Information Unraveling Revisited: Disclosure of Horizontal Attributes

Levent Celik
CERGE-EI†
P.O. Box 882
Politickyh veznu 7
111 21, Praha 1, Czech Rep.
celik@cerge-ei.cz

May 20, 2011

Abstract
A large literature has analyzed verifiable information disclosure when consumers are uncertain about vertical attributes of a good. This paper complements the literature by considering uncertainty about horizontal attributes. A seller who is privately informed about the variety of the good he sells meets a buyer who is privately informed about her ideal taste for variety. Prior to a possible transaction, the seller chooses the optimal level of information to disclose about the variety. I characterize an equilibrium in which the seller fully reveals variety when the buyer’s preference for her ideal taste is sufficiently strong. Otherwise, he reveals how the variety fits the expected ideal taste of the buyer. The set of fully revealed locations monotonically shrinks from all to (almost) none as the buyer’s preference for her ideal taste becomes weaker. From a policy perspective, mandating full information disclosure is socially harmful.

Keywords: Verifiable information disclosure, asymmetric information, product differentiation.
JEL Classification: D82, D83, M37.

*I would like thank Simon Anderson, Devrim Demirel, Rick Harbaugh, Stepan Jurajda, Bilgehan Karabay, Peter Katuscak, Claudio Mezzetti, Regis Renault, Avner Shaked, Evangelia Vourvachaki and Kresimir Zigic for helpful comments and suggestions. All errors are mine.
†CERGE-EI is a joint workplace of the Center for Economic Research and Graduate Education, Charles University, and the Economics Institute of the Academy of Sciences of the Czech Republic.
1 Introduction

A large literature has analyzed how much information a privately-informed seller voluntarily reveals when consumers are unable to tell the quality of a product prior to purchase. In their seminal papers, Grossman (1981), Grossman and Hart (1980) and Milgrom (1981) show that the seller fully reveals quality as long as there is a credible and costless means of conveying it. The primary driver of this finding is the fact that consumers’ willingness-to-pay is strictly increasing in perceived quality. Therefore, a high-quality seller would always reveal its quality and distinguish itself from its own lower-quality images. As this reasoning applies to all seller types, if quality information is withheld, then it can only be the lowest-quality seller. Thus, information “unravels.” Accordingly, mandatory disclosure rules are redundant because disclosure is costless and the seller voluntarily reveals the quality of the product regardless of its value.

Many goods are characterized by several attributes some of which are horizontal. However, very little attention has been paid to verifiable information disclosure when consumers are unable to observe horizontal attributes of a good. The main objective of this paper is to characterize the extent of information disclosure and the resulting social efficiency in such environments, and compare the results with those of the quality disclosure literature. In contrast to a vertical attribute such as quality, consumers rank different varieties of a horizontal attribute differently. Geographical location of a real estate property, expertise area of a researcher or sweetness of a wine are a few examples. It is a priori unclear to what extent the unraveling argument works, if at all, when consumer uncertainty concerns a horizontal attribute. The seller may choose to provide only partial information, thereby bringing the perceived attribute closer to the ideal taste of the average buyer. For instance, the seller of a house may choose to say in an advertisement that the house is located within a certain distance from a central square rather than revealing its exact location if the average buyer has a higher willingness-to-pay for a house closer to that square.

To answer these questions, I consider a simple sales encounter for a good in which there is a single seller (he) and a single buyer (she). The good is characterized by a single horizontal attribute, which I call variety. The seller is privately informed about
the variety of the good while the buyer is privately informed about her ideal taste for the variety. Traditionally, markets with goods that have horizontal attributes have been analyzed using spatial models, and I continue in this tradition. Accordingly, the variety of the good and the buyer’s ideal taste for it are represented by particular locations along a unit line à la Hotelling (1929), and the buyer strictly prefers a variety that is closer to her ideal taste. Prior to a possible transaction, the seller chooses a price and makes a report about the variety of the good. The only restriction I impose on the report is that it must be truthful. In other words, possible reports range from being very precise (revealing the exact variety) to very vague (staying silent). The buyer observes the price and the report, and responds by purchasing one unit of the good or none.

I characterize a perfect Bayesian equilibrium (PBE) in which the information regarding the distance between the variety of the good and the expected ideal taste of the buyer (from the seller’s point of view) fully unravels. The seller fully reveals the exact variety only when the buyer’s preference for her ideal taste is sufficiently strong. Moreover, the set of fully revealed varieties monotonically shrinks (from all to (almost) none) as the buyer’s preference for her ideal taste becomes weaker. Hence, information unraveling is still in effect, but not to the fullest extent.

The intuition for this finding is as follows. From the seller’s point of view, the probability of a purchase is higher when the variety of the good is closer to the expected ideal taste of the buyer. When it is not sufficiently close, the seller is tempted to disclose only partial information so as to bring the buyer’s perceived variety (i.e., the expected variety conditional on the report received) closer to her expected ideal taste. However, such a report leaves some uncertainty regarding the true variety. The buyer is uncertainty-averse in the sense that her willingness-to-pay would be higher had the seller made a precise report indicating the same perceived variety without any uncertainty. Thus, there are two opposing factors the seller takes into account when deciding what report to make: (i) eradicating buyer uncertainty by fully revealing the variety, and (ii) bringing the buyer’s perceived variety closer to the expected ideal taste of the buyer by disclosing partial information. The buyer understands that her expected ideal taste acts as a reference point for the seller. This leads her to adopt a pessimistic posture in which she associates a
partially-revealing report with the variety that is farthest away from this reference point. Therefore, in situations when the seller discloses partial information, he never includes in his report varieties that are more distant from the expected ideal taste of the buyer than the true variety is. Since the seller employs the same strategy for all possible varieties, the distance between the true variety of the good and the expected ideal taste of the buyer fully unravels.

It may be easier to see the unraveling result with an example. As described before, the variety as well as the buyer’s ideal taste are represented by locations over the unit line [0, 1]. Suppose that the expected ideal taste of the buyer is $\frac{1}{2}$ and that, in equilibrium, the seller fully reveals the varieties in [.4, .6] and makes a report in the form $[x, 1 - x]$ for each other variety $x$. Consider the case when the seller makes a report saying that the variety belongs to [.3, .7]. In this case, the buyer rationally infers that the true variety must be either .3 or .7, because the seller would not make such a report had the true variety been closer to $\frac{1}{2}$. Hence, the degree of mismatch between the variety and the expected ideal taste of the buyer fully unravels.

The strength of the buyer’s preference for her ideal taste plays an important role in the determination of which varieties are fully revealed. When it is weak, the buyer perceives different varieties as close substitutes, so uncertainty about the variety does not lower her willingness-to-pay too much. In this case, the seller’s incentive to disclose partial information is higher. Similarly, when it is strong, the seller has a higher incentive to make a precise report since uncertainty significantly lowers the buyer’s willingness-to-pay. This relationship is monotonic in the strength of the buyer’s preference for her ideal taste, and therefore, the set of fully revealed varieties expands as it becomes stronger.

Returning back to the example of disclosing the geographical location of a house, most buyers (or renters) are typically very picky for the neighborhood they want to live in, but not so much for the precise location within a neighborhood. Indeed, many house advertisements fully reveal the neighborhood the house is located in without specifying its precise location. Again, many say that the house is within a certain distance from a central location. This is in line with the predictions of this paper. Accordingly, buyers should rationally infer that the house is in fact exactly at that distance from the specified
central location. Similar observations can be obtained from other markets. In classified personal advertisements (commonly known as “personals”), people typically fully reveal certain characteristics (such as gender, where they live, etc.) while presenting other characteristics in a way to make an average person in a target group (sharing similar demographics) be interested in them. For instance, in order to increase the probability of a successful match, a 60-year old, 6'5" tall man may fully reveal the city where he lives while saying that he is in the range [50, 60] in terms of age and [6', 6'5"] in terms of height.

Whether mandatory disclosure rules are beneficial or not has been an important question. According to the literature on quality disclosure, mandatory rules are redundant because the seller voluntarily reveals the quality of the good regardless of its value. This finding is confirmed in the current paper, too. I find that a social planner cannot improve welfare by mandating the seller to fully reveal a bigger set of varieties than the seller voluntarily does, while such a policy is often socially harmful. The intuition for this finding is as follows. The uncertainty effect typically induces the seller to charge a lower price when he reveals the distance between the variety and the expected ideal taste of the buyer compared to when he fully reveals it. Therefore, if the seller chooses to disclose partial information rather than fully revealing the variety, the expected demand he faces in the former case must be larger than the expected demand he faces in the latter. Since price is simply a transfer between the buyer and the seller, the demand enlargement effect of disclosing partial information improves total welfare. Thus, forcing the seller to fully reveal a variety that he voluntarily does not is often socially harmful.

The basic model allows several extensions. I discuss these in section 5. Most importantly, buyer uncertainty about a vertical attribute (say, quality) can easily be incorporated. In this case, the usual unraveling story applies with respect to quality disclosure. Thus, regardless of the buyer’s prior beliefs for it, quality would be fully revealed in every PBE. Accordingly, all the main results about variety disclosure remain the same.

Many authors have studied verifiable information disclosure in different contexts. However, most of them focus on vertical attributes. Examples include Jovanovic (1982) in which information disclosure is costly; Matthews and Postlewaite (1985) who allow
the seller to decide whether to acquire quality information or not; Fishman and Hagerty (1990) who analyze how much discretion a seller should be allowed in choosing how much information to disclose about quality; Shin (1994) who incorporates uncertainty about the degree of information the seller possesses about quality; Board (2009), Cheong and Kim (2004), Hotz and Xiao (2007), Levin, Peck and Ye (2009), Milgrom and Roberts (1986) and Stivers (2004) who analyze quality disclosure in competitive environments;\(^1\) Jin (2005), Jin and Leslie (2003) and Lewis (2010) who examine quality disclosure empirically; Daughety and Reinganum (2008) who incorporate the possibility of signaling quality by price into the standard disclosure framework; and Kamenica and Gentzkow (2009), Kartik (2009) and Rayo and Segal (2009) who study unified models of verifiable information disclosure and cheap talk à la Crawford and Sobel (1982).

