Relative Performance Evaluation and Managerial Outside Options

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Abstract

This paper provides a theoretical explanation of the relative performance evaluation (RPE) puzzle. In a simple model of moral hazard with limited commitment where aggregate shocks affect both reservation utilities and firm's technology, we find that RPE is optimal under very specific circumstances. We consider different technologies: unaffected by aggregate shocks, pro-cyclical, counter-cyclical and different demand for managerial services: neutral, pro-cyclical and produce interesting predictions for executive pay. The effect of aggregate shocks on the optimal choice of effort is also analyzed.

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1 Introduction

Since the seminal paper of Jensen and Murphy [15], there has been a big debate in both academic and media circles about whether observed compensation packages provide the proper incentives to firms' executives. An important theoretical prediction that dates back to Holmstrom [12] is that common uncertainty should be filtered out of the agent's evaluation. This requires that the executive is paid according to performance relative to a benchmark which is referred to as relative performance evaluation (hereafter, RPE). A common interpretation is that CEO compensation should be increasing in own firm's performance and decreasing in industry or market performance.[1] However, there is little if any empirical evidence of RPE.[2]

There has been some work towards explaining the RPE puzzle. One strand of the literature focuses on the possibility that the executive may insulate himself/herself from the market risk by adjusting his/her total financial wealth, thus making RPE irrelevant.[3] Salas Fumas [22] argues that under imperfectly competitive product markets an executive pay that increases in the benchmark may in fact soften competition. Celentani and Loveira [5] show that compensation is not monotonically decreasing in the benchmark if the marginal return of effort depends on the aggregate state. In order to address the wide use of broad-based stock option plans, Oyer [20] ignores the incentive role of the compensation package and focuses on the participation constraint instead. He demonstrates that if agent's outside opportunities are positively correlated with firm's performance and adjusting compensation is costly, the best linear contract may involve indexing to market. Himmelberg and Hubbard [11] show that the scheme solving a second-order Taylor approximation of the principal's problem may not exhibit RPE if aggregate shocks affect the marginal return of executive services and the manager's reservation utility.

In the current paper, we address the RPE puzzle by a simple model of moral hazard where a risk-neutral principal (a proxy for firm's shareholders) hires a risk-averse agent (the CEO) to operate a stochastic technology mapping actions (executive effort) to outcomes (firm's performance). Since the exerted effort remains unobservable by the principal, the optimal contract conditions execu-

[1] See, for example, Rappaport [21].

[2] See, for example, Antle and Smith [1], Lambert and Larcker [18], Gibbons and Murphy [10], Barro and Barro [4], Janakiraman, Lambert and Larcker [14], Garen [8], Joh [17], Aggarwal and Samwick [2, 3].

[3] See, for example, Feltham and Xie [7], Maug [19], Jin [16], Garvey and Milbourn [9].

tive pay on the observed outcome instead. We assume limited commitment and embrace Oyer [20]'s idea that agent's outside opportunities vary with market conditions by relating the CEO's reservation utility to an aggregate outcome observed by both parties prior to contracting. Unlike Oyer [20], however, we fully analyze the role of the contract in the provision of managerial incentives which affects the extent of indexing to the market. Similarly to Celentani and Loveira [5], the probability distribution over firm's outcomes depends on the aggregate outcome. Unlike Himmelberg and Hubbard [11], our model is discrete and does not require any approximations.[4] Furthermore, we distinguish between pro-cyclical and counter-cyclical firms, which results in some interesting theoretical predictions. Unlike the previous literature, we also analyze the effect of the aggregate outcome on the optimally induced level of effort.

The model is single-period, but can easily be extended to multiple periods or infinite horizon. The timing is as follows. An aggregate outcome is realized according to a known distribution. The outcome is observed by both the principal and the manager and determines latter's reservation utility. The principal offers the manager a contract recommending some level of effort and specifying a compensation scheme mapping outcomes to wages. If the manager rejects, both parties enjoy their reservation utilities. If the manager accepts, (s)he exerts some level of effort which is unobservable by the principal. An individual outcome is realized according to a known distribution conditional on the aggregate outcome and the exerted effort. The principal pays the respective wage to the manager. Both parties enjoy their respective utilities.

Consider the case where the firm's outcome is independent of the aggregate outcome. Since the manager is risk-averse, a variation in the value of his/her outside options increases the cost of implementing a particular level of effort. In case of log-utility, higher reservation utilities reinforce low effort, while lower reservation utilities reinforce high effort. With fixed reservation utility there is no reason to index the CEO's pay to the market. However, if the demand for managerial services increases with the aggregate outcome (and the optimal effort choice remains unaffected), the executive will enjoy a pay rise for any individual outcome, and therefore, a rise in his/her average wage. A marginal increase in the aggregate outcome may potentially switch the optimal effort from high to low for a particular class of utility functions, which would lead to an effective

[4] We do consider a version with continuous-valued shocks as an illustration. There, we work with the best linear contract. Numerical simulations suggest that this is a good approximation for the optimal contract.

pay cut if the firm performs well and a pay rise if the firm performs badly.

Now, assume that the firm's performance depends on the aggregate outcome. As before, the pay implementing low effort increases in the aggregate outcome. Regarding the cost of implementing high effort, we need to distinguish between pro-cyclical firms the performance of which depends positively on the aggregate outcome and counter-cyclical firms for which the dependence is negative. A marginal increase in the aggregate outcome will increase the wage compensation implementing high effort for a counter-cyclical firm and would increase/does not affect/decrease it for a pro-cyclical firm depending on whether the increase in reservation utilities dominates/cancels/is dominated by the decrease in the probability of failure (low outcome) weighted by the additional disutility of high over low effort divided by its contribution to the probability of success (high outcome). If high effort is indeed optimal, RPE will only be observed for pro-cyclical firms where the reservation utility effect of aggregate effort is weaker than its direct effect on the probability of success. If CEO's reservation utility remains fixed, the first effect would disappear, so RPE would always hold for pro-cyclical firms.

