-wise, the net lending (borrowing) item reflects either the part of the savings not spent on fixed capital and inventories accumulation (at $NCr^t > 0$), or the use of resources accumulated in the previous periods to finance current expenditures if the incomes received in the reporting period are insufficient to cover them (at $NCr^t < 0$). At the same time, there can be a situation when some institutional sectors will have to resort to spending the previously accumulated resources, at the time the other ones—to their saving. In 1999, the households sector was observed to be in the situation of a fairly significant lack of current incomes to finance their expenditures on end consumption and savings, at the time the public institutions and corporate sectors had positive savings, the net lending (borrowing) indicator (net savings) for the economy as a whole being positive. This means that in 1999 net savings also served as resources to finance the current period expenditures (for households) and expenditures on consumption and savings of the future periods (for the state and corporations). This inhomogeneity of the analyzed flow calls for the need to account for the opportunities for different economic agents to simultaneously borrow resources from their own savings of the previous periods and accumulate savings, which implies disjoint the lending and borrowing items from the single "net lending" item.

In the model under study, the net income flow from ROW (here we mean only incomes from property and current transfers) is also divided into two components, reflecting the receipt and transfer of resources by domestic economy, and may be regarded both as resources, and expenditures simultaneously. As in the case of dividing the net lending (borrowing) item, the reason for this is inhomogeneity of the flow by economic agent ñ for some agents (as a rule, for corporations) ñ these are expenditures, for others (as a rule, households) ñ additional incomes.

We should point out some specific features of the model related to inventory dynamics. Unlike other indicators, this one can be both positive and negative, i.e., depict, depending on its sign, either the economic agents' expenditures, or their resources. A negative change in inventories was registered in 1998, which motivates a dual reflection of inventories in the model. In the use part of the X^t matrix, the changes in inventories are shown in column $\mathrm{Inv}^t_{\mathrm{exp}}$, in the resource part ñ in the row $\mathrm{Inv}^t_{\mathrm{res}}$. For all the years, except for 1998, $\mathrm{Inv}^t_{\mathrm{res}}$ is assumed equal to zero (as well all the X^t_{ij} elements in the respective row). For 1998, on the contrary, the indicator $\mathrm{Inv}^t_{\mathrm{exp}}$ together with all the X^t_{ij} elements of the respective column is assumed a priori equal to zero. Thus, we ensure non-negativeness of all the X^t_{ij} variables considered in the model, and the bounding sums.

To describe the dynamic links between the X^t_{ij} elements, we suggest considering the following model ($\forall i = \overline{1, I}$, $j = \overline{1, J}$, $t = \overline{2, T}$), where I, J are the numbers of rows and columns of the matrix X^t respectively; T is the number of years in the time period under review:

$$X^t_{ij} = X^{t-1}_{ij} + \alpha^+_{ij}(D^t_i)^+(A^t_i - A^{t-1}_i) + \alpha^-_{ij}(D^t_j)^-(A^t_i - A^{t-1}_i) + A^t_i\varepsilon^t_{ij}, \quad (1)$$

where ε^t_{ij} is the relative error, while α^+_{ij} and α^-_{ij} are model coefficients not changing with time, and $(D^t_i)^+$ and $(D^t_i)^-$ are logical variables of the type:

$$(D^t_i)^+ = \begin{cases} 1, \text{ if } (A^t_i - A^{t-1}_i) \geq 0 \\ 0, \text{ if } (A^t_i - A^{t-1}_i) < 0 \end{cases}, \quad (D^t_i)^- = 1 - (D^t_i)^+. \quad (2)$$

Let us comment on the selected specification of equation (1). The indicators analyzed in the model represent macroeconomic time series, which in the absence of global changes (for example, transformation of the Russian economy in early 1990s) are, as a rule, stationary in the first differences. Taking into consideration the extremely small length of the available sample, within the framework of the current research it does not make sense to conduct any stationarity analysis, moreover, the model is built according to a determined statement. However, it may be useful to apply some a priori knowledge on the type of difference equation suitable for describing the analyzed flows, in particular, its representation as a difference process, a series stationary in the first differences. This representation implies, first, including into the right part of equation (1) an autoregression variable with the unit coefficient, and, second, temporal stability of the income flow distribution coefficients by main items of end expenditures, not in levels but in increments.

A specific feature of model (1) is introduction of different coefficients for those cases when the income flow of the certain type A^t_i (in the current market prices) is either increasing, or decreasing, in the reporting period as compared to the previous one. This is caused by the fact that the economic agents' behavior is, generally speaking, different when their financial resources are increasing or decreasing, therefore, the left and right derivatives of the flow X^t_{ij} by the increment A^t_i (positive or negative) may be different. This is reflected by the introduction of two coefficient sets: (α^+_{ij}) and (α^-_{ij}). Since these coefficients are interpreted as unit weights of changes in the elements X^t_{ij} (ΔX^t_{ij}) in the increments of income flows (ΔA^t_i), for them fulfilled should be the limitations of non-negativity and equality to one of their sums by row ($\forall i = \overline{1, I}$, $j = \overline{1, J}$).

$$\alpha^+_{ij} \geq 0, \quad \alpha^-_{ij} \geq 0, \quad \sum_j \alpha^+_{ij} = 1, \quad \sum_j \alpha^-_{ij} = 1. \quad (3)$$

Note that the function of the error $A^t_i\varepsilon^t_{ij}$ possesses a rather useful property: *the sum of the elements $A^t_i\varepsilon^t_{ij}$ by row equals zero*, which can be easily shown by summing up the equation of type (1) by j.

The necessary requirement to the model is balancing the elements X^t_{ij} and their bounding sums, which is reflected as limitations:

$$\sum_j X^t_{ij} = A^t_i, \quad \forall i = \overline{1, I}, \quad t = \overline{1, T};$$
$$\sum_j X^t_{ij} = B^t_j, \quad \forall j = \overline{1, J}, \quad t = \overline{1, T}. \quad (4)$$

Let us introduce into the model also the size of the maximum relative error ε^t_{ij} as an independent variable (and denote it as μ). Introduction of such variable implies, by definition, fulfillment of the following set of limitations:

$$\left|\varepsilon_{ij}^t\right| \le \mu, \quad \forall i = \overline{1, I}, \quad j = \overline{1, J}, \quad t = \overline{2, T}. \qquad (5)$$

Let us consider further the criterion of the optimization problem assumed as a basis for the empirical model. The traditional criterion is that of the least-squares method (LSM)—the sum of the error squares. We propose to modify for this problem the LSM-criterion and consider the target function of the type:

$$F = \lambda\mu^2 + (1-\lambda)\sum_{t=2}^{T}\sum_{i=1}^{T}\sum_{j=1}^{T}(\varepsilon_{ij}^t)^2, \qquad (6)$$

where the parameter $\lambda \in [0, 1]$ is exogenous. Note that the function F is more general optimization criterion as compared to that of LSM and can easily reduced to the latter at $\lambda = 0$. The selected target function, as a more flexible one, gives the opportunity to choose between reducing the relative error size, minimizing the sum of squared errors, or applying both criteria together. In fact, the chosen criterion represents a convolution of the target functions of the double-criteria problem, where λ is interpreted as significance of one of them.