To the best of my knowledge, Sun (2011) is the only paper that considers verifiable disclosure of horizontal attributes. In her model, a monopolist sells a good that has both horizontal and vertical attributes, and consumers are uncertain about both. However, the monopolist is constrained to choose one of the two strategies; either fully disclose both attributes or stay silent. Therefore, Sun’s findings apply to situations when the seller cannot provide partial information.

The remainder of the paper is organized as follows. In the next section, I introduce the basic model. In sections 3 and 4, I characterize the equilibrium level of information disclosure and investigate its social efficiency properties, respectively. In section 5, I discuss possible extensions to the main model. Finally, in section 6, I conclude. All proofs are relegated to the Appendix.

## 2 Model

A profit-maximizing seller (S) offers a good (G) for sale which is characterized by a location over the unit interval, denoted by \(x \in [0,1]\). The location here indicates the variety of G, such as color, sweetness, etc. S is privately informed about \(x\). I will use masculine pronouns for S and sometimes refer to \(x\) as S’s type. The production costs do not depend on \(x\), and without loss of generality, are assumed to be zero.

\(^1\)Hotz and Xiao (2007) and Levin, Peck and Ye (2009) also allow for horizontal product attributes. However, they assume that these are commonly known by consumers.
On the other side of the market, there is a single potential buyer (B) who has a unit demand for G. B’s ideal taste, which describes the particular variety of G that she ideally wants to consume, is described by a location $\lambda \in [0, 1]$. This is private knowledge of B. Similarly, I will use feminine pronouns for B and sometimes refer to $\lambda$ as B’s type. If B buys a unit of G at a price $P$, then her net utility is $v - t(\lambda - x)^2 - P$, where $v$ is the gross utility B enjoys when the variety of G perfectly matches with her ideal taste (i.e., when $x = \lambda$) and $t$ measures the degree of disutility B incurs when $x$ and $\lambda$ differ from each other.\(^2\) Not buying G yields zero utility. If B buys a unit of G, then S’s payoff is $P$. Otherwise, S gets zero payoff.

The timing of the game is as follows. First, Nature selects a value for $x \in [0, 1]$ from a strictly positive density function $f(x)$ which is symmetric around $\frac{1}{2}$, and a value for $\lambda$ from a uniform density function defined over $[0, 1]$. Hence, the ex-ante expected value of both the location of G and the ideal taste of B is $\frac{1}{2}$. S privately observes $x$ while B privately observes $\lambda$. After observing $x$, S sends a truthful and costless message $M \subset [0, 1]$, and chooses a price $P$ to which he commits thereafter.\(^3\) In case S is indifferent between two or more messages, I assume that he sends each with a strictly positive probability. I will discuss this assumption in more detail later. B observes $M$ and $P$, and then decides whether to buy G or not. Finally, the payoffs are realized. All aspects of the game are common knowledge.

It is necessary to make a few remarks about the model. First, note that B’s utility function implies uncertainty-aversion with respect to $x$. In other words, B dislikes uncertainty about the location of G. For instance, a precise message $M = \{\frac{1}{2}\}$ is more favorable for B than a message $M = [\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$ which implies a conditional expected value of $\frac{1}{2}$ for $x$. Second, although I assume a single buyer with a privately known ideal taste, the results are identical with a continuum of buyers whose ideal tastes are uniformly distributed over the unit line. These two specifications are equivalent. Third, B has a unit demand in my model. This is without loss of generality, because, as it will be clear later, the probability of a purchase declines with price. In other words, despite the unit demand assumption, S faces a downward-sloping expected demand function.

\(^2\)Alternatively, $v$ can be interpreted as the quality of G. See section 5 for further discussion.

\(^3\)A message is truthful when $x \in M$. 
Fourth, I assume that S makes his reporting and pricing decisions simultaneously and that price is observed by B prior to purchase. The simultaneity assumption is not crucial; S may make his reporting and pricing decisions in any order. However, it is crucial that B observes the price prior to purchase and S commits to the price he chooses. Finally, in line with the quality disclosure literature, I focus on truthful and costless messages.

The location of the good, \( x \), is exogenously given in this paper. However, it is possible to allow S to influence it. Consider a production process in which the choice of location is subject to an error. For instance, sweetness of a wine crucially depends on the climate which is difficult to predict beforehand. In this scenario, S chooses a target location for G. The realized value of the error then determines the final location of G. Assuming that B knows the distribution of the error, her prior beliefs for the final location will be defined over a subset of \([0, 1]\). In fact, if the error term has a zero-mean symmetric distribution, S chooses a target location of \( \frac{1}{2} \) since this is the expected ideal taste of B from his point of view. In this case, B’s prior beliefs for \( x \) will be symmetric around \( \frac{1}{2} \).

I use the concept of perfect Bayesian equilibrium (PBE) to solve the model. Let \( m(x) \) describe the reporting strategy of S which is a mapping from \([0, 1]\) to all subsets of \([0, 1]\) such that \( x \in m \). This determines what message S sends as a function of his private information. Let \( p(x \mid M) \) denote the pricing strategy of S when the message he sends is M. Similarly, let \( b(\lambda, M, P) \) describe the buying strategy of B, where \( b = 1 \) if she buys G and \( b = 0 \) if she does not. Finally, let \( \pi \) describe how B updates her beliefs based on the message and the price chosen by S. Thus, \( \pi(z \mid M, P) \) is the probability density B assigns to \( x = z \) when S sends a message \( M \) and chooses a price \( P \). A PBE for this game is then defined as follows.

**Definition 1** A PBE for this game is a quadruple \((b, p, m, \pi)\) which is characterized by the following four conditions:

\begin{align}
(D.1) \quad \text{For all } M \text{ and } P, b \text{ is B's best buying decision:} \\
&= \begin{cases} 
1, & \int_0^1 (v - t(\lambda - x)^2 - P)\pi(x \mid M, P)dx \geq 0 \\
0, & \text{otherwise}
\end{cases} 
\end{align}
(D.2) Given (D.1), \( p \) is the price that maximizes \( S \)'s expected revenue when he sends a message \( M \):

\[
p(x \mid M) = \arg\max_p \int_0^1 b(\lambda, M, p) Pd\lambda.
\]

(D.3) Given (D.1) and (D.2), \( m \) is the message that maximizes \( S \)'s expected revenue subject to \( x \in m \):

\[
m(x) = \arg\max_{M \supseteq \{x\}} \int_0^1 b(\lambda, M, p(x \mid M)) p(x \mid M)d\lambda.
\]

(D.4) Let \( \Omega \) describe the set of locations that induce \( S \) to send a message \( M \) and choose a price \( P \), i.e., \( \Omega = \{x \mid m = M, p = P\} \). Then, for all \( M \) and \( P \) such that \( \Omega \neq \emptyset \), \( B \) updates her beliefs in the following way:

\[
\pi(x \mid M, P) = \begin{cases} 
\frac{f(x)}{\int_{x \in \Omega} f(x) dx}, & x \in \Omega \\
0, & \text{otherwise}
\end{cases}.
\]

(D.1) states that, for any observed message \( M \) and price \( P \), \( B \) decides to buy a unit of \( G \) only if, given her updated beliefs, her expected net utility is non-negative. \( S \) rationally anticipates \( B \)'s best response to any given \( M \) and \( P \), and chooses the best price and message that maximize his expected revenue, \( \int_0^1 bPd\lambda \). These are stated in (D.2) and (D.3). Finally, (D.4) states that \( B \) rationally anticipates the price and the message \( S \) chooses for each \( x \), and updates her beliefs about \( x \) in a Bayesian way for any observed \( M \) and \( P \).

### 3 Equilibrium information disclosure

In this section, I investigate the properties of equilibrium information disclosure. I first describe \( B \)'s optimal behavior for a given message and price. I then describe the optimal message and the price \( S \) chooses for each \( x \), taking \( B \)'s optimal behavior given. The main result is stated in Proposition 2 which provides a description of equilibrium information disclosure.

\( B \)'s optimal behavior is summarized by (D.1) and (D.4). Given a message \( M \) and a price \( P \), she updates her beliefs about \( x \), as described in (D.4), and buys \( G \) if and only
if her net expected surplus from buying is non-negative, as described in (D.1). Thus,

\[ b(\lambda, M, P) = 1 \iff v - tE [(\lambda - x)^2 \mid x \in \Omega] - P \geq 0, \tag{5} \]

where \( \Omega \) is, as described in (D.4), the set of locations that induce S to send a message \( M \) and choose a price \( P \). Solving expression (5) for \( \lambda \) yields

\[ \lambda^L = \max \left\{ 0, E[x \mid x \in \Omega] - \sqrt{\frac{v - P}{t} - Var[x \mid x \in \Omega]} \right\}, \tag{6} \]

\[ \lambda^H = \min \left\{ 1, E[x \mid x \in \Omega] + \sqrt{\frac{v - P}{t} - Var[x \mid x \in \Omega]} \right\}, \tag{7} \]

where \( \lambda^L \) (\( \lambda^H \)) is the lowest (highest) type of B that buys G when S sends a message \( M \) and chooses a price \( P \).

S takes B’s optimal buying behavior as given and maximizes his expected revenue. Since S is uncertain about \( \lambda \), the price he chooses for a given \( M \) maximizes the expected demand \( E[bP] \) as described in (D.2). For notational convenience, let \( D = E[bP] \). The expected demand is simply the probability that \( \lambda \) lies between \( \lambda^L \) and \( \lambda^H \). Since S’s priors for \( \lambda \) are uniform over \([0, 1]\), the expected demand S faces is given by

\[ D(P; x, v, t) = \lambda^H - \lambda^L. \]

As mentioned in the previous section, \( t \) measures how strong B’s preference for her ideal taste is, and \( v \) can be interpreted as the quality of G. When \( t \) is high, a mismatch between the variety of G and B’s ideal taste reduces B’s willingness-to-pay badly. Similarly, when \( v \) is high, consumption of G offers a high utility. Therefore, the expected demand S faces at a given price is increasing in the value of \( \frac{v}{t} \).

