JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY S 0894-0347(06)00522-4 Article electronically published on February 6, 2006

CAYLEY GROUPS

NICOLE LEMIRE, VLADIMIR L. POPOV, AND ZINOVY REICHSTEIN

Contents

- 1. Introduction
- 1.1. The classical Cayley map
- 1.2. Basic definitions, main problem, and examples
- 1.3. Notational conventions
- 1.4. Main results
- 1.5. Application to Cremona groups
- 2. Preliminaries
- 2.1. Γ -fields and Γ -varieties
- 2.2. Γ -lattices
- 2.3. Stable equivalence and flasque resolutions
- 2.4. Speiser's Lemma
- 2.5. Homogeneous fiber spaces
- 3. Cayley maps, generic tori, and lattices
- 3.1. Restricting Cayley maps to Cartan subgroups
- 3.2. Generic tori
- 3.3. Proof of Theorem 1.27
- 4. Reduction theorems
- 4.1. Unipotent normal subgroups
- 4.2. The Levi decomposition
- 4.3. From reductive to semisimple
- 4.4. From semisimple to simple
- 5. Proof of Theorem 1.28: An overview
- 6. The groups $\mathbf{SL}_n/\boldsymbol{\mu}_d$ and their character lattices
- 6.1. Lattices $Q_n(d)$
- 6.2. Properties of $Q_n(d)$
- 7. Stably Cayley groups of type A_n
- 7.1. Restricting $Q_n(d)$ to some subgroups
- 7.2. Lattices Λ_p and Λ_{2p}
- 7.3. Completion of the proof of Proposition 5.1
- 8. Stably Cayley groups of type D_n
- 8.1. Root system of type D_n
- 8.2. Lattices Y_{2m} and Z_{2m}

Received by the editors January 14, 2005.

2000 Mathematics Subject Classification. Primary 14L35, 14L40, 14L30, 17B45, 20C10.

Key words and phrases. Algebraic group, Lie algebra, reductive group, algebraic torus, Weyl group, root system, birational isomorphism, Cayley map, rationality, cohomology, permutation lattice.

The first and last authors were supported in part by NSERC research grants. The second author was supported in part by ETH, Zürich, Switzerland, Russian grants $P\Phi\Phi M 05-01-00455$, HIII-123.2003.01, and a (granting) program of the Mathematics Branch of the Russian Academy of Sciences.

O2006 American Mathematical Society Reverts to public domain 28 years from publication