Joining together the above described limitations and target function (6), let us formulate the general framework of the optimization problem one whose basis we intend to estimate the elements of the matrices X^t and their dynamic equations coefficients:

$$\begin{cases} \lambda\mu^2 + (1-\lambda)\sum_{t=2}^{T}\sum_{i=1}^{T}\sum_{j=1}^{T}(\varepsilon_{ij}^t)^2 \xrightarrow[X, E, A^+, A^-, \mu]{} \min \\ X_{ij}^t = X_{ij}^{t-1} + \alpha_{ij}^+ D_i^+(A_i^t - A_i^{t-1}) + \alpha_{ij}^- D_i^-(A_i^t - A_i^{t-1}) \\ \quad + A_i^t\varepsilon_{ij}^t, \ \forall i = \overline{1, I}, \ j = \overline{1, J}, \ t = \overline{2, T} \\ \sum_j X_{ij}^t = A_i^t, \quad \forall i = \overline{1, I}, \quad t = \overline{1, T} \\ \sum_i X_{ij}^t = B_j^t, \quad \forall j = \overline{1, J}, \quad t = \overline{1, T} \\ \sum_j \alpha_{ij}^+ = 1, \quad \forall i = \overline{1, I}, \quad j = \overline{1, J} \\ \sum_j \alpha_{ij}^- = 1, \quad \forall i = \overline{1, I}, \quad j = \overline{1, J} \\ \left|\varepsilon_{ij}^t\right| \le \mu, \quad \forall i = \overline{1, I}, \ j = \overline{1, J}, \ t = \overline{2, T} \\ \alpha_{ij}^+ \ge 0, \quad \forall i = \overline{1, I}, \quad j = \overline{1, J} \\ \alpha_{ij}^- \ge 0, \quad \forall i = \overline{1, I}, \quad j = \overline{1, J} \\ X_{ij}^t \ge 0, \quad \forall i = \overline{1, I}, \quad j = \overline{1, J}, \quad t = \overline{1, T}, \end{cases} \qquad (7)$$

As can be seen from system (7), the independent variable set by which optimization is conducted consists of the sets X, E, A^+, A^- and maximal relative error μ, where $X = \{X_{ij}^t, i = \overline{1, I}, j = \overline{1, J}, t = \overline{1, T}\}$, $E = \{\varepsilon_{ij}^t, i = \overline{1, I}, j = \overline{1, J}, t = \overline{2, T}\}$, $A^+ = \{\alpha_{ij}^+, i = \overline{1, I}, j = \overline{1, J}\}$, $A^- = \{\alpha_{ij}^-, i = \overline{1, I}, j = \overline{1, J}\}$.

IMPOSITION OF STATISTICAL LIMITATIONS

As noted above, a necessary stage in model building is the imposition of additional limitations based on the theoretical scheme [1], and accessible statistical information enabling one to set the range of changes of the elements X_{ij}^t. The need for an additional limitation is motivated by the necessity to, first, account for actually existent differences in the dynamics of the elements in consideration, and, second, attract new information on the modeled financial flows in order to reach, in terms of content, maximal plausibility of the model solution. The system (7) may be used to assess the elements X_{ij}^t and their dynamic equations coefficients α_{ij}^+ and α_{ij}^-; however, the experience of solving the model has shown that this optimization problem has too many "degrees of freedom" since one needs to determine the values of many unknowns from little source information. Use of additional limitations of the set type allows one to alleviate this imbalance through involvement of exogenous data on the changes of the analyzed flows not included in the model.

Let us look successively at the limitations on the dynamics of the elements X_{ij}^t (let us call them *statistical limitations*).

1. Limitations on the part (share) of compensation X_{11}^t ($t = \overline{1, T}$) spent on end consumption of households. The item "compensation" (by the interindustry balance as well as national accounts methodology) includes social contribution to the state budget and state nonbudgetary funds and individual income tax. These contributions transferred to the state before the employees receive the compensations for their labor cannot be spent by the latter on end consumption. Moreover, the households also pay to the public sector the property taxes. The latter can be paid not only from the wages but also, for example, from the mixed income, or incomes received by households in the process financial resources redistribution. But as the amount of property taxes paid by the population was relatively small for most years of the analyzed time interval, let us assume that this type of compulsory payments is financed from the wages. Thus, the absolute value of the share of compensation X_{ij}^t spent on end consumption of households cannot exceed the difference between the wage indicators and contributions paid to public authorities[3]. To account for all of them, one will

[3] Note that in the analyzed model the wage indicator includes concealed wages.

have to involve data on the social contributions and taxes on incomes and properties of non-artifical persons, which are included in national accounts. In view of the specific features of methodology and calculations discussed in detail in [1], the compensation of employees indicators in the national accounts and inter-industry balance do not match, therefore, in the limitation on X^t_{ij} we used, instead of the absolute values of the contributions, their shares in the compensations of employees by the national accounts methodology (net of concealed wages W^t_{hidden}). Mathematically described, the limitation can be expressed as ($\forall t = \overline{1, T}$).

$$X^t_{11} \le (1 - \varphi^t)(A^t_1 - W^t_{\text{hidden}}) + W^t_{\text{hidden}}, \tag{8}$$

$$\varphi^t = \frac{(\text{deduct. soc.insur})^t + (\text{indiv. income and prop.tax})^t}{W^t_{HC} - W^t_{\text{hidden}}}, \tag{9}$$

where W^t_{HC} is compensation by the national accounts methodology.

2. Limitations on the share of compensation X^t_{12} ***(t =*** $\overline{1, T}$***) spent by public authorities on end consumption.*** Social contributions and individual income and property taxes transferred to the public sector represent part of its incomes and can be spent on covering the public institutions' expenditures on end consumption. This means that one can introduce for the flows X^t_{12} the limitation of the type ($\forall t = \overline{1, T}$):

$$X^t_{12} \le \varphi^t(A^t_1 - W^t_{\text{hidden}}). \tag{10}$$

Inequality (10) shows that one can spend, of course, less money on end consumption financed from public resources then it is shown in its right part, since the government also has other types of income. However, one cannot spend more resources out of the wages W^t then it is indicated in the right part of inequality (10).

3. Constrains on the GVA share X^t_{32} ***(t =*** $\overline{1, T}$***) spent on end consumption of public institutions and nonprofit organizations serving households.*** In contrast to the officially registered wages, one does not levy social contributions from the incomes received by household from their entrepreneurial activity, or gross mixed income. Different should be the situation with income taxes. By the current legislation, household members have to pay taxes from GVA to the public sector. In the Russian Statistical Yearbook, the sum of the taxes imposed on incomes on non-artifical persons is not separated by source of income (wages or mixed income). Moreover, in practice, households tend most cases to conceal their mixed incomes not to pay on them; therefore, the lion's share of taxes on members of the household incomes is levied from actually registered compensation of employees. In this respect, within the framework of the analyzed model, we make an assumption that all the individual income taxes are transferred to the state only from compensation of employees, and not GVA. This assumption can be expressed as a limitation ($\forall$t = $\overline{1, T}$):

$$X^t_{32} = 0. \tag{11}$$

4. Limitations on the share of gross profit X^t_{21} ***(t =*** $\overline{1, T}$***) spent on the end consumption of households.*** Incomes from property are an important item of financial resources redistribution in economy. A considerable part of households' incomes from property are financial flows received from the corporate sector: dividends, interest on deposits and corporate securities. Data on these types of incomes can be obtained from the Balance of Population's Money Incomes and Expenditures (BMIE). Incomes from property also include rental incomes, however, in the publicly available statistics there are no data on the share of the corporate sector in the population's rental incomes. Moreover, the biggest part of these incomes is formed, most probably, in the households sector, through renting the housing space by one group of households to another. Therefore, one may neglect the population's rental incomes received by the corporate sector.

