Fuzzy-rough data mining

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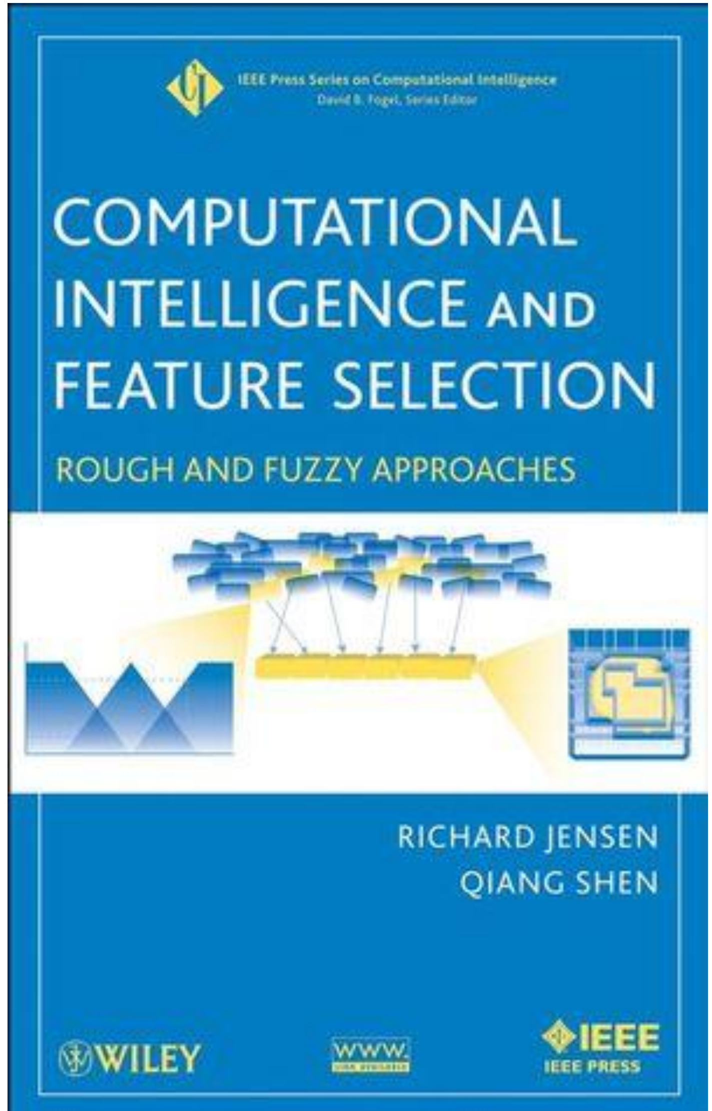
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Outline

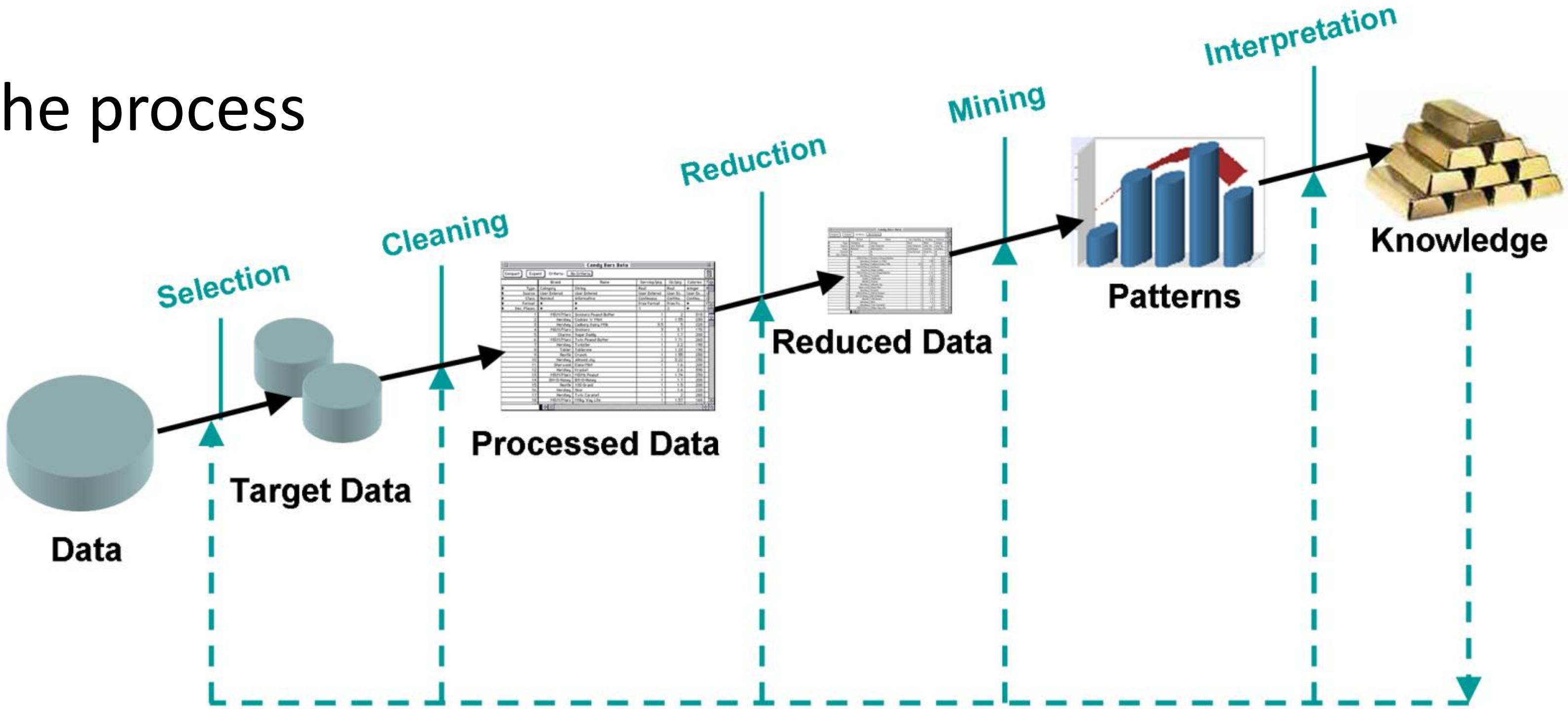
- Introduction to knowledge discovery/data mining
- Feature selection and rough set theory
- Fuzzy-rough feature selection and extensions
- Fuzzy-rough instance selection
- Fuzzy-rough classification/prediction
- Practical session with Weka

Data mining

- Process of semi-automatically analyzing large databases to find patterns (or models) that are:
  - **valid**: hold on new data with some certainty
  - **novel**: non-obvious to the system
  - **useful**: should be possible to act on the item
  - **understandable**: humans should be able to interpret the pattern/model

Knowledge discovery

- The process



- The problem of too much data
  - Requires storage
  - Intractable for data mining algorithms
  - Noisy or irrelevant data is misleading/confounding

Results of Data Mining include:

- *Forecasting* what may happen in the future
- *Classifying* people or things into groups by recognizing patterns
- *Clustering* people or things into groups based on their attributes
- *Associating* what events are likely to occur together
- *Sequencing* what events are likely to lead to later events

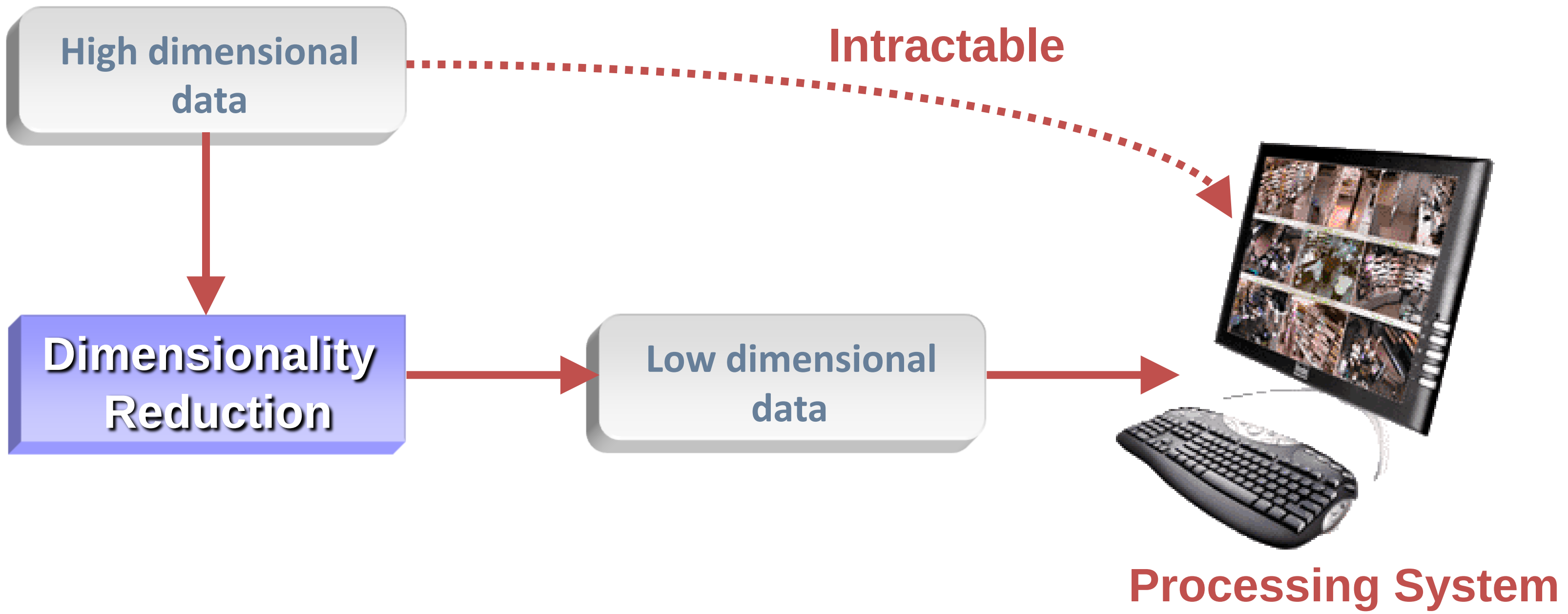
Applications

- Banking: loan/credit card approval
  - predict good customers based on old customers
- Customer relationship management:
  - identify those who are likely to leave for a competitor
- Targeted marketing:
  - identify likely responders to promotions
- Fraud detection: telecommunications, financial transactions
  - from an online stream of events identify fraudulent events
- Medicine: disease outcome, effectiveness of treatments
  - analyze patient disease history: find relationship between diseases

Feature Selection

Feature selection

- Why dimensionality reduction/feature selection?



