Marketing via Friends: Strategic Diffusion of Information in Social Networks with Homophily

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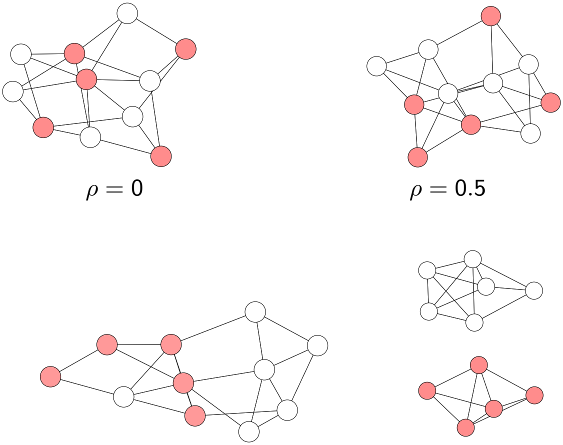

$\rho = 0$ $\rho = 0.5$ $\rho = 0.75$ $\rho = 1.0$

Literature review

- **WOM literature:**
  - Theoretical: Campbell (2010), Goyal and Galeotti (2009), Lopez-Pintado & Watts (2009)
  - Empirical: Leskovec et al. (2008), Iribarren and Moro (2007)

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- **Homophily:**
  - Friendship and segregation: Currarini, Jackson & Pin (2009)
  - Learning and diffusion: Jackson & Golub (2010)
  - Social norms and preferences: Christakis & Fowler (2007), Fiore and Donath (2005)

Model

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  - Vector $(\boldsymbol{\rho}^{\mathbf{A}}, \boldsymbol{\rho}^{\mathbf{B}})$ identifies proportion of consumers of the same type in the neighborhood of a randomly chosen consumer of type A and B.

Model cont'd

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    - Threshold level of the product's characteristic $\mathbf{\bar{w}_j}$, s.t. induces a consumer to buy the product
- Consumers can buy the product only if they learn about it from:
  - Direct advertisement.
  - Observing a neighbor who has already acquired the product.

Model cont'd

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- Cost of production is 0.
- To induce sales the monopolist advertises product to infinitesimal part of the population.

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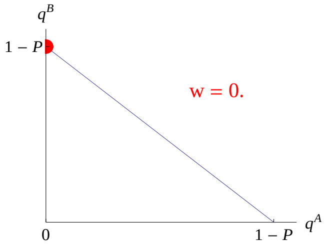
- Monopolist chooses optimally characteristic w and price P to maximize profits:



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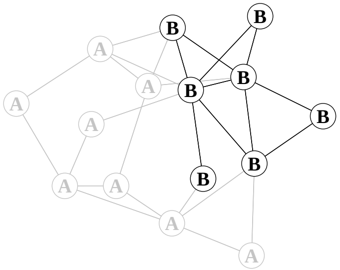

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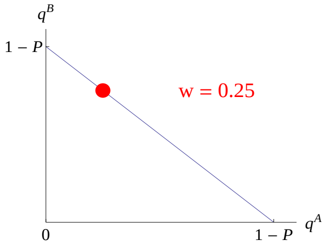

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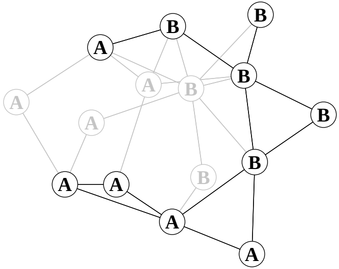

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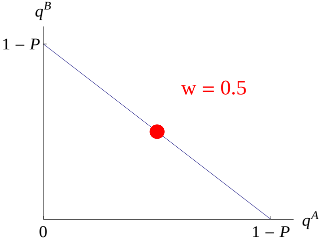

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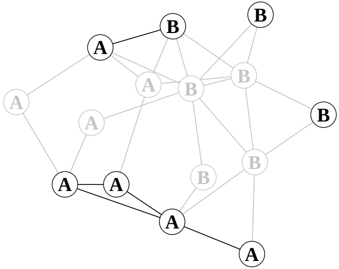

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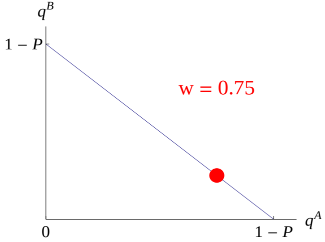