In addition to incomes from property, households also receive from the corporate sector the revenues recorded under the items "incomes from enterprises and organizations, except for compensation of employees", "social transfers" (including those from nonprofit organizations serving households included by the methodology of national accounts into the corporate sector), and "other current transfers". According to the methodological principles (see [6, pp. 95–96]), incomes from enterprises and organizations, except for compensation of employees include social payments (insurance payments, pecuniary aid, dismissal wages, etc. paid out of the enterprise funds) and other payments (travel expenditures, author's honoraria, etc.). These data can also be obtained from BMIE. The information on social and current transfers[4] is available in the integrated table of national accounts.

Thus, the resources paid from gross profits for households consumption should not exceed the sums of dividends, interest payments on deposits and securities, incomes from enterprises and organizations, except for compensation of employees, as well as social and other current transfers from the corporate sector ($\forall t = \overline{1, T}$):

$$\begin{aligned} X_{21}^t \le & (\text{div})^t + (\% \text{ on dep. and securities})^t \\ & + (\text{inc. from ent. save wages})^t \\ & + (\text{soc. transfers from corp.})^t \\ & + (\text{other cur. transfers from corp.})^t. \end{aligned} \quad (12)$$

Unfortunately, the interests on governmental and private securities are not divided in BMIE. Moreover, other current transfers received by households are not distributed by source. But since inequality (12) represents the upper limit, one can include in its right part the interests on all the securities, not only corporate ones, and all other current transfers paid to households in the reporting period. Of course, this is a simplification; however, in this case there is not enough statistical information to impose a more exact limitation.

5. Limitations on the shares of net taxes on products X_{51}^t and other net taxes X_{41}^t ($t = \overline{1, T}$) spent on end consumption of households. Net taxes on production and import including net taxes on products and other net taxes on production, being one of government's income sources, may be used for paying out the state social transfers to the population (other current transfers, as one can see from [6, 8], are not paid by the state to the population). Social transfers, in their turn, may be spent by households, for example, on end consumption. Note that the categories of population receiving social transfers from the state spend them mostly on end consumption and save only an insignificant part of them. The source of information on these financial flows is national accounts (the account of secondary distribution of incomes). The main items included into social transfers are pensions, allowances and social assistance, stipends, reimbursements of expenditures to disabled persons and damage to rehabilitated persons, and insurance compensations from governmental social insurance funds. On this basis, we can set the upper limit for the sum of the flows X_{41}^t and X_{51}^t ($\forall t = \overline{1, T}$):

$$X_{41}^t + X_{51}^t \le Tr_g^t, \quad (13)$$

[4] On the structure of the items "social transfers" and "other transfers" please see the publication [7].

where Tr_g^t is the amount of governmental social transfers by the national accounts data.

In the right part of inequality (13), we use absolute values of the transfer payments; shares of the analyzed income flow of the economy—net taxes on production and import—we not introduced in this case. This is motivated by the fact that the data on net other taxes on production in the national accounts of the years 2001, 2004, and 2005 (see. [9–11]) [5] are practically the same as the analogous indicators of the interindustry balance, and the information on net taxes on products is available in the national accounts only.

6. Limitations on the share of gross profit X_{22}^t ($t = \overline{1, T}$) spent on end consumption of public institutions and nonprofit organizations serving households. As it is known, the gross profit indicator reflects net incomes of all the enterprises that are legal persons, including those financed from the state budget. Therefore, a part of gross profits, more exactly the share of budget enterprises' profits, may be spent on covering the expenditures on end consumption of the public authorities. Moreover, according to the national accounts methodology, nonprofit organizations serving households are included into the corporate sector, and the expenditures on consumption of nonprofit organizations serving households are accounted for in this model in the item "end consumption of public institutions". Hence, these expenditures can also be financed from gross profits.

One also should take into account the items of financial resources redistribution from corporations to the state: corporate profits taxes, incomes from property, and other current transfers received by public institutions from the corporate sector. These payments can also be made from gross profits. Note that the accessible statistics has data only on the total sum of money the public sector has received in the form of incomes from property and other current transfers; one cannot track the share of corporations in the indicated flows. Nevertheless, given the upper limit on the element X_{22}^t, the available information would be enough, though the limitation is not that effective in this case.

Including the expenditures of nonprofit organizations serving households into end consumption of public institutions enables one to also set the lower limit on the element X_{22}^t, since these expenditures should necessarily be financed by the corporate sector.

[5] It is these publications that were used as information base when imposing the limitations, since the most recent issue of 2006 [12] demonstrates rather significant differences with the interindustry balance.

Thus, the two side limitation on X_{22}^t can be expressed as ($\forall t = \overline{1, T}$)

$$\text{(end cons. of nonprofit organizations serving households)}^t \le X_{22}^t$$
$$\le \text{(end cons. nonprofit organizations serving households)}^t + \text{(corp. prof. tax)}^t \quad (14)$$
$$+ \text{(other cur. transf. and inc. from prop. received by public sec)}^t.$$

Data on all the indicators used in (14) are available in the integrated tables of national accounts.

7–8. Limitations on the shares of wages X_{15}^t and GVA X_{35}^t as well as on the shares of net taxes on products X_{55}^t and other net taxes X_{45}^t ($t = \overline{1, T}$) spent on paying out the incomes from property and current transfers to ROW. In constructing the limitations that allow one to specify the range of changes of the elements X_{ij}^t, the problem arises systematically consisting in that it is impossible to separate the financial flow received by an institutional sector, by agents who transfer the flow. A similar problem also arises with respect to the incomes from property and current transfers paid by the residents to ROW (in the analyzed model these flows are aggregated and presented in the column PC_f^t of the matrix X^t). However, the available information one the amount of money paid by each agent-resident within this item (including payments to other agents-residents) allows one to introduce "upper" limits on the shares of the residents' primary incomes paid to ROW. These limits are relevant because for households and, during a number of years, for the government, the amount of incomes from property paid by these sectors to all economic agents was lower than that of incomes from property received by ROW. As regards current transfers, we cannot set this kind of limit. Taking into account the relationships between the elements of GVA and end product, we can introduce two sets of limitations: on the shares of wages and GVA (households' primary incomes) and on the shares of taxes on products and other taxes on production (public sector's primary incomes) paid to ROW in the form of incomes from property and current transfers:

$$X_{15}^t + X_{35}^t \le \text{(incomes from prop. paid by households)}^t + \text{(all ther cur. transfers paid to ROW)}^t; \quad (15)$$

$$X_{45}^t + X_{55}^t \le \text{(incomes from prop. paid by the gov)}^t + \text{(all other cur. transfers paid to ROW)}^t. \quad (16)$$

Limitation (15) makes sense at $\forall t = \overline{1, T}$, while limitation (16)—at $t \ne \overline{2, 5}$ (at $t = \overline{2, 5}$, i.e., for 1996–1999 limitation (16) in this model is a priori ineffective). In the right part of inequalities (15) and (16), incomes from property paid by households and public institutions respectively also include all other current transfers received by ROW. This is motivated by the fact that in this case it does not make sense to add other current transfers paid by these sectors to other ones (including residents) ñ the limitations are a fortiori less effective.