- Growth of information - need to manage this effectively
- Curse of dimensionality - a problem for machine learning and data mining
- Data visualisation - graphing data

Why do it?

- Case 1: We're interested in *features*
  - We want to know which are relevant
  - If we fit a model, it should be *interpretable*

- Case 2: We're interested in *prediction*
  - Features are not interesting in themselves
  - We just want to build a good classifier (or other kind of predictor)

Case 1

- We want to know which features are relevant; we don’t necessarily want to do prediction

- *E.g. what causes lung cancer?*
  - Features are aspects of a patient’s medical history
  - Decision feature: did the patient develop lung cancer?
  - Which features best predict whether lung cancer will develop?

- *E.g. what stabilizes protein structure?*
  - Features are structural aspects of a protein
  - Real-valued decision feature—protein energy
  - Features that give rise to low energy are stabilizing

Case 2

- We want to build a good predictor

- *E.g. text classification*
  - Features for all English words, and maybe all word pairs
  - Common practice: throw in every feature you can think of, let feature selection get rid of useless ones
  - Training too expensive with all features

- *E.g. disease diagnosis*
  - Features are outcomes of expensive medical tests
  - Which tests should we perform on the patient?

Aspects of features

- Correlation
  - The extent to which one subset of features depends on another

- So ideally we want:
  - **High relevancy**: high correlation with the decision feature
  - **Low redundancy**: very little correlation between features within a subset

Problems

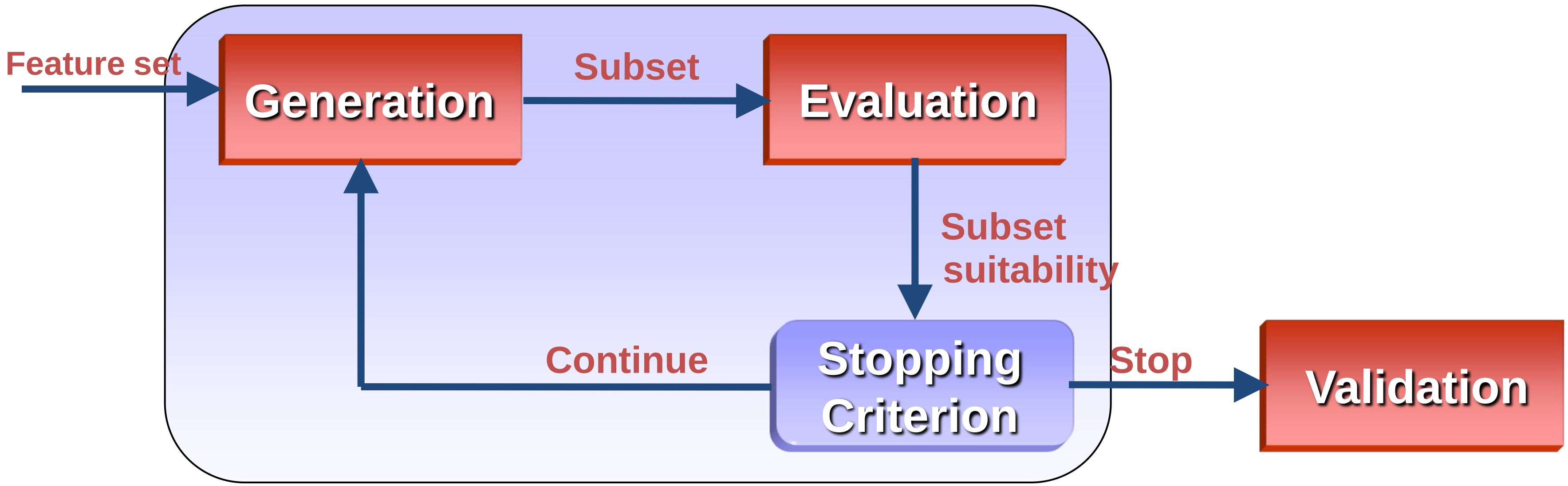
- Noisy data
  - Due to measurement inaccuracies
  - Can also affect decision feature values

- Inconsistent data

	f_1	f_2	class
instance 1	a	b	c1
instance 2	a	b	c2

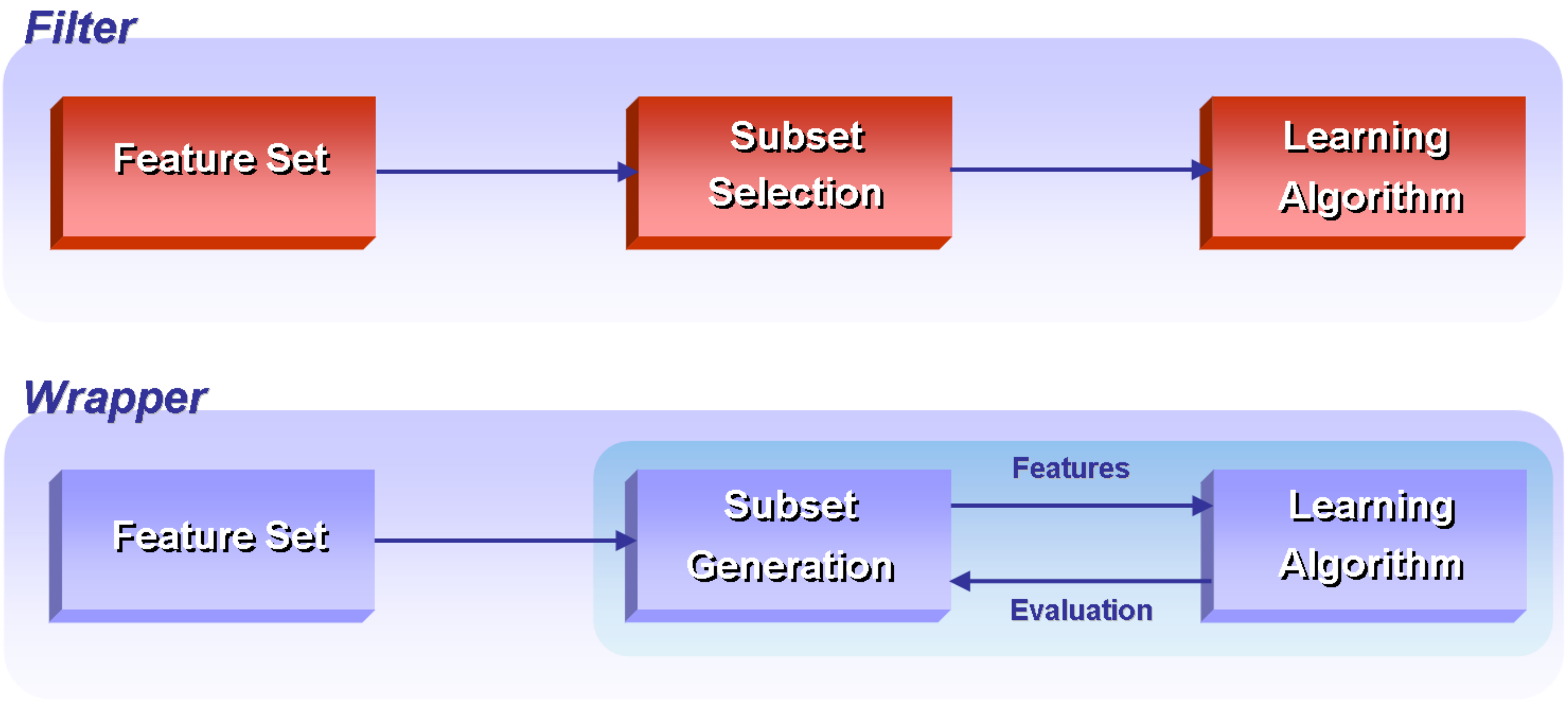
Feature selection process

- Feature selection (FS) preserves data semantics by **selecting** rather than **transforming**



- **Subset generation**: forwards, backwards, random…
- **Evaluation function**: determines 'goodness' of subsets
- **Stopping criterion**: decide when to stop subset search

Types of FS



Filter
Feature Set
Subset Selection
Learning Algorithm
Wrapper
Feature Set
Subset Generation
Features
Learning Algorithm
Evaluation

Search in FS

- We have an evaluation function Eval
  - Task: find a subset of features, F, that maximises Eval(F)
  - Often want to minimise |F|

  - Filter, if Eval = subset evaluation measure
  - Wrapper, if Eval = evaluation via classifier

- Brute-force approach impractical, so search is required
  - Often greedy hill-climbing is used
  - Other search techniques can be used...