Analyzing the expected demand function, \( D(P; x, v, t) \), leads to two important observations. On the one hand, S wants to bring the perceived location of G (i.e., \( E[x \mid x \in \Omega] \)) as close to the expected ideal taste of B (which is \( \frac{1}{2} \)) as possible by sending a partially-revealing message that pools the actual location of G with more central ones. This strictly raises the expected demand S faces when \( x \) is close to 0 or 1. On the other hand, S wants to keep uncertainty (captured by \( Var[x \mid x \in \Omega] \)) as low as possible due to B’s uncertainty-aversion. At times \( \lambda^L \) and \( \lambda^H \) do not bind (i.e., not equal to 0 and 1,
respectively), a higher uncertainty lowers the expected demand. These two factors work
against each other, so S’s optimal decision depends on which factor dominates.

First, consider the situation when $x$ is commonly known (or, equivalently, when S fully
reveals it). Letting a subscript 1 indicate this situation, equations (6) and (7) reduce to

$$\lambda^L_1 = \max \left\{ 0, x - \sqrt{\frac{v - P}{t}} \right\},$$ (8)

$$\lambda^H_1 = \min \left\{ 1, x + \sqrt{\frac{v - P}{t}} \right\}.$$ (9)

Let $p_1$ and $R_1$ denote, for a given $(v, t)$, the optimal price S chooses and the resulting
equilibrium expected revenue he makes when $x$ is known. For a given location $x$, the
revenue-maximizing price is

$$p_1(x, v, t) = \arg \max_P PD_1(P; x, v, t),$$

which leads to equilibrium expected revenue S makes

$$R_1(x, v, t) = p_1 D_1(p_1, x, v, t).$$

**Proposition 1** $R_1(x, v, t)$ is strictly increasing for $x < \min \{ \sqrt{\frac{v}{t}}, \frac{1}{2} \}$, constant for
\(\min \{ \sqrt{\frac{v}{t}}, \frac{1}{2} \} \leq x \leq \max \left\{ \frac{1}{2}, 1 - \sqrt{\frac{v}{t}} \right\} \) and strictly decreasing for $\min \left\{ \frac{1}{2}, 1 - \sqrt{\frac{v}{t}} \right\} < x \leq 1$.

From S’s point of view, the likelihood B buys G is higher the closer the location of G
is to the expected ideal taste of B. That is why the revenue S expects under full location
information increases as $x$ gets closer to $\frac{1}{2}$. When $\frac{v}{t}$ is not too high, neither $\lambda^L_1$ nor $\lambda^H_1$
binds (i.e., is not equal to 0 and 1, respectively) at the optimal price S chooses for the
values of $x$ between $\min \{ \sqrt{\frac{v}{t}}, \frac{1}{2} \} \) and $\max \left\{ \frac{1}{2}, 1 - \sqrt{\frac{v}{t}} \right\} \). Therefore, for these locations,
S is effectively unconstrained and is able to achieve the highest revenue he can. When
$x$ is closer to the edges, on the other hand, either $\lambda^L_1$ or $\lambda^H_1$ becomes binding and the
expected demand S faces goes down. Therefore, S earns a lower revenue as $x$ is farther
away from $\frac{1}{2}$. When $\frac{v}{t}$ is sufficiently high, either $\lambda^L_1$ or $\lambda^H_1$ become binding for all locations
and therefore $R_1$ has unique maximum at $x = \frac{1}{2}$.

\footnote{The equilibrium value of $p_1$ for all $(x, v, t)$ can be found in Appendix A.}
Proposition 1 has an important implication: when B is uncertain about \( x \), S’s optimal information disclosure strategy calls for fully revealing all locations \( \min \{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \} \leq x \leq \max \{ \frac{1}{2}, 1 - \sqrt{\frac{v}{3t}} \} \). This is because doing so leads to a revenue of \( R_1(\frac{1}{2}, v, t) \). Since neither \( \lambda^L_1 \) nor \( \lambda^H_1 \) is binding at the optimal price S chooses for these locations, sending a partially-revealing message cannot improve the expected demand S faces. Note that \( x = \frac{1}{2} \) is fully revealed in every PBE regardless of the value of \( \frac{v}{t} \).

This observation plays a key role in the characterization of equilibrium information disclosure. Suppose that S fully reveals the locations that lie in \((z, 1 - z)\) in equilibrium and consider the case when \( x = z \). S knows that, regardless of the message he sends, B will never assign a positive probability to the values of \( x \) between \( z \) and \( 1 - z \) because S would normally fully reveal these locations. In other words, the usual unraveling story is at work here. In case S chooses not to fully reveal \( x = z \), his problem is to choose a message that brings the perceived location as close to \( \frac{1}{2} \) as possible while keeping uncertainty as low as possible. Since \( f(x) \) is symmetric around \( \frac{1}{2} \), S can induce a perceived location of exactly \( \frac{1}{2} \) by sending, for instance, a message \( M = [z, 1 - z] \). This message also leads to the lowest uncertainty that S can induce. In fact, as Proposition 2 describes, S’s equilibrium choice reduces to either fully revealing \( x = z \) or sending a partially-revealing message that would induce B to think that \( x \) is equal to either \( z \) or \( 1 - z \).

**Proposition 2** There exists a PBE in which the value of \( \left| \frac{1}{2} - x \right| \) fully unravels, whereas \( x \) is fully revealed if and only if \( \frac{v}{t} \) is sufficiently low. Moreover, the set of fully revealed locations monotonically shrinks as \( \frac{v}{t} \) is higher.

Proposition 2 describes how the information unraveling result extends to markets with goods that have horizontal attributes. In equilibrium, B understands that her expected ideal taste acts as a reference point for S. This leads her to adopt a pessimistic posture in which she associates a partially-revealing message with the location that is farthest away from this reference point. Therefore, in case S chooses to send a partially-revealing message, he pools the true location only with the ones that are equally or less distant from \( \frac{1}{2} \). Since S employs the same strategy for all possible locations, the distance between the true location of G and the expected ideal taste of B fully unravels.
Which locations are fully revealed in equilibrium depends on how large $\frac{v}{t}$ is. Recall that for an observed message $M$ and price $P$, B buys G only if her location is at most $\sqrt{\frac{v-P}{t}} - \text{Var}[x | x \in \Omega]$ units away from $E[x | x \in \Omega]$. When $\frac{v}{t}$ is sufficiently low, consumption of G offers a low utility and/or B's preference for her ideal taste is sufficiently strong. In this case, leaving B with a high uncertainty significantly lowers her likelihood of a purchase. Therefore, S fully reveals all values of $x$, thereby having $\text{Var}[x | x \in \Omega]$ equal to 0. When $\frac{v}{t}$ is high, on the other hand, B perceives different varieties of G as close substitutes. So, uncertainty does not bother her too much. Therefore, the adverse effect of uncertainty is lower, and as such, S prefers bringing the perceived location to $\frac{1}{2}$ by sending a partially-revealing message. When $\frac{v}{t}$ is sufficiently high, S fully reveals only the most central location, $x = \frac{1}{2}$, while sending a partially-revealing message for the rest.

It is important to note that there are many messages that lead to the same equilibrium outcome. For example, a message $M = [z, 1-z]$ or simply $M = \{z, 1-z\}$ induces B to conclude that the true location is either $z$ or $1-z$. Multiplicity of equilibrium messages is typical in verifiable information disclosure games. However, since all equilibria are payoff-equivalent, it does not change any of the results. It is also important to note that, in case S sends a partially-revealing message, the price he would choose for either of the two locations that are inferred by B would be the same since, otherwise, price would signal the location. So, in equilibrium, price is not informative about location.

Similar with the earlier notation, let a subscript 0 indicate a partially-revealing message whereby $p_0$ and $R_0$ denote the optimal price S chooses and the equilibrium expected revenue he sends a partially-revealing message. If $R_0 > R_1$ for a particular $x$, then S chooses to send a partially-revealing message. The revenue S expects to earn in this case can be found as follows. Suppose $x = z$ and S sends a message $M = [z, 1-z]$. B's
inference is $\Omega = \{z, 1 - z\}$ where she assigns equal probability to each possibility. So,

$$E[x | x \in \Omega] = \frac{1}{2},$$

$$Var[x | x \in \Omega] = \left(\frac{1}{2} - z\right)^2,$$

and thus equations (6) and (7) reduce to

$$\lambda^L_0 = \max \left\{ 0, \frac{1}{2} - \sqrt{\frac{v - P}{t}} - \left(\frac{1}{2} - z\right)^2 \right\}, \quad \text{(10)}$$

$$\lambda^H_0 = \min \left\{ 1, \frac{1}{2} + \sqrt{\frac{v - P}{t}} - \left(\frac{1}{2} - z\right)^2 \right\}. \quad \text{(11)}$$

Expressing these expressions for a generic $x$, the revenue-maximizing price is

$$p_0(x, v, t) = \arg \max_P PD_0(P; x, v, t),$$

where $D_0(P; x, v, t) = \lambda^H_0 - \lambda^L_0$. This leads to the equilibrium expected revenue $S$ makes

$$R_0(x, v, t) = p_0 D_0(p_0, x, v, t).$$

A comparison of $R_1$ and $R_0$ yields the set of locations that are fully revealed in equilibrium. This is graphically illustrated in Figure 1 for $v = 0.6$ and $t = 1$. The horizontal axis indicates the values of $x$. The solid curve is the expected revenue $S$ earns when $x$ is fully revealed, $R_1$, while the dashed curve is the expected revenue he earns when he sends a partially-revealing message, $R_0$. As seen in the figure, a set of central locations (i.e., $x \in [x_H, 1 - x_H]$) is fully revealed because, as described earlier, $S$ can achieve a revenue of $R_1(\frac{1}{2}, v, t)$ by fully revealing these locations. Focusing on $x \leq \frac{1}{2}$, as $x$ gets

---

Note that Bayes’ rule does not work since both are ex-ante zero-probability events. In this case, updating proceeds as follows:

$$\text{Prob}(x = z | x \in \{z, 1 - z\}) = \lim_{\varepsilon \to 0} \frac{F(z + \varepsilon) - F(z)}{F(z + \varepsilon) - F(z) + F(1 - z) - F(1 - z - \varepsilon)}.$$

Using l’Hôpital’s rule,

$$\text{Prob}(x = z | x \in \{z, 1 - z\}) = \lim_{\varepsilon \to 0} \frac{f(z + \varepsilon)}{f(z) + f(1 - z) - f(1 - z - \varepsilon)} = \frac{f(z)}{f(z) + f(1 - z)} = \frac{1}{2}.$$

The equilibrium value of $p_0$ for all $(x, v, t)$ can be found in Appendix A.
more distant from $\frac{1}{2}$, $\lambda^L_1$ becomes binding in case S fully reveals $x$, so S prefers sending a partially-revealing message. The adverse effect of uncertainty is minimal for locations close to $\frac{1}{2}$ but increases quickly as the true location gets closer to the edges. Therefore, the locations below $x_L$ (symmetrically those above $1 - x_L$) are also fully revealed.