9–11. Limitations on shares of wages X_{16}^t and GVA X_{36}^t, on the shares of gross profits X_{26}^t as well as shares of net taxes on products X_{56}^t and other net taxes X_{46}^t ($t = \overline{1, T}$) used for net savings (the item "lending" in Table 1). During the analyzed period (1995–2003), the net lending (borrowing) indicator (net savings indicator) was positive. This means that lending prevailed in the economy. However, in certain years different institutional sectors-residents resorted to borrowings: corporate sector—in 1995–1999, public institutions—in 1997–1998, households—in 1999. Statistical information on net lending (borrowing) by institutional sector is available in the integrated tables of national accounts. However, taking into account the inconsistency of this source with the interindustry balance as well as the fact that this item is a balancing item both in the model and in SNA, it would be ill-considered to directly apply this information. Therefore, the data on lending and borrowing were calculated by the authors on their own, on the basis of the accessible information.

Let give a brief description of the algorithm of calculating these indicators. At first we estimated the indicators of net lending (borrowing) (NCr_k^t) for the corporate sector, households and public institutions by the formula:

$$NCr_k^t = S_k^t - K_k^t - \mathrm{Inv}_k^t, \quad (17)$$

where S_k^t is gross savings of the sector k; K_k^t—for gross fixed capital accumulation by the sector k; Inv_k^t— for the expenditures on purchasing (or incomes from reducing) inventories.

The calculations by formula (17) we conducted using the data of the integrated tables of national accounts. Then, the calculated indicators were used to estimate the shares of lending (borrowing) of the sectors in the lending (borrowing) in economy as a whole. The last step was to calculate the indicators of lending and borrowing for the sectors on the basis of the above shares and information on net lending in economy as a whole estimated by the model using the balance method.

Thus obtained data were used to set "upper" limits on the shares of gross profits X_{26}^t as well as shares of net taxes on products X_{56}^t and other net taxes X_{46}^t ($t = \overline{1, T}$).

$$X_{26}^t \le Cr_{\text{corp}}^t, \quad (18)$$

$$X_{46}^t + X_{56}^t \le Cr_g^t, \quad (19)$$

where Cr_{corp}^t and Cr_g^t are lending provided by the corporate and public sector respectively. Note that in 1999 (t = 5) the households sector was a "net" borrower, therefore the parts of wages and GVA that went on lending in this year may be assumed equal to zero:

$$X_{16}^5 = X_{36}^5 = 0. \quad (20)$$

12. Limitations on the shares of wages X_{13}^t and GVA X_{33}^t ($t = \overline{1, T}_{\max}$) spent on gross fixed capital accumulation. Gross fixed capital accumulation represents an item of end expenditures of all the economic agents, which poses certain problems in introducing limitations on the respective elements X_{ij}^t. The problems consist in the fact that, strictly speaking, on the shares of wages and GVA spent on gross fixed capital accumulation (GFCA) cannot have an "upper" limit be limited of the resources spent by households on GFCA, because other institutional sectors can finance these expenditures from the resources received from households in the process of redistribution. According to the theoretical scheme described in the publication [1], the households pay to the benefit of other economic agents the income and property taxes, other current transfers and incomes from property transferred to other economic agents. On this basis, we may set the limitation ($\forall t = \overline{1, T}$):

$$\begin{aligned} X_{13}^t + X_{33}^t \le K_{hh}^t & \\ + (\text{incomes from prop. paid by households})^t & \\ + (\text{other cur. transfers paid by households})^t & \\ + (\text{phys. pers. income and prop. taxes})^t, & \end{aligned} \quad (21)$$

where K_{hh}^t is GFCA by households. All the data required for constructing limitation (21) were obtained from the integrated table of national accounts.

13. Additional limitations. Let us impose limitations on the elements X_{ij}^t and coefficients α_{ij}^+ and α_{ij}^- having auxiliary character and based on certain assumptions.

It is assumed in the model that borrowings made by economic agents do not transform into lending (net savings). Moreover, incomes from property received from ROW are not sent back abroad in the form of "other world's" incomes from property. One can express it in the form of limitations in the following way ($\forall t = \overline{1, T}$):

$$X_{65}^t = 0 \quad \text{and} \quad X_{76}^t = 0. \quad (22)$$

Within the framework of the model, we also assume that fulfilled is the limitation of the form ($\forall j = \overline{1, J}$):

$$\alpha_{2j}^+ = \alpha_{2j}^-. \quad (23)$$

Imposition of limitation (23) is motivated by the fact that at $i = 2$ (the row refers to the gross profit indicator) there is only one observation (at t = 4, i.e., in 1998), which uses the coefficients α_{ij}^+, and not α_{ij}^-. Generally speaking, it does not make sense to estimate a coefficient by one observation only; therefore we suggest not distinguishing for row $i = 2$ the coefficients of the dynamic equations of elements X_{2j}^t depending on the sign of increase of gross profits A_2^t.

To achieve the most correct interpretation of the model solutions, we revealed in the course of estimation the need to impose limitations on the shares of wages and GVA spent on end consumption of households ($\forall t = \overline{1, T}_{\max}$):

$$X_{11}^t / A_1^t = X_{31}^t / A_3^t. \quad (24)$$

Compensation of employees and GVA are primary income flows of households and, probably, are spent on this sector's end consumption. On the basis of the existing statistical information one cannot separate the shares of the indicated income types in financing the population's end consumption; therefore we accept the hypothesis of their equality.

As already noted, in 1998, inventories were decreasing (in other years they were increasing), hence, could have served as additional source of income. At the same time, we suggest making the assumption that this source of income did not transform into a lending flow and was spent on consumer or production needs. In the form of the limitation this assumption is written as ($\forall t = \overline{1, T}$):

$$X_{86}^4 = 0. \quad (25)$$

Analysis of the Empirical Model Solution

The basic optimization problem (7) and set of statistical limitations enabling one to specify the variation limits of the elements of Quadrant IV of the interindustry balance, described in detail in the previous section, fully specify the empirical model of the relationships between the elements of GVA and end product. The model is an optimization problem of quadratic programming on the set described by linear limitations with criterion (6). The conditional optimization problem set within the framework of the model has a comparatively higher dimensionality because it contains 1301 variable and 1049 limitations.

The solution was obtained at the parameter $\lambda = 0.5$. Note that the principle of selecting the parameter λ turns into content-based analysis of the solution obtained at different numeric values of this parameter. In the authors' opinion, at $\lambda = 0.5$ the model solution has an acceptable maximal relative error and the best economic interpretation.

The primary analysis of the obtained model solution has revealed the need to introduce some logical variables (*dummy*), which helped reduce the value of the maximal relative error of the model and improve the quality of forecast. *Dummy*-variables were added to the dynamic equations for elements X^t_{46} and X^t_{56} reflecting the parts of net other taxes on production and net taxes on products used for lending (net savings). Let us determine the indicated variables in the following way ($\forall t = \overline{2, T}$):

$$d1^t = \begin{cases} 1, \text{ if } \Delta Cr^t < 0 \text{ and } \Delta Cr^{t-1} \geq 0, \\ 0, \text{ in other cases}, \end{cases} \quad (26)$$

$$d2^t = \begin{cases} 1, \text{ if } \Delta Cr^t \geq 0 \text{ and } \Delta Cr^{t-1} < 0, \\ 0, \text{ in other cases}. \end{cases} \quad (27)$$

We introduce *dummy*-variables into dynamic equations (1) for variables X^t_{46} and X^t_{56} in the additive manner ($\forall i = \{4, 5\}, j = 6, t = \overline{2, T}$):

$$X^t_{ij} = X^{t-1}_{ij} + \alpha^+_{ij}(D^t_i)^+ \Delta A^t_i + \alpha^-_{ij}(D^t_i)^- \Delta A^t_i + \beta^1_{ij} d1^t + \beta^2_{ij} d2^t + A^t_i \varepsilon^t_{ij}, \quad (28)$$

where β^1_{ij} and β^2_{ij} are the coefficients of the logical variables.