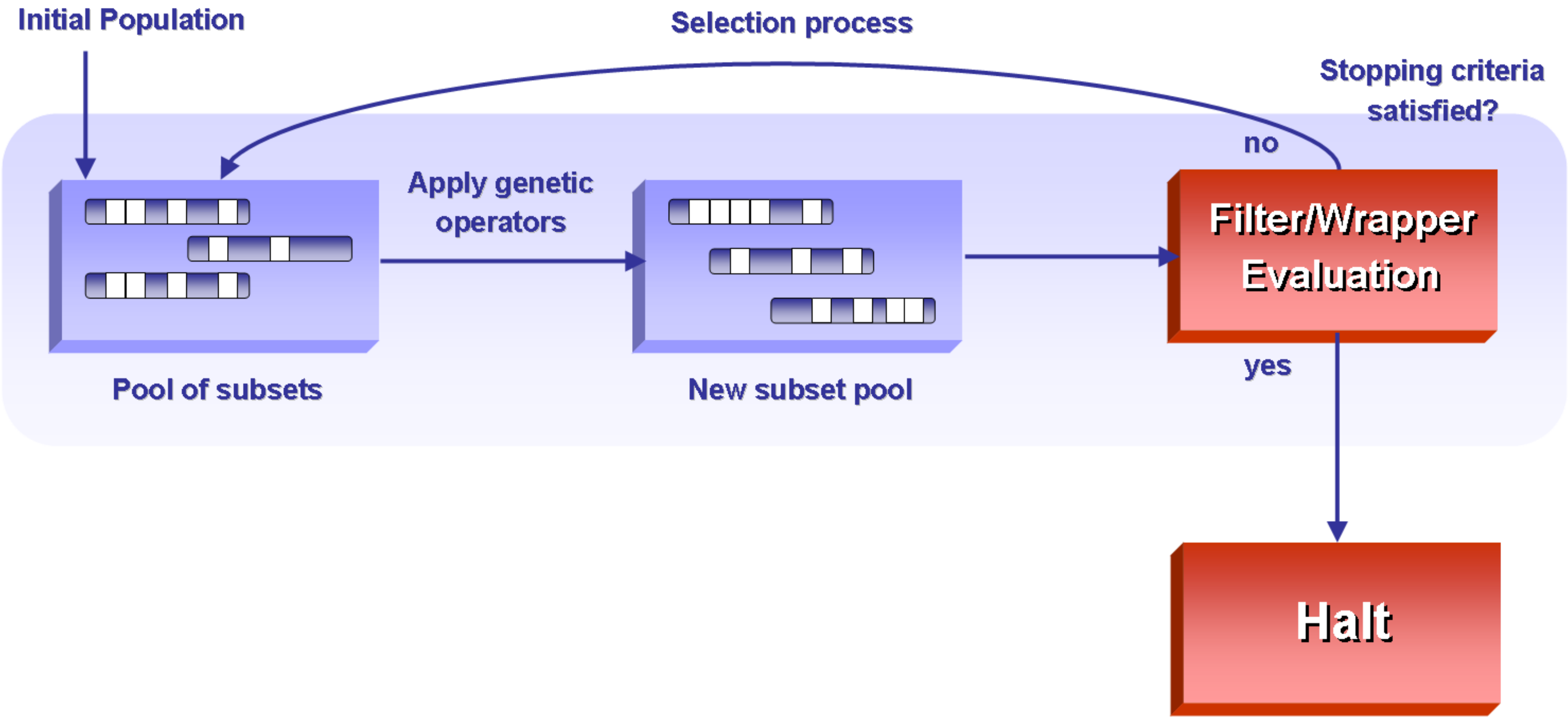
Sequential forward selection

- Algorithm:
  - Begin with zero attributes
  - Evaluate all feature subsets w/ exactly 1 feature
  - Select the one with the best performance
  - Add to this subset the feature that yields the best performance for subsets of next larger size
  - Repeat this until stopping criterion is met

Sequential backward selection

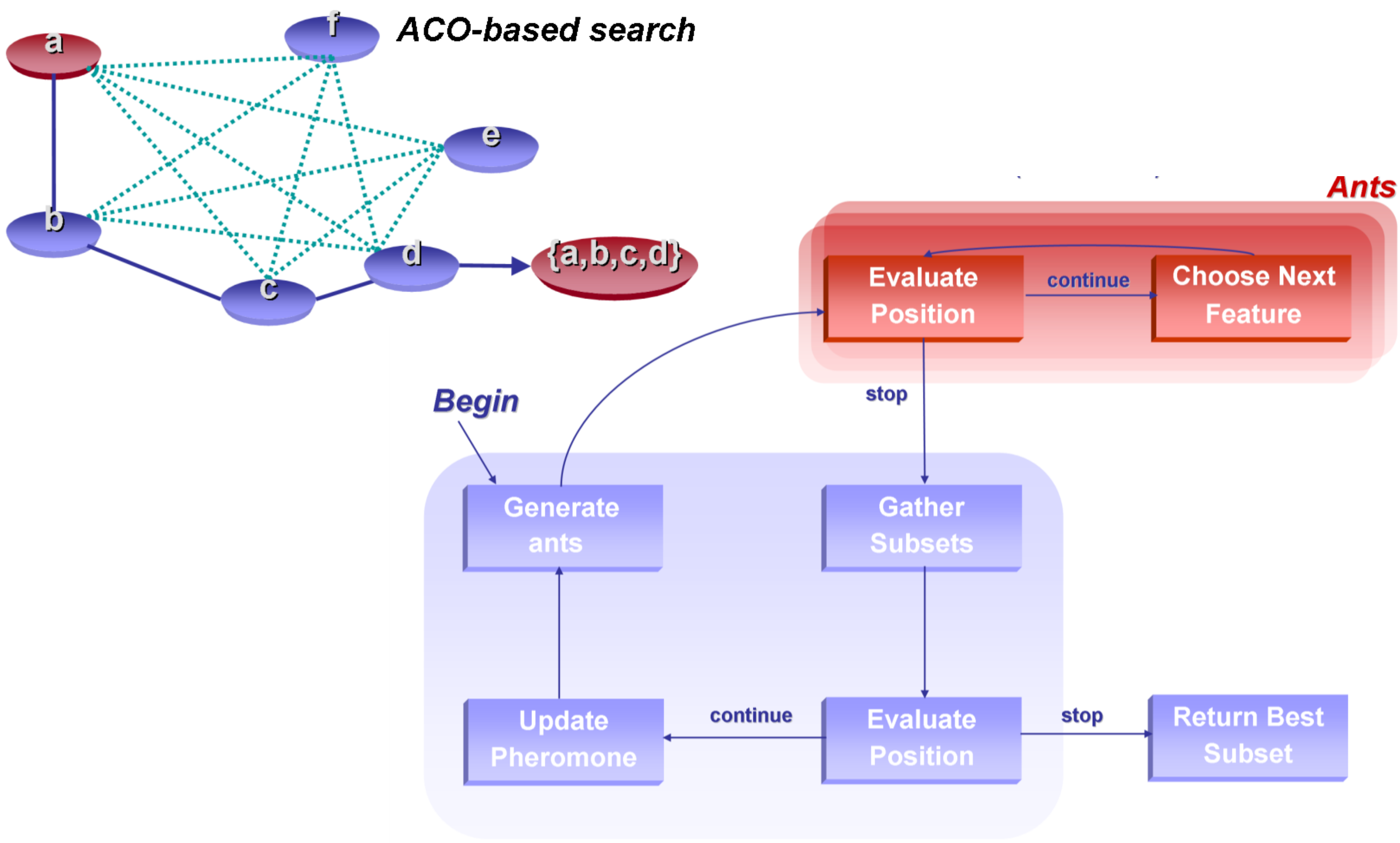
- Begins with all features
- Repeatedly removes a feature whose removal yields the maximal performance improvement
  - Can be filter or wrapper

GA-based FS



Initial Population
Selection process
Stopping criteria
satisfied?
no
Apply genetic
operators
Filter/Wrapper
Evaluation
yes
Pool of subsets
New subset pool
Halt

Ant-based FS



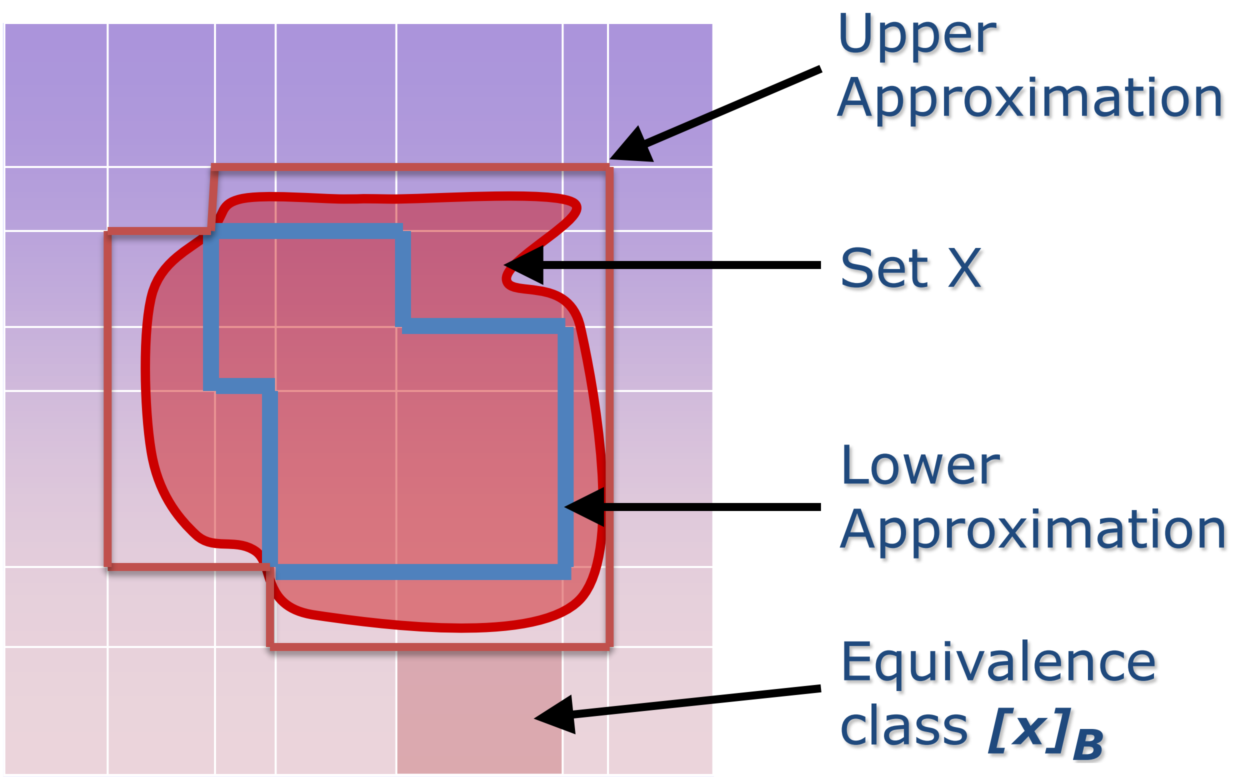
ACO-based search
a
f
e
b
d
c
{a,b,c,d}
Ants
Evaluate Position
continue
Choose Next Feature
stop
Begin
Generate ants
Gather Subsets
Update Pheromone
continue
Evaluate Position
stop
Return Best Subset