Note that changes in the aggregate outcome may provoke changes in the level of effort induced under the optimal contract. With fixed reservation utilities, low effort remains optimal for marginal increases in the aggregate outcome if the firm is pro-cyclical (optimal pay cut) and for marginal decreases if it is counter-cyclical (optimal pay cut), while high effort remains optimal for marginal decreases in the aggregate outcome if the firm is pro-cyclical (optimal pay rise) and for marginal increases if it is counter-cyclical (optimal pay rise). Note that for a pro-cyclical firm, a marginal decrease in the aggregate output may change the optimal effort from low to high, while a marginal increase may switch it from high to low. Then, the decrease may result in an optimal pay cut if the firm performs badly and pay rise if it performs well, while the increase may lead to an optimal pay rise under a bad performance and pay cut under a good performance. Alternatively, for a counter-cyclical firm, a marginal increase in the aggregate output may change the optimal effort from low to high, while a marginal decrease may switch it from high to low. Then, the increase may result in an optimal pay cut after a bad performance and pay rise after a good performance, while the decrease may produce an optimal pay rise for bad performance and pay cut for good performance.

We also consider a model with normally distributed additive shocks and analyze the best linear contract. With fixed reservation utility, the aggregate shock affects the executive salary, but not his/her share of the firm's profit. In particular, a higher realization of the aggregate shock has a negative effect on the fixed component of the compensation received by an executive working for a pro-cyclical firm. In case of a counter-cyclical firm, the effect may become positive if the individual shock is sufficiently noisy and sufficiently correlated to the aggregate shock. In any case, the effect is proportional to the slope of the linear scheme.

If a marginal increase in the aggregate shock increases agent's reservation utility, it also increases his/her share of the firm's profit. While the effect on the fixed salary becomes ambiguous for a pro-cyclical firm, any positive effect for an executive running a counter-cyclical firm would now be reinforced.

The rest of the paper is structured as follows. Section 2 analyzes RPE in a general model of optimal contracting under moral hazard. Section 3 considers a model with normally distributed idiosyncratic and aggregate shocks and derives the optimal linear compensation scheme. Section 4 concludes.

2 General Model

A principal (she) needs an agent (him) to operate a firm (alternatively, a stochastic technology) for one period. The probability distribution of outcomes is conditional on an aggregate outcome realized in the beginning of the period and on the effort exerted by the agent. Both parties know the conditional distribution, but the principal cannot observe the actual level of effort. However, she can condition the remuneration of the agent on the individual outcome which is publicly observable. Both the principal and the agent have some outside options the value of which depends on the aggregate outcome. The set of possible actions is denoted by A and is a compact subset of $\mathbb{R}_+$. We assume that the individual outcome and the aggregate outcome are elements of Y and Y_A respectively which are both subsets of $\mathbb{R}$. The distribution of individual outcome conditional on effort and aggregate outcome is described by a probability mass function $\pi : Y \times A \times Y_A \to (0, 1)$. The aggregate outcome is distributed according to a probability mass function $p : Y_A \to [0, 1]$.[5] Given an outcome y, effort a, and wage w, the agent's utility is $v(w) - a$, where v is twice continuously differentiable, strictly increasing and strictly concave. The respective utility of the principal is given by $y - w$. The reservation utilities of the agent and the

[5] π and p can be considered as density functions if y or respectively y_A are continuous random variables.

principal are $\underline{V}$ and $\underline{U}$ respectively. Each of them is a real-valued function defined on Y_A. The timing is as follows. At the beginning of the period, nature picks an aggregate outcome y_A from Y_A according to p. The aggregate outcome is observed by both parties and determines their reservation utilities: $\underline{V}(y_A)$ and $\underline{U}(y_A)$. The principal proposes a contract to the agent which recommends an effort level a and specifies an outcome-contingent wage scheme w. The agent may accept or reject the contract depending on whether his expected utility under the contract is higher or lower than his reservation utility $\underline{V}(y_A)$. We assume that he will accept if the utilities are the same. If the agent rejects the contract, both parties enjoy their respective reservation utilities. Then, nature picks up an outcome y according to the respective conditional distribution. The outcome is observed by both parties. The principal pays the agent $w(y)$ and the relationship ends. Given the unobservability of effort, the principal offers an incentive compatible contract guaranteeing that the effort recommended by the contract is actually implemented. The contract also needs to be individually rational since otherwise both parties can revert to their respective outside options.

Then, if an aggregate outcome y_A is initially observed, the optimal contract solves the following problem:

$$\max_{a,w(.)}\sum_{y\in Y}(y-w)\pi(y,a,y_A) \text{ s.t.:}$$

$$a \in A \tag{1}$$

$$\sum_{y\in Y}(v(w)-a)\pi(y,a,y_A) \geq \underline{V}(y_A) \tag{2}$$

$$a \in \arg\max_{a'\in A}\sum_{y\in Y}(v(w)-a')\pi(y,a',y_A) \tag{3}$$

Note that if the above contract is not individually rational for the principal, she would not have proposed it and both parties would have enjoyed their respective reservation utilities. That is, if the value function of the above problem is $U^*(y_A)$, then the principal actually chooses between $U^*(y_A)$ and $\underline{U}(y_A)$, whichever is bigger.