The logical variables are not exogenous but determined endogenously on the basis of a certain statistical indicator dynamics. This method of introducing indicative variables is the most appropriate, because, first, it makes sense in term of economics, and, second, when forecasting by the model one does not encounter the problem of choosing the value of the *dummy*-variable. Let us comment on the last point. To determine the values of both logical variables specified according to (27) and (28), one should know the amount of lending for the forecasted period, moreover, the lending indicator should be forecasted on the basis of the model. Therefore, at the first step one can predict the structure of resource end use (including the lending indicator) without *dummy*-variables in order to select the sign of increase/decrease in lending in the future period. Then, the obtained information can be used to set logical variables and construct a more accurate forecast.

Taking into account the logical variables (27) and (28), we have obtained the final solution of the empirical model describing the relationship between the elements of GVA and end product (see *Appendix*). The solution is presented in the form of filled-in tables of the dynamic equation coefficients for the elements X^t_{ij} of the modified Quadrant IV of the interindustry balance (Table A1) and tables representing the values of these elements for all the years of the analyzed period (Tables A2–A10).

Among all the elements of the modified Quadrant IV of the interindustry balance estimated by the model of greatest interest are those describing the distribution of the basic income sources—wages, gross profits and mixed incomes, net taxes by the most extensive and important items of expenditures: end consumption of households and the public sector, and gross fixed capital accumulation. The main part of the households' primary incomes from production activity in the reporting period is spent on their end consumption—on average about 62%. At the same time, the households' primary incomes are the main source of financing end consumption in this sector. According to the dynamic equations coefficients estimates for the shares of wages and GVA, the most part of the increase in these incomes (more than 70%) is pent on increasing the end consumption of the population, while the growth of primary incomes is not reflected on the dynamics of net savings (lending). This result can be explained by the fact that during the transition period the general level of consumption in the country was not high; therefore the increase in the population's incomes was largely transformed in the growth of end consumption. On average, the households spent on net savings, according to the solution obtains, about 10% of their primary incomes from production.

The share of wages spent on fixed capital accumulation as well as the respective increase is comparatively small. At the same time, the respective shares and increase in GVA are noticeably higher. It appears to be due to the fact that entrepreneurs owing non-corporate enterprises have to pay for these enterprises' fixed capital accumulation from their own resources and, hence, spend a larger part of their own incomes within this item of expenditures as compared to hired workers. According to SNA methodology, GFCA also includes the populations' expenditures on purchasing and repairs of houses and apartments. At the same time, the model coefficients show that in the conditions of insufficient consumption, the expenditures on housing purchasing and repairs, on the one hand, might not have been very high, on the other hand, were financed largely not from the wages but from other sources.

The main source of covering the expenditures on fixed capital accumulation are gross profits (covers on average about 70% of fixed capital costs). This result is natural because fixed capital accumulation is done mostly by corporations whose main income source is gross profits. We should also point out that gross profits include incomes from production activity not only of private corporations but also budget enterprises that may cover fixed capital accumulation from their profits (if they exist). However, the most part of expenditures on the increase in the fixed capital of budget enterprises comes from net taxes on products.

A significant part of gross profits is spent on end consumption of households and government authorities, which is mostly the result of financial resources redistribution process [1]. The corporate sector pays directly only the expenditures on end consumption of the nonprofit organizations serving households, the remaining part of gross profits spent on consumption is transferred to households and the public sector in the form of dividends, current transfers, corporate profit taxes, etc., who, in their turn, use them for buying consumer goods and paying for the services.

We should point out that almost 20% of the increase in gross profits is spent on the increase of incomes from property and current transfers received by ROW. In general, during the whole analyzed period, the government authorities and households were recipients of *net* incomes from property and other current transfers, while the corporate sector was transferring to other sectors (both residents, and ROW) significantly more incomes from property and other current transfers than received from them. As the calculations by the model showed, this regularity is observed for the corporate sector as a whole but also for non-residents, since it was the corporations who paid out more than half of the resources in these items to ROW.

Within the framework of the empirical model, we made an attempt to distinguish the distribution of net taxes on products and other net taxes on production by elements of end use, though in reality it is rather difficult because both sources of income belong to the government. According to the model, the indicated types of net taxes are spent largely on end consumption of the households. It is clear that taxes can be used in this item of expenditures not *directly*, but only as a result, of financial resources redistribution. Significant shares of net taxes are largely in accord with their use on paying out social transfers, which are, as a rule, spent by households on purchasing consumer good and paying for the service.

We should note that the public sector expenditures on end consumption receive a small share of resources obtained as net taxes on products and net other taxes on production, though this item of expenditures is relatively significant and should be financed by the government only. The apparent contradiction can be easily solved if one takes into account that the government authorities end consumption receive significant shares of wages and gross profits. These shares reflect the charges on the payroll, individual income and property taxes, and corporate profit taxes, i.e., it represents the public sector's incomes and may be spent on covering the government authorities' expenditures on end consumption.

In contrast to net taxes on products, the indicator of other taxes on production was characterized during the analyzed period both by positive and negative increments; therefore we estimated for the elements X^t_{4j} that correspond to this indicator the coefficients α^+_{4j} and α^-_{4j}. The estimates of the coefficients α^+_{4j} and α^-_{4j} (see Appendix, Table A1) are significantly different, therefore the hypothesis on the need to introduce different (depending on the increment sign) income flows A^t_i coefficients in the dynamic equations X^t_{ij} cannot be rejected. We should point out the nature of the differences in the estimates of the coefficients α^+_{41} and α^-_{41}. If the increment of net other taxes on production is positive, almost two thirds of this increment go on the increase in the end consumption of the households. In case of a negative increment, the end consumption of the population decreases only by one third of the value of the net other taxes of products. The obtained outcome gives an apt illustration of the fact that when the government's incomes are growing, it can raise the payments to the population only to the amount that the responsible bodies consider necessary, but it is rather difficult to reduce the already not lavish social transfers in case of decreasing tax revenues.

There is also a significant difference between the coefficients α^+_{46} and α^-_{46} of the dynamic equations for the parts of other taxes on production spent on net savings. However, in this case difference is of opposite nature as compared to the above case. If net other taxes on production are increasing, one third of their increment goes to net lending, reducing of this government income flow results in a decrease of net savings almost by two thirds of the value of the decrease in the tax revenues in the category under consideration. One can interpret this as follows. The increase in tax revenues, as one can see from the obtained estimates of the model coefficients, are mostly used to finance the also increasing government expenditures (the permanent increase in expenditures took place already because we used nominal values of the indicators, which do not eliminate the impact of inflation). In case of decrease in the value of one type of its revenues, the government has to save less in the conditions of increasing expenditures.