Rough set theory

Rough set theory

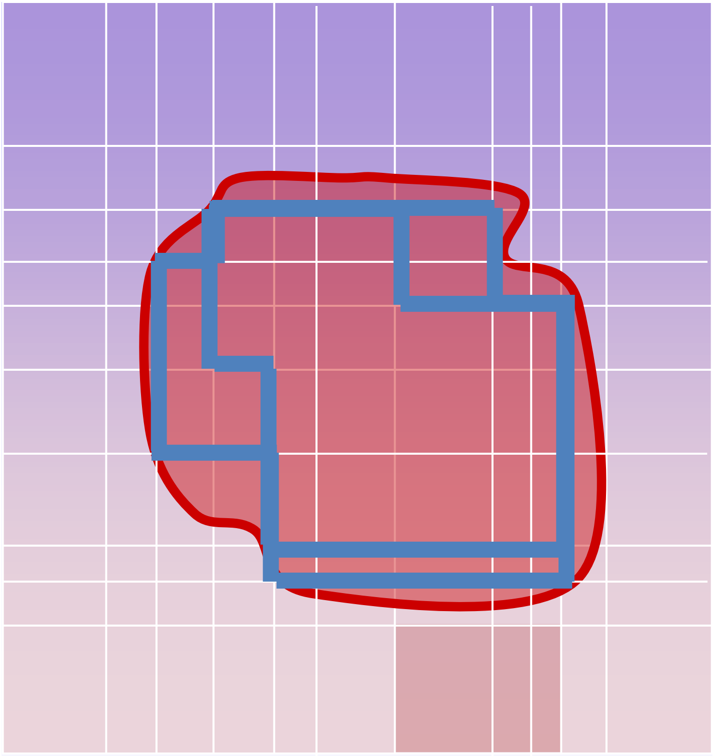
- Invented in 1982, but took off in the 1990s
- Based on set theory
- Approximates (estimates) a concept via two sets:
  - Lower approximation (containing elements that definitely belong to the concept)
  - Upper approximation (containing elements that possibly belong)

Rough set theory



Rough set feature selection

- By considering more features, concepts become easier to define…



Recap: datasets

	Headache	*Muscle pain*	*Temp.*	*Flu*
U1	Yes	Yes	Normal	No
U2	Yes	Yes	High	Yes
U3	Yes	Yes	Very-high	Yes
U4	No	Yes	Normal	No
U5	No	No	High	No
U6	No	Yes	Very-high	Yes

Objects, instances

Each column is a feature/symptom/measurement

The final column is the decision feature

Example dataset

	Headache	*Muscle pain*	*Favourite TV program*	*Temp.*	*Flu*
U1	Yes	Yes	Lost	Normal	No
U2	Yes	Yes	24	High	Yes
U3	Yes	Yes	24	Very-high	Yes
U4	No	Yes	A-Team	Normal	No
U5	No	No	24	High	No
U6	No	Yes	A-Team	Very-high	Yes

Equivalence classes

	Headache	*Muscle pain*	*Favourite TV program*	*Temp.*	*Flu*
U1	**Yes**	**Yes**	**Lost**	**Normal**	**No**
U2	**Yes**	**Yes**	**24**	**High**	**Yes**
U3	**Yes**	**Yes**	**24**	**Very-high**	**Yes**
U4	**No**	**Yes**	**A-Team**	**Normal**	**No**
U5	**No**	**No**	**24**	**High**	**No**
U6	**No**	**Yes**	**A-Team**	**Very-high**	**Yes**

$[U1]_{R_H} = R_{\{H\}}\, U1 = \{U1, U2, U3\}$

$[U5]_{R_H} = R_{\{H\}}\, U5 = \{U4, U5, U6\}$

$R_{\{H\}} = \{\{U1, U2, U3\},\{U4, U5, U6\}\}$

$R_{\{H,M\}} = \{\{U1,U2,U3\}, \{U4,U6\},\{U5\}\}$

$X = \{U1, U2, U3, U4, U5, U6\}$

Rough set theory

- Approximating a concept A using knowledge in feature subset B
  - Lower approximation: contains objects that definitely belong to A

$$R_B{\downarrow}A \quad = \quad \{x \in X \mid [x]_{R_B} \subseteq A\}$$

  - Upper approximation: contains objects that possibly belong to A

$$R_B{\uparrow}A \quad = \quad \{x \in X \mid [x]_{R_B} \cap A \neq \emptyset\}$$

Rough set theory

	Headache	*Muscle pain*	*Temp.*	*Flu*
U1	**Yes**	**Yes**	**Normal**	**No**
U2	**Yes**	**Yes**	**High**	**Yes**
U3	**Yes**	**Yes**	**Very-high**	**Yes**
U4	**No**	**Yes**	**Normal**	**No**
U5	**No**	**No**	**High**	**No**
U6	**No**	**Yes**	**Very-high**	**Yes**

Set B={H,M}
R_B = {{U1,U2,U3}, {U4,U6}, {U5}}

$$R_B{\downarrow}A \quad = \quad \{x \in X | [x]_{R_B} \subseteq A\}$$

For concept Flu=No
A={U1,U4,U5}

$$R_B{\downarrow}A \quad =$$

Rough set feature selection

- Positive region
  - Find the lower approximation for each decision concept
  - Take the union of these
  - Summarises the information contained in a subset for the full dataset

$$POS_B = \bigcup_{x \in X} R_B{\downarrow}[x]_{R_d}$$

- Dependency function
  - Take the cardinality of the positive region (number of elements) and divide by the number of objects in the dataset
  - This is the evaluation measure
  - When this reaches 1, search can stop

$$\gamma_B = \frac{|POS_B|}{|X|}$$

Reducts

Given (X,A∪{d}), B $\subseteq$ A

B is a decision reduct if B satisfies

$$\gamma_B = \gamma_{\mathcal{A}}$$

and no proper subset of B satisfies it

- Core = set of all features that appear in *every* reduct

Rough set feature selection

- One approach: QuickReduct
- Attempts to remove unnecessary or redundant features
  - **Evaluation**: function based on rough set concept of lower approximation
  - **Generation**: greedy hill-climbing algorithm employed
  - **Stopping criterion**: when maximum evaluation value is reached (= reduct or superreduct)

Rough set feature selection

	Headache	*Muscle pain*	*Favourite TV program*	*Temp.*	*Flu*
U1	Yes	Yes	Lost	Normal	No
U2	Yes	Yes	24	High	Yes
U3	Yes	Yes	24	Very-high	Yes
U4	No	Yes	A-Team	Normal	No
U5	No	No	24	High	No
U6	No	Yes	A-Team	Very-high	Yes

consider each feature individually at first

Rough set feature selection

	Headache	*Flu*
U1	Yes	No
U2	Yes	Yes
U3	Yes	Yes
U4	No	No
U5	No	No
U6	No	Yes

Set B={H}
Lower approximation for Flu=No is {}
Lower approximation for Flu=Yes is {}
Positive region is {}

Rough set feature selection

	Temp.	*Flu*
U1	Normal	No
U2	High	Yes
U3	Very-high	Yes
U4	Normal	No
U5	High	No
U6	Very-high	Yes

Set B={T}
Lower approximation for Flu=No is {U1,U4}
Lower approximation for Flu=Yes is {U3,U6}
Positive region is {U1,U3,U4,U6}

Rough set feature selection

	Favourite TV program	*Temp.*	*Flu*
U1	**Lost**	**Normal**	**No**
U2	**24**	**High**	**Yes**
U3	**24**	**Very-high**	**Yes**
U4	**A-Team**	**Normal**	**No**
U5	**24**	**High**	**No**
U6	**A-Team**	**Very-high**	**Yes**

Set B={TV, T}
Lower approximation for Flu=No is {U1,U4}
Lower approximation for Flu=Yes is {U3,U6}
Positive region is {U1,U3,U4,U6} (unchanged)

Rough set feature selection

	Headache	*Temp.*	*Flu*
U1	**Yes**	**Normal**	**No**
U2	**Yes**	**High**	**Yes**
U3	**Yes**	**Very-high**	**Yes**
U4	**No**	**Normal**	**No**
U5	**No**	**High**	**No**
U6	**No**	**Very-high**	**Yes**

Set B={H,T}
Lower approximation for Flu=No is {U1,U4,U5}
Lower approximation for Flu=Yes is {U2,U3,U6}
Positive region is {U1,U2,U3,U4,U5,U6} = X

Example 2

	Diploma	*Experience*	*French*	*Reference*	*Decision*
x_1	MBA	Medium	Yes	Excellent	Accept
x_2	MBA	Low	Yes	Neutral	Reject
x_3	MCE	Low	Yes	Good	Reject
x_4	MSc	High	Yes	Neutral	Accept
x_5	MSc	Medium	Yes	Neutral	Reject
x_6	MSc	High	Yes	Excellent	Accept
x_7	MBA	High	No	Good	Accept
x_8	MCE	Low	No	Excellent	Reject

$$R_B{\downarrow}A \quad = \quad \{x \in X | [x]_{R_B} \subseteq A\}$$

Fuzzy-rough feature selection

Fuzzy-rough set theory

- Problems:
  - Rough set methods (usually) require data discretization beforehand
  - Also no flexibility in approximations

