

HOMEWORK 1

1. (a) Prove the identity $(1 + q + \cdots + q^9)(1 + q^{10} + \cdots + q^{90})(1 + q^{100} + \cdots + q^{900})\dots = \frac{1}{1-q}$ (for q such that the left-hand side of the equation converges).
(b) Prove that any positive integer N can be written in the decimal number system, i.e., can be uniquely represented in the form $N = a_0 + 10a_1 + 100a_2 + \cdots + 10^k a_k$, for a suitable k and $a_0, \dots, a_k \in \{0, 1, \dots, 9\}$, $a_k \neq 0$.
(c) What is the relation between questions 1a and 1b ?
2. Write explicitly
(a) the generating function $P_{3,10}(x)$ for the number of 3 + 3-digit lucky tickets,
(b) the generating function $P_{r,q}(x)$ for the number of $r + r$ -digit lucky tickets in the number system with base q .
3. A player throws three dice a) in succession, b) simultaneously. How many different results may occur ?