Labor market and search through personal contacts.

Roman Chuhay,

ICEF, CAS,
Higher School of Economics

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Literature review

- **Social Networks and Labor markets:**
  - Calvo-Armengol and Jackson (2004) study the correlation of employment statuses and wages of connected workers.
  - Calvo-Armengol & Zenou (2005) consider regular network in the framework of Mortensen-Pissarides model.
  - Ioannides and Soetevent (2006) perform a numerical analysis for the case of Poisson random network.
Motivation

- Previous literature imposes various simplifying assumptions:
  - Only one worker initially becomes aware about a job offer.
  - Job offers can be transmitted only to immediate contacts.
  - Offer is relayed at random.
  - Firm behavior and wages are exogenous.

- The structure of job contact network is also an open question and varies from one study to another.
# The model

**Workers:**

- A large number $N$ of ex-ante identical workers that are embedded into an undirected network of personal contacts.

- The network is characterized by a socialization level $s$ of workers that has cost $c_s$.

- With some probability a worker learns about a vacancy directly from an employer. The worker may accept the offer or pass it to her contacts.

- An employed worker produces output $y$ and receives wage $w_t$.

- With probability $\delta$ an employed worker looses a job.
Firms and wage:

- A firm can open a vacancy. We refer to $v_t = V_t/N$ as vacancy rate.
- The cost of having an unfilled vacancy is $\gamma$.
- A wage is bargained according to the Nash bargaining process.
Assumptions on Matching Function

- Matching function \( m(s, \nu, \mu) \) depends on socialization level \( s \), vacancy rate \( \nu \) and unemployment rate \( \mu \).

- We require the resulting matching function to satisfy the following four properties:

  (A1) \( m(s, \nu, \mu) \) is positive and increasing in both \( \mu \) and \( \nu \).

  (A2) \( m(s, \nu, \mu) \leq \min(\mu, \nu) \), \( m(s, 1, \mu) = \mu \) and \( m(s, \nu, 1) = \nu \).

  (A3) \( \frac{m(s, \nu, \mu)}{\nu} \) is decreasing in \( \nu \) and \( \frac{m(s, \nu, \mu)}{\mu} \) is decreasing in \( \mu \).

  (A4) \( m(s, \nu, \mu) \) is increasing in the socialization level \( s \).
Worker’s Problem

The stream of discounted utility of employed worker $I_{E,t}$ and of unemployed $I_{U,t}$ are given by:

\[
I_{E,t} = w_t - cs + \frac{1}{1+r}[(1 - \delta)I_{E,t+1} + \delta I_{U,t+1}]
\]

\[
I_{U,t} = -cs + \frac{1}{1+r} \left[ \left(1 - \frac{1}{u_t} m(s, v_t, u_t) \right) I_{U,t+1} + \frac{1}{u_t} m(s, v_t, u_t) I_{E,t} \right]
\]

where $r$ is the discount factor.
Firm’s Problem

We denote by $I_{F,t}$ and $I_{V,t}$ the expected inter-temporal profits generated by a filled job, and a vacancy respectively:

$$I_{F,t} = y - w_t + \frac{1}{1+r} [(1 - \delta)I_{F,t+1} + \delta I_{V,t+1}]$$

$$I_{V,t} = -\gamma + \frac{1}{1+r} \left[ (1 - \frac{1}{v_t} m(s, v_t, u_t)) I_{V,t+1} + \frac{1}{v_t} m(s, v_t, u_t) I_{F,t} \right]$$
Labor Market Turnover

- At the beginning of each period $t$, the proportion $m(s, v_{t-1}, u_{t-1})$ of workers start to work.

- At the end of each period with probability $\delta$ an employed worker loses a job and becomes unemployed.

\[ u_t = u_{t-1} - m(s, v_{t-1}, u_{t-1}) + \delta(1 - u_{t-1} + m(s, v_{t-1}, u_{t-1})) \]

- In the steady state:

\[ m(s, v, u) = \frac{\delta}{1 - \delta}(1 - u) \]
Wage

- Workers’ wage is determined according to the generalized Nash bargaining process, with worker’s bargaining power being denoted by $\beta \in [0, 1]$: 

$$w = \arg \max_{w} \left\{ (I_E - I_U)^\beta (I_F - I_V)^{1-\beta} \right\}$$

- One can show that in the steady state:

$$w = \beta \left( y + \gamma \frac{v}{u} \right)$$
Existence and Uniqueness of the Equilibrium.

Proposition

For any $s$ there is a unique labor market equilibrium $\{u^*(s), v^*(s), w^*(s)\}$ if \(\gamma \frac{(r+\beta+\delta)}{(1-\beta)} < Y < \frac{\gamma (r+\beta+\delta)}{\delta (1-\beta)}\). Moreover, functions $u^*(s), v^*(s),$ and $w^*(s)$ are continuous.

- The first part of the condition, \(\frac{\gamma(r+\beta+\delta)}{(1-\beta)} < Y\) implies that $Y$ is sufficiently high and firms want to hire workers when $u = 1$.
- Part $Y < \frac{\gamma (r+\beta+\delta)}{\delta (1-\beta)}$ puts upper bound on productivity not allowing $v$ to explode.
We relate $u^*(s)$ and $v^*(s)$ to workers' socialization level and productivity:

\begin{itemize}
\item[(i)] $u^*(s)$ decreases in socialization level of workers $s$ and productivity $y$.
\item[(ii)] $v^*(s)$ increases in the productivity $y$, while $v^*(s)$ decreases in $s$ if $u^*(s) < \bar{u}_v$ and increases otherwise, where $\bar{u}_v = \frac{\sqrt{\beta \delta (1-\delta)(\delta+r)-\beta \delta}}{(1-\delta)(\delta+r)-\beta \delta}$.
\end{itemize}
Market Tightness and Wage

- Market tightness is the ratio of the number of vacancies to number of unemployed workers.
- The market tightness indicates which side of the market is better-off.

**Proposition**

The equilibrium market tightness $\frac{v^*(s)}{u^*(s)}$ and the wage $w^*(s)$ are increasing in the socialization level $s$. 

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An Example of Matching Function

- To illustrate an application of our model we consider the network formation mechanism a la Galeotti and Merlino (2010).

- Each worker $i$ selects a socialization level, $s_i \geq 0$. Let $s = (s_1, \ldots, s_n)$ be a profile of socialization levels.

- A probability that a link between $i$ and $j$ is present at time period $t$ is given by:

$$g_{ij}(s) = \rho(s)s_i s_j,$$

where

$$\rho(s) = \begin{cases} 
\left(\sum_{j=1}^{n} s_j\right)^{-1}, & \text{if } s \neq 0 \\
0, & \text{otherwise}
\end{cases}$$
Probability to get at least one job-offer through contacts.

The matching function in this case is:

\[ m(s, v, u) = u[v + (1 - v)P^s(s, v, u)] = u \left[ 1 - (1 - v)e^{-\frac{(1-u)v}{u}}(1-e^{-us}) \right] \]

**Lemma**

The matching function \( m(s, v, u) \) satisfies conditions (A1)-(A4) and is concave in \( u, v \) and \( s \).
Conclusion

- We formulated four properties that a matching function should satisfy.
- Using these properties we showed that result obtained in previous studies about unemployment rate holds in more general setup.
- Our framework allowed us to get new results concerning the impact of network of personal contacts on vacancy rate, market tightness and wage.