- Example
  - Objects either belong fully to the lower (or upper) approximation, or not at all

Hybridizing rough and fuzzy sets

- Two lines of thought in hybridization
  - **Axiomatic** approach – investigate mathematical properties
  - **Constructive** approach – generalize lower and upper approximations

- The fuzzy-rough tools described in this tutorial are built on the definition in:
  - *A.M. Radzikowska, E.E. Kerre, A comparative study of fuzzy rough sets, Fuzzy Sets and Systems, vol. 126, no. 2, pp. 137-155, 2002.*

Fuzzy rough sets

Rough set

$$R_B\uparrow A = \{x \in X \mid \boxed{[x]_{R_B} \cap A \neq \emptyset}\}$$

$$R_B\downarrow A = \{x \in X \mid \boxed{[x]_{R_B} \subseteq A}\}$$

Fuzzy-rough set

t-norm

$$R\uparrow A(y) = \sup_{x \in X} \mathcal{T}(R(x,y), A(x))$$

$$R\downarrow A(y) = \inf_{x \in X} \mathcal{I}(R(x,y), A(x))$$

implicator

FRFS (old)

- Based on Dubois and Prade's definitions
  - Fuzzy lower approximation:

$$\mu_{\underline{P}X}(x) = \sup_{F \in U/P} \min(\mu_F(x), \inf_{y \in U} I(\mu_F(y), \mu_X(y)))$$

  - Fuzzy positive region:

$$\mu_{POS_P(Q)}(x) = \sup_{X \in U/Q} \mu_{\underline{P}X}(x)$$

  - Evaluation function:

$$\gamma'_P(Q) = \frac{|\mu_{POS_P(Q)}(x)|}{|U|} = \frac{\sum_{x \in U} \mu_{POS_P(Q)}(x)}{|U|}$$

FRFS (new)

- Based on fuzzy similarity

$$R_a(x,y) = 1 - \frac{|a(x) - a(y)|}{|a_{\max} - a_{\min}|}$$

$$R_P(x,y) = \bigcap_{a \in P} \{R_a(x,y)\}$$

- Lower/upper approximations

$$\begin{aligned} R{\uparrow}A(y) &= \sup_{x \in X} \mathcal{T}(R(x,y), A(x)) \\ R{\downarrow}A(y) &= \inf_{x \in X} \mathcal{I}(R(x,y), A(x)) \end{aligned}$$

FRFS: evaluation function

- Fuzzy positive region #1

$$POS_B(y) = \left(\bigcup_{x \in X} R_B \downarrow R_d x \right)(y)$$

- Fuzzy positive region #2 (weak)

$$POS_B(y) = (R_B \downarrow R_d y)(y)$$

- Dependency function

$$\gamma_B = \frac{|POS_B|}{|POS_{\mathcal{A}}|}$$

FRFS: evaluation function

- Alternative measure: delta function

$$\delta_B = \frac{\min_{x \in X} POS_B(x)}{\min_{x \in X} POS_{\mathcal{A}}(x)}$$

- Properties

Proposition 2: For subsets B_1, B_2 of $\mathcal{A}$,

$$B_1 \subseteq B_2 \Rightarrow \begin{cases} \gamma_{B_1} \leq \gamma_{B_2} \\ \delta_{B_1} \leq \delta_{B_2} \end{cases}$$

Proposition 3: $\gamma_{\mathcal{A}} = \delta_{\mathcal{A}} = 1$

FRFS: finding reducts

- Fuzzy-rough QuickReduct
  - **Evaluation**: use the dependency function (or other fuzzy-rough measure)
  - **Generation**: greedy hill-climbing
  - **Stopping criterion**: when maximal evaluation function is reached (or to degree α)

FRFS example

Object	a	b	c	q
1	-0.4	-0.3	-0.5	no
2	-0.4	0.2	-0.1	yes
3	-0.3	-0.4	-0.3	no
4	0.3	-0.3	0	yes
5	0.2	-0.3	0	yes
6	0.2	0	0	no

$$R_a(x,y) = \max\left(\min\left(\frac{a(y)-a(x)+\sigma_a}{\sigma_a}, \frac{a(x)-a(y)+\sigma_a}{\sigma_a}\right), 0\right)$$

FRFS example

- If A={1,3,6}, calculate $(R_a{\downarrow}A)(3)$

$$
\begin{array}{cc}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
R_a(x,y) = & \begin{pmatrix}
1.0 & 1.0 & 0.699 & 0.0 & 0.0 & 0.0 \\
1.0 & 1.0 & 0.699 & 0.0 & 0.0 & 0.0 \\
0.699 & 0.699 & 1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 0.699 & 0.699 \\
0.0 & 0.0 & 0.0 & 0.699 & 1.0 & 1.0 \\
0.0 & 0.0 & 0.0 & 0.699 & 1.0 & 1.0
\end{pmatrix}
\end{array}
$$

$$A(x) = (1,0,1,0,0,1)$$

$$\mathcal{I}_{\mathcal{S}_{\mathrm{W}}}(x,y) = \min(1-x+y,1)$$

$$(R{\downarrow}A)(y) = \inf_{x \in X} \mathcal{I}(R(x,y), A(x))$$

FRFS example

$$\begin{aligned}(R{\downarrow}A)(y) &= \inf_{x\in X} \mathcal{I}(R(x,y), A(x)) \\ &= \inf\{I(0.699,1), I(0.699,0), I(1,1), \\ &\qquad I(0,0), I(0,0), I(0,1)\} \\ &= 0.301\end{aligned}$$

$$\begin{aligned}\gamma_{\{a\}} &= \frac{0.602}{6} \\ &= 0.1003\end{aligned}$$

FRFS example

- The search continues...

$$\gamma_{\{b\}} = 0.3597 \qquad \gamma_{\{c\}} = 0.4078$$

- Feature *c* looks more promising, so choose this and continue searching

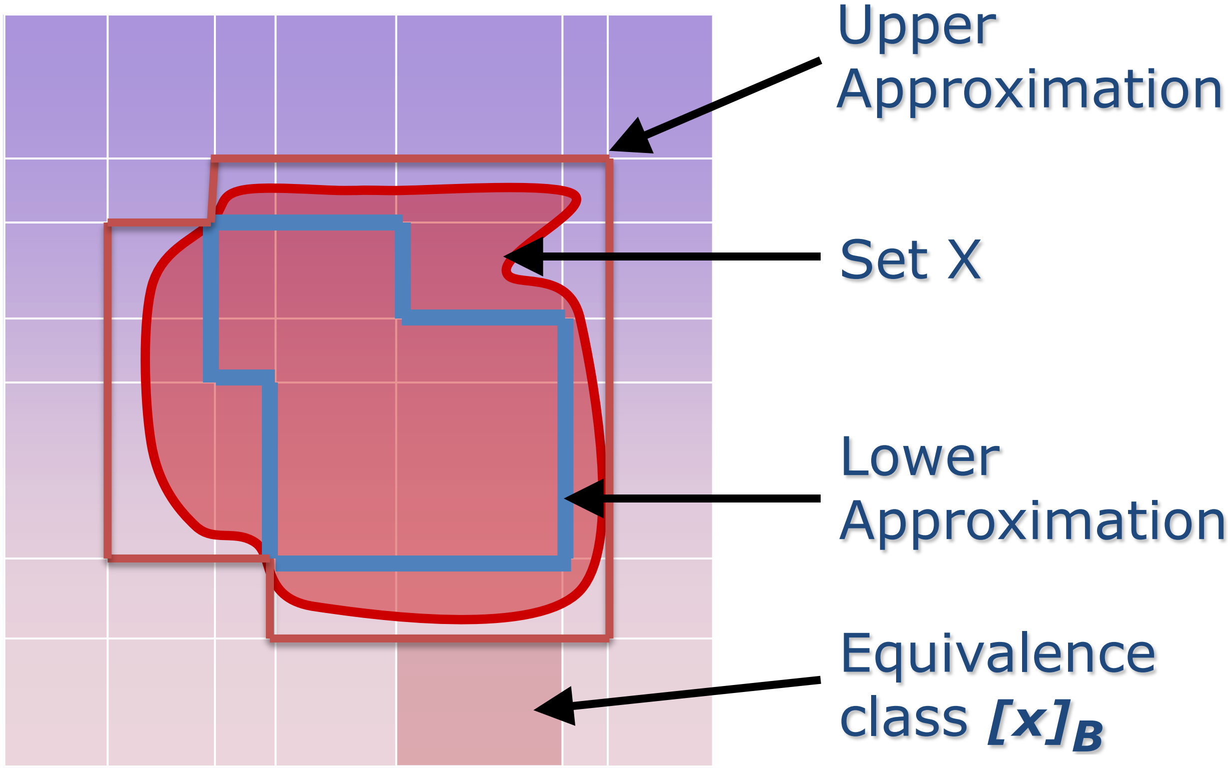
$$\gamma_{\{a,c\}} = 0.5501 \qquad \gamma_{\{b,c\}} = 1.0$$

FRFS

- Other subset generation methods
  - GAs
  - ACO
  - Backward elimination

- Other subset evaluations
  - Fuzzy boundary region
  - Fuzzy entropy
  - Fuzzy discernibility functions

Boundary region



FRFS: boundary region

- Fuzzy lower and upper approximation define fuzzy boundary region

$$R{\uparrow}A(y) - R{\downarrow}A(y)$$

- For each concept, minimise the boundary region
  - (also applicable to crisp RSFS)
- Results seem to show this is a more informed heuristic (but slower to calculate)

FRFS: issues

- Problem – noise tolerance!