Figure 1: The set of fully revealed locations (indicated by double arrows) when $v = 0.6$ and $t = 1$.

Holding $v$ constant, for lower (higher) values of $t$, both curves in Figure 1 shift upwards (downwards). The magnitude of the shift is higher for $R_0$ compared to $R_1$. Therefore, the set of fully revealed locations shrinks (grows). In other words, $x_H$ increases (decreases) while $x_L$ decreases (increases) as $\frac{v}{t}$ becomes higher. When $\frac{v}{t}$ is below a certain threshold (approximately 0.521), S fully reveals all values of $x$. When it is sufficiently high (higher than 0.75), S fully reveals only the most central location, $x = \frac{1}{2}$, while sending a partially-revealing message for the remaining locations.
4 Social Planner’s Problem

In this section, I analyze the social welfare properties of equilibrium information disclosure. I focus attention on policies in which a social planner may mandate S to fully reveal a given set of locations. When full disclosure is not mandatory for a particular location \( x \), S may choose to fully reveal it or send a partially-revealing message as described in the previous section (i.e., pool it with \( 1 - x \)). Thus, if the total expected welfare (S’s revenue plus B’s net utility) evaluated under full disclosure is higher than the expected welfare evaluated under a partially-revealing message for a particular location \( x \), then the social planner mandates S to fully reveal it (unless S voluntarily does so).

Even though the classical information disclosure literature typically finds excessive information disclosure, this finding critically depends on the assumption that consumers have unit demands with identical reservations prices. In this case, since disclosure does not change equilibrium demand, it is purely redistributive. In the current model, on the other hand, the expected demand S faces is downward-sloping. Although S charges a higher price under full location information, B makes a better-informed decision. So, while it is clear that S’s expected revenue goes down by mandating him to fully reveal a location which he would normally not reveal, B’s net expected utility may increase. Therefore, it is a priori unclear whether there is any need for intervention.

When \( x \) is fully revealed, S chooses a price \( p_1 \) and B buys G if her location is at most \( \sqrt{\frac{v - p_1}{t}} \) units away from \( x \) (in other words, if \( \lambda \in [\lambda_L, \lambda_H] \)). Thus, the total expected welfare when the true location is \( x \) and is fully revealed, \( W_1 \), can be expressed as

\[
W_1(x, v, t) = R_1(x, v, t) + \int_{\lambda_L}^{\lambda_H} (v - p_1(x, v, t) - t(\lambda - x)^2) d\lambda.
\] (12)

Note that price is simply a transfer between S and B, so the total expected welfare is equal to the gross expected utility of B. Similarly, when S sends a partially-revealing message, he chooses a price \( p_0 \) and B buys G if her location is at most \( \sqrt{\frac{v - p_0}{t} - \left(\frac{1}{2} - x\right)^2} \)

---

7The first-best is to set the price equal to the marginal cost of production (which is 0) and force S to fully reveal the variety at all times.
8If disclosure is sufficiently costly, a monopoly seller may under-provide full quality information when demand is downward-sloping. See Daughety and Reinganum (2008) and Celik (2011) for further details.
units away from $\frac{1}{2}$. The total expected welfare in this case is

$$W_0(x, v, t) = R_0(x, v, t) + \int_{\lambda_0^R(p_0, x, v, t)}^{\lambda_0^F(p_0, x, v, t)} \left( v - p_0(x, v, t) - t(\lambda - x)^2 \right) d\lambda.$$  \hfill (13)

If $W_1 > W_0$ for some $x$ for which $R_1 < R_0$ (so that $S$ normally sends a partially-revealing message), then the social planner mandates $S$ to fully reveal it. Proposition 3 establishes that there is actually no value of $x$ for which this is true.

**Proposition 3** Mandating $S$ to fully reveal a location does not improve social welfare, while it is often socially harmful.

The intuition for this finding is as follows. Compared to full disclosure, sending a partially-revealing message leaves $B$ with some uncertainty regarding the true location of $G$. This typically induces $S$ to charge a lower price. Therefore, if $S$ chooses to send a partially-revealing message rather than a fully-revealing one, the expected demand he faces in the former case must be larger than the expected demand he faces in the latter. Since price is simply a transfer, the demand enlargement effect of disclosing partial information improves the total welfare. Thus, forcing $S$ to fully reveal a variety that he voluntarily does not is often socially harmful.

## 5 Discussion

In this section, I discuss several points related to possible extensions of the model. The first point regards incorporating uncertainty about quality. As noted before, the parameter $v$ can be interpreted as the quality of the good. If the buyer is also uncertain about the quality, it can easily be shown that the seller’s expected revenue is strictly increasing in the perceived quality of the buyer. This is true even when the seller is assumed to be uncertain about the buyer’s taste for quality.\(^9\) Thus, regardless of her prior beliefs for it, quality would be fully revealed in every PBE. In other words, the usual unraveling

\(9\)Consider the utility function $\theta v - p - t(\lambda - x)^2$, where $\theta > 0$. Here, $\theta$ measures the buyer’s taste for quality. It is easy to show that the seller’s expected revenue is strictly increasing in the perceived quality when the seller is uncertain about $\theta$ and $\lambda$ while the buyer is uncertain about $v$ and $x$. 

16
story applies with respect to quality disclosure. Accordingly, all of the main results about location disclosure remain valid.

Second, I have considered a general message technology whereby the seller could send any message that includes the true location of the good. If the seller is somehow constrained to either fully reveal the location or stay silent, then the structure of equilibrium information disclosure substantially changes. When the buyer’s preference for her ideal taste is sufficiently strong, the seller fully reveals all locations. When it is weak, the seller fully reveals a set of central locations while staying silent for the remaining ones. Depending on the shape of the buyer’s prior beliefs, there may be multiple PBE. In this case, each PBE is characterized with a different set of fully revealed locations. The set of fully revealed locations shrinks in every PBE as the buyer’s preference for her ideal taste becomes weaker, but is always non-empty. Sun (2011) analyzes this problem when the buyer’s disutility due to a mismatch increases linearly with the value of the mismatch (i.e., when the transportation cost function is linear). Her findings are very similar with one major difference. She finds that the seller stays silent for all locations if the buyer’s preference for her ideal is sufficiently strong.

The third point is about the shape of the transportation cost function. I have assumed a quadratic function because it implies an uncertainty-averse buyer. In other words, at a given price, the buyer strictly prefers a fully revealing message than a partially-revealing message that implies the same perceived location. The class of PBE described in Proposition 2 remains the same for any strictly convex transportation function. When the transportation function is linear as in Sun (2011), on the other hand, the indifferent buyer type may be uncertainty-neutral. This is most pronounced when the seller optimally serves all types of the buyer (i.e., when \( \frac{v}{t} \) is sufficiently large) so that the indifferent buyer is either \( \lambda = 0 \) or \( \lambda = 1 \) (or both). In this case, all messages that lead to the same perceived location are equivalent from the indifferent buyer’s point of view. Therefore, the seller chooses any (truthful) partially-revealing message that leads to a perceived location of \( \frac{1}{2} \), thereby achieving the same expected revenue as he would achieve when \( x = \frac{1}{2} \). As a result, there are many possible equilibria which are payoff-equivalent for the seller, but are substantially different in terms of the buyer’s equilibrium inferences.
The class of PBE described in Proposition 2 is still valid. Another possibility is a fully pooling PBE where all seller types send a message $M = [0, 1]$, which is equivalent to staying silent.

The fourth point is related to the assumption of costless information disclosure. If disclosure is costly, on the contrary, then the seller may prefer staying silent when the location is close to the edges. However, provided that it is not too costly, the structure of information disclosure stays the same for more central locations. If it is too costly, the seller stays silent for all locations. In this case, it may be socially beneficial to mandate the seller to fully reveal a set of central locations. See Daughety and Reinganum (2008) and Celik (2011) for a similar result in a quality-disclosure framework when the seller faces a downward-sloping demand.

A final point is about the prior beliefs of the buyer about the location of the good. Even though I have assumed that the prior beliefs are symmetric around $\frac{1}{2}$, the class of PBE described in Proposition 2 is valid for any prior beliefs. Consider the following off-equilibrium beliefs. When the seller includes many locations in his message, the buyer associates the good with the location that is farthest away from $\frac{1}{2}$. In case there are two such locations, the buyer assigns a positive probability to both.\(^{10}\) Under these beliefs, the seller never sends a message that includes locations farther away from the center than the good’s true location. Therefore, the class of PBE described in Proposition 2 remains valid. However, it is generally not the unique PBE. Unless $f(x)$ is symmetric around $\frac{1}{2}$, sending a partially-revealing message as described in Proposition 2 does not lead to a perceived location of $\frac{1}{2}$. As such, the seller may choose to send a message that would bring the perceived location closer to $\frac{1}{2}$ unless the adverse effect of uncertainty is too high.