The first constraint is a feasibility constraint for the exerted effort. It simply says that the recommended effort is an element of A. The second constraint is the individual rationality constraint for the agent, namely that his expected utility under the contract is not less then his reservation utility. The third constraint is the incentive compatibility constraint that guarantees that the agent chooses to exert the recommended effort.

Let us assume that $A = \{\underline{a}, \overline{a}\}$, where $\underline{a} < \overline{a}$ and that the distribution conditional on high effort stochastically dominates the distribution conditional on low effort for any $y_A \in Y$. Also, assume that the principal's problems has a solution $a^*, w^*(.)$. Then, if the Lagrange multipliers of the individual rationality constraint (2) and the incentive-compatibility constraint (3) are λ and μ respectively, the following first order condition should be satisfied:

$$\frac{1}{v'(w^*(y))} = \lambda + \mu \left(1 - \frac{\pi(y, a', y_A)}{\pi(y, a^*, y_A)}\right). \tag{4}$$

Here it should be noted that the optimal contract as well as the Lagrange multipliers depend on the realized aggregate effort y_A.

Let us also assume that strong monotonicity of the likelihood ratio (SMLR) holds for any $y_A \in Y_A$. We focus on the case where there are two possible levels of effort: high effort and low effort. Namely, $A = \{\underline{a}, \overline{a}\}$, $\underline{a} < \overline{a}$.

Proposition 1 *For any $y_A \in Y_A$, the principal optimally implements low effort by a fixed wage $w_{\underline{a}}^{y_A} = v^{-1}(\underline{V}(y_A) + \underline{a})$, $\forall y$, while optimal implementation of high effort requires a strictly increasing compensation scheme such that $\lambda(y_A) > 0$ and $\mu(y_A) > 0$, i.e., with both individual rationality and incentive compatibility constraints binding.*

The proposition shows that the optimal contract guarantees the agent just his reservation utility. The principal uses fixed pay to implement low effort and non-constant (strictly increasing under SMLR) pay that is just incentive-compatible to implement high effort.

Before we proceed, we introduce some further notation. Let E_p and E_{a,y_A} denote the mathematical expectations under p and $\pi(., a, y_A)$ respectively. Note that we will often drop the second subscript from the latter expectation referring to it simply as E_a. Finally, let E_{y_A} denote the expectation under $\pi(., a^*(y_A), y_A)$.

In what follows we will ignore principal's reservation utilities assuming they are low enough and do not affect the optimal contract. As for the agent, we will consider two cases: varying reservation utilities (VRU) versus constant reservation utilities (CRU). Under VRU, the reservation utility of the agent varies across aggregate outcomes, i.e., $\inf_{y_A \in Y_A} \underline{V}(y_A) < \sup_{y_A \in Y_A} \underline{V}(y_A)$. Let $V = E_p \underline{V}$ be his mean reservation value. Under CRU, the reservation utility of the agent is constant at V for all aggregate outcomes, i.e., $\underline{V}(y_A) = V$, $\forall y_A \in Y_A$. From now on, we will denote the dependence of a particular variable on y_A by a corresponding super- or sub-index. The CRU case is respectively denoted by a super- or sub-index c. For example, the wage that optimally implement low effort under CRU is denoted as $w^c_{\underline{a}}$.

From Proposition 1, the wage that optimally implements low effort is increasing and strictly convex in $\underline{V}(y_A)$. Therefore, $w^{y_A}_{\underline{a}} \lesseqgtr w^c_{\underline{a}}$ iff $\underline{V}(y_A) \lesseqgtr V$. In particular, the wage implementing low effort when agent's reservation utility is highest is strictly greater than the respective wage under CRU which in turn is strictly greater than the respective wage when agent's reservation utility is lowest. Moreover, the cost of implementing low effort averaged over realizations of the aggregate outcome is higher than the cost of implementing low effort under CRU as the following proposition shows.

Proposition 2 $E_p w_{\underline{a}} > w^c_{\underline{a}}$.

Proof. $E_p w_{\underline{a}} = E_p v^{-1}(\underline{V} + \underline{a}) > v^{-1}(E_p(\underline{V} + \underline{a})) = v^{-1}(V + \underline{a}) = w^c_{\underline{a}}$, where the inequality follows from the fact that v is strictly concave, and so v^{-1} is strictly convex. ■

In order to proceed with the analysis, we introduce more structure. Assume that there are two possible outcomes: high and low. Namely, $Y = \{\underline{y}, \overline{y}\}$, $\underline{y} < \overline{y}$. Let $\underline{\pi}^{y_A} := \pi(\underline{y}, \underline{a}, y_A)$ be the probability of low outcome conditional on low effort and $\bar{\pi}^{y_A} := \pi(\underline{y}, \bar{a}, y_A)$ be the probability of low outcome conditional on high effort given an aggregate outcome y_A. Note that stochastic dominance requires that $\underline{\pi}^{y_A} > \bar{\pi}^{y_A}$, which also guarantees that SMLR holds. Low effort is optimally implemented by the flat scheme $v^{-1}(\underline{V}(y_A) + \underline{a})$. High effort requires an increasing compensation scheme such that individual rationality and incentive compatibility are binding. Denote by $\underline{w}^{y_A}_{\bar{a}} := w^{y_A}_{\bar{a}}(\underline{y})$ and

$\overline{w}_{\overline{a}} := w_{\overline{a}}^{y_A}(\overline{y})$ the wages associated with low and respectively high outcome, and by $\underline{v}_{\overline{a}}^{y_A} = v(\underline{w}_{\overline{a}}^{y_A})$ and $\overline{v}_{\overline{a}}^{y_A} = v(\overline{w}_{\overline{a}}^{y_A})$ the respective utility the agent receives from consuming them. We obtain:

$$\underline{v}_{\overline{a}}^{y_A} = \underline{V}(y_A) + \frac{(1-\overline{\pi}^{y_A})\,\underline{a} - (1-\underline{\pi}^{y_A})\,\overline{a}}{\underline{\pi}^{y_A} - \overline{\pi}^{y_A}}, \tag{5}$$

$$\overline{v}_{\overline{a}}^{y_A} = \underline{V}(y_A) + \frac{\underline{\pi}^{y_A}\overline{a} - \overline{\pi}^{y_A}\underline{a}}{\underline{\pi}^{y_A} - \overline{\pi}^{y_A}}, \tag{6}$$

where $\overline{v}_{\overline{a}}^{y_A} > \underline{v}_{\overline{a}}^{y_A}$. Each of these utilities, and therefore, the respective wages are increasing in $\underline{V}(y_A)$.