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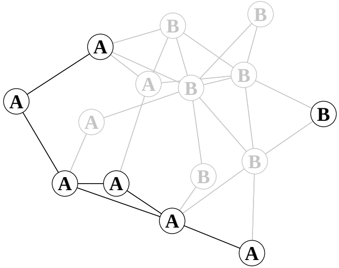

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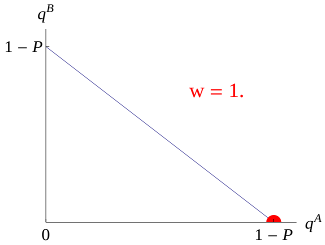

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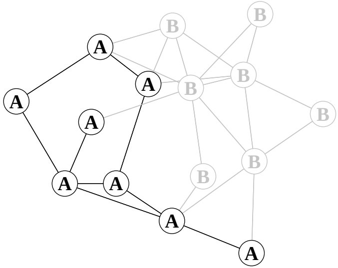

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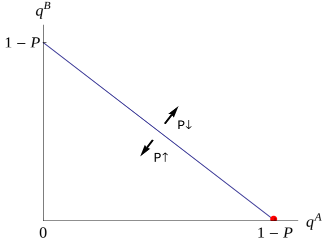

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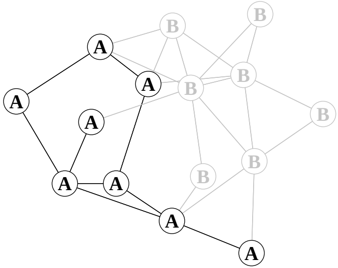

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Cascade of sales per advertisement

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- Expected size of cascade of sales per advertisement:

$$s(q^A, q^B, \rho, z_1, z_2, \gamma) =$$

$$(\gamma \;\; 1-\gamma)\left[\mathbf{I} + \frac{z_1^2}{z_2}\left(\left[\mathbf{I} - \frac{z_2}{z_1}\begin{pmatrix} q^A\rho & q^A(1-\rho) \\ q^B(1-\rho) & q^B\rho \end{pmatrix}\right]^{-1} - \mathbf{I}\right)\right]\begin{pmatrix} q^A \\ q^B \end{pmatrix}$$

where z_1 and z_2 are expected numbers of first and second neighbors.

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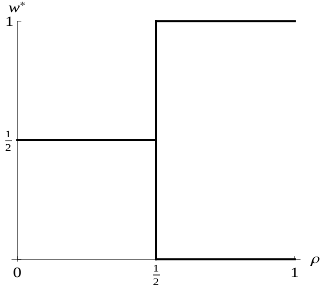
- Necessary condition for existence of the giant component of connected consumers, $\frac{z_2}{z_1} > 1$.
- The existence of the global cascade in the case when $\frac{z_2}{z_1} < 2$ hinges on the homophily level.

Optimal design strategy:

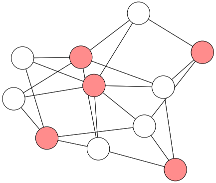


Proposition

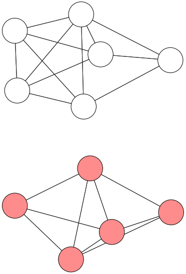
- *The optimal characteristic of the product is the following correspondence:*

$$w^* = \begin{cases} [0,1], & \rho = \frac{1}{2} \\ 1/2, & \rho < \frac{1}{2} \\ \{0,1\}, & \rho > \frac{1}{2} \end{cases}$$

Optimal design strategy: Intuition



$\rho = 0$



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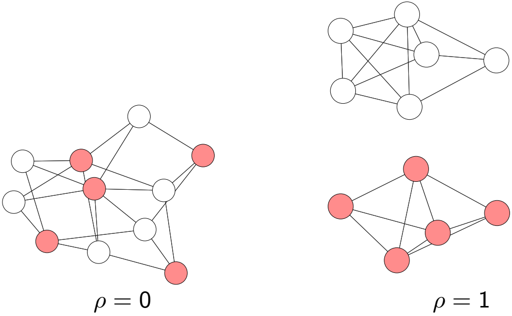

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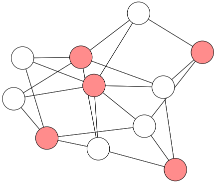

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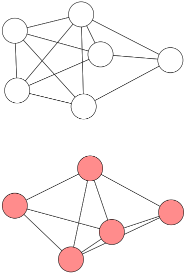

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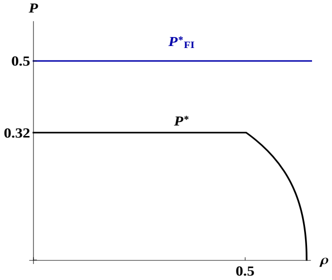
- Case $\rho = 1$:
  - Two clusters of consumers of the same type.
  - Specialized design is optimal.