6 Conclusion

In this paper, I analyze the level of information a privately-informed monopoly seller voluntarily reveals about the horizontal attribute of the good he sells. The horizontal

\(^{10}\)There is an exception for messages such that $M \subset [\sqrt{\frac{1}{3t}}, 1 - \sqrt{\frac{1}{3t}}]$. These are the locations for which $S$ earns a revenue of $R_1(\frac{1}{2}, v, t)$. In this case, assume the buyer believes the message as it is. Since such an inference introduces a positive variance, $S$ would never deviate from fully revealing $x \subset [\sqrt{\frac{1}{3t}}, 1 - \sqrt{\frac{1}{3t}}]$. 

18
attribute is captured by a location over the unit line. I consider a single buyer with a privately known ideal taste which is also captured by a location (although the findings would be the same if there is a continuum of buyers with different ideal tastes). Although information unraveling does not apply to the fullest extent, it is still at work. I characterize an equilibrium in which the degree of mismatch between the true location of the good and the expected ideal taste of the buyer fully unravels. The driving force for this finding is the (optimal) skepticism of the buyer; any partially-revealing message induces her to believe that the true location of the good is the one in the message that is farthest away from her expected ideal taste. The seller fully reveals the true location only when the buyer’s preference for her ideal taste is sufficiently strong. As it becomes weaker, the set of fully revealed locations monotonically shrinks and when it is sufficiently weak, the seller fully reveals only the location that corresponds to the expected ideal taste of the buyer.

From a social point of view, I find that it is never welfare-improving, but is often socially harmful, to mandate the seller to fully reveal a location that he voluntarily does not. The reason for this finding is the demand enlargement effect of a partially-revealing message whereby the seller typically charges a lower price compared to what he would charge under full disclosure. This is in line with the classical information disclosure literature which also finds excessive disclosure.

I have assumed that horizontal attributes of a good can be described by a single location. Future work may consider multiple horizontal and vertical attributes and analyze the incentives of a monopoly seller to provide information on multiple dimensions. Moreover, such an extension would enable an empirical test of the model. An example is the market for real estate where there is typically a limited number of characteristics sellers may reveal in advertisements.

**Appendix A: Equilibrium price**

In this part of the appendix, I derive the equilibrium price $S$ chooses under the two possible scenarios: when $S$ fully reveals $x$ and when he sends a partially-revealing message. This will later be helpful in the proofs of propositions. Note that since $S$’s beliefs for $\lambda$
are uniform over [0, 1], the probability B buys G at a given price is symmetric around \( \frac{1}{2} \) with respect to \( x \). So, it will be sufficient to characterize equilibrium price for \( x \leq \frac{1}{2} \) only.

**Case 1 When S fully reveals \( x \)**

Since S’s beliefs for \( \lambda \) are uniform over [0, 1], the probability B buys G at some given price is symmetric around \( \frac{1}{2} \) with respect to \( x \). So, it will be sufficient to characterize equilibrium price for \( x \leq \frac{1}{2} \) only. By equations (6) and (7), for a given \((P, v, t)\), if S chooses a price such that \( \frac{v-P}{t} < \frac{1}{4} \), then

\[
D_1(P; x, v, t) = \begin{cases} 
  x + \sqrt{\frac{v-P}{t}}, & x < \sqrt{\frac{v-P}{t}} \\
  2\sqrt{\frac{v-P}{t}}, & \sqrt{\frac{v-P}{t}} \leq x \leq \frac{1}{2} 
\end{cases}
\]

If, on the other hand, \( \frac{1}{4} \leq \frac{v-P}{t} < 1 \), then

\[
D_1(P; x, v, t) = \begin{cases} 
  x + \sqrt{\frac{v-P}{t}}, & x < 1 - \sqrt{\frac{v-P}{t}} \\
  1, & 1 - \sqrt{\frac{v-P}{t}} \leq x \leq \frac{1}{2} 
\end{cases}
\]

Finally, when \( \frac{v-P}{t} \geq 1 \), all types of B buy G, so \( D_1(P; x, v, t) = 1 \). Maximization of \( P \left( x + \sqrt{\frac{v-P}{t}} \right) \) with respect to \( P \) leads to a price of \( \frac{2v}{9} \left( \frac{3v}{t} - x^2 + x \sqrt{\frac{3v}{t} + x^2} \right) \), while the same for \( 2P \sqrt{\frac{v-P}{t}} \) leads to a price of \( \frac{2v}{3} \).

Checking for corner solutions leads to the following equilibrium price (tedious but otherwise straightforward algebra).

- **If \( \frac{v}{t} < \frac{3}{4} \),**
  \[
p_1(x, v, t) = \begin{cases} 
  \frac{2v}{9} \left( \frac{3v}{t} - x^2 + x \sqrt{\frac{3v}{t} + x^2} \right), & x < \sqrt{\frac{v}{5t}} \\
  \frac{2v}{3} \left( \frac{v}{t} - x^2 \right), & \sqrt{\frac{v}{5t}} \leq x \leq \sqrt{\frac{v}{3t}} \\
  \frac{2v}{3}, & \sqrt{\frac{v}{3t}} \leq x \leq \frac{1}{2} 
\end{cases}
\]

- **If \( \frac{3}{4} \leq \frac{v}{t} < \frac{5}{4} \),**
  \[
p_1(x, v, t) = \begin{cases} 
  \frac{2v}{9} \left( \frac{3v}{t} - x^2 + x \sqrt{\frac{3v}{t} + x^2} \right), & x < \sqrt{\frac{v}{5t}} \\
  \frac{2v}{t} \left( \frac{v}{t} - x^2 \right), & \sqrt{\frac{v}{5t}} \leq x \leq \frac{1}{2} 
\end{cases}
\]

- **If \( \frac{5}{4} \leq \frac{v}{t} < 3 \),**
  \[
p_1(x, v, t) = \begin{cases} 
  \frac{2v}{9} \left( \frac{3v}{t} - x^2 + x \sqrt{\frac{3v}{t} + x^2} \right), & x < 2 - \sqrt{\frac{1 + \frac{v}{t}}{t}} \\
  \frac{2v}{t} \left( \frac{v}{t} - (1-x)^2 \right), & 2 - \sqrt{\frac{1 + \frac{v}{t}}{t}} \leq x \leq \frac{1}{2} 
\end{cases}
\]
• If \( \frac{v}{t} \geq 3 \),

\[ p_1(x, v, t) = t \left( \frac{v}{t} - (1 - x)^2 \right) \text{ for all } x \leq \frac{1}{2}. \]

Note that \( p_1 \) is non-monotonic in \( x \) (as \( x \) goes from 0 to \( \frac{1}{2} \)). For \( \frac{v}{t} < \frac{5}{4} \), when \( x \) is sufficiently close to 0, \( S \) prefers to keep the price low in order to increase the probability of a purchase, thereby leaving a positive surplus to the \( \lambda = 0 \) type B. So, in this region, \( S \) effectively chooses the highest type of B that he wants to serve. Therefore, as \( x \) gets closer to \( \frac{1}{2} \), the price \( S \) optimally sets increases. When \( \frac{v}{t} \geq \frac{5}{4} \), the real question \( S \) faces is whether to sell or not to the \( \lambda = 1 \) type B. This particular buyer type is willing to pay more for values of \( x \) closer to 1. Therefore, the equilibrium price is increasing over \( x \in [0, \frac{1}{2}] \) when \( \frac{v}{t} \) is large.

**Case 2 When \( S \) sends a partially-revealing message**

When \( S \) sends a partially-revealing message, say \( M = [x, 1 - x] \), B infers that the true variety must be either \( x \) or \( 1 - x \). Hence, when \( S \) charges a price \( P \), equations (6) and (7) reduce to

\[ \lambda_0^L = \max \left\{ 0, \frac{1}{2} - \sqrt{\frac{v - P}{t} - \left( \frac{1}{2} - x \right)^2} \right\}, \]

\[ \lambda_0^H = \min \left\{ 1, \frac{1}{2} + \sqrt{\frac{v - P}{t} - \left( \frac{1}{2} - x \right)^2} \right\}. \]

For a given \((P, v, t)\), if \( S \) chooses a price such that \( \frac{v - P}{t} < \frac{1}{4} \),

\[ D_0(P; x, v, t) = \begin{cases} 0, & x < \frac{1}{2} - \sqrt{\frac{v - P}{t}} \\ 2\sqrt{\frac{v - P}{t} - \left( \frac{1}{2} - x \right)^2}, & \frac{1}{2} - \sqrt{\frac{v - P}{t}} \leq x \leq \frac{1}{2} \end{cases}. \]

Similarly, if \( \frac{1}{4} \leq \frac{v - P}{t} < \frac{3}{4} \),

\[ D_0(P; x, v, t) = \begin{cases} 2\sqrt{\frac{v - P}{t} - \left( \frac{1}{2} - x \right)^2}, & x < \frac{1}{2} - \sqrt{\frac{v - P}{t} - \frac{1}{4}} \\ 1, & \frac{1}{2} - \sqrt{\frac{v - P}{t} - \frac{1}{4}} \leq x \leq \frac{1}{2} \end{cases}. \]

Finally, when \( \frac{v - P}{t} \geq \frac{3}{4} \), all types of B buy G, so \( D_0(P; x, v, t) = 1 \). Maximizing \( PD_0(P; x, v, t) \), with respect to \( P \) leads to the following equilibrium price (when the expected demand equals 0 for any \( P \geq 0 \), I assume that the equilibrium price is 0).
• If $\frac{v}{t} < \frac{3}{4}$, 

$$p_0(x, v, t) = \begin{cases} 
0 & , x < \frac{1}{2} - \frac{\sqrt{\frac{v}{t}}}{3} \\
\frac{2}{3}(\frac{v}{t} - \frac{1}{3} - x)^2 & , \frac{1}{2} - \frac{\sqrt{\frac{v}{t}}}{3} \leq x \leq \frac{1}{2} 
\end{cases}.$$

• If $\frac{1}{4} \leq \frac{v}{t} < \frac{3}{4}$, 

$$p_0(x, v, t) = \frac{2t}{3} \left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2\right), \text{ for all } x \leq \frac{1}{2}.$$

• If $\frac{3}{4} \leq \frac{v}{t} < 1$, 

$$p_0(x, v, t) = \begin{cases} 
\frac{2}{3}(\frac{v}{t} - \frac{1}{3} - x)^2 & , x < \frac{1}{2} - \frac{\sqrt{\frac{v}{t}} - \frac{3}{4}}{2} \\
t \left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right) & , \frac{1}{2} - \frac{\sqrt{\frac{v}{t}} - \frac{3}{4}}{2} \leq x \leq \frac{1}{2} 
\end{cases}.$$

• If $\frac{v}{t} \geq 1$, 

$$p_0(x, v, t) = t \left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right), \text{ for all } x \leq \frac{1}{2}.$$

In this scenario, when $\frac{v}{t}$ is small, $S$ cannot generate any demand for $G$ unless it is located sufficiently close to $\frac{1}{2}$. So, in this case, the choice of price is random. I assume, for simplicity, that $S$ charges a price of 0 in such a case. In all other cases, $p_0$ is strictly positive and it strictly increases as $x$ gets closer to $\frac{1}{2}$. When $\frac{v}{t}$ sufficiently large, $S$ serves all types of $B$, so in this case, the equilibrium price is the one that leaves zero surplus to $\lambda = 0$ (or, equivalently, $\lambda = 1$) type B.