$$R{\uparrow}A(y) = \boxed{\sup_{x \in X}} \mathcal{T}(R(x,y), A(x))$$

$$R{\downarrow}A(y) = \boxed{\inf_{x \in X}} \mathcal{I}(R(x,y), A(x))$$

Vaguely quantified rough sets

Pawlak rough set

y belongs to the lower approximation of A iff
all elements of Ry belong to A

y belongs to the upper approximation of A iff
at least one element of Ry belongs to A

VQRS

y belongs to the lower approximation of A iff
most elements of Ry belong to A

y belongs to the upper approximation of A iff
at least some elements of Ry belong to A

VQRS

$$R{\uparrow}_{Q_l}A(y) \;=\; Q_l\left(\frac{|Ry \cap A|}{|Ry|}\right)$$

$$R{\downarrow}_{Q_u}A(y) \;=\; Q_u\left(\frac{|Ry \cap A|}{|Ry|}\right)$$

R, A: crisp *or* fuzzy
R↓A, R↑A: fuzzy

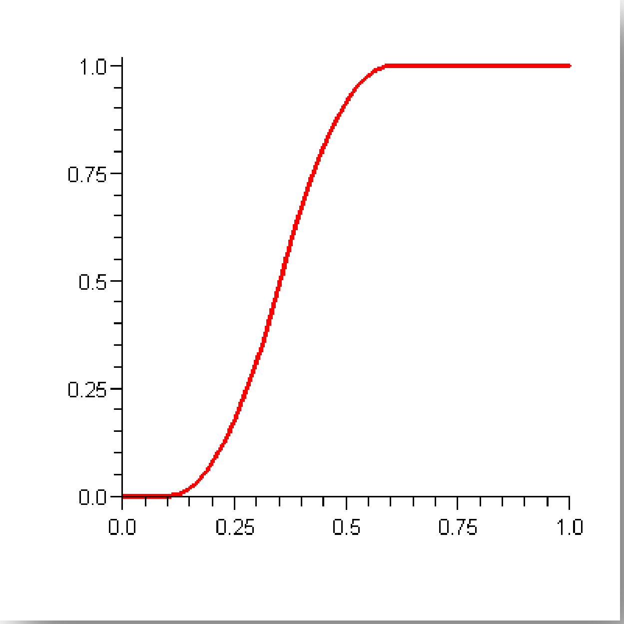
y belongs to the lower approximation *to the extent that* **most elements** of Ry belong to A

y belongs to the upper approximation *to the extent that* **some elements of** Ry belongs to A

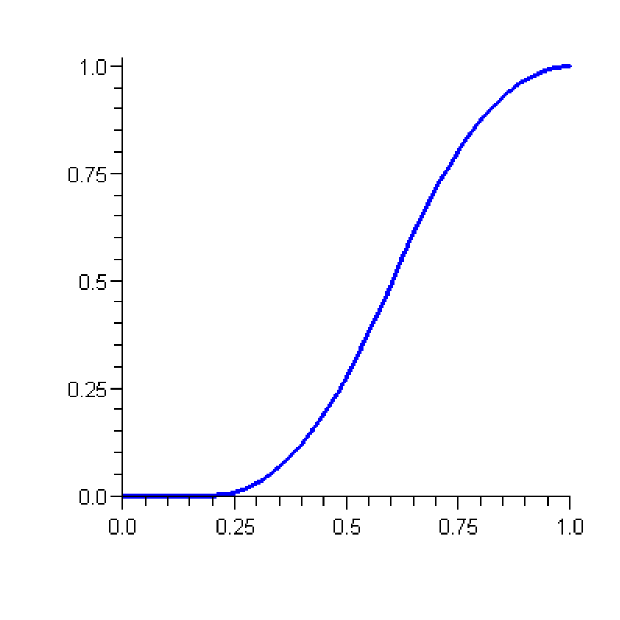
Fuzzy quantifiers: examples

Fuzzy quantifier (Zadeh):
[0,1]→[0,1] mapping Q

Q_l: some



Q_u: most

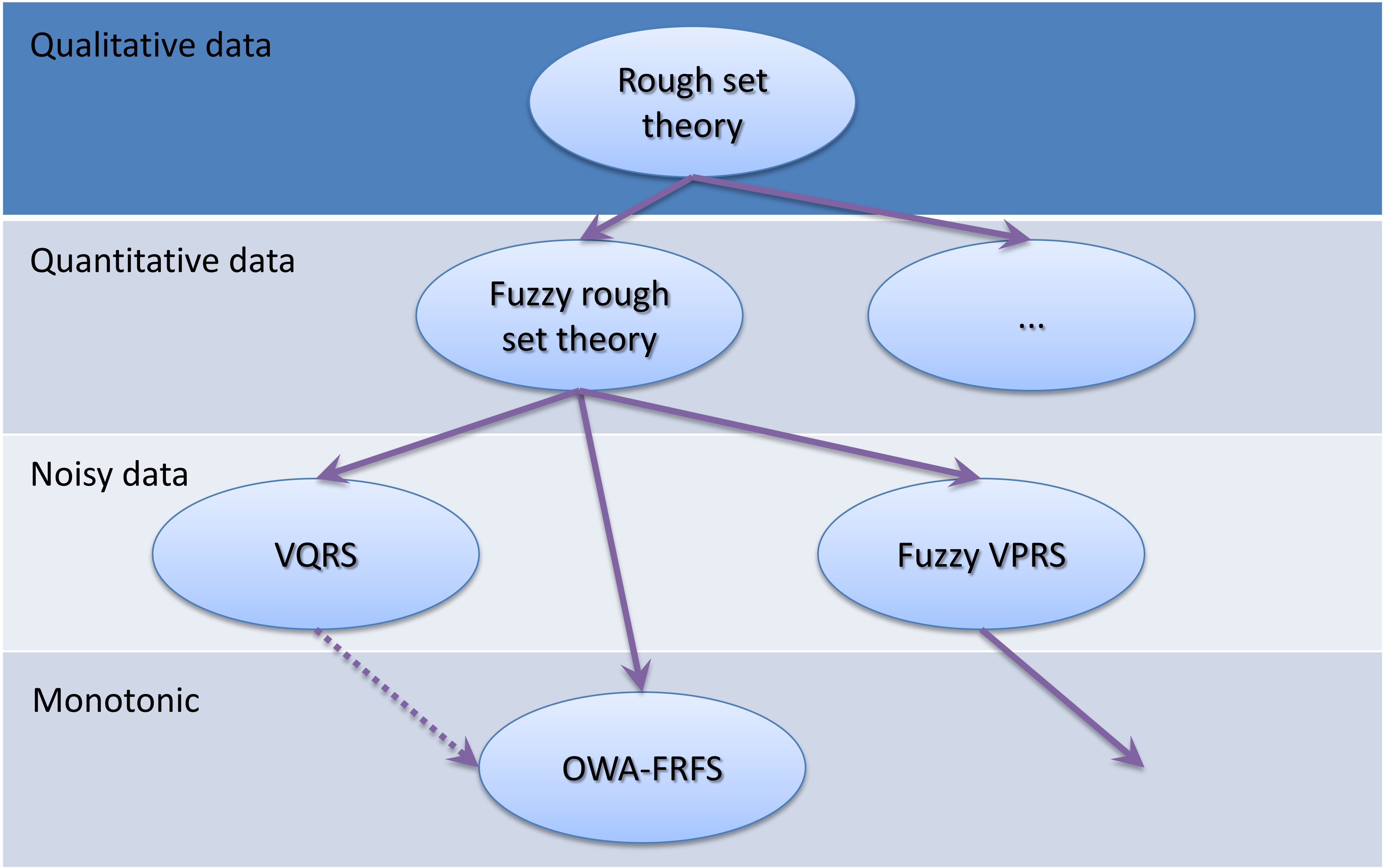


VQRS-based feature selection

- Use the quantified lower approximation, positive region and dependency degree
  - **Evaluation**: the quantified dependency (can be crisp or fuzzy)
  - **Generation**: greedy hill-climbing
  - **Stopping criterion**: when the quantified positive region is maximal (or to degree α)

- Should be more noise-tolerant, but is non-monotonic

Progress


Qualitative data
Rough set theory
Quantitative data
Fuzzy rough set theory
...
Noisy data
VQRS
Fuzzy VPRS
Monotonic
OWA-FRFS

OWA-FRFS

- E.g. values {3, 5, 2, 7, 4}
  - (Ordered values = 7 5 4 3 2)
  - OWA modelling of sup and inf:
    - sup-weights 1 0 0 0 0 = 7
    - inf-weights 0 0 0 0 1 = 2
  - OWA relaxation of sup and inf:
    - sup-relax-weights 0.7 0.2 0.1 0.0 0.0 = 6.3
    - inf-relax-weights 0.0 0.0 0.1 0.2 0.7 = 2.4

OWA-FRFS

- New lower and upper approximations

$$(R{\downarrow}_{W_l}A)(y) = OWA_{W_l}\langle \mathcal{I}(R(x_i,y),A(x_i))\rangle$$
$$(R{\uparrow}_{W_u}A)(y) = OWA_{W_u}\langle \mathcal{T}(R(x_i,y),A(x_i))\rangle$$

- Feature selectors can be built on this

- Vague quantifiers can be modelled with this

More issues...

- Problem #1: how to choose fuzzy similarity?
- Problem #2: how to handle missing values?