A tangible share of all the net taxes on production and import, especially net taxes on products is saved by the government. Here also high is the share of the increment in the specified incomes of the public sector that is used to increase net savings (34% for net other taxes on production and 42% for net taxes on products). Here we can also suggest a rather interesting interpretation.

Taxes on products include taxes on export and import, i.e., the government's incomes from oil export and duties levied when good are imported on the country's territory [3, p. 240]. These resources are fairly significant and, as a rule, saved by the government. At present, this kind of savings makes up the most part of the Stabilization Fund of the Russian Federation.

One cannot but point out that according to the outcomes of the empirical model; more than one fourth, on average, of the income from property transferred to ROW and current transfers were financed from net taxes on production and import. These expenditures reflect mainly the payments on servicing the Russian Federation's external debt to foreign creditors. At the same time, the coefficients reflecting the relationship between the dynamics of net taxes on production and import, and transfers to ROW, are relatively small. This outcome is explained by the fact that the model was built on the time interval 1995–2003, while the most active external debt repayment took place in the later years.

Incomes from property and current transfers received by residents from ROW were spent, according to the calculations, mostly on end consumption of the households—the major recipient of net incomes from property over the whole period under study. Of this also speak the rather shares of gross profits spent on covering the population's expenditures on end consumption. As model calculations have shown, the households were recipients of net incomes from property not only from corporate residents, but also from ROW. This conclusion is confirmed by the fact that the dynamics of wages and GVA—the primary incomes of the population from production activity—show a relatively weak correlation with the incomes from property and current transfers paid to ROW. Let us also note that a significant share of the revenues from ROW is transformed into residents' savings.

Concluding the brief analysis of the most interesting outcomes of the constructed model, let us underscore that the main purpose of the empirical model is to develop a methodologically transparent toolkit enabling one to analyze the distribution and consistency of incomes and expenditures in the economy. In-depth analysis of the economic situation on the basis of the developed model represents an independent research to be conducted at the nest stage of investigating the problematic in focus.

Empirical Model Quality Assessment. Predicting of Final Product Structure with this Model

Assessment of the quality of the constructed empirical model describing the relationships between the elements of GVA and final product represents a rather complicated problem. The developed model is the first attempt to estimate quantitatively and describe the dynamics of the shares of the institutional sectors primary incomes flows in the domestic economy financing the elements of final demand. Therefore it does not seem possible to compare the obtained results with any other estimates. In these conditions, the model can be considered acceptable if its solution does not contradict the common sense and can be interpreted in terms of economics.

A content-based interpretation of the main properties of the model's optimal solution, presented above, gives grounds to assume that, in general, the proposed model gives an adequate description of the analyzed relationships. Moreover, one can also formulate quantitative criteria of assessing the quality of the empirical model. Among these we should mention the characteristics of the distribution of the relative errors[6] ε_{ij}^t and the final product structure prediction error by the given values of primary incomes from production and property.

First of all, to assess the quality of the constructed model we calculated the time averaged modules of the errors $\bar{e}_{ij}$ and maximum in time error modules of m_{ij} in the dynamic equations for elements X_{ij}^t ($\forall i = \overline{1, I}$, $j = \overline{1, J}$):

$$\bar{e}_{ij} = \frac{1}{T-1}\sum_{t=2}^{T}\left|\varepsilon_{ij}^t\right|, \quad m_{ij} = \max_{t=\overline{2,T}}\{\left|\varepsilon_{ij}^t\right|\}. \qquad (29)$$

The estimates of the parameters $\bar{e}_{ij}$ and m_{ij} are given in the *Appendix* in Tables A11 and A12 respectively. It follows from the table data though the modulus of the maximal relative error $\mu = \max_{i,j}\{m_{ij}\}$ amounted to 12.9% (which not so much, though), in most cases the value of m_{ij} was significantly lower. To this testifies the shift of the mean value of the indicators m_{ij} to the zero: $\bar{m}_{ij} \approx 3{,}4\%$. Even more convincing seem to be estimates of the time averaged modules of the errors $\bar{e}_{ij}$, whose maximum value amounts to 5,2%, and the mean one ñ to 1,3%. Thus, the level errors in the analyzed system of dynamic equations can in general be considered acceptable.

Let us now pass over to the predicting capabilities of the model. In general, conducting a serious testing of the quality of the forecast obtained on the basis of the model requires estimation of the consistency between the structure of end expenditures to that of the primary incomes on a certain set of the previously coordinated scenario plans. This implies the need to include this model into a larger macrostructural model. However, at the current stage, this kind of testing is impossible; therefore we shall go a simpler way. As we noted above, national accounts are published with a smaller time lag than the input-output tables. Therefore the information on

[6] Within the framework of the present research, the errors ε_{ij}^t are introduced in the determined statement; therefore in this case we mean empirical and not probabilistic error distribution.

Table 2. Relative errors in forecasting end use elements

Year	B_j^t					
	C_{hh}^t	C_g^t	K^t	Inv_{exp}^t	PC_f^t	Cr^t
	1	2	3	4	5	6
2004	0.042	0.001	–0.085	–0.138	0.186	–0.094
2005	0.055	–0.050	–0.093	–0.132	0.106	–0.017

Table 3. Relative errors in forecasting end use structure

Year	B_j^t					
	C_{hh}^t	C_{hh}^t	C_{hh}^t	C_{hh}^t	C_{hh}^t	C_{hh}^t
	1	2	3	4	5	6
2004	0.020	0.000	–0.015	–0.003	0.009	–0.010
2005	0.025	–0.009	–0.017	–0.004	0.005	–0.002

the boundary sums of the matrix X^t for the years 2004 and 2005 is accessible. Therefore, we can construct a forecast of the end use structure based on the income data, and then compare the result with the national accounts data. Of course, due to inconsistency of the national accounts and interindustry balance, the forecast would a priori have some error not related anyhow to the suitability of the model for predicting, however, in the present situation we cannot offer anything better.

Using the national accounts statistics for forecast poses a problem: for the year 2005 the indicators of gross profits and GVA were not separated. To overcome this difficulty, the share of mixed income in the sum of two indicators in 2005 was predicted using a simple autoregression model and estimated as 23.4%.

When predicting the structure of end use of financial resources, there may be difficulties with inventories change indicator, which can be either positive and represent an expenditure item, or negative, i.e., be part of the resources. In other words, there is some ambiguity in the structure of the prognosticated matrix X^t. This problem has a dual solution. If the model is used to assess the consistency the given scenario plans of the primary incomes and end use structures, this problem does not arise at all. In other cases, the flow of inventories can be combines with gross fixed capital accumulation into the item "investment expenditures", which would also allow to avoid ambiguity in the structure of the predicted X_{ij}^t elements matrix.

Note that the analyzed model gives the opportunity to predict by the set for the future period values of aggregated income flows of economic agents-residents, not only the aggregated structure of end use, but also elements of the modified Quadrant IV of the interindustry balance X_{ij}^t. At the same time, the full-balance feature of the model expressed as equality of the sums of all incomes and expenditures in the economy (including net savings) is also observed in the forecast. This means that in fact the model prognosticates not individual elements of end use of financial resources, but the end use structure. Therefore it is most correct to estimate the forecast error not relative to individual end use elements, but to all the used resources. It in this way one can most correctly determine to what extent the predicted structure of end expenditures is correct.