Optimal pricing strategy:



Proposition

- *The optimal price P^* is lower than in the case of full information and for $\rho > \frac{1}{2}$ is strictly decreasing function in the level of homophily.*
- *For two degree distributions p(k) and p'(k) and corresponding optimal prices P^* and P'^* if p(k) is a mean preserving spread of p'(k) then $P^* < P'^*$.*

Demand function

$$Q(P,\rho,z_1,z_2) = \begin{cases} \frac{1-P}{2}\left(1+\frac{z_1(1-P)}{2-z_2/z_1(1-P)}\right), & \rho \le \frac{1}{2} \\ \frac{1-P}{2}\left(1+\frac{z_1(1-P)}{\frac{1}{\rho}-z_2/z_1(1-P)}\right), & \rho > \frac{1}{2} \end{cases}$$

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Proposition

The demand function $Q(P,\rho,z_1,z_2)$ has following properties:

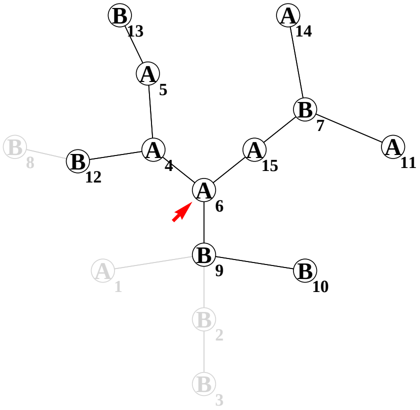
1. *Decreasing and convex in price P.*
2. *Increasing and convex in homophily level ρ, for $\rho > \frac{1}{2}$.*
3. *The absolute value of the price elasticity of demand is:*

$$\frac{P}{1-P}\left(1+z_1\left(\frac{1}{z_1-(1-P)z_2\rho}-\frac{1}{z_1+(1-P)(z1^2-z2)\rho}\right)\right),$$

which is higher than price elasticity in the case of full information $\frac{P}{1-P}$ and is increasing in homophily level ρ, for $\rho > \frac{1}{2}$.

Demand: Intuition

- Demand is decreasing and convex in P.



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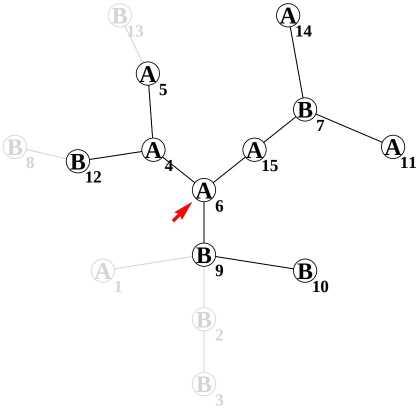

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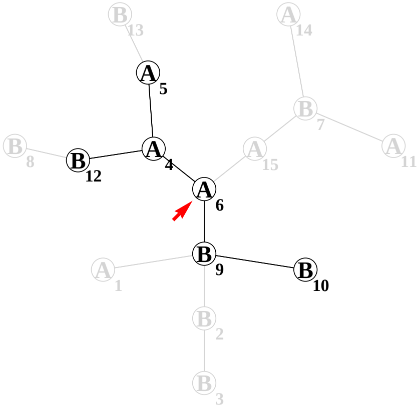

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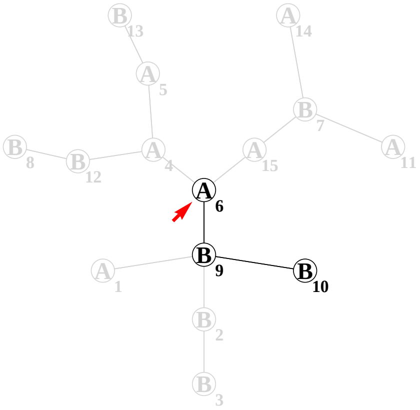