2.1 Case 1. The distribution of individual outcomes does not depend on the aggregate outcome

In this case, we have $\overline{\pi}^{y_A} = \overline{\pi}$ and $\underline{\pi}^{y_A} = \underline{\pi}$ for any $y_A \in Y_A$.

Conditioning on a particular individual outcome realization, the wage implementing high effort when agent's reservation utility is highest is strictly greater than the respective wage under CRU which in turn is strictly greater than the respective wage when his reservation utility is lowest (since now only the first term on the right-hand side of (5) and (6) depends on y_A). Note that for any realized individual outcome, the expected utility of agent from consuming his wage equals his utility from consuming his respective wage under CRU, where the expectation is taken over the realizations of the aggregate outcome. For example, $E_p\overline{v}_{\overline{a}} = E_p\underline{V} + \frac{\underline{\pi}\overline{a} - \overline{\pi}\underline{a}}{\underline{\pi} - \overline{\pi}} = V + \frac{\underline{\pi}\overline{a} - \overline{\pi}\underline{a}}{\underline{\pi} - \overline{\pi}} = \overline{v}_{\overline{a}}^{c}$.

The following proposition shows that implementing high effort is more costly under VRU than under CRU.

Proposition 3 $E_p(E_{\overline{a}}w_{\overline{a}}) > E_{\overline{a}}w_{\overline{a}}^{c}$.

Proof. Fix $y \in Y$. Then, $E_p w_{\overline{a}}(y) = E_p v^{-1}(v_{\overline{a}}(y)) > v^{-1}(E_p v_{\overline{a}}(y)) = v^{-1}(v_{\overline{a}}^{c}(y)) = w_{\overline{a}}^{c}(y)$. Consequently, $E_p(E_{\overline{a}}w_{\overline{a}}) = E_{\overline{a}}(E_p w_{\overline{a}}(y)) > E_{\overline{a}}w_{\overline{a}}^{c}$. ■

We saw that both implementing low and high effort is more costly under VRU than under CRU.

For each recommended level of effort, the agent on average receives the same utility under VRU as under CRU. However, since the agent is risk averse, he needs to be compensated for the additional risk stemming from the variation in the value of his outside option under VRU. His wage averaged across variations in his reservation utility is therefore higher than under CRU, which raises the average cost born by the principal. It is interesting to see how this additional cost will influence the principal's decision as to which effort level to implement. After all, the optimal contract specifies an optimal effort level and a related compensation scheme. It does not regard the implementation of suboptimal effort levels.

It appears that even with two possible outcomes, we cannot obtain a clear relationship between the variation in agent's reservation utilities and the level of effort recommended by the principal. The difficulty is illustrated below. Assume, for example, that low effort is optimal under y_A. Is it also optimal for y'_A such that $V(y'_A) > V(y_A)$? From the optimality of $\underline{a}$ under y_A, it follows that $E_{\underline{a}}y - E_{\overline{a}}y \geq w_{\underline{a}}^{y_A} - E_{\overline{a}}w_{\overline{a}}^{y_A}$. In order to have $\underline{a}$ optimal under y'_A, we need $E_{\underline{a}}y - E_{\overline{a}}y \geq w_{\underline{a}}^{y'_A} - E_{\overline{a}}w_{\overline{a}}^{y'_A}$. A sufficient condition for that is $w_{\underline{a}}^{y_A} - E_{\overline{a}}w_{\overline{a}}^{y_A} > w_{\underline{a}}^{y'_A} - E_{\overline{a}}w_{\overline{a}}^{y'_A}$. This is equivalent to $f\left(v_{\underline{a}}^{y_A}\right) - E_{\overline{a}}f\left(v_{\overline{a}}^{y_A}\right) > f\left(v_{\underline{a}}^{y_A} + \Delta V\right) - E_{\overline{a}}f\left(v_{\overline{a}}^{y_A} + \Delta V\right)$, where $f := v^{-1}$ and $\Delta V = \underline{V}(y'_A) - \underline{V}(y_A) > 0$. A sufficient condition is $F := f\left(v_{\overline{a}} + \Delta V\right) - f\left(v_{\underline{a}} + \Delta V\right) + f\left(v_{\underline{a}}\right) - f\left(v_{\overline{a}}\right) > 0$, $\forall y \in Y$, where the superindexes and the dependence on y have been suppressed. Consider $y = \overline{y}$. Then, $v_{\underline{a}} < v_{\overline{a}}$. Note that $\exists x_1 \in \left(v_{\overline{a}}, v_{\overline{a}} + \Delta V\right)$ and $x_2 \in \left(v_{\underline{a}}, v_{\underline{a}} + \Delta V\right)$ such that $F = \Delta V\left(f'(x_1) - f'(x_2)\right)$. If we assume that $v_{\underline{a}} + \Delta V \leq v_{\overline{a}}$, then we will have $x_1 > x_2$, from where $F > 0$. Therefore, assume $v_{\underline{a}} + \Delta V > v_{\overline{a}}$. Note that $\exists x_3 \in \left(v_{\underline{a}} + \Delta V, v_{\overline{a}} + \Delta V\right)$ and $x_4 \in \left(v_{\underline{a}}, v_{\overline{a}}\right)$ such that $F = \left(v_{\overline{a}} - v_{\underline{a}}\right)\left(f'(x_3) - f'(x_4)\right)$. Since $v_{\underline{a}} + \Delta V > v_{\overline{a}}$, we have $x_3 > x_4$, and so $F > 0$. However, for $y = \underline{y}$, $v_{\underline{a}} > v_{\overline{a}}$, and we obtain that $F < 0$. So, the result depends on the sign of $\overline{\pi}F\left(\underline{y}\right) + (1 - \overline{\pi})F(\overline{y})$, which is not so straight-forward to establish.