### Appendix B: Proofs of the Propositions

In this part of the appendix, I present the proofs. As before, I will consider only the values of $x$ over $[0, \frac{1}{2}]$ unless otherwise noted.

**Proof of Proposition 1.** Using Envelope Theorem, over the values of $x$ for which $p_1 = \frac{2t}{9} \left(\frac{3v}{t} - x^2 + x\sqrt{\frac{3v}{t} + x^2}\right)$, we have $\frac{dR_1}{dx} = P\frac{\partial D_1(P;x,v,t)}{\partial x}$ evaluated at $P = p_1$. In this range, $D_1 = x + \sqrt{\frac{v-P}{t}}$, so $\frac{\partial D_1}{\partial x} = 1$. Since $p_1 > 0$, it follows that $\frac{dR_1}{dx} = p_1 > 0$ for these values of $x$. For the values of $x$ for which $\lambda = 0$ or $\lambda = 1$ type B is made indifferent between buying and not, Envelope Theorem is not applicable (because it is a corner solution). When $\frac{v}{t} < \frac{5}{4}$, this happens for $\sqrt{\frac{v}{3t}} \leq x < \min\left\{\sqrt{\frac{v}{3t}}, \frac{1}{2}\right\}$ in which case $S$ charges a price $p_1 = t \left(\frac{v}{t} - x^2\right)$ and faces an expected demand $D_1 = x + \sqrt{\frac{v-P}{t}} = 2x$. So, the
equilibrium revenue is simply $R_1 = 2t \left( \frac{x}{t} - x^2 \right)x$, which is strictly increasing in $x$ for all $x < \min \left\{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \right\}$. Similarly, when $\frac{v}{t} \geq \frac{3}{4}$, it happens for $\max \left\{ 0, 2 - \sqrt{1 + \frac{v}{t}} \right\} \leq x \leq \frac{1}{2}$ in which case $S$ charges a price $p_1 = t \left( \frac{x}{t} - (1 - x)^2 \right)$ and serves all types of $B$ (since $x + \sqrt{\frac{v}{t} - \frac{P}{t}} = 1$), so $R_1 = t \left( \frac{x}{t} - (1 - x)^2 \right)$. This is again strictly increasing in $x$. Finally, when $\frac{v}{t} < \frac{3}{4}$, $S$ charges a price $p_1 = \frac{4v}{3} \sqrt{\frac{v}{3t}}$ for all $x < \sqrt{\frac{v}{3t}}$, and faces an expected demand $D_1 = 2\sqrt{\frac{v}{3t}} = 2\sqrt{\frac{v}{3t}}$. Hence, the revenue $R_1 = \frac{4v}{3} \sqrt{\frac{v}{3t}}$ is constant for these values of $x$.

**Proof of Proposition 2.** I start with showing that there exists a PBE in which the value of $\mid \frac{1}{2} - x \mid$ fully unravels. I then proceed with showing that the set of fully revealed locations shrinks as $\frac{v}{t}$ is higher. To make the latter easier, I present two lemmas below. Finally I argue that there are values of $\frac{v}{t}$ for which $R_0 < R_1$ for all $x$ and for which $R_0 > R_1$ for all $x$. This concludes the proof.

Before proceeding, I make the following two important observations. First, if two messages lead to the same perceived location, $S$ strictly chooses the message associated with a lower implied variance. Formally, suppose there are two messages $M$ and $M'$ such that $E \left[ x \mid x \in \Omega \right] = E \left[ x \mid x \in \Omega' \right]$ and $Var \left[ x \mid x \in \Omega \right] < Var \left[ x \mid x \in \Omega' \right]$. Then, $M$ leads to a strictly higher expected revenue than $M'$. Second, off-equilibrium beliefs cannot be randomly chosen in verifiable disclosure games. After observing an off-equilibrium message, $B$ will not assign a positive probability to any $x \not\in M$. For example, if $S$ unexpectedly fully reveals $x$, then $B$ believes $S$ because lying is ruled out.

Now, suppose $\frac{v}{t} < \frac{3}{4}$ so that the region $\sqrt{\frac{v}{3t}} \leq x \leq 1 - \sqrt{\frac{v}{3t}}$ is non-empty. By Proposition 1, this is where the expected revenue $S$ earns is constant and is equal to $R_1 \left( \frac{1}{2}, v, t \right)$. The locations in this region must be fully revealed in every PBE, because any partially-revealing PBE implies a positive variance, $Var \left[ x \mid x \in \Omega \right] > 0$, and $S$ can profitably deviate by fully revealing $x$ thereby achieving $R_1(x, v, t) = R_1 \left( \frac{1}{2}, v, t \right)$. For $x < \sqrt{\frac{v}{3t}}$, by Proposition 1, $R_1(x, v, t)$ is strictly increasing in $x$. Given that $S$ fully reveals all $x \in \left[ \sqrt{\frac{v}{3t}}, 1 - \sqrt{\frac{v}{3t}} \right]$, then it is best for $S$ to either fully reveal $x$ or reveal $\mid \frac{1}{2} - x \mid$, say by sending $M = [x, 1 - x]$, for all $x < \sqrt{\frac{v}{3t}}$ (symmetrically, for $x > 1 - \sqrt{\frac{v}{3t}}$). The latter strategy is associated with the lowest variance among all possible inferences $S$ may induce $B$ to make. This is because pooling with locations that are farther away
from $\frac{1}{2}$ than the good’s true location simply raises $\text{Var} [x \mid x \in \Omega]$.

The only complication may arise with extreme off-equilibrium beliefs. Suppose after observing a partially-revealing message, B assigns a probability of 1 to the location that is farthest away from $\frac{1}{2}$. In case there are two such locations, assume that B assigns a probability of 1 to the location that is higher than $\frac{1}{2}$. Under these beliefs, S is indifferent between sending any message $M \subset [\sqrt{\frac{3}{3t}}, 1 - \sqrt{\frac{3}{3t}}]$ for $\sqrt{\frac{3}{3t}} \leq x \leq 1 - \sqrt{\frac{3}{3t}}$, because any such message leads to an expected revenue of $R_1 \left(\frac{1}{2}, v, t\right)$. Remember that I assumed, in case S is indifferent between two or more messages, he sends each with a strictly positive probability. Hence, under the described beliefs, S may choose a partiallyrevealing message for $\sqrt{\frac{3}{3t}} \leq x \leq 1 - \sqrt{\frac{3}{3t}}$ with a positive probability, which induces B to misinterpret the true location. Thus, these extreme off-equilibrium beliefs cannot be part of a PBE for $\sqrt{\frac{3}{3t}} \leq x \leq 1 - \sqrt{\frac{3}{3t}}$. An example of consistent off-equilibrium beliefs upon observing $M \subset [\sqrt{\frac{3}{3t}}, 1 - \sqrt{\frac{3}{3t}}]$ is believing the message as it is. Since this introduces a positive variance, S would never deviate.

Similarly, these extreme off-equilibrium beliefs cannot be part of a PBE for $x < \sqrt{\frac{3}{3t}}$, either. To see this, fix a value of $x < \sqrt{\frac{3}{3t}}$ and suppose that S is indifferent between fully revealing $x$ and revealing $|\frac{1}{2} - x|$. But then, even under the extreme off-equilibrium beliefs described above, S chooses the latter strategy with a positive probability. Hence, B’s inference would again be inconsistent. This means that, when B observes a partially-revealing message $M \not\subset [\sqrt{\frac{3}{3t}}, 1 - \sqrt{\frac{3}{3t}}]$, she assigns a probability of 1 to the location that is farthest away from $\frac{1}{2}$, but in case there are two such locations, she must assign a positive probability to both of them.

When $\frac{v}{t} \geq \frac{3}{4}$, $R_1 (x, v, t)$ is strictly increasing in $x$ for all $x \leq \frac{1}{2}$. Again, given that S fully reveals $x = \frac{1}{2}$ in every PBE, it is best for S to either fully reveal $x$ or reveal $|\frac{1}{2} - x|$, say by sending $M = [x, 1 - x]$, for all $x \neq \frac{1}{2}$. Hence, all of the arguments I made for $x < \sqrt{\frac{3}{3t}}$ above are precisely applicable in this case. This means that, even under the most skeptical off-equilibrium beliefs, S follows the strategy of either fully revealing $x$ or revealing $|\frac{1}{2} - x|$, whichever is more profitable. Hence, when S sends a message $M = [x, 1 - x]$, B optimally assigns a positive probability to both $x$ and $1 - x$. This completes the first part of the proof.
Next, I show that the set of fully revealed locations shrinks as \( \frac{v}{t} \) is higher. This is substantially eased by the following two lemmas. The first one establishes that, under both strategies S may choose (i.e., either fully reveal \( x \) or reveal \( \frac{1}{2} - x \)), the derivative of equilibrium revenue divided by \( t \) with respect to \( \frac{v}{t} \) is equal to the corresponding expected demand. The second lemma shows that whenever a partially-revealing message is more profitable than fully revealing \( x \), the expected demand under the former is at least as large as the one under latter. Before proceeding with the lemmas, note from equations (6) and (7) that, under both strategies, price enters the expected demand function as \( \frac{E}{t} \). Moreover, the equilibrium prices I find in Appendix A are multiples of \( t \). Thus, both \( \frac{R_{1}}{t} \) and \( \frac{R_{2}}{t} \) (\( j = 0, 1 \)) are functions of only \( x \) and \( \frac{v}{t} \).