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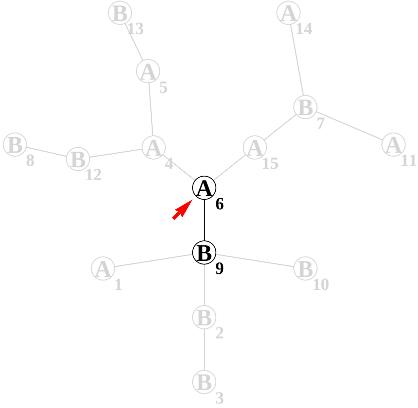

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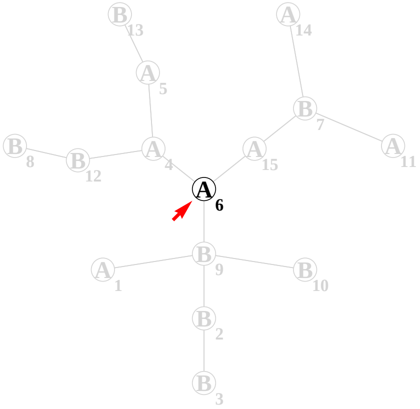

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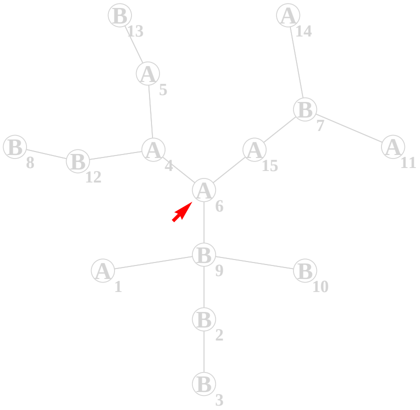

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- **Consumer surplus is increasing in the level of homophily**
  - Demand is increasing - more consumers buy the product.
  - The optimal price is decreasing in the level of homophily.

$$CS(P^*(\rho), \rho, z_1, z_2) = \int_{P^*(\rho)}^{1} Q(P, \rho, z_1, z_2) dP$$

Model Extensions

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- **Non-linear probability frontier.**
  - Bend inward frontier - results are the same.
  - Bend outward frontier - similar shape, but design is continuous.
- **Monopolist benefits from one group.**
  - Low levels of homophily - compromise design is still optimal.
  - High levels of homophily - compromise design is optimal when audience is small.

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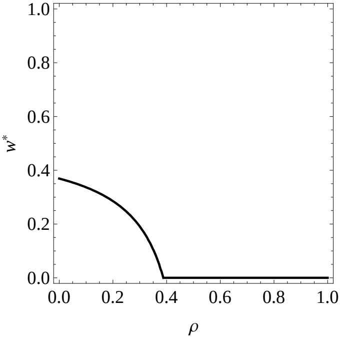

Proposition

There is threshold value $\hat{\rho}_1(z_1, z_2)$ such that if $\rho < \hat{\rho}_1(z_1, z_2)$ the optimal characteristic w_1^* belongs to the interval $\left(0, \frac{1}{2}\right]$, while if $\rho > \hat{\rho}_1(z_1, z_2)$ then $w_1^* = 0$

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- Consumers of type A constitute 80% of the population ($\gamma = 0.8$). The monopolist maximizes sales to consumers of type B.
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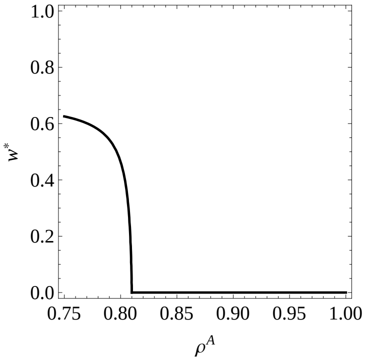

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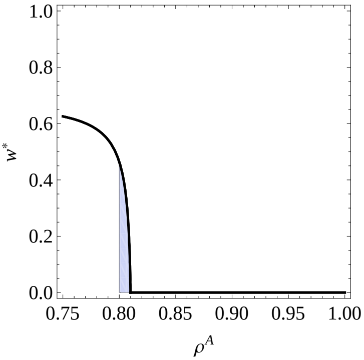

Conclusions

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- A product designed to attract both types of consumers may be optimal even if a monopolist benefits only from one group of consumers.