Some intuition may be obtained from the case of log utility.

Proposition 4 *Assume $v(w) = \log(w)$. Then, the following holds:*
(a) If $\underline{a} \in a_{y_A}^*$, then $a_{y'_A}^* = \underline{a}$, $\forall y'_A \in Y : V(y'_A) > V(y_A)$
(b) If $\overline{a} \in a_{y_A}^*$, then $a_{y'_A}^* = \overline{a}$, $\forall y'_A \in Y : V(y'_A) < V(y_A)$.

Proof. We present the proof of part (a). The proof of part (b) is analogous. We need $w_{\underline{a}}^{y_A} - E_{\overline{a}}w_{\overline{a}}^{y_A} > w_{\underline{a}}^{y'_A} - E_{\overline{a}}w_{\overline{a}}^{y'_A}$. This is equivalent to $E_{\underline{a}}e^{v_{\overline{a}}+\Delta V} - e^{v_{\underline{a}}+\Delta V} +$

$e^{v_{\underline{a}}} - E_{\underline{a}} e^{v_{\overline{a}}} > 0$, where $\Delta V = V(y'_A) - V(y_A) > 0$. We need $E_{\underline{a}} e^{v_{\overline{a}}} > e^{v_{\underline{a}}}$. This is indeed the case since $E_{\underline{a}} e^{v_{\overline{a}}} > e^{E_{\underline{a}} v_{\overline{a}}} > e^{v_{\underline{a}}}$. ■

Part (a) of the Proposition says that if low effort is optimal under a particular reservation utility of the agent, then it is also optimal for all higher reservation utilities. In particular, if low effort is optimal under CRU, then it is also optimal under VRU for reservation utilities above the mean, V. Part (b) says that if high effort is optimal for a particular reservation utility of the agent, then it is also optimal for all lower reservation utilities. In particular, if high effort is optimal under CRU, then it is also optimal under VRU for reservation utilities below the mean, V. In other words, lower reservation utilities reinforce the implementation of high effort, while higher reservation utilities reinforce the implementation of low effort. Then, we can conjecture that lower reservation utilities would on average lead to higher outcomes (by reinforcing high effort).

While this result demonstrates the increased cost of inducing high effort under higher values for the agent's outside option, it cannot be generalized for utilities different from the logarithmic one.

Another important question is how changes in the aggregate outcome affect agent's pay. Since the individual firm's outcome is independent from the aggregate, there seems to be no reason to relate the agent's compensation to y_A. This is true for the case of CRU, where no component of the optimal contract is affected by the aggregate outcome. However, agent's pay does depend on y_A under VRU. Due to binding individual rationality, the contract depends on the agent's reservation utility which is in turn a function of the aggregate outcome. Indeed, both the fixed wage implementing low effort and the monotonic scheme implementing high effort are increasing in agent's reservation utility. Then, since we expect the demand for managerial services, and so the agent's reservation utility, to increase with the aggregate outcome, the agent's compensation designed to implement a particular effort level will also increase in y_A. If the increase in the aggregate outcome does not change the optimal level of effort induced by the contract, the agent will enjoy a pay rise for any individual outcome, i.e., $\frac{\partial w^*_{y_A}}{\partial y_A} > 0$, $\forall y$ and since the distribution of individual outcomes is unaffected by the increase in y_A, he should also have his average wage increase, i.e., $\frac{\partial E_{y_A} w^*_{y_A}}{\partial y_A} > 0$.

We have previously seen that there is not a straight-forward rule about how a change in the aggregate outcome may affect the optimal effort. When agent's utility of consumption is logarithmic, we can use the result of Proposition 4 to say a bit more about the behavior of optimal pay. If low effort is optimal at y_A, it will remain optimal for any increase in y_A, from where the optimal wage remains fixed but increases if y_A increases. Moreover, if high effort is optimal at y_A, it will remain optimal for any decrease in y_A, from where the optimal wage will decrease for any individual outcome, and so on average, if y_A decreases.