Interval-valued FRFS

- Answer #1: Model uncertainty in fuzzy similarity by interval-valued similarity

IV fuzzy rough set

$$\mu_{\underline{\widetilde{R_P X}}}(x) = \inf_{y \in \mathbb{U}} \mathcal{I}(\mu_{\widetilde{R_P}}(x,y), \mu_{\widetilde{X}}(y))$$

$$\mu_{\overline{\widetilde{R_P X}}}(x) = \sup_{y \in \mathbb{U}} \mathcal{T}(\mu_{\widetilde{R_P}}(x,y), \mu_{\widetilde{X}}(y))$$

IV fuzzy similarity

$$\mu_{R_{a*}}(x,y) = 1 - \left(\frac{|a(x) - a(y)|}{|a_{\max} - a_{\min}|}\right)^m$$

$$\mu_{R_a^*}(x,y) = 1 - \frac{|a(x) - a(y)|}{|a_{\max} - a_{\min}|}$$

Interval-valued FRFS

- When comparing two object values for a given attribute – what to do if at least one is missing?
- Answer #2: Model missing values via the unit interval

$$\mu_{\widetilde{R_a}}(x,y) = \begin{cases} \mu_{\widetilde{R_a}}(x,y) & \text{if } a(x), a(y) \neq *, \\ [0,1] & \text{otherwise} \end{cases}$$

Other measures

- Boundary region

$$\mu_{\widetilde{BND_P(X)}}(x) \quad = \quad \mu_{\widetilde{\overline{R_P}X}}(x) - \mu_{\widetilde{\underline{R_P}X}}(x)$$

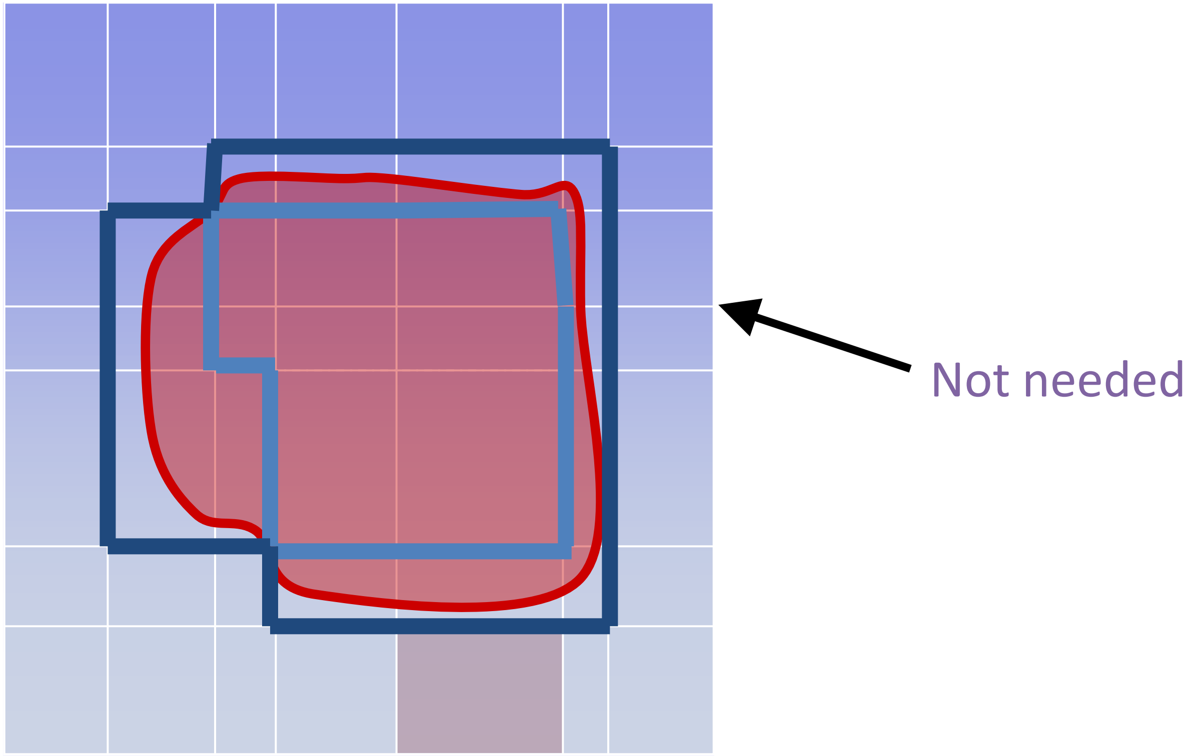
- Discernibility function

$$\widetilde{c_{ij}}(P) = \mathcal{I}(\underbrace{\mathcal{T}(\mu_{\widetilde{R_a}}(x_i, x_j))}_{a \in P}, \mu_{\widetilde{R_{\mathbb{D}}}}(x_i, x_j))$$

$$\widetilde{g}(P) = \frac{2. \sum\limits_{1 \leq i < j \leq n} \widetilde{c_{ij}}(P)}{n(n-1)}$$

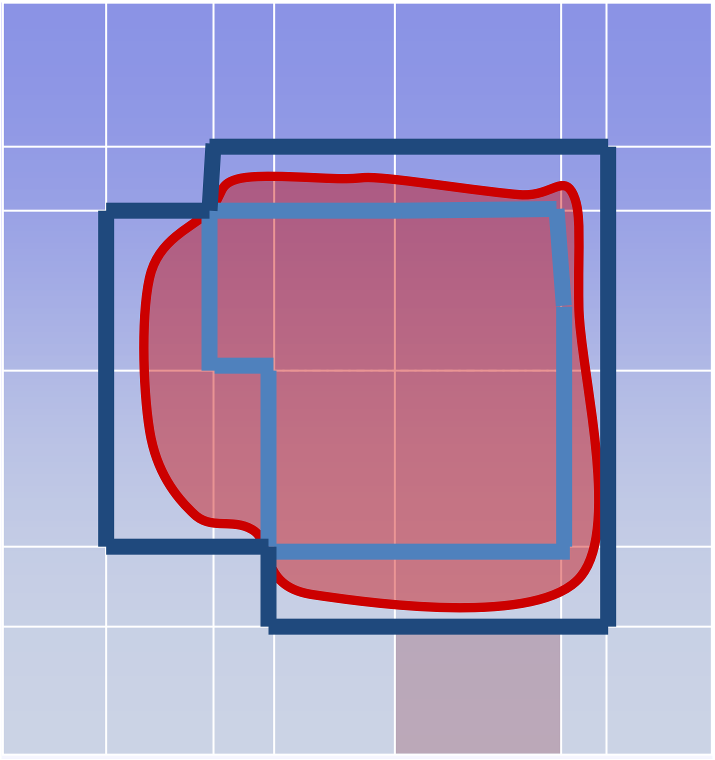
Instance Selection

Instance selection: basic idea



Remove objects to keep the underlying approximations unchanged, or to improve them

Instance selection: basic idea



Remove objects to keep the underlying approximations unchanged, or to improve them

Fuzzy-rough sets

- Parameterized relation

$$R_a^\alpha(x,y) = \max\left(0, 1 - \alpha\frac{|a(x)-a(y)|}{l(a)}\right)$$

$$R_B^\alpha(x,y) = \mathcal{T}(\underbrace{R_a^\alpha(x,y)}_{a\in B})$$

- Fuzzy-rough definitions:

$$(R_B^\alpha{\downarrow}^S A)(y) \quad = \quad \inf_{x\in S} \mathcal{I}(R_B^\alpha(x,y), A(x))$$

$$POS_B^{\alpha,S}(y) = (R_B^\alpha{\downarrow}^S R_d^\alpha y)(y)$$

$$\gamma_B^{\alpha,S} = \frac{\sum_{y\in S} POS_B^{\alpha,S}(y)}{|S|}$$

FRIS-I

FRIS-I(S,α,τ).
S, the set of objects to be reduced;
α, the granularity parameter;
τ, a selection threshold.

(1) $Y \leftarrow S$
(2) **foreach** $x \in S$
(3) **if** $(POS_{\mathcal{A}}^{\alpha,S}(x) < \tau)$
(4) $Y \leftarrow Y - \{x\}$
(5) **return** Y

FRIS-II

FRIS-II(S,α).
S, the set of objects to be reduced;
α, the granularity parameter.

while (`true`)
    $z \leftarrow \emptyset$, $\rho_z \leftarrow 1$
    foreach $x \in S$
        if ($POS_{\mathcal{A}}^{\alpha,S}(x) < \rho_z$)
            $z \leftarrow x$
            $\rho_z \leftarrow POS_{\mathcal{A}}^{\alpha,S}(x)$
    if ($z \neq \emptyset$)
        $S \leftarrow S - \{z\}$
    else return S

FRIS-III

FRIS-III(S,α).
S, the set of objects to be reduced;
α, the granularity parameter.

(1) $\rho \leftarrow \gamma_{\mathcal{A}}^{\alpha,S}$
(2) **while** $(\rho \neq 1)$
(3) $z \leftarrow \emptyset,\ \rho_z \leftarrow 0$
(4) **foreach** $x \in S$
(5) **if** $(\gamma_{\mathcal{A}}^{\alpha,S-\{x\}} > \rho_z)$
(6) $z \leftarrow x,\ \rho_z \leftarrow \gamma_{\mathcal{A}}^{\alpha,S-\{x\}}$
(7) $S \leftarrow S - z$
(8) $\rho \leftarrow \rho_z$
(9) **return** S

Fuzzy-rough classification and prediction

Nearest neighbour algorithm

- 1-NN:

  Given a test instance x_m,

  - First locate the nearest training example x_n
  - Then $f(x_m) := f(x_n)$

- *k*-NN:

  Given a test instance x_m,

  - First locate the *k* nearest training examples
  - **If** target function = discrete **then** take vote among its *k* nearest neighbours
    else take the mean of the *f* values of the *k* nearest neighbours (prediction)

Fuzzy NN

Input: X, the training data; $\mathcal{C}$, the set of decision classes; y, the object to be classified; K, the number of nearest neighbours

Output: Classification for y

begin

  $N \leftarrow$ getNearestNeighbours(y,K)

  foreach $C \in \mathcal{C}$ **do**

    $C'(y) = \sum_{x \in N} R(x, y)C(x)$

  end

  output $\underset{C \in \mathcal{C}}{\arg\max}\,(C'(y))$

end

Fuzzy-rough NN

Input: X, the training data; $\mathcal{C}$, the set of decision classes; y, the object to be classified