Let us look at the results of model forecast for end use elements for the years 2004 and 2005 and compare them with the actual national accounts data. The respective matrices of the income flow parts used to finance the end demand elements for 2004 and 2005 are given in the *Appendix* (Tables A13-A14). Information on the proposed model forecast errors in comparison to the statistical data of the national accounts is reflected in Tables 2 and 3.

In Table 2, there are relative errors of the forecast of the values of end use elements in comparison to the actual values of these elements: in Table 3—relative errors of the forecast of the end use structure (i.e., measured in relation to the total amount of expenditures). It follows from these data that the end use structure is prognosticated at an acceptable level—the maximal relative error amounted to 2.5%. At the same time, in relation to the actual values of the end use elements, to most successful prognoses are obtained for the expenditures on end consumption of households and government authorities. These flows are the biggest and of greatest interest, therefore the possibility to obtain an acceptable forecast of these indictors is a positive characteristic of the model. The least accurate is the forecast of the changes in inventories, incomes from property, and current transfers paid by the residents to ROW. As we noted in [1], the indicator of changes in inventories is rather roughly estimated in the statistics and changes too greatly when we try to determine it more exactly. Moreover, the estimates of this indictor in the "input-output" tables and national accounts often show multifold divergence (even the sign may be different!) Given this quality of the source information, it would be unrealistic, in the authors' opinion, to expect a more accurate forecast. In view of the above, a significant divergence between the predicted changes in the inventories and the actual value should not be considered as error caused by an incorrectness of the model construction.

We should also point out that the empirical model was built first of all to describe the relationships between the elements of GVA and end product. The forecast of the end demand elements (except for the inventories) is fairly acceptable, especially in the conditions of source data inconsistency. As regards the flows of expenditures that are not parts of the end product, one can aggregate these indicators to diminish their forecast error. The forecast in case of aggregating the transfers to ROW and lending item is given in Tables 4 and 5.

Table 4. Relative errors in forecasting end use elements under the aggregation of transfers to ROW and net savings

Year	B_j^t				
	C_{hh}^t	C_g^t	K^t	$\mathrm{In}v_{\exp}^t$	$PC_f^t + Cr^t$
	1	2	3	4	5, 6
2004	0.042	0.001	–0.085	–0.138	–0.006
2005	0.055	–0.050	–0.093	–0.132	0.020

Table 5. Relative errors in forecasting end use structure under the aggregation of transfers to ROW and net savings

Year	B_j^t				
	C_{hh}^t	C_g^t	K^t	$\mathrm{In}v_{\exp}^t$	$PC_f^t + Cr^t$
	1	2	3	4	5, 6
2004	0.020	0.000	–0.015	–0.003	–0.001
2005	0.025	–0.009	–0.017	–0.004	0.003

As one can see from Tables 4 and 5, the aggregating techniques turned out to be quite successful, because the forecast error was leveled almost to zero. The results of the forecast of expenditure items that are not part of the end product testify to the fact that their dynamics should, perhaps, be accounted for by a more complex method than it has been done within the framework of the present empirical model.

Completing the analysis of the constructed forecast of the end use elements for the years 2004 and 2005, let us emphasize that the main purpose of constructing the empirical model from the point of view of the forecast was not to predict the absolute values of the end demand elements as such but in the opportunity to analyze the consistency of the primary incomes and end use structures. In this sense the prognostication results can be considered fairly successful, which can also be considered as a quality criterion of the model in consideration.

Note that the mathematical models of the suggested type are in some way a universal tool, as they enable one to solve the problem inverse to the described one—to prognosticate the primary incomes structure using a given structure of end use. To do this, the model should apparently be modified, however, in general, there is such a possibility and it can be easily implemented.

CONCLUSION

The empirical mathematical model constructed on the basis of the theoretical scheme enabled one to *quantitatively* describe the relationships between the elements of GVA and end product for the period 1995–2003. The model contains a transparent, detailed mechanism, by means of which we were able to estimate the shares of the aggregated agents' primary income flows spent on financing various end use items. We suggested interpreting the calculated shares as elements of the modified Quadrant IV of the interindustry balance (or the tables of using goods and services). The obtained results can be considered as an doubtless contribution to the improvements of the analytical toolkit for studying the economic reproduction processes. Of special note is the fact that the empirical model is so constructed that it can be joined with the interindustry balance model in order to complement this tool giving a detailed description of the relationship between the material flows and information on financial flows. At the same time, the model can be used independently of the interindustry balance.

Apart from the filled-in tables describing the income parts in the end-use elements, the empirical model estimates a system of dynamic equations for all these shares. At the same time, the system was constructed in the form easy for prognosticating. An important specific feature of the empirical mathematical model is the opportunity to predict these shares as well as the end use structure given the flows of primary incomes. Within the framework of the research, we made a forecast of the expenditures structure for the years 2004 and 2005, which was compared to the national accounts data. Taking into account that the national accounts data are not consistent with the interindustry balance and are significant refined with time, the obtained forecast can be considered fairly acceptable.

Note that the empirical model is of greatest use when it is included into a macrostructural model of the Russian economy. In this case the model of the relationship between the elements of GVA and end product enables one to coordinate the prognoses or scenario plans of the primary incomes and expenditures given within the framework of the macrostructural model.

The constructed empirical model can be considered a full-fledged tool of macroeconomic analysis and prognostication, since this model represents a not once estimated system but a recurrent algorithm, which can be realized by an external researcher without participation of the authors of the paper.

Finally, we should note that the proposed developments not intended initially to describe all the details of the relationships between the elements of GVA and end product, may serve as a necessary basis for conducting further investigations in this area. One of the lines of improvement is inclusion of the constructed model into the macrostructural model in order to conduct content-based analysis of incomes and expenditures analysis on the basis of using the estimates of the elements of the modified Quadrant IV of the interindustry balance and their dynamic relationships. A second possible line of research is seen by the authors in detailed elaboration of the empirical model in order to approximate the theoretical scheme, which would allow one to conduct a more complex qualitative analysis.

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APPENDIX

Empirical Model Solution Results

Table A1. Estimates of empirical model coefficients

Incomes \ Expenditures				B_j^t					
				C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
				1	2	3	4	5	6
A_i^t	W^t	α_{ij}^+	1	0.714	0.270	0.014	0	0.002	0
	Pr^t	$\alpha_{ij}^+ = \alpha_{ij}^-$	2	0.131	0.122	0.320	0.030	0.184	0.212
	MI^t	α_{ij}^+	3	0.762	0	0.238	0	0	0
	NOT^t	α_{ij}^+	4	0.657	0	0	0.004	0	0.339
		α_{ij}^-		0.354	0	0	0	0.030	0.615
		β_{ij}^1		–	–	–	–	–	-2.427×10^7
		β_{ij}^2		–	–	–	–	–	3.283×10^7
	NPT^t	α_{ij}^+	5	0.256	0.018	0.187	0.108	0.009	0.421
		β_{ij}^1		–	–	–	–	–	-3.798×10^7
		β_{ij}^2		–	–	–	–	–	18.919×10^7
	PI_f^t	α_{ij}^+	6	1.000	0	0	0	0	0
	Bor^t	α_{ij}^+	7	0.037	0.225	0.630	0.109	0	0
		α_{ij}^-		0.648	0.080	0.232	0.040	0	0
	Inv_{res}^t	α_{ij}^+	8	0.211	0	0	0	0.789	0

Note: 0 the coefficient was estimated and found equal to zero, "–" the coefficient was not estimated.

