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STABILITY CRITERIA FOR THE ACTION OF A SEMISIMPLE GROUP ON A FACTORIAL MANIFOLD

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Abstract. In this work it is proved that, for the regular action of a semisimple irreducible algebraic group G on an affine space, the existence of a closed orbit of maximum dimension is equivalent to the existence of an invariant open set at any point of which the stationary subgroup is reductive. This result is established for the action of G on manifolds of a special type (the so-called factorial manifolds). There are given several other conditions equivalent to the existence of a closed orbit of maximum dimension for the action of G on an arbitrary affine manifold.

Let k be an algebraically closed field of characteristic zero, which will be viewed in the sequel as the universal domain.

Definition 1. An irreducible affine manifold X is called *factorial* if the ring $k[X]$ of regular functions defined on it is factorial and every invertible regular function is a constant.

Let G be an irreducible semisimple algebraic group acting regularly on X. This action induces k-automorphisms of the ring $k[X]$ and of the field $k(X)$ of rational functions on X. Let O_x denote the orbit of the point $x \in X$. Let m_G be the maximum dimension of the orbits of the action of G on X. It is not hard to show that each orbit is a smooth quasi-projective manifold. Points with $\dim O_x = m_G$ are called points of general position. Standard arguments connected with tangent spaces show that points of general position form an open subset in X. Let us denote it by Ω. Let $\Pi = X - \Omega$. As follows from the definition, Π is a G-invariant submanifold in X, for every point x of which $\dim O_x < m_G$.

Definition 2. The action is called *locally transitive* if $\dim X = m_G$.

Clearly, if the action is locally transitive then there exists in X a unique orbit of dimension m_G, and it is the set Ω. Since $\dim G \geq m_G$, if $\dim X > \dim G$ the action cannot be locally transitive.

Definition 3. The action is called *stable* if X contains a G-invariant open set such that every orbit lying within it is closed in X (see [7]).