Next, we perform some local analysis to investigate the effect of marginal changes in the aggregate outcome on the optimal choice of effort. For the sake of illustration, assume that there exists an aggregate outcome y_A for which the principal is indifferent between choosing high or low effort, i.e., $E_{\overline{a}}y - E_{\underline{a}}y = E_{\overline{a}}v^{-1}(v) - v^{-1}(v^c)$. The left-hand side of the equality is unaffected by changes in the aggregate outcome. Then, if the right-hand side increases, the principal will choose to implement low effort, while if it decreases, she will prefer to induce high effort. Consider a marginal change in the aggregate outcome, dy_A. Note that the resulting change in the right-hand side equals $\left(E_{\overline{a}}\frac{1}{v'(v^{-1}(v))} - \frac{1}{v'(v^{-1}(v^c))}\right)\frac{\partial \underline{V}}{\partial y_A}dy_A$.The term in brackets has the same sign as $v'(v^{-1}(E_{\overline{a}}v)) - E_{\overline{a}}v'(v^{-1}(v))$. Since v is strictly concave, we have $v'(v^{-1}(E_{\overline{a}})) > v'(E_{\overline{a}}v^{-1}(v))$. If $v''' \leq 0$, $v'(E_{\overline{a}}v^{-1}(v)) \geq E_{\overline{a}}v'(v^{-1}(v))$, from where $v'(v^{-1}(E_{\overline{a}}v)) - E_{\overline{a}}v'(v^{-1}(v)) > 0$. Therefore, if the third derivative of agent's utility of consumption is non-positive, low effort remains the principal's optimal choice for marginal increases in the aggregate outcome (optimal pay rise), while high effort remains optimal for marginal decreases in the aggregate outcome (optimal pay cut).[6] Unfortunately, we cannot derive a similar result for the case when $v''' > 0$, since the sign of the term in brackets remains ambiguous. Note that if $v''' \leq 0$, a marginal decrease in the aggregate output may change the optimal effort from low to high, while a marginal increase may switch it from high to low. Then, the decrease may result in an optimal pay rise for $\overline{y}$ (if $\frac{\partial \underline{V}}{\partial y_A}$ small) and pay cut for $\underline{y}$, while the increase may produce an optimal pay cut for $\overline{y}$ (if $\frac{\partial \underline{V}}{\partial y_A}$ small) and pay rise for $\underline{y}$.

2.2 Case 2. The distribution of individual outcomes depends on the aggregate outcome

In this case, both $\overline{\pi}^{y_A}$ and $\underline{\pi}^{y_A}$ vary across Y_A. It is interesting to see how changes in the aggregate outcome y_A affect the wages. As before, the pay implementing low effort will unequivocally increase in the aggregate outcome (given

[6] We have essentially shown that the global result of Proposition 4 holds locally for a class of utility functions.

that managerial services are more demanded in a boom than in a through). What about the wage scheme implementing high effort? Here, the answer is not so straight-forward. Let us again consider the case of two possible individual outcomes. High effort is implemented by the utility scheme $(\underline{v}_{\overline{a}}^{y_A}, \overline{v}_{\overline{a}}^{y_A})$. Now, however, both terms on the right-hand side of (5) and respectively (6) depend on the aggregate outcome: the first term through the reservation utility and the second through the conditional probabilities. Let's consider the effect of a marginal increase of y_A:

$$\frac{\partial \underline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \underline{V}}{\partial y_A} + \frac{\overline{a}-\underline{a}}{(\underline{\pi}-\overline{\pi})^2}\left((1-\overline{\pi})\frac{\partial \underline{\pi}}{\partial y_A} - (1-\underline{\pi})\frac{\partial \overline{\pi}}{\partial y_A}\right)$$
$$\frac{\partial \overline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \underline{V}}{\partial y_A} + \frac{\overline{a}-\underline{a}}{(\underline{\pi}-\overline{\pi})^2}\left(\underline{\pi}\frac{\partial \overline{\pi}}{\partial y_A} - \overline{\pi}\frac{\partial \underline{\pi}}{\partial y_A}\right)$$

If we assume that the aggregate outcome has the same marginal effect on both distributions (the distribution conditional on low and the distribution conditional on high effort), i.e., $\frac{\partial \overline{\pi}}{\partial y_A} = \frac{\partial \underline{\pi}}{\partial y_A}$, then we will have:

$$\frac{\partial \underline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \overline{v}_{\overline{a}}}{\partial y_A} = \frac{\partial \underline{V}}{\partial y_A} + \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}}\frac{\partial \overline{\pi}}{\partial y_A}$$

with the first term positive (rising demand for managerial services in a boom) and the second negative for a pro-cyclical and positive for a counter-cyclical firm. Therefore, a marginal increase in the aggregate outcome would increase the wage compensation implementing high effort for a counter-cyclical firm and would increase/does not affect/decrease it for a pro-cyclical firm depending on whether the increase in reservation utilities dominates/cancels/is dominated by the decrease in the probability of failure (low outcome) weighted by the additional disutility of high over low effort divided by its contribution to the probability of success (high outcome). Then, if high effort is indeed optimal, RPE will only be observed for pro-cyclical firms where the reservation utility effect of aggregate effort is weaker than its direct effect on the probability of success. Note that under CRU, the first effect would disappear, so RPE would always hold for pro-cyclical firms.

When discussing the relevance of RPE, we have implicitly assumed that the change in the aggregate outcome does not affect the optimal choice of effort. Now, we will analyze when such an assumption is indeed justified. As in the previous section, let us assume that there is an aggregate outcome y_A for which the principal is indifferent between choosing low or high income. Then, we have $E_{\overline{a}}y - E_{\underline{a}}y = E_{\overline{a}}v^{-1}(v) - v^{-1}(v^c)$. Consider a marginal change in the aggregate outcome dy_A. If we assume $\frac{\partial \overline{\pi}}{\partial y_A} = \frac{\partial \underline{\pi}}{\partial y_A}$ as before, the left-hand side is unaffected by dy_A. The change in the right-hand side equals $(A+B)dy_A$, where $A = \left(E_{\overline{a}} \frac{1}{v'(v^{-1}(v))} - \frac{1}{v'(v^{-1}(v^c))} \right) \frac{\partial \underline{V}}{\partial y_A} dy_A$ and $B = (v^{-1}(\underline{v}) - v^{-1}(\overline{v})) \frac{\partial \overline{\pi}}{\partial y_A} dy_A$. A rise in the aggregate outcome would naturally increase the respective probability of success, $(1-\overline{\pi})$ if the firm is pro-cyclical and decrease it if the firm is counter-cyclical. Therefore, B will be positive for a pro-cyclical firm and negative for a counter-cyclical one.