Output: Classification for y

begin

$N \leftarrow \text{getNearestNeighbours}(y, K)$

$\tau \leftarrow 0$, $Class \leftarrow \emptyset$

foreach $C \in \mathcal{C}$ **do**

if $((R{\downarrow}C)(y) + (R{\uparrow}C)(y))/2 \geq \tau$ **then**

$Class \leftarrow C$

$\tau \leftarrow ((R{\downarrow}C)(y) + (R{\uparrow}C)(y))/2$

end

end

output $Class$

end

Fuzzy-rough NN

Input: X, the training data; d, the decision feature; y, the object for which to find a prediction

Output: Classification for y

begin

$N \leftarrow$ getNearestNeighbours(y,K)

$\tau_1 \leftarrow 0$, $\tau_2 \leftarrow 0$

foreach $z \in N$ **do**

$M \leftarrow ((R{\downarrow}R_d z)(y) + (R{\uparrow}R_d z)(y))/2$

$\tau_1 \leftarrow \tau_1 + M * d(z)$

$\tau_2 \leftarrow \tau_2 + M$

end

if $\tau_2 > 0$ **then**

output τ_1/τ_2

else

output $\sum_{z \in N} d(z)/|N|$

end

end

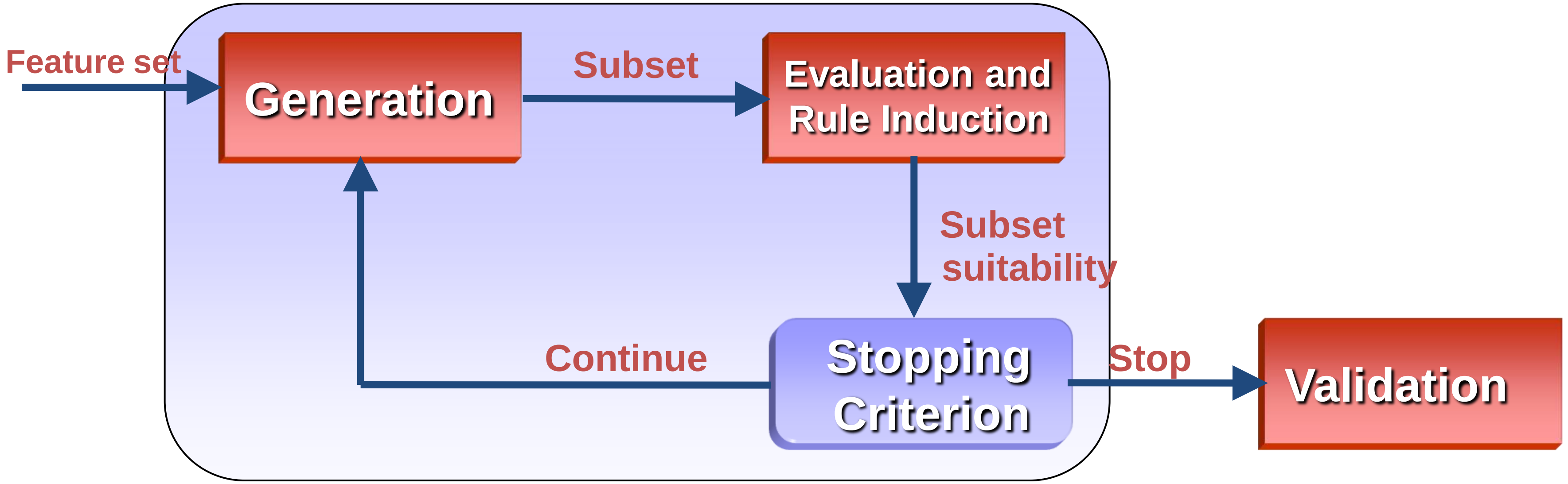
Discovering rules via RST

- Equivalence classes
  - Form the antecedent part of a rule
  - The lower approximation tells us if this is predictive of a given concept (*certain* rules)

- Typically done in one of two ways:
  - Overlaying reducts
  - Building rules by considering individual equivalence classes (e.g. LEM2)

Framework

- The fuzzy tolerance classes used during this process can be used to create fuzzy rules



QuickRules

$B := \{\}, Rules := \{\}, Cov := \{\}$
do
    $T := B$
    foreach $a \in (\mathcal{A} \setminus B)$
        foreach $y \in X \setminus covered(Cov)$
            if $POS_{B \cup \{a\}}(y) = POS_{\mathcal{A}}(y)$
                $\textsc{Check}(B \cup \{a\}, R_{B \cup \{a\}}y, R_d y)$
        if $\gamma_{B \cup \{a\}} > \gamma_T$
            $T := B \cup \{a\}$
    $B := T$
until $\gamma_B = \gamma_{\mathcal{A}}$
return $B, Rules$

Check

CHECK(B, C, D).

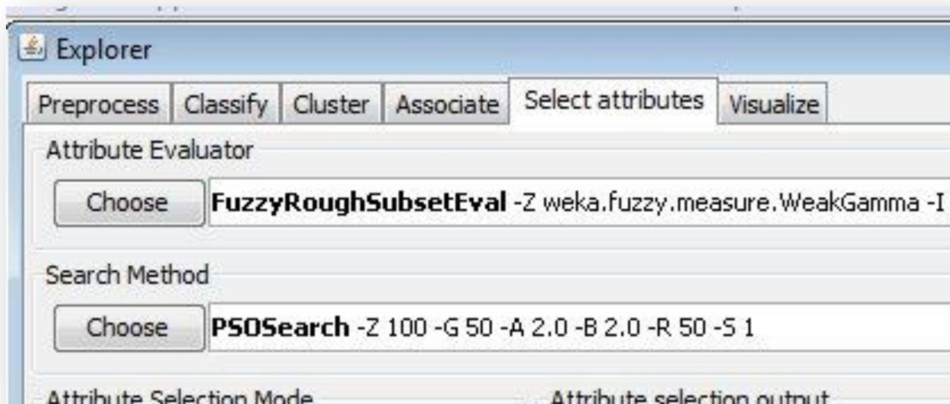
$Add := true$
foreach $Rule \in Rules$
    if $C \subseteq Rule.C$
        Add := false; **break**
    elseif $Rule.C \subset C$
        $Rules := Rules \setminus Rule$
if Add = true
    $Rules := Rules \cup (B, C, D)$
    $Cov := Cov \cup C$
return

Weka

Try the algorithms!

FR methods in Weka

- Weka implementations of all fuzzy-rough methods can be downloaded from:



http://users.aber.ac.uk/rkj/book/wekafull.jar

Other developments

- Fuzzy Discernibility Matrices for FRFS
  - Extends the DMs for crisp rough set feature selection
  - Also employs similar simplification schemes

- Fuzzy-rough semi-supervised learning
  - For mixtures of labelled and unlabelled data

Papers

- Fuzzy-rough feature selection
  - R. Jensen and Q. Shen. **New Approaches to Fuzzy-Rough Feature Selection**. IEEE Transactions on Fuzzy Systems, vol. 17, no. 4, pp. 824-838, 2009.
  - R. Jensen and Q. Shen. **Computational Intelligence and Feature Selection: Rough and Fuzzy Approaches.** IEEE Press/Wiley & Sons, 2008.
  - C. Cornelis, R. Jensen, G. Hurtado Martin, D. Slezak. **Attribute Selection with Fuzzy Decision Reducts.** Information Sciences, vol. 180, no. 2, pp. 209-224, 2010.
  - G.C.Y. Tsang, D. Chen, E.C.C. Tsang, J.W.T. Lee, and D.S. Yeung. **On attributes reduction with fuzzy rough sets.** Proc. 2005 IEEE International Conference on Systems, Man and Cybernetics, vol. 3, pp. 2775–2780, 2005.
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  - Q. Hu, D. Yu, and Z. Xie. **Information-preserving hybrid data reduction based on fuzzy-rough techniques**. Pattern Recognition Letters, vol. 27, no. 5, pp. 414–423, 2006.
  - Q. Hu, P. Zhu, J. Liu, Y. Yang, D. Yu. **Feature Selection via Maximizing Fuzzy Dependency.** Fundamenta Informaticae, vol. 98 (2-3): 167-181, 2010.
  - E.C.C. Tsang, D. Chen, D.S. Yeung, X. Wang, J. Lee. **Attributes Reduction Using Fuzzy Rough Sets.** IEEE T. Fuzzy Systems, vol. 16, no. 5, pp. 1130-1141, 2008.
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- FRFS extensions
  - R. Jensen and Q. Shen. **Interval-valued Fuzzy-Rough Feature Selection in Datasets with Missing Values.** Proceedings of the 18th International Conference on Fuzzy Systems (FUZZ-IEEE'09), pp. 610-615, 2009.
  - C. Cornelis, N. Verbiest and R. Jensen. **Ordered Weighted Average Based Fuzzy Rough Sets.** Proceedings of the 5th International Conference on Rough Sets and Knowledge Technology (RSKT2010), pp. 78-85, 2010.
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- FR instance selection
  - R. Jensen and C. Cornelis. **Fuzzy-rough instance selection.** Proceedings of the 19th International Conference on Fuzzy Systems (FUZZ-IEEE'10), pp. 1776-1782, 2010.

- FR classification/prediction
  - R. Jensen and C. Cornelis. **A New Approach to Fuzzy-Rough Nearest Neighbour Classification.** Transactions on Rough Sets XIII, LNCS 6499, pp. 56-72, 2011.
  - R. Jensen, C. Cornelis and Q. Shen. **Hybrid Fuzzy-Rough Rule Induction and Feature Selection.** Proceedings of the 18th International Conference on Fuzzy Systems (FUZZ-IEEE'09), pp. 1151-1156, 2009.
  - R. Jensen and Q. Shen**. Fuzzy-Rough Feature Significance for Fuzzy Decision Trees.** Proceedings of the 2005 UK Workshop on Computational Intelligence, pp. 89-96. 2005.