Table A2. Estimates of elements of modified Quadrant IV of the 1995 interindustry balance, bln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{In}v_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	288746	108195	–	39097	2408	267360
	Pr^t	2	–	191295	258455	17425	–	–
	MI^t	3	78136	–	47947	12065	1081	51765
	NOT^t	4	46629	4596	6086	–	9372	–
	NPT^t	5	85908	31727	–	–	19358	–
	PI_f^t	6	10839	55	130	22	–	10654
	Bor^t	7	250123	–	8606	1756	2489	

Table A3. Estimates of elements of modified Quadrant IV of the 1996 interindustry balance, bln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{In}v_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	474036	207135	–	20411	–	329057
	Pr^t	2	14234	225565	310729	29128	22224	–
	MI^t	3	104835	–	59038	10213	–	53845
	NOT^t	4	73872	5189	6400	–	9566	13147
	NPT^t	5	97999	33936	9366	699	19418	22765
	PI_f^t	6	14018	61	122	11	–	10690
	Bor^t	7	251915	24716	72544	13210	104	–

Table A4. Estimates of elements of modified Quadrant IV of the 1997 interindustry balance, bln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{In}v_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	683452	318210	12061	33858	–	171313
	Pr^t	2	41901	242414	323555	25909	33814	–
	MI^t	3	156078	–	76047	9877	–	36354
	NOT^t	4	103563	10897	11324	2645	14945	–
	NPT^t	5	116848	43410	22316	4873	27878	–
	PI_f^t	6	15250	100	135	29	–	10557
	Bor^t	7	145358	11347	32833	5801	–	–

Table A5. Estimates of elements of modified Quadrant IV of the 1998 interindustry balance, bln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	852521	335369	3835	–	9651	84332
	Pr^t	2	70318	224774	300934	–	63342	3001
	MI^t	3	215598	–	86470	–	–	23080
	NOT^t	4	130635	12600	12747	–	20155	–
	NPT^t	5	133498	45634	28927	–	36615	–
	PI_f^t	6	21824	93	118	–	–	10495
	Bor^t	7	54104	–	–	–	67	–
	Inv_{res}^t	8	5257	–	–	–	19672	–

Table A6. Estimates of elements of modified Quadrant IV of the 1999 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	1419617	490173	–	4529	19526	–
	Pr^t	2	211738	263271	517570	–	261842	334717
	MI^t	3	389983	–	138049	2556	659	–
	NOT^t	4	183830	–	–	–	10884	75303
	NPT^t	5	161436	–	25085	–	–	308840
	PI_f^t	6	101444	–	–	–	–	10189
	Bor^t	7	55719	6839	19929	3453	–	–

Table A7. Estimates of elements of modified Quadrant IV of the 2000 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	2017113	721773	22603	14511	–	160393
	Pr^t	2	225546	421633	872416	63844	327168	508171
	MI^t	3	462755	–	182107	8843	1734	18212
	NOT^t	4	271926	1044	1996	1730	8673	128814
	NPT^t	5	228334	17484	103530	45757	–	437285
	PI_f^t	6	132171	–	73	43	–	10621

Table A8. Estimates of elements of modified Quadrant IV of the 2001 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	2663879	959003	95558	61634	–	68718
	Pr^t	2	342894	565437	1148439	122429	340773	369536
	MI^t	3	578994	–	228693	9927	–	18920
	NOT^t	4	235857	5403	6779	5973	8378	38331
	NPT^t	5	315931	45854	181971	91792	3250	455886
	PI_f^t	6	190212	15	271	142	–	10405

Table A9. Estimates of elements of modified Quadrant IV of the 2002 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	3501598	1282446	96613	–	–	166889
	Pr^t	2	423373	684175	1321095	98705	409383	256455
	MI^t	3	688108	–	270747	4599	–	28453
	NOT^t	4	230109	7550	6877	5514	7951	28753
	NPT^t	5	368695	68238	228334	110127	13956	452293
	PI_f^t	6	196314	52	275	–	–	10602

Table A10. Estimates of elements of modified Quadrant IV of the 2003 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	4219517	1525230	86972	–	69918	285563
	Pr^t	2	547857	868522	1697524	144785	705922	113257
	MI^t	3	783432	–	313506	4203	8716	38912
	NOT^t	4	192115	25	1390	–	–	–
	NPT^t	5	440507	67199	289181	131998	18685	643810
	PI_f^t	6	382462	–	360	–	–	11272

Table A11. Estimates of time average moduli or the errors ε_{ij}^t in the dynamic equations of elements of X_{ij}^t

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	Inv_{exp}^t	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	0.040	0.012	0.007	0.011	0.004	0.052
	Pr^t	2	0.021	0.027	0.030	0.015	0.021	0.048
	MI^t	3	0.031	0.000	0.009	0.008	0.002	0.025
	NOT^t	4	0.021	0.019	0.018	0.010	0.018	0.021
	NPT^t	5	0.028	0.027	0.026	0.019	0.021	0.039
	PI_f^t	6	0.002	0.000	0.001	0.000	0.000	0.002
	Bor^t	7	0.003	0.002	0.001	0.001	0.001	0.000
	Inv_{res}^t	8	0.000	0.000	0.000	0.000	0.000	0.000

Table A12. Estimates of time maximum moduli of the errors ε_{ij}^t in the dynamic equations of elements of X_{ij}^t

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	Inv_{exp}^t	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	0.094	0.049	0.016	0.026	0.011	0.129
	Pr^t	2	0.044	0.047	0.050	0.039	0.046	0.085
	MI^t	3	0.073	0.000	0.018	0.030	0.008	0.063
	NOT^t	4	0.046	0.047	0.047	0.029	0.038	0.063
	NPT^t	5	0.073	0.101	0.103	0.055	0.079	0.091
	PI_f^t	6	0.005	0.002	0.001	0.001	0.000	0.005
	Bor^t	7	0.009	0.007	0.005	0.004	0.007	0.000
	Inv_{res}^t	8	0.000	0.000	0.000	0.000	0.000	0.000

Table A13. Forecast of elements of modified Quadrant IV of the 2004 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	Inv_{exp}^t	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	5341158	1949595	109006	–	72407	285563
	Pr^t	2	642164	956760	1928289	166747	838409	266372
	MI^t	3	1080528	–	406109	4203	8716	38912
	NOT^t	4	527400	25	1390	2045	–	173385
	NPT^t	5	587706	77572	397144	194368	24040	886387
	PI_f^t	6	298474	–	360	–	–	11272

Table A14. Forecast of elements of modified Quadrant IV of the 2005 interindustry balance, mln rub.

Incomes \ Expenditures			B_j^t					
			C_{hh}^t	C_g^t	K^t	$\mathrm{Inv}_{\exp}^t$	PC_f^t	Cr^t
			1	2	3	4	5	6
A_i^t	W^t	1	6473026	2377830	131242	–	74918	285563
	Pr^t	2	823581	1126504	2372209	208996	1093273	560916
	MI^t	3	1348191	–	489538	4203	8716	38912
	NOT^t	4	798258	25	1390	3698	–	313453
	NPT^t	5	816827	93717	565191	291449	32376	1263968
	PI_f^t	6	453360	–	360	–	–	11272