**Lemma 1** \( \frac{d(R_{1}/t)}{dv/v(t)} = D_{1}(p_{1}, x, v, t) \) and \( \frac{d(R_{0}/t)}{dv/v(t)} = D_{0}(p_{0}, x, v, t) \) for all \( x \).

**Proof of Lemma 1.** I start with the case when S fully reveals \( x \). First, take the values of \( x \) and \( \frac{v}{t} \) for which \( p_{1} = \frac{2}{3} \left( \frac{3}{t} - x^{2} + x \sqrt{\frac{3}{t} + x^{2}} \right) \). In this region, \( D_{1} = x + \frac{\sqrt{v-P}}{t} \). By Envelope Theorem, \( \frac{d(R_{1}/t)}{dv/v(t)} = \frac{P}{t} \frac{\partial D_{1}}{\partial v/v(t)} \) evaluated at \( P = p_{1} \). Since \( \frac{\partial D_{1}}{\partial v/v(t)} = -\frac{\partial D_{1}}{\partial P/v(t)} \), and the revenue maximization problem implies \( D_{1} + \frac{p}{t} \frac{\partial D_{1}}{\partial P/v(t)} = 0 \) evaluated at \( P = p_{1} \), we have \( \frac{d(R_{1}/t)}{dv/v(t)} = D_{1} \). When \( \frac{v}{t} < \frac{5}{4} \), for \( \sqrt{\frac{2}{3t}} \leq x \leq \min \left\{ \sqrt{\frac{2}{3t}}, \frac{1}{2} \right\} \), S charges a price \( p_{1} = t \left( \frac{v}{t} - x^{2} \right) \) and faces an expected demand \( D_{1} = x + \frac{\sqrt{v-P}}{t} = 2x \). The equilibrium revenue is simply \( R_{1} = 2t \left( \frac{v}{t} - x^{2} \right) x \), and thus, \( \frac{d(R_{1}/t)}{dv/v(t)} = 2x \), which equals the equilibrium expected demand. Similarly, when \( \frac{v}{t} \geq \frac{5}{4} \), for max \{0, 2 - \sqrt{1 + \frac{v}{t}}\} < \( x \leq \frac{1}{2} \), S charges a price \( p_{1} = t \left( \frac{v}{t} - (1 - x)^{2} \right) \) and serves all types of B, so \( R_{1} = t \left( \frac{v}{t} - (1 - x)^{2} \right) \).

Again, \( \frac{d(R_{1}/t)}{dv/v(t)} = 1 \) which equals the equilibrium expected demand. Finally, when \( \frac{v}{t} < \frac{3}{4} \), for \( \sqrt{\frac{2}{3t}} \leq x \leq \frac{1}{2} \), S charges a price \( p_{1} = \frac{2v}{3} \) and faces an expected demand \( D_{1} = 2\sqrt{\frac{v-P}{t}} = 2\sqrt{\frac{2}{3t}} \). Hence, \( \frac{R_{1}}{t} = 4 \left( \frac{2}{3t} \right)^{3/2} \), and thus, \( \frac{d(R_{1}/t)}{dv/v(t)} = 2\sqrt{\frac{2}{3t}} \) which, again, equals the equilibrium expected demand.

When S sends a partially-revealing message, there are three prices he can possibly charge, as given in Appendix A. When \( x < \frac{1}{2} - \sqrt{\frac{7}{t}} \), the expected demand equals 0 for all values of the price, so the result is trivial for this case. In the range where \( p_{0} = \frac{2t(\frac{t}{2} - (\frac{1}{2} - x)^{2})}{3} \), the expected demand is \( D_{0} = 2\sqrt{\frac{v-P}{t}} - (\frac{1}{2} - x)^{2} \). By Envelope Theorem, \( \frac{d(R_{0}/t)}{dv/v(t)} = \frac{P}{t} \frac{\partial D_{0}}{\partial v/v(t)} \) evaluated at \( P = p_{0} \). Since \( \frac{\partial D_{0}}{\partial v/v(t)} = -\frac{\partial D_{0}}{\partial P/v(t)} \), and the revenue maximization problem implies \( D_{0} + \frac{P}{t} \frac{\partial D_{0}}{\partial P/v(t)} = 0 \) evaluated at \( P = p_{0} \), we have \( \frac{d(R_{0}/t)}{dv/v(t)} = \frac{\sqrt{v-P}}{t} \).
Finally, in the range where \( p_0 = t \left( \frac{v}{t} - \frac{1}{2} - x \right)^2 - \frac{1}{4} \), the expected demand is \( D_0 = 1 \). Hence, \( \frac{R_0}{t} = \frac{p_0}{t} \), and thus, \( \frac{d(R_0/t)}{d(v/t)} = 1 = D_0 \).}

**Lemma 2** If \( R_0 \geq R_1 \) for some \( x \), then \( D_0 \geq D_1 \) for the same \( x \).

**Proof of Lemma 2.** When \( x = \frac{1}{2} \), two regimes are equivalent, so the following analysis applies to \( x < \frac{1}{2} \). If \( R_0 \geq R_1 \) for some \( x \) at which \( D_0 = 1 \), the result is trivial. From Appendix A, this happens for \( \frac{1}{2} - \sqrt{\frac{v}{t} - \frac{3}{4}} \leq x \leq \frac{1}{2} \) when \( \frac{3}{4} \leq \frac{v}{t} < 1 \), and for all \( x \) when \( \frac{v}{t} \geq 1 \). For values of \( x \) for which \( p_0 = 0 \) or for which \( p_1 = \frac{2v}{3} \), it is always true that \( R_0 < R_1 \), so, again, the result is trivial. For the remaining configurations, it is enough to simply compare the equilibrium values of \( p_0 \) and \( p_1 \) for the same \( x \). When \( x < \sqrt{\frac{v}{t}} \), for all \( x \) in the range,

\[
p_1 = \frac{2t}{9} \left( \frac{3v}{t} - x^2 + x \sqrt{\frac{3v}{t} + x^2} \right) \geq \frac{2v}{3} > \frac{2t}{3} \left( \frac{v}{t} - \frac{1}{2} - x \right)^2 = p_0.
\]

When \( \sqrt{\frac{v}{t}} \leq x < \min \{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \} \),

\[
p_1 \left( \sqrt{\frac{v}{3t}}, v, t \right) = \frac{2v}{3} > \frac{2t}{3} \left( \frac{v}{t} - \frac{1}{2} - \sqrt{\frac{v}{3t}} \right)^2 = p_0 \left( \sqrt{\frac{v}{3t}}, v, t \right).
\]

Since \( p_1 \) is decreasing and \( p_0 \) is increasing in \( x \) in this range, it follows that \( p_1 > p_0 \) for all \( x \) here, too. Hence, if \( R_0 \geq R_1 \) for some \( x < \min \{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \} \), then it must be that \( D_0 \geq D_1 \) for the same \( x \).

Finally, I argue that there are values of \( \frac{v}{t} \) for which \( R_0 < R_1 \) for all \( x \) and for which \( R_0 > R_1 \) for all \( x \) (except for \( x = \frac{1}{2} \) where \( R_0 = R_1 \)). Together with the two lemmas, this concludes the proof. Note that \( R_0 < R_1 \) for all \( x \geq \sqrt{\frac{v}{t}} \) (since \( R_1 = R_1 \left( \frac{1}{2}, v, t \right) \) in this region) and for all \( x \leq \frac{1}{2} - \sqrt{\frac{v}{t}} \) (since \( R_0 = 0 \) in this region). \( \sqrt{\frac{v}{t}} = \frac{1}{2} - \sqrt{\frac{v}{t}} \) when \( \frac{v}{t} = \frac{3}{8(2+\sqrt{3})} \). Thus, when \( \frac{v}{t} \leq \frac{3}{8(2+\sqrt{3})} \), \( R_0 < R_1 \) for all \( x \). Similarly, when \( \frac{v}{t} \) is sufficiently high, \( R_0 > R_1 \) for all \( x \). For instance, when \( \frac{v}{t} > 3 \), from Appendix A, \( R_0 = t \left( \frac{v}{t} - \frac{1}{2} - x \right)^2 - \frac{1}{4} \) and \( R_1 = t \left( \frac{v}{t} - (1-x)^2 \right) \). A comparison yields that \( R_0 > R_1 \) when \( \frac{1}{2} - x > 0 \), which is true for all \( x < \frac{1}{2} \).\(^{11}\) Thus, for each value of \( x \), there is a value of \( \frac{v}{t} \) such that \( R_0 > R_1 \). Then, by Lemmas 1 and 2, if \( R_0 > R_1 \) at some \( x \), then \( R_0 > R_1 \) at the same value of \( x \) for all higher values of \( \frac{v}{t} \).