Under CRU, A disappears, and we can conclude that low effort remains optimal for marginal increases in the aggregate outcome if the firm is pro-cyclical (optimal pay cut) and for marginal decreases if it is counter-cyclical (optimal pay cut), while high effort remains optimal for marginal decreases in the aggregate outcome if the firm is pro-cyclical (optimal pay rise) and for marginal increases if it is counter-cyclical (optimal pay rise). Note that for a pro-cyclical firm, a marginal decrease in the aggregate output may change the optimal effort from low to high, while a marginal increase may switch it from high to low. Then, the decrease may result in an optimal pay cut for $\underline{y}$ (if $\left| \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} \frac{\partial \overline{\pi}}{\partial y_A} \right|$ small) and pay rise for $\overline{y}$, while the increase may lead to an optimal pay rise for $\underline{y}$ (if $\left| \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} \frac{\partial \overline{\pi}}{\partial y_A} \right|$ small) and pay cut for $\overline{y}$. Alternatively, for a counter-cyclical firm, a marginal increase in the aggregate output may change the optimal effort from low to high, while a marginal decrease may switch it from high to low. Then, the increase may result in an optimal pay cut for $\underline{y}$ (if $\left| \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} \frac{\partial \overline{\pi}}{\partial y_A} \right|$ small) and pay rise for $\overline{y}$, while the decrease may produce an optimal pay rise for $\underline{y}$ (if $\left| \frac{\overline{a}-\underline{a}}{\underline{\pi}-\overline{\pi}} \frac{\partial \overline{\pi}}{\partial y_A} \right|$ small) and pay cut for $\overline{y}$.

Under VRU, we should also consider the sign and magnitude of A. From the previous section, we know that if $v''' \leq 0$, A will be positive for both pro- and counter-cyclical firms. Therefore, the local results related to the effect of aggregate outcome on a pro-cyclical firm's optimal effort choice under CRU are reinforced under VRU. For a counter-cyclical firm, however, A and B will have opposite signs and the effect on the optimal effort choice will be governed by whichever is dominant. If A dominates B, the results from the previous subsection apply. If $A = -B$, the optimal effort choice will never be affected. If B dominates A, the results from the previous paragraph apply.

3 Model with Continuous-Valued Shocks

3.1 General analysis

Let η denote an aggregate shock that determines the aggregate outcome y_A. Then, we can write the reservation utility of the agent as a function of the aggregate shock: $\underline{V}(\eta)$. We assume that the individual outcome is an increasing function of managerial action and also depends on the aggregate shock η and an idiosyncratic shock ε. In particular, we assume the following additive form:

$$y = g(a) + \eta + \varepsilon,$$

where $g'(a) > 0$. We assume a bivariate normal distribution for the shocks, namely:

$$\begin{bmatrix} \eta \\ \varepsilon \end{bmatrix} \backsim N\left(\begin{bmatrix} \mu_\eta \\ \mu_\varepsilon \end{bmatrix}, \begin{bmatrix} \sigma_\eta^2 & \rho\sigma_\eta\sigma_\varepsilon \\ \rho\sigma_\eta\sigma_\varepsilon & \sigma_\varepsilon^2 \end{bmatrix}\right).$$

In this setting, we will refer to a firm as pro-cyclical if the correlation between the firm's idiosyncratic shock and the aggregate shock is positive. A firm will be called counter-cyclical if the correlation is instead negative.

Given that η is observed before the contract is signed, we are interested in the distribution of the individual outcome conditional on the aggregate shock. We have $\varepsilon|\eta \backsim N\left(\mu_\varepsilon + \rho\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\eta - \mu_\eta\right), \left(1-\rho^2\right)\sigma_\varepsilon^2\right)$ and so, $y|\eta \backsim N\left(\mu(a), \sigma^2(a)\right)$, where $\mu(a) := g(a) + \eta + \mu_\varepsilon + \rho\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\eta - \mu_\eta\right)$, while $\sigma^2(a) := \left(1-\rho^2\right)\sigma_\varepsilon^2$.[7]

We also assume that the utility of the agent from exercising action a and consuming wage w is $-e^{-rw(y)} - a$, where $r > 0$ is agent's absolute risk aversion. Consider that there are two feasible actions: $\underline{a}$ and $\overline{a}$, with $\underline{a} < \overline{a}$. Then, after observing an aggregate shock η, the principal's problem becomes:

$$\max_{a,w(.)} \mu(a) - \int w(y) \frac{1}{\sqrt{2\pi\sigma^2(a)}} e^{-\frac{(y-\mu(a))^2}{2\sigma^2(a)}} dy \text{ s.t.:}$$

[7] Note that only the conditional mean of y depends on a. This is because the idiosyncratic shock affects the individual outcome additively. However, if we assume a multiplicative form, e.g., $y = g(a)\varepsilon + \eta$, both the conditional mean and variance of the individual outcome would depend on the action: $\mu(a) = \eta + g(a)\left[\mu_\varepsilon + \rho\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\eta - \mu_\eta\right)\right]$ and $\sigma^2(a) = g(a)^2\left(1-\rho^2\right)\sigma_\varepsilon^2$.