\(^{11}\)In fact, it can be shown that \( R_0 < R_1 \) for all \( x \) when \( \frac{v}{t} \leq 0.521 \) and \( R_0 > R_1 \) for all \( x \) when \( \frac{v}{t} \geq 0.75 \). However, the derivation is long and tedious, but otherwise, straightforward algebra. Since it is not important for the results, I skip it here. It is available upon request.
Proof of Proposition 3. From equations (12) and (13),

\[
W_1 = R_1 + \int_{\lambda_1^U}^{\lambda_1^L} (v - p_1 - t(\lambda - x)^2) d\lambda = \int_{\lambda_1^U}^{\lambda_1^L} (v - t(\lambda - x)^2) d\lambda,
\]

\[
W_0 = R_0 + \int_{\lambda_0^U}^{\lambda_0^L} (v - p_0 - t(\lambda - x)^2) d\lambda = \int_{\lambda_0^U}^{\lambda_0^L} (v - t(\lambda - x)^2) d\lambda.
\]

When \( \frac{v}{t} \geq 1 \), all types of B are served for any \( x \) in case S sends a partially-revealing message, and \( v - t(\lambda - x)^2 \geq 0 \) for each type of B. Thus, when \( \frac{v}{t} \geq 1 \), \( W_0 \geq W_1 \) for all \( x \) (with equality when all types of B are served under full disclosure, too). So, the proof is trivial in this case; mandating full disclosure brings no extra gain while it is often harmful. Similarly, for parameter values where the expected demand is zero under the partially-revealing strategy (this happens for \( x \leq \frac{1}{2} - \sqrt{\frac{v}{t}} \) when \( \frac{v}{t} \leq \frac{1}{4} \)), full disclosure is welfare superior to sending a partially-revealing message. However, since S voluntarily reveals all \( x \) for these parameter values, there is no need for mandating full disclosure.

For the remainder of the proof, I will focus on the remaining situations (i.e., \( \frac{v}{t} < 1 \) and \( D_0 > 0 \)).

First, note from the demand functions in Appendix A that the welfare functions can equivalently be represented as

\[
W_1(x, v, t) = R_1(x, v, t) + \int_{p_1}^{v} D_1(P; x, v, t) dP,
\]

\[
W_0(x, v, t) = R_0(x, v, t) + \int_{p_0}^{v-t(\frac{1}{2}-x)^2} D_0(P; x, v, t) dP,
\]

where the last term in each line is the corresponding consumer surplus. For the following, let \( CS_1 \) and \( CS_0 \) denote these terms. The upper bounds of the integrals are the prices at which the corresponding demands become zero.

Case 1: \( x < \sqrt{\frac{v}{5t}} \)

In this case, if S sends a partially-revealing message for \( x \), the resulting consumer surplus
is expressed as

\[ CS_0 = \int_{p_0}^{v-tx^2} 2\sqrt{\frac{v-P}{t} - \left(\frac{1}{2} - x\right)^2} dP = \frac{4t}{3} \left(\frac{v-p_0}{t} - \left(\frac{1}{2} - x\right)^2\right)^{3/2}. \]

In the region where \( p_0 = \frac{2t(\frac{v}{t} - (\frac{1}{2} - x)^2)}{3} \), this is equal to \( CS_0 = \frac{4t}{3} \left(\frac{\frac{v}{t} - (\frac{1}{2} - x)^2}{3}\right)^{3/2}. \) Note that this is exactly \( \frac{1}{3} \) of the revenue \( S \) earns for the same \( x \), so \( CS_0 = \frac{1}{3} R_0 \) in this region.

In the region where \( p_0 = t \left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right) \), on the other hand, \( CS_0 = \frac{4t}{3} \left(\frac{1}{4}\right)^{3/2} = \frac{4}{6}. \)

When \( S \) fully reveals \( x \), the resulting consumer surplus is

\[ CS_1 = \int_{p_1}^{v-tx^2} \left(x + \sqrt{\frac{v-P}{t}}\right) dP + \int_{v-tx^2}^{v} 2\sqrt{\frac{v-P}{t}} dP. \]

Evaluated at \( x = 0 \), \( p_1 = \frac{2v}{3} \), and \( CS_1 = \int_{2v/3}^{v} \sqrt{\frac{v-P}{t}} dP = \frac{2t}{3} \left(\frac{v}{3}\right)^{3/2}. \) Note that this is exactly \( \frac{1}{3} \) of the revenue \( S \) earns for the same \( x \) when he fully reveals it, so \( CS_1(0, v, t) = \frac{1}{3} R_1(0, v, t) \). Next, observe that

\[ \frac{d \left(CS_1 - \frac{R_1}{3}\right)}{dx} = (v - tx^2 - p_1) - \left(\frac{x + \sqrt{\frac{v-p_1}{t}}}{t}\right) \frac{dp_1}{dx} - \frac{p_1}{3}. \]

The equilibrium price in this region can be rewritten as \( p_1 = \frac{2t}{9} \left(\sqrt{\frac{3v}{t} + x^2 + 2x} \right) \left(\sqrt{\frac{3v}{t} + x^2} - x\right) \) and the resulting demand as \( x + \sqrt{\frac{v-p_1}{t}} = \frac{1}{3} \left(\sqrt{\frac{3v}{t} + x^2 + 2x}\right) \). Taking the derivative of \( p_1 \) with respect to \( x \) and then multiplying the result with the expected demand gives

\[ \left(x + \sqrt{\frac{v-p_1}{t}}\right) \frac{dp_1}{dx} = \frac{1}{3} \left(1 - \frac{x}{\sqrt{\frac{3v}{t} + x^2}}\right) p_1. \]

Plugging this back into \( \frac{d\left(CS_1 - \frac{R_1}{3}\right)}{dx} \) leads to

\[ \frac{d \left(CS_1 - \frac{R_1}{3}\right)}{dx} = v - tx^2 - \left(\frac{5}{3} - \frac{1}{3} \frac{x}{\sqrt{\frac{3v}{t} + x^2}}\right) p_1 \]

Note that \( \frac{x}{\sqrt{\frac{3v}{t} + x^2}} \) is increasing in \( x \), so the maximum value it can take in this region is \( \frac{\sqrt{3}}{\sqrt{\frac{3v}{t} + x^2}} = \frac{1}{4} \). Similarly, \( tx^2 \geq 0 \) and \( p_1 \geq \frac{2v}{3} \). Thus,

\[ \frac{d \left(CS_1 - \frac{R_1}{3}\right)}{dx} \leq v - \left(\frac{5}{3} - \frac{1}{12}\right) \frac{2v}{3} = \frac{v}{18} < 0. \]
So, \( R_{13} \) rises more quickly than \( CS_{1} \) as \( x \) increases, which means that \( CS_{1} < \frac{1}{3}R_{1} \) for all \( x \) in this region. Thus, in the region where \( p_0 = \frac{2\left((\frac{v}{t} - (\frac{1}{2} - x)^2\right)}{3} \), we have

\[
CS_0 - CS_1 > \frac{1}{3}(R_0 - R_1).
\]

This condition means that if \( R_0 \geq R_1 \) for a particular \( x \), then \( CS_0 \geq CS_1 \) and, in turn, \( W_0 \geq W_1 \) for the same \( x \). Hence, mandating full disclosure is harmful.

When \( \frac{3}{4} \leq \frac{v}{t} < 1 \), if \( S \) sends a partially-revealing message, he charges a price \( p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right) \) for \( \frac{1}{2} - \sqrt{\frac{v}{t} - \frac{3}{4}} < \frac{x}{t} \leq \frac{1}{2} \). For these parameter values, I first show that \( CS_{1t} \) is increasing in \( \frac{v}{t} \), and then show that \( \max_x \frac{CS_{1t}}{t} \) evaluated at \( \frac{v}{t} = 1 \) is less than \( \frac{1}{6} \). First, note that \( CS_{1} \) can be rewritten as

\[
CS_{1} = (v - tx^2 - p_1)x - \frac{2t}{3}x^3 + \frac{2t}{3} \left(\frac{v - p_1}{t}\right)^{3/2} + \frac{4t}{3}x^3
\]

Next, observe that

\[
\frac{d \left(\frac{v - p_1}{t}\right)}{d \left(\frac{v}{t}\right)} = 1 - \frac{2}{9} \left(3 + \frac{3x}{2\sqrt{\frac{3v}{t} + x^2}}\right) = \frac{1}{3} - \frac{x}{3\sqrt{\frac{3v}{t} + x^2}} > 0.
\]

Since \( \frac{CS_{1t}}{t} \) is increasing in \( \frac{v - p_1}{t} \), it is also increasing in \( \frac{v}{t} \). When \( \frac{v}{t} = 1 \), it is easy to show that \( \frac{v - p_1}{t} = \frac{1}{3} \left(\sqrt{3 + x^2} - x\right) \). Plugging this back into \( CS_{1} \) and maximizing it with respect to \( x \) leads to \( \arg\max_x CS_{1} \approx 0.2256 \) and \( \max_x \frac{CS_{1t}}{t} \approx 0.141 \). This is less than \( \frac{1}{6} \), which means that \( CS_0 > CS_1 \) for all parameter values for which \( p_0 = t\left(\frac{v}{t} - \left(\frac{1}{2} - x\right)^2 - \frac{1}{4}\right) \).

**Case 2:** \( \sqrt{\frac{v}{t}} \leq x < \min \left\{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \right\} \)

In this case, \( p_1 = v - tx^2 \) and thus,

\[
CS_{1} = \int_{v-tx^2}^{v} 2\sqrt{\frac{v - P}{t}} dP = \frac{4t}{3}x^3.
\]

The revenue \( S \) earns under when he fully reveals \( x \) is \( R_1 = 2t\left(\frac{v}{t} - x^2\right) \). The condition \( \sqrt{\frac{v}{3t}} \leq x < \min \left\{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \right\} \) can equally be represented as \( 3x^2 < \frac{v}{t} \leq 5x^2 \), so \( 4tx^3 < R_1 \leq 8tx^3 \). Hence, \( CS_{1} < \frac{1}{3}R_1 \), which implies that

\[
CS_0 - CS_1 > \frac{1}{3}(R_0 - R_1).
\]
Again, mandating S to fully reveal $x$ in situations when he voluntarily does not is socially harmful.

**Case 3:** $\min \left\{ \sqrt{\frac{v}{3t}}, \frac{1}{2} \right\} \leq x \leq \frac{1}{2}$

This case is relevant only when $\frac{v}{t} < \frac{3}{4}$. In this region, $R_1 > R_0$ for all $x$ (except for $x = \frac{1}{2}$ where two regimes are equivalent). So, mandatory disclosure rules are unnecessary. ■
References