$$
\begin{gathered}
a \in \{\underline{a}, \overline{a}\} \\
-\int e^{-rw(y)} \frac{1}{\sqrt{2\pi\sigma^2(a)}} e^{-\frac{(y-\mu(a))^2}{2\sigma(a)^2}} dy - a \geq \underline{V}(\eta) \\
-\int e^{-rw(y)} \left[\frac{1}{\sqrt{2\pi\sigma^2(a)}} e^{-\frac{(y-\mu(a))^2}{2\sigma(a)^2}} - \frac{1}{\sqrt{2\pi\sigma^2(a')}} e^{-\frac{(y-\mu(a'))^2}{2\sigma(a')^2}} \right] dy \geq a - a',
\end{gathered}
$$

where the second constraint guarantees individual rationality while the third imposes incentive compatibility. We denote their respective Lagrange multipliers by $\lambda_1(\eta)$ and $\lambda_2(\eta)$. The first order condition with respect to $w(y)$ leads to:

$$
\frac{1}{re^{-rw(y)}} = \lambda_1(\eta) + \lambda_2(\eta) \left(1 - \sqrt{\frac{\sigma^2(a)}{\sigma^2(a')}} e^{-\frac{\sigma^2(a)\left(y-\mu(a')\right)^2 - \sigma^2(a')(y-\mu(a))^2}{2\sigma^2(a)\sigma^2(a')}} \right).
$$

Plugging for $\mu(.)$ and $\sigma^2(.)$, we obtain:

$$
\begin{aligned}
w(y) = \frac{1}{r}\log r + \frac{1}{r}\log \Bigg(\lambda_1(\eta) + \\
\lambda_2(\eta) \left(1 - e^{-\frac{(g(a)-g(a'))\left(2y-g(a)-g(a')-2\left(\eta+\mu_\varepsilon+\rho\frac{\sigma_\varepsilon}{\sigma_\eta}(\eta-\mu_\eta)\right)\right)}{2(1-\rho^2)\sigma_\varepsilon^2}} \right) \Bigg)
\end{aligned}
\tag{7}
$$

Unfortunately, the Lagrange multipliers are unknown functions of the aggregate shock, so we are not able to analyze how changes in the aggregate affect the resulting wage scheme.

3.2 Best linear wage

In this subsection, we compute the optimal linear scheme. If we think of the individual outcome y as the firm's profit, then a linear wage $w_l(y)$ consists of a fixed salary component α and a profit share β, i.e., $w_l(y) = \alpha + \beta y$.

It is straight-forward to show that low effort is optimally implemented by a fixed wage such that individual rationality binds, i.e., by offering the agent a compensation of $-\frac{1}{r}\log\left(-\underline{V}(\eta) - \underline{a}\right)$ at any contingency. If the agent's reservation utility is unaffected by the aggregate shock, this fixed wage does not depend on η. If the reservation utility increases with the aggregate shock, then the fixed wage rises with η.

Now, consider the implementation of high effort. If both individual rationality and incentive compatibility bind, the optimal linear scheme has:

$$\beta = \frac{\log\left(\frac{\underline{V}(\eta)+\underline{\alpha}}{\underline{V}(\eta)+\overline{a}}\right)}{r(g(\overline{a})-g(\underline{\alpha}))},$$

$$\alpha = -\beta\left(\eta+\mu_\varepsilon+\rho\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\eta-\mu_\eta\right)\right)+\frac{1}{2}r\beta^2\left(1-\rho^2\right)\sigma_\varepsilon^2+ \frac{g(\underline{\alpha})\log(-\underline{V}(\eta)-\overline{a})-g(\overline{a})\log(-\underline{V}(\eta)-\underline{\alpha})}{r(g(\overline{a})-g(\underline{\alpha}))}.$$

The scheme has a positive slope which approximately equals the agent's return (in terms of expected utility of consumption) from choosing low over high effort divided by the respectively foregone contribution to the individual outcome weighted by agent's absolute risk aversion.

If agent's reservation utility remains constant across aggregate shocks, then η affects α, but not β. In particular, a higher realization of the aggregate shock has a negative effect on the fixed component of the compensation received by an executive working for a pro-cyclical firm. In case of a counter-cyclical firm, the effect may become positive if the individual shock is sufficiently noisy and sufficiently correlated to the aggregate shock. In any case, the effect is proportional to the slope of the linear scheme β.

If the agent's reservation utility increases with the aggregate shock, then η also affects β. In particular, $\frac{\partial\beta}{\partial\eta} = \frac{\overline{a}-\underline{\alpha}}{r(g(\overline{a})-g(\underline{\alpha}))(\underline{V}(\eta)+\overline{a})(\underline{V}(\eta)+\underline{\alpha})}\frac{\underline{V}(\eta)}{\partial\eta} > 0$. A marginal increase in the aggregate shock has now an ambiguous effect on the fixed component of the compensation of an agent working for a pro-cyclical firm because of its positive effect on the last two terms of α. Moreover, if a marginal increase in η had a positive effect on α under constant reservation utility and a counter-cyclical firm, this effect would now be reinforced through the increase in $\underline{V}(\eta)$.

It is interesting to see whether the best linear wage can indeed be optimal under some conditions. Holmstrom and Milgrom [13] show that this is the case in a continuous time model where the agent has a constant absolute risk aversion and consumes his payoff in the final period. Alternatively, Edmans, Gabaix, Sadzik and Sannikov [6] obtain log-linear compensation rules in a similarly set-up discrete model where the agent has a constant relative risk aversion. Numerical experiments for our single-period model suggest that while the linear scheme may not be optimal in general, it provides a good approximation for the optimal contract.

4 Conclusion

This paper builds a simple moral hazard problem to address the optimal provision of incentives in environments where aggregate shocks affect both firm's technology and the demand for managerial services. We find that RPE is optimal under very special circumstances, so its scarce use in actual compensation packages is hardly surprising. We produce some interesting theoretical predictions about executive compensation related to firm's technology and the market for executives. The next logical step is taking the model